Subloops, Barkhausen noise, and disorder induced critical behavior

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Hysteresis loops are often seen in experiments at first order phase transformations when the system goes out of equilibrium, such as in the supercooling of liquids and in magnets. The nonequilibrium, zero-temperature random-field Ising model has been studied as a model for the hysteretic behavior of these transformations. As disorder is added, one finds a transition where the jump in the saturation hysteresis loop (corresponding to an infinite avalanche) decreases to zero. At this transition the model exhibits power law distributions of noise (avalanches), universal behavior, and a diverging length scale [O. Perković, K. Dahmen, and J. P. Sethna, Phys. Rev. B **59**, 6106 (1999)]. Interestingly, not only the saturation loops but also subloops reflect this critical point, and at the critical disorder one finds history-induced critical scaling. We present simulation results for histories in systems of almost 14 million spins. Concentric inner subloops are found to resemble rescaled saturation loops at effectively higher (possibly correlated) disorder. In addition, avalanche size distributions for the inner subloops are collapsed using Widom scaling methods. The resulting exponents and scaling functions are shown to differ from those corresponding to the saturation loop.
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I. INTRODUCTION

Many physical systems that are far from thermal equilibrium show hysteresis in response to an external force or field (''the response lags the force''). A subset of these systems responds with collective or crackling noise to a change in the driving force. Examples range from Barkhausen noise in magnets,[1] to acoustic emission in martensitic shape-memory alloys,[2] to earthquakes. The noise in many of these systems can be understood as being due to avalanches—a region transforming under an external force, triggering coupled regions to transform as well, and so on. In magnets, Barkhausen noise is due to collective domain reorientation in response to a change in the external magnetic field.

Recently, hysteresis and avalanches in disordered magnetic materials have been modeled using several variants of the nonequilibrium, zero-temperature random-field Ising model (RFIM), one of the simplest models of magnetism with other applications far beyond magnetic systems.[3]

In Secs. II and III we briefly review the model and some previous results for the saturation hysteresis loop and the associated Barkhausen noise. In Sec. IV we report new results on subloops that reveal disorder-induced critical behavior. Conclusions are given in Sec. V.

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II. MODEL

In the zero-temperature RFIM magnetic domains are represented by spins s_i on a (hyper)cubic lattice, which can take two values: $s_i = \pm 1$. The spins interact ferromagnetically with their nearest neighbors with a strength J and are coupled to a uniform external magnetic field H, which is directed along the spins. Disorder is simulated by a random field h_i associated with each site, and taken from a Gaussian random distribution function

$$\rho(h_i) = \frac{1}{\sqrt{2\pi}R} \exp\left(-\frac{h_i^2}{2R^2}\right) \quad (1)$$

with standard deviation (''disorder'') R. The Hamiltonian is then given by

$$\mathcal{H} = -J\sum_{\langle ij \rangle} s_i s_j - \sum_i (H + h_i) s_i\,, \quad (2)$$

where the sum extends over nearest neighbors. To sweep through the saturation hysteresis loop the external magnetic field is adiabatically ramped from $H=-\infty$ to $H=+\infty$ and back. In the initial state at $H=-\infty$ all spins are pointing down. As the field is slowly increased, each spin flips when its local field, $H + h_i + \Sigma_j s_j$, changes sign. A spin flip can trigger neighboring spins to flip as well, thus creating an avalanche of spin flips, which is the analog of a Barkhausen pulse. At zero temperature, this model is completely deterministic. The simulations are based on the code published on the web[4] which has been modified to allow for subloops in the history. For an in depth description of the base code implementation see Ref. 3.

III. REVIEW OF RESULTS FOR SATURATION HYSTERESIS LOOP

Below a critical disorder $R_c = 2.16\,\mathrm{J}$ the hysteresis loop looks rather square with a jump ΔM in the magnetization corresponding to an infinite avalanche that flips a finite fraction of the system's spins. The jump in the magnetization scales to zero as $\Delta M \sim (R_c - R)^{\beta}$, where $\beta \simeq 0.035 \pm 0.028$ is a ''universal'' prediction of the model for three-dimensional magnets.[5] In this context universal means independent of the microscopic details and identical for a large class of materials that must share merely some basic properties, such as symmetries, dimensions, etc. At R_c the magnetization is described by a power law of the form $M(H) - M_c \sim (H - H_c)^{1/\delta}$, with $H_c = 1.435\,\mathrm{J}$, where again δ is a universal prediction for experiments ($\beta\delta \simeq 1.81 \pm 0.32$ in three dimensions).[5] Above the critical disorder the hysteresis loop is smooth, corresponding to mostly small avalanches. The exact values of R_c, H_c, and M_c are nonuniversal.

