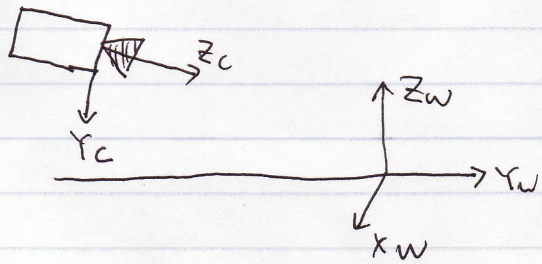
Example of projective transformation
(collineation, plane homography)



Transformation from ground plane $Z_w = 0$ to image plane is

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} \sim \underbrace{\begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \vec{r}_1 & \vec{r}_2 & \vec{T} \end{pmatrix}} \begin{pmatrix} X \\ Y \\ W \end{pmatrix}$$

$$\det(A) \neq 0 \quad \text{iff} \quad (\vec{r}_1 \times \vec{r}_2)^T \vec{T} \neq 0$$

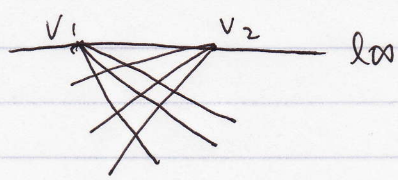
Vanishing point in X_w-direction $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $\quad \begin{pmatrix} f\,v_{11} \\ f\,v_{21} \\ v_{31} \end{pmatrix} \sim V_1$

Y_w-direction $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $\quad \begin{pmatrix} f\,v_{21} \\ f\,v_{22} \\ v_{23} \end{pmatrix} \sim V_2$

Equation of the horizon line is

$$(V_1 \times V_2)^T \begin{pmatrix} u \\ v \\ w \end{pmatrix} = 0$$

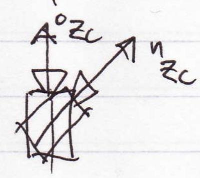
Coefficients of horizon line $l_\infty \sim V_1 \times V_2$



Example of projective transformation: Mosaicking (photoshop)

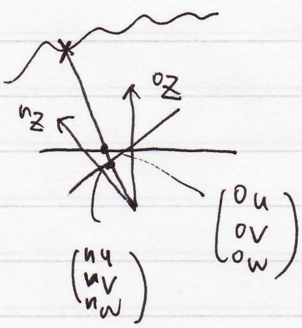
Main assumption of mosaicking is pure rotation:

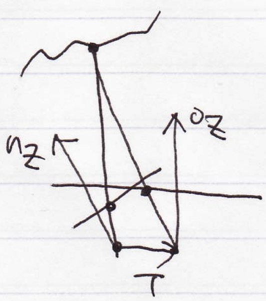
$$\begin{pmatrix} {}^{o}X_c \\ {}^{o}Y_c \\ {}^{o}Z_c \end{pmatrix} = R \begin{pmatrix} {}^{n}X_c \\ {}^{n}Y_c \\ {}^{n}Z_c \end{pmatrix}$$



What is mosaicking?

Given $\begin{pmatrix} {}^{n}u \\ {}^{n}v \\ {}^{n}w \end{pmatrix}$ coordinate in n-th frame transform them in the 0th-frame: $\begin{pmatrix} {}^{o}u \\ {}^{o}v \\ {}^{o}w \end{pmatrix}$





"I have to know the depth"

* pure rotation vs * rotation + translation

Pure rotation effects are depth invariant

$$\begin{pmatrix} {}^{o}u \\ {}^{o}v \\ {}^{o}w \end{pmatrix} \sim \begin{pmatrix} {}^{o}f & 0 & 0 \\ 0 & {}^{o}f & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} {}^{o}X_c \\ {}^{o}Y_c \\ {}^{o}Z_c \end{pmatrix}$$

