1. (Sipser 1.20) Given alphabet \( \Sigma = \{a, b\} \), for each of the following languages provide two strings that are in the language and two that are not. 10 pts

   (a) \( a^*b^* \)
   (b) \( a(ba)^*b \)
   (c) \( a^* \cup b^* \)
   (d) \( (aaa)^* \)
   (e) \( \Sigma^*a\Sigma^*b\Sigma^*a\Sigma^* \)

2. Given alphabet \( \Sigma \), a homomorphism is a function \( h: \Sigma \rightarrow \Sigma'^* \) that maps symbols of \( \Sigma \) to strings in another alphabet \( \Sigma' \). We can extend homomorphisms from symbols to strings by \( h(w_1w_2\cdots w_n) = h(w_1)h(w_2)\cdots h(w_n) \).

   Prove that, if \( L \) is a regular language, then

   \[ h(L) = \{ h(w) \mid w \in L \} \]

   is also a regular language (hint: regular expressions).

3. Prove or disprove the following claims regarding languages \( L \) and \( M \) over alphabet \( \Sigma = \{0, 1\} \). 12 pts

   (a) If \( L \) is regular then \( L \cup M \) is also regular.
   (b) If \( L \) is regular then \( L \cap M \) is also regular.
   (c) If neither \( L \) nor \( M \) is regular then \( L \cup M \) is not regular.
   (d) If neither \( L \) nor \( M \) is regular then \( L \cap M \) is not regular.

4. (Sipser 1.28) Convert the following regular expressions into equivalent NFAs. You don’t need to prove the correctness of the NFAs. 10 pts

   (a) \( a(abb)^* \cup b \)
   (b) \( aa^* \cup ab(ab)^* \)

5. Given alphabet \( \Sigma = \{a, b, c\} \), is language \( L = \{ w = a^k b^m c^n \mid k + m + n \text{ is even} \} \) regular? Either way, prove it. 10 pts

6. Given alphabet \( \Sigma = \{1\} \), prove that the language

   \[ L = \{1^p \mid p \text{ is prime} \} \]

   is not regular.
7. Given alphabet $\Sigma = \{1\}$, prove that the language

$$L = \{1^n \mid n \text{ is not a perfect square}\}$$

is not regular.

8. (Sipser 1.46) Given alphabet $\Sigma = \{0, 1\}$, prove that the following languages are not regular.

(a) $L = \{0^m1^n \mid m, n \geq 0\}$

(b) $L = \{w \mid w \text{ is not a palindrome}\}$ where a palindrome is any string $w$ that is the same backwards and forwards, e.g. 0100010.

9. Log into Automata Tutor and complete the regular expression construction problems.