An assignment is a sequence \( i_1, i_2 \ldots i_{20} \), where \( i_j \) is the job to which person \( j \) is assigned. Since only one person can be assigned to a job, it follows that the sequence is a permutation of the numbers 1, \ldots, 20 and so there are 20! different possible assignments.

There are 4! possible arrangements. By assigning instruments to Jay, Jack, John and Jim, in that order, we see by the generalized basic principle that there are 2x1x2x1=4 possibilities.

\[
\frac{12!}{6!4!} = 27,720
\]

a) 30^5 

b) 30\cdot29\cdot28\cdot27\cdot26

\[
\binom{20}{2}
\]

The first gift can go to any of the 10 children, the second to any of the remaining 9 children. Hence there are 10\cdot9\cdots5\cdot4 =604,800 possibilities.

a) There are \( \binom{8}{3} \binom{4}{3} + \binom{8}{3} \binom{2}{1} \binom{4}{2} = 896 \) possible committees.

There are \( \binom{8}{3} \binom{4}{3} \) that do not contain either of the 2 men, and there are \( \binom{8}{3} \binom{2}{1} \binom{4}{2} \) that contain exactly 1 of them.
b) There are \[ \binom{6}{3} + \binom{2}{1} \binom{6}{2} = 1000 \] possible committees.

c) There are \[ \binom{7}{3} + \binom{7}{2} + \binom{7}{3} = 910 \] possible committees. There are \[ \binom{7}{3} \] in which neither feuding party serves; \[ \binom{7}{2} \] in which the feuding woman serves; and \[ \binom{7}{3} \] in which the feuding man serves.

1-23 \[ 3! \cdot 2^3 = 48 \]

1-31 a) The number of non-negative integer solutions of \( x_1 + x_2 + x_3 + x_4 = 8 \) is \[ \binom{11}{3} = 165 \]

b) Here it is the number of positive solutions – hence the answer is \[ \binom{7}{3} = 35 \]

1-33 a) \( x_1 + x_2 + x_3 + x_4 = 20, \forall x_1 \geq 2, x_2 \geq 2, x_3 \geq 3, x_4 \geq 4 \).

Let \( y_1 = x_1 - 1, y_2 = x_2 - 1, y_3 = x_3 - 2, y_4 = x_4 - 3 \)

Then, \( y_1 + y_2 + y_3 + y_4 = 13, y_i > 0 \).

Hence, there are \[ \binom{12}{3} = 220 \] possible strategies.

b) There are \[ \binom{15}{2} \] investments only in 1,2,3

There are \[ \binom{14}{2} \] investments only in 1,2,4

There are \[ \binom{13}{2} \] investments only in 1,3,4

There are \[ \binom{13}{2} \] investments only in 2,3,4

\[ \binom{15}{2} + \binom{14}{2} + 2 \binom{13}{2} + \binom{12}{3} = 572 \] possibilities.
Chapter 1 Theoretical Exercises

1-8 There are \( \binom{n+m}{r} \) groups of size \( r \). As there are \( \binom{n}{i} \binom{m}{r-i} \) groups of size \( r \) that consist of \( i \) men and \( r-i \) women, we see that

\[
\binom{n-m}{r} = \sum_{i=0}^{r} \binom{n}{i} \binom{m}{r-i}.
\]

Chapter 2 Problems

2-4 \( A = \{1, 0001, 0000001, \ldots\} \quad B = \{01, 00001, 0000001, \ldots\} \quad (A \cup B)' = \{00000.., 001, 000001, \ldots\} \)

2-5 a) \( 2^5 = 32 \)

b) \( W = \{(1, 1, 1, 1, 1), (1, 1, 1, 0, 1), (1, 1, 0, 1, 1), (1, 1, 1, 0, 0), (1, 1, 0, 0, 1), (1, 1, 0, 0, 0), (1, 0, 1, 1, 1), (1, 0, 1, 1, 0), (1, 0, 1, 0, 1), (1, 0, 1, 0, 0), (0, 0, 1, 1, 0), (0, 0, 1, 1, 1), (0, 0, 1, 0, 1), (0, 0, 0, 1, 1), (0, 0, 0, 0, 1), (0, 0, 0, 0, 0)\} \)

c) \( 8 \)

d) \( AW = \{(1, 1, 1, 0, 0), (1, 1, 0, 0, 0)\} \)

2-9 Choose a customer at random. Let \( A \) denote the event that this customer carries an American Express card and \( V \) the event that he or she carries a VISA card.

\[
P(A \cup V) = P(A) + P(V) - P(AV) = .24 + .61 - .11 = .74
\]

Therefore, 74 percent of the establishment’s customers carry at least one of the two types of credit cards that it accepts.

2-18 \( \frac{2 \cdot 4 \cdot 16}{52 \cdot 51} \)

2-25 \( P(E_n) = \left( \frac{26}{36} \right)^{n-1} \left( \frac{4}{36} \right), \quad \sum_{n=1}^{\infty} P(E_n) = \frac{2}{5} \)

2-30 a) \( \frac{\binom{7}{3} \binom{8}{3}}{\binom{8}{4} \binom{9}{4}} = 1/18 \)
b) \( \frac{\binom{7}{3} \binom{8}{3}}{\binom{8}{4} \binom{9}{4}} = 1/18 = 1/6 \)
c) \( \frac{\binom{7}{3} \binom{8}{4} + \binom{7}{8} \binom{8}{4}}{\binom{8}{4} \binom{9}{4}} = 1/2 \)
2-31 \[ P\{\text{complete}\} = \frac{3 \cdot 2 \cdot 1}{3 \cdot 3 \cdot 3} = \frac{2}{9} \]
\[ P\{\text{same}\} = \frac{3}{27} = \frac{1}{9} \]

2-33 \[ \binom{5}{2} \binom{15}{2} = \frac{70}{323} \]

2-45 \( \frac{1}{n} \) if discard, \( \frac{(n-1)^{k-1}}{n^k} \) if do not discard.

2-52 a) \[ \frac{20 \cdot 18 \cdot 16 \cdot 14 \cdot 12 \cdot 10 \cdot 8 \cdot 6}{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13} \]

b) \[ \frac{\binom{10}{1} \binom{9}{2} \cdot 2^6}{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13} \]

Chapter 2
Theoretical Exercises

2-7 a) \( E \)
b) \( EF \)
c) \( EG \cup F \)

2-11 \[ 1 \geq P(E \cup F) = P(E) + P(F) - P(EF) \]