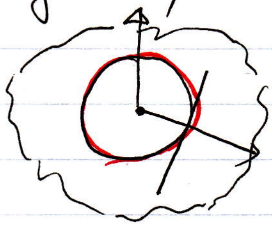
$$\sim \underbrace{\begin{pmatrix} {}^{o}f & & \\ & {}^{o}f & \\ & & 1 \end{pmatrix} R \begin{pmatrix} {}^{n}f & & \\ & {}^{n}f & \\ & & 1 \end{pmatrix}^{-1}} \begin{pmatrix} {}^{n}u \\ {}^{n}v \\ {}^{n}w \end{pmatrix}$$

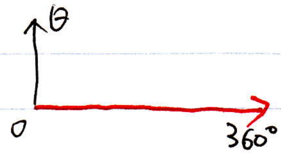
$$\det(A) = \frac{{}^{o}f^2}{{}^{n}f^2}$$

eye motion "efferent copy"

Because it is a projective transformation we can compute A from 4 corresponding points without knowing ${}^{o}f$, ${}^{n}f$ and R

Singularity of perspective panorama I





Subgroups of projective transformations

If $A_{31} = A_{32} = 0$ then you do not need $\mathbb{P}^2$

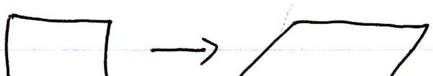
$$\begin{pmatrix} u' \\ v' \\ w' \end{pmatrix} \sim \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} \Leftrightarrow \begin{pmatrix} u'/w' \\ v'/w' \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} u/w \\ v/w \end{pmatrix} + \begin{pmatrix} A_{13} \\ A_{23} \end{pmatrix}$$

Points at infinity stay at infinity

$$\begin{pmatrix} u' \\ v' \\ 0 \end{pmatrix} \sim \begin{pmatrix} & & \\ & & \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ v \\ 0 \end{pmatrix}$$

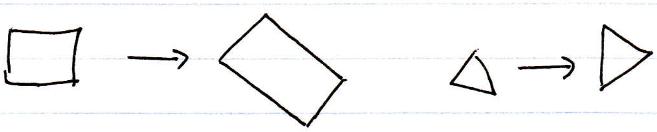
$\boxed{\text{The horizon remains the same } w=0}$

Transformation is called affine.



$$\begin{pmatrix} s\cos\theta & s\sin\theta & t_x \\ -s\sin\theta & s\cos\theta & t_y \\ 0 & 0 & 1 \end{pmatrix}$$

<u>similitude or similarity</u>

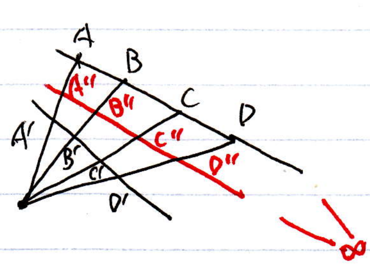


$s=1$, rigid transformation

<u>What is invariant in a projective transformation?</u>

Cross ratio $\{A, B; C, D\}$

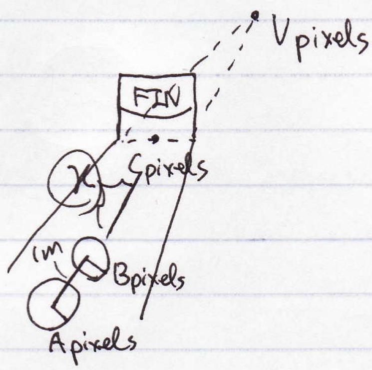
$$= \frac{AC}{AD} : \frac{BC}{BD}$$



$$\frac{AC}{A\infty} : \frac{BC}{B\infty} = \frac{A''C''}{A''\infty} : \frac{B''C''}{B''\infty}$$

