CIS 262: Fall 2008: Midterm 1, October 7, 12.00–1.20pm

Please write your answers succinctly and rigorously.

1. Let $\Sigma = \{0, 1\}$. Consider the language L consisting of words that contain 010 (that is, the language given by $(0 + 1)^*010(0 + 1)^*$).

   (a) Draw a DFA A for L. Note: you don't need to use translations from regular expressions to ε-NFAs, and then to DFAs, just figure out the DFA directly. 10pts

   (b) For each state q of the DFA A in part (a), precisely describe the set of words that lead the DFA from its initial state to q. 20pts

2. Let $\Sigma = \{a, b\}$. Let L_1 be the set of words w that contain an even number of a's. Let L_2 be the set of words w that end with b. Let $L_3 = L_1 \cap L_2$.

   (a) Draw a 2-state DFA A_1 for L_1. 3pts

   (b) Draw a 2-state DFA A_2 for L_2. 3pts

   (c) Use the product construction on the DFAs A_1 and A_2 to get a DFA A_3 for L_3. 8pts

   (d) For the DFA A_3, which states are equivalent (or indistinguishable) to each other? You may want to use the table-filling algorithm of Section 4.4, but you do not need to show all the intermediate steps. 8pts

   (e) Use the answer in part (d), to produce the minimal DFA for L_3. 8pts

3. For each of the statements below, state whether the statement is True or False. In either case, give a short justification (one or two sentences). 30pts

   (a) The language of a DFA is empty if and only if the set of its final states is empty.

   (b) If A and B are DFAs such that $L(A) = L(B)$ then A and B must have the same number of states.

   (c) Regular languages are closed under the operation of set difference.

4. A word w is a *palindrome* if it is its own reverse (that is, $w = w^R$). For example, 010 is a palindrome, but 001 is not. Let $\Sigma = \{0, 1\}$. Prove that the set of palindromes is not regular using the Pumping Lemma. 30pts

5. For two words y and w, we say that y is a *subword* of w, if $w = xyz$ for some words x and z. For example, the word 011 has following subwords: ε, 0, 1, 01, 11, and 011. For a language L, the set of its subwords, denoted *subwords*(L), contains all the subwords of the words in L. That is, a word y belongs to *subwords*(L) precisely when there exists a word w in L such that y is a subword of w.

   (a) If $L = 0^+10^*$, write the regular expression that captures *subwords*(L). 10pts

   (b) Let L be a language, and let A be a DFA for L with states Q, initial state q_0, final states F, and transition function δ. Define an automaton B that accepts *subwords*(L). The automaton B can be a DFA or an NFA or an ε-NFA. 10pts

   (c) Prove that B accepts a word if and only if it belongs to *subwords*(L). 10pts