For $R \geqslant R_c$ the general scaling form for the distribution $D_{\mathrm{int}}(S,R)$ of avalanche sizes for the saturation loop is

$$D_{\mathrm{int}}(S,R) \sim S^{-(\tau+\sigma\beta\delta)} \bar{\mathcal{D}}_{+}^{\mathrm{int}}(S^{\sigma}(R-R_c)), \tag{3}$$

where $\bar{\mathcal{D}}_{+}^{\mathrm{int}}$ is a universal scaling function shown as a dotted line in the inset of Fig. 2, with $\tau + \sigma\beta\delta = 2.03 \pm 0.03$ and $1/\sigma = 4.2 \pm 0.3$ both universal exponents.[5] As the critical disorder is approached from above, the avalanche size distribution $D_{\mathrm{int}}(S,R)$ shows more and more decades of self-similar (power law) scaling behavior, up to an exponential cutoff size which grows as $S_{\max} \sim (R - R_c)^{-1/\sigma}$, and reaches the system size near the critical point $R = R_c$, as is expected at a *second order phase transition*.[6] There are many related quantities that show similar scaling behavior near the critical point, for example avalanche durations, power spectra, various correlation functions, and the magnetization curve itself, (see Refs. 1, 3, and 7). Generally we found that the model predictions from simulations with up to a billion spins lie well within the error bars of the experimentally observed scaling exponents,[5] although controlled experiments with tuned disorder to systematically test these ideas are still to be done.[10]

IV. RESULTS FOR SUBLOOPS

Interestingly, at (or very close to) the critical disorder $R = R_c$ the avalanche distributions of subloops scale as well. Several example magnetization curves for the saturated hysteresis loop with subloops are shown in Fig. 1. In (a) the saturated loop was run at the value $R = 2.225\,\mathrm{J}$. The subloops were chosen to be concentric, symmetric in their maximum magnetization values $M_{\max}$, and equally spaced in magnetization with $\Delta M_{\max} = 0.1$. Examining the subloops shows that as one moves inwards, that is to later histories, the subloops resemble rescaled saturation loops at disorders increasingly larger than the system disorder $R = 2.225\,\mathrm{J}$.[5] In fact, pre-

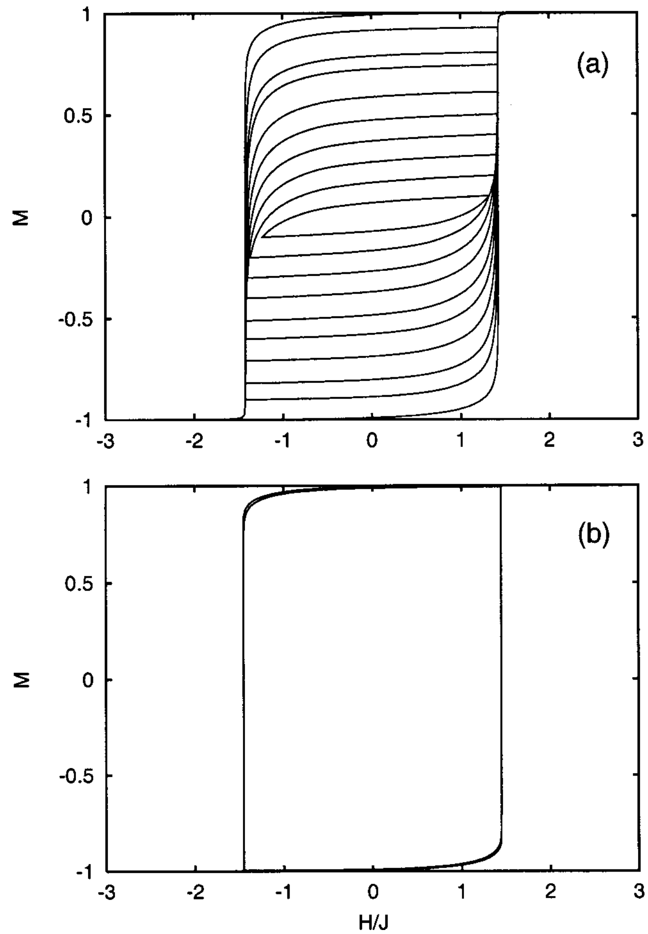

FIG. 1. Saturation hysteresis loops with concentric inner subloops for a 240^3 system size. (a) The saturation loop was run at a disorder of $R = 2.225\,\mathrm{J}$ which is above the infinite system $R_c = 2.16\,\mathrm{J}$. Subloops, on average, are spaced evenly in magnetization by $M = 0.25$ and appear similar to rescaled saturation loops at higher disorder as one moves further inward (to later histories). (b) The saturation loop was run at a disorder of $R = 2.1\,\mathrm{J}$, which is below the infinite system R_c. Unlike the case in (a), only a few subloops are unique after which, due to the large infinite avalanche below R_c, all later subloops are congruent.

flipped spins that are not participating in the inner subloop may well act as an added, possibly correlated, ''effective disorder.'' To test this hypothesis, one may start the outer loop at a disorder below R_c. Then, if the history does increase the effective disorder, one should be able to observe a transition from square to smooth loops as seen for the saturation loops. In Fig. 1(b) such a history is shown for a system disorder with $R < R_c$. Instead of resulting in subloops evenly spaced in $M_{\max}$ as in Fig. 1(a), almost all subloops collapse onto a single subloop: almost all spins of the system flip in the single large avalanche that turns over a finite fraction of the spins. Thus for $R < R_c$, states inside the outer loop are not accessible through adiabatic histories that start from the saturated spin state, and it is not possible to tune the system through the phase transition this way.[8] However, by examining scaling collapses of various quantities on the accessible side of the critical point one may ascertain its existence.[5] To this end we present scaling collapses of integrated avalanche distributions and correlation functions for subloops in systems at the critical disorder $R_c(L)$ for the simulated system

