Assignment I
Solutions

Chapter 1  Problems

1-3  An assignment is a sequence \( i_1, i_2 \ldots i_{20} \), where \( i_j \) is the job to which person \( j \) is assigned. Since only one person can be assigned to a job, it follows that the sequence is a permutation of the numbers 1,…,20 and so there are 20! different possible assignments.

1-4  There are 4! possible arrangements. By assigning instruments to Jay, Jack, John and Jim, in that order, we see by the generalized basic principle that there are 2x1x2x1=4 possibilities.

1-7  a) 6!=720  
b) 2 ·3! ·3!=72  
c) 4!3!=144  
d) 6·3·2·2·1·1=72

1-9  \[ \frac{12!}{6!4!} = 27,720 \]

1-12  a) 30^5  
b) 30·29·28·27·26

1-13  \[ \binom{20}{2} \]

1-17  The first gift can go to any of the 10 children, the second to any of the remaining 9 children. Hence there are 10·9·8….5·4 =604,800 possibilities.

1-19  a) There are \[ \binom{8}{3} + \binom{8}{3} \binom{2}{1} = 896 \] possible committees.
There are \( \binom{8}{3} \binom{4}{3} \) that do not contain either of the 2 men, and there are
\( \binom{8}{3} \binom{2}{1} \binom{4}{2} \) that contain exactly 1 of them.

b) There are \( \binom{6}{3} \binom{6}{3} + \binom{2}{1} \binom{6}{2} \binom{6}{3} = 1000 \) possible committees.

c) There are \( \binom{7}{3} \binom{5}{3} + \binom{7}{2} \binom{5}{3} + \binom{7}{3} \binom{5}{2} = 910 \) possible committees. There are
\( \binom{7}{3} \binom{5}{3} \) in which neither feuding party serves; \( \binom{7}{2} \binom{5}{3} \) in which the feuding woman serves; and \( \binom{7}{3} \binom{5}{2} \) in which the feuding man serves.

1-23 \( 3! \cdot 2^3 = 48 \)

1-31 a) The number of non-negative integer solutions of \( x_1 + x_2 + x_3 + x_4 = 8 \) is
\( \binom{11}{3} = 165 \)

b) Here it is the number of positive solutions – hence the answer is \( \binom{7}{3} = 35 \)

1-33 a) \( x_1 + x_2 + x_3 + x_4 = 20, \forall x_1 \geq 2, x_2 \geq 2, x_3 \geq 3, x_4 \geq 4. \)
Let \( y_1 = x_1 - 1, y_2 = x_2 - 1, y_3 = x_3 - 2, y_4 = x_4 - 3 \)
Then, \( y_1 + y_2 + y_3 + y_4 = 13, y_i > 0. \)
Hence, there are \( \binom{12}{3} = 220 \) possible strategies.

b) There are \( \binom{15}{2} \) investments only in 1,2,3
There are \( \binom{14}{2} \) investments only in 1,2,4
There are \( \binom{13}{2} \) investments only in 1,3,4
There are \( \binom{13}{2} \) investments only in 2,3,4
\( \binom{15}{2} + \binom{14}{2} + 2 \binom{13}{2} + \binom{12}{3} = 552 \) possibilities.
Chapter 1 Theoretical Exercises

1-8 There are \( \binom{n+m}{r} \) groups of size \( r \). As there are \( \binom{n}{i} \binom{m}{r-i} \) groups of size \( r \) that consist of \( i \) men and \( r-i \) women, we see that
\[
\binom{n-m}{r} = \sum_{i=0}^{r} \binom{n}{i} \binom{m}{r-i}.
\]

Chapter 2 Problems

2-4 \( A = \{1, 0001, 0000001, \ldots\} \)
\( B = \{01, 00001, 00000001, \ldots\} \)
\((A \cup B)^c = \{00000..., 001, 000001, \ldots\}\)

2-5 a) \( 2^5 = 32 \)
b) \( W = \{(1, 1, 1, 1, 1), (1, 1, 1, 0, 1), (1, 1, 0, 1, 1), (1, 1, 0, 1, 0), (1, 1, 0, 1, 0), (1, 1, 0, 1, 0), (1, 1, 0, 1, 0), (1, 1, 1, 1, 1), (1, 1, 1, 0, 1), (0, 1, 1, 1, 1), (0, 1, 1, 0, 1), (1, 0, 1, 1, 1), (0, 1, 1, 0, 1), (0, 0, 1, 1, 1)\}\)
c) 8
d) \( AW = \{(1, 1, 1, 0, 0), (1, 1, 0, 0, 0)\}\)

