

CIS 262 Fall 2009: Homework 3, Due October 27

Please write your answers succinctly and rigorously.

1. Consider the grammar G with a single variable R , which is also the start variable, over the alphabet $\{0, 1, +, *, (,)\}$: 10pts

$$R \rightarrow 0 \mid 1 \mid R* \mid R + R \mid (R)$$

- (a) Show a left-most derivation of the word $0 + (1 * + 0)$
 - (b) Show a parse tree for the word $(0 + 1) * + 1$
 - (c) Describe in words the language $L(G)$.
2. Consider the language $L = \{0^i 1^j \mid i \leq j \leq 2i\}$. Give a context-free grammar G for L . Prove that your construction is correct. That is, prove: a word w belongs to L if and only if it belongs to $L(G)$. 15pts
 3. Prove that every regular language can be captured by a context-free grammar. For this, consider a DFA $A = (Q, q_0, F, \delta)$, define a grammar G that accepts the same language as A . For each state q of A , G will have a corresponding nonterminal. Figure out the start symbol and productions of G , and prove that $L(G) = L(A)$. 10pts
 4. Consider the grammar G over the alphabet $\{i, e\}$ with a single non-terminal S and productions given by: 15pts

$$S \rightarrow iS \mid iSeS \mid \varepsilon$$

- (a) What is the language generated by G ?
- (b) Show that G is ambiguous.
- (c) Find a grammar G' such that $L(G) = L(G')$ and G' is unambiguous.