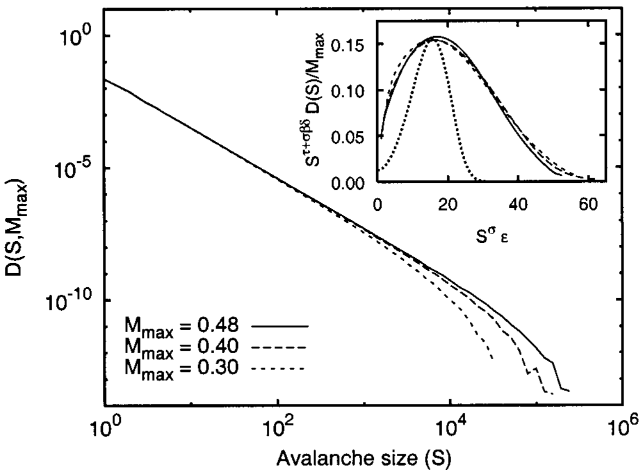

FIG. 2. Integrated avalanche size distribution curves for 240^3 systems at $R=2.225$ J and averaged over 40 random seeds. Curves are given for subloops starting at values of $M_{\max}=0.48$, 0.40, and 0.30. The inset contains a collapse of the three respective distributions. The dotted curve in the inset is the scaling function $\overline{\mathcal{D}}_{+}^{\text{int}}$ (Ref. 5) for the saturation loop and has been scaled to match the maximum of the subloop curves. Used in the collapse was $\epsilon=(M_c-M_{\max})/M_{\max}$, where M_c was taken to be one. The collapse yields the exponents $\tau+\sigma\beta\delta=2.04\pm0.05$ and $\alpha=1/\sigma=3.2\pm0.5$.

size L^3. The system size dependent critical disorder $R_c(L)$, with $R_c(L)\rightarrow R_c$ for $L\rightarrow\infty$, is defined as the disorder which gives the maximum number of system spanning avalanches in the saturation loop.[5]

First, the integrated avalanche size distribution $D(S)$ was measured for subloops spaced by $\Delta M_{\max}=0.025$ in magnetization. The avalanche distributions are plotted in Fig. 2. The curves display a behavior similar to those of the saturation loop with a cutoff size decreasing as the history-induced disorder is increased. These avalanche distributions were collapsed using the scaling ansatz $D(S)\sim M_{\max}S^{-(\tau+\sigma\beta\delta)}\mathcal{D}(S\epsilon^{\alpha})$, with $\epsilon=(1-M_{\max})/M_{\max}$. The factor $M_{\max}$ compensates for the reduced number of spins that flip in successive subloops. According to scaling theory one again expects $\tau+\sigma\beta\delta$, α, and the scaling function $\mathcal{D}$ to be universal. The collapse is shown in the insert of Fig. 2 as the thin lines. To remove the effects of the finite system size, only subloops that did not contain any spanning avalanches were used in the collapse.

Comparing the exponents from the collapse of the subloop data and that of the outer saturation loop reveals that the values of $\tau+\sigma\beta\delta$ are identical within error, as is to be expected, since $\epsilon=0$ corresponds to the saturation loop. The exponent $\alpha=3.2\pm0.5$, however, is significantly smaller than its saturation counterpart $1/\sigma=4.2$, with no overlap in the error estimates. Now one would expect the value $\alpha=1/\sigma$, for example, if the flipped spins at $M=M_{\max}$ were uniformly distributed at random: $(1-M_{\max})$ is the density of flipped spins, and hence a measure of the history induced disorder. However the difference is not surprising since the history induced disorder may be a correlated disorder, as opposed to the uncorrelated system disorder $R_c(L)$. For comparison, the outer loop scaling function[5] is also given in the inset of Fig. 2 as a dotted line. It is clear that the scaling functions for the two types of disorders are indeed different, with that of the saturation hysteresis loop being much more sharply peaked. Similar conclusions can be drawn from collapses of the avalanche correlation functions[5] for the same histories as used in Fig. 2.[9]

V. CONCLUSIONS

We have shown that for systems with a critical amount of disorder, not only the saturation loop, but also subloops, reflect history induced critical scaling: a universal scaling function relates the Barkhausen noise of the inner subloops to the noise in the saturation loop, so that it becomes possible to predict certain aspects of the long length scale behavior of the inner subloops merely from measurements of the long length scale behavior of the saturation loop. A systematic experimental test of these predictions would be very interesting. Some aspects of the related disorder induced phase transition predicted for the $M(H)$ curve of the saturation loop have recently been verified experimentally.[10] A corresponding test for the associated Barkhausen noise remains to be done.

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