Abstract

A simulated quantum annealer takes advantage of the adiabatic theorem to solve problems by transitioning from a simple to complex quantum state while maintaining minimal energy. We simulate these quantum states, energies, and transitions on classical machines by mathematical abstractions for proof of concept and to circumvent the complications of physical implementation.

Goals

1. Simulate quantum state on a classical machine
2. Proof of concept of the adiabatic theorem and its applications to quantum computation

Quantum Computing

• Classical bits can take on values 1 or 0.
• Qubits can take on any value of a unit vector in complex space

Simulated Quantum Annealer

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System Design

Simulation Under Quantum Gate Model

We multiply vectors by matrix representations of linear transformations to simulate quantum gates acting on qubits.

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Quantum Annealing

Simple Hamiltonian with easily found ground quantum state and minimal energy

Slow transition to more complex Hamiltonians, while maintaining minimal energy

Complex Hamiltonian, still with minimal energy, whose ground state encodes solution

Apply Gate Model Abstraction Technique to Annealing

Hamiltonian energy function

$$H|\psi\rangle = \lambda|\psi\rangle$$

Linear transformation on quantum state vector

System Implementation

Simulated Quantum Annealing Algorithm

1. Start at simple $H_B$
2. Define complex $H_C$
3. Define slow moving transition function, $H_T$, from $H_B$ to $H_C$
4. Transition slowly over time $s$ until minimal gap is reached

Using Simulated Quantum Annealing to Solve 3SAT

Given $(x_1 \lor x_2 \lor x_3) \land \cdots \land (x_i \lor x_j \lor x_k)$, find assignments for all $x_i$ such that the entire expression evaluates to true. Under classical computing, 3SAT is in NPC.

<table>
<thead>
<tr>
<th>Step</th>
<th>Implementation</th>
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<tbody>
<tr>
<td>$H_B$ simple</td>
<td>Every qubit in equally weighted superposition of 0 and 1</td>
</tr>
<tr>
<td>$H_C$ complex</td>
<td>All clauses that evaluate to true have energy 0; all other clauses have energy 1; Will have minimal energy when the entire expression is satisfied</td>
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<tr>
<td>Transition Function</td>
<td>Transition slowly from $H_B$ to $H_C$ by making small changes to $H_B$ to a neighboring state</td>
</tr>
<tr>
<td>Solution</td>
<td>Stop when minimal gap between current state and $H_C$ is reached; Eigenvector for minimal eigenvalue encodes the satisfying assignments</td>
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Conclusion and Further Work

Current quantum computers use annealing, but their ability to provide exponential speedup and computational robustness are not yet certain. Simulation of quantum annealing allows us to bypass the hurdles of physical implementation and study annealing’s potential uses.