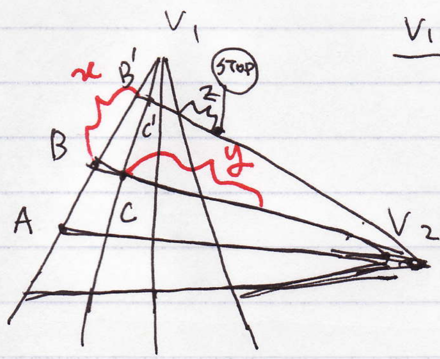
$= 1$ $\qquad = 1$

Single view metrology



$$\lambda = \underbrace{\frac{AC}{AV} : \frac{BC}{BV}}_{\text{all known}} = \frac{A_wC_w}{A_wV_w} : \frac{B_wC_w}{B_wV_w} = \frac{1+x}{x} \Rightarrow x$$

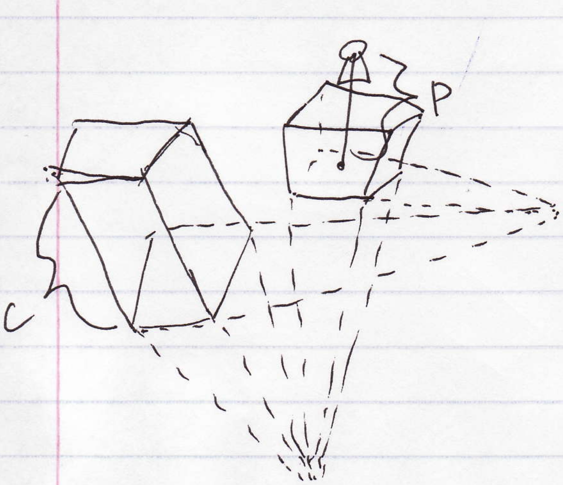
$\boxed{A_wV_w \quad B_wV_w} = 1m$

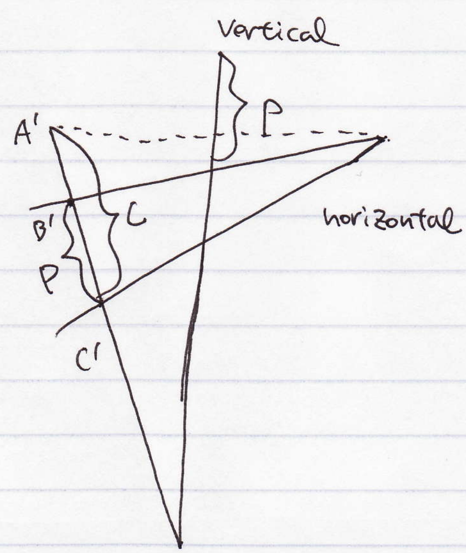


V_1, V_2 Known

If $AB \perp BC$ $B'C' = BC$ in world $\Rightarrow z$

If $AB = BC = 1\,m$ in world then all distances x and y are computable.





Multi-camera

Zisserman ECCV 1996
"Video metrology"

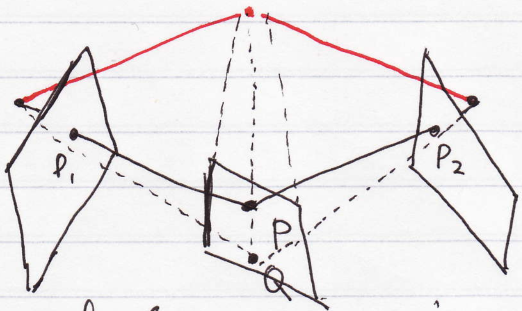


image plane 1 image plane 2

① Assume that we can find the vertical vanishing point
② Assume that we know the projective transformation of field $p_1 \sim A p_2$

If points transform line $p_1 \sim A p_2$ then lines transform $l_1 \sim A^{-T} l_2$

$l_2 \sim v_2 \times P_2$
$l'_2 \sim A^{-1} l_2$

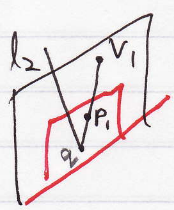


image plane 1

$\underset{\text{intersection}}{q} \sim l_2' \times (\cancel{V_2}\, v_1 \times P_1)$

projection of the intersection of the two vertical planes with the ground