2-9 Choose a customer at random. Let \( A \) denote the event that this customer carries an American Express card and \( V \) the event that he or she carries a VISA card.
\[
P(A \cup V) = P(A) + P(V) - P(AV) = .24 + .61 - .11 = .74
\]
Therefore, 74 percent of the establishment’s customers carry at least one of the two types of credit cards that it accepts.
2-15

\[
a) \quad 4 \cdot \binom{13}{5} / \binom{52}{5} \\
b) \quad 13 \cdot \binom{4}{2} \cdot \binom{12}{3} \cdot \binom{4}{1} / \binom{52}{5} \\
c) \quad \binom{13}{4} \cdot \binom{4}{2} \cdot \binom{44}{2} / \binom{52}{5} \\
d) \quad \binom{13}{4} \cdot \binom{12}{3} \cdot \binom{4}{2} \cdot \binom{4}{1} / \binom{52}{5} \\
e) \quad \binom{13}{4} \cdot \binom{48}{1} / \binom{52}{5}
\]

2-18 \quad \frac{2 \cdot 4 \cdot 16}{52 \cdot 51}

2-25 \quad P(E_n) = \left( \frac{26}{36} \right)^{n-1} \cdot \frac{4}{36}, \quad \sum_{n=1}^{\infty} P(E_n) = \frac{2}{5}

2-28 \quad P\{\text{Same}\} = \binom{5}{3} + \binom{6}{3} + \binom{8}{3} / \binom{19}{13}

\[
P\{\text{Different}\} = \binom{5}{1} \cdot \binom{6}{1} \cdot \binom{8}{1} / \binom{19}{3}
\]

If sampling is done with replacement,

\[
P\{\text{Same}\} = \frac{5^3 + 6^3 + 8^3}{(19)^3}
\]

\[
P\{\text{Different}\} = P\{\text{RBG}\} + P\{\text{BRG}\} + P\{\text{RGB}\} + \ldots + P\{\text{GRB}\}
\]

\[
= \frac{6 \cdot 6 \cdot 6}{(19)^3} = 0.21
\]

2-30

\[
a) \quad \frac{\binom{7}{3} \cdot \binom{8}{3} \cdot \binom{3!}{3!}}{\binom{8}{4} \cdot \binom{9}{4} \cdot \binom{4!}{4!}} = 1/18 \\
b) \quad \frac{\binom{7}{3} \cdot \binom{8}{9} \cdot \binom{3!}{3!}}{\binom{8}{4} \cdot \binom{9}{4} \cdot \binom{4!}{4!}} = 1/18 = 1/6 \\
c) \quad \frac{\binom{7}{3} \cdot \binom{8}{4} + \binom{7}{3} \cdot \binom{8}{4} \cdot \binom{3}{3}}{\binom{8}{4} \cdot \binom{9}{4}} = 1/2
\]

2-31 \quad P\{\text{complete}\} = \frac{3 \cdot 2 \cdot 1}{3 \cdot 3 \cdot 3} = \frac{2}{9}

\[
P\{\text{same}\} = \frac{3}{27} = \frac{1}{9}
\]
2-33 \[
\frac{\binom{5}{2} \binom{15}{2}}{\binom{20}{4}} = \frac{70}{323}
\]

2-37

a) \[
\frac{\binom{7}{5}}{\binom{10}{5}} = \frac{1}{12} \approx 0.0833
\]

b) \[
\frac{\binom{7}{3}}{\binom{10}{5}} + \frac{1}{12} = \frac{1}{2}
\]

2-45 \(1/n\) if discard, \(\frac{(n-1)^{i-1}}{n^k}\) if do not discard.

2-49 \[
\binom{6}{3} \binom{12}{3} / \binom{12}{6}
\]

2-52 a) \[
\frac{20 \cdot 18 \cdot 16 \cdot 14 \cdot 12 \cdot 10 \cdot 8 \cdot 6}{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13}
\]

b) \[
\frac{\binom{10}{1} \binom{9}{6} \cdot 8! \cdot 5^6}{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13}
\]

Chapter 2
Theoretical Exercises

2-7 a) E
b) EF
c) EG \(\cup\) F

2-11 \[1 \geq P(E \cup F) = P(E) + P(F) - P(EF)\]