

CIS 262 Fall 2009: Homework 1, Due September 29

Please write your answers succinctly and rigorously.

1. Let $\Sigma = \{a, b\}$. For each of the languages below, draw a DFA that accepts the language. Explain your answer.
 - (a) The set of words w such that w contains the subword aba . 3pts
 - (b) The set of words w such that the number of occurrences of the subword ab in w equals the number of occurrences of the subword ba in w . For instance, $aabbba$ has one occurrence each of ab and ba , and is in the language, while $abbab$ has two occurrences of ab , but only one occurrence of ba , and hence, is not in the language. 7pts
2. Let $\Sigma = \{a, b\}$. Consider the language L consisting of words w such that w has *odd* number of a symbols and *even* number of b symbols.
 - (a) Give a DFA that accepts L . 3pts
 - (b) *Prove* that the DFA of part (a) accepts L . That is, show that for every word w , $\hat{\delta}(q_0, w)$ belongs to the accepting set precisely when w satisfies the desired requirement. For such a proof you need to characterize for every state, the set of words that lead to that state. Then, you can use induction on w . 10pts
3. Let $\Sigma = \{a, b, c\}$. For natural numbers m and n , consider the parameterized language $L_{m,n}$ consisting of words w such that w contains at least m occurrences of the symbol a and at most n occurrences of the symbol b .
 - (a) Draw the DFA for the language $L_{2,3}$, that is, DFA to accept the set of words that contain at least 2 a 's and at most 3 b 's. 2pts
 - (b) Generalize the construction in part (a) to describe the DFA to accept $L_{m,n}$. How many states does it have, as a function of m and n ? 5pts
 - (c) If A is a DFA accepting $L_{m,n}$, give a precise *lower* bound (as a function of m and n) on the number of states of A . Prove your answer. 10pts
4. For $\Sigma = \{0, 1 \dots 9\}$, consider the set L of words w such that the last digit of w has not appeared before. For example, the words 54052, 9980, 7, and ε are in L , but the words 878 and 56292 are not in L . Give a *nondeterministic* finite automaton that accepts L . Try to take advantage of nondeterminism as much as possible. Explain your answer. 10pts