Lecture #6 – Compression

# ESE 1500 – Digital Audio Basics

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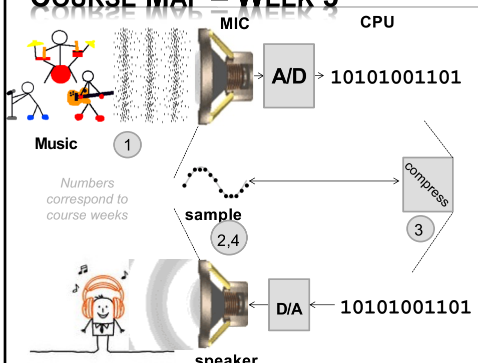
## Lecture Topics

- Where are we on course map?
- Preclass
- Compression: Lossy and Lossless
- Lossless Compression
  - Probability-based lossless compression
    - Huffman Encoding
- Part 2:
  - Common case
  - Entropy
    - Shannon Limits
- Next Lab
- References



## Course Map – Week 3






## Preclass



## Preclass

- **Tell me and I forget, teach me and I may remember, involve me and I learn**
  - -- Benjamin Franklin
- **73 symbols** (fancy, more general term for letters)
- **19 unique (ignoring case)**
  - (A, B, C, D, E, F, G, H, I, L, M, N, O, R, T, V, Y, space, comma)
  - How many bits to represent each symbol?
- **How many bits to encode quote?**



## Preclass

- **Tell me and I forget, teach me and I may remember, involve me and I learn**
  - -- Benjamin Franklin
- **73 symbols**
- **19 unique (ignoring case)**
- **If symbols occurrence equally likely, how many occurrences of each symbol should we expect in quote?**
- **How many E's are there in the quote?**

## Preclass

- Tell me and I forget, teach me and I may remember, involve me and I learn
  - -- Benjamin Franklin
- 73 symbols
- 19 unique (ignoring case)
- Conclude
  - Symbols do not occur equally
  - Symbol occurrence is not uniformly random

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## Preclass

- Tell me and I forget, teach me and I may remember, involve me and I learn
  - -- Benjamin Franklin
- Using uniform encoding (from question 1)
  - How many bits to encode first 24 symbols?
- How many bits using encoding given (Q5a)?

$$TotalBits = \sum_{1}^{24} bits[quote[i]]$$

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## Preclass

- Tell me and I forget, teach me and I may remember, involve me and I learn
  - -- Benjamin Franklin
- Using uniform encoding (question 1)
  - How many bits to encode all 73 symbols?
- How many bits using encoding given (Q5c)?

$$TotalBits = \sum_{1}^{73} bits[quote[i]]$$

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## Conclude

- Can encode with (on average) fewer bits than $\log_2$(unique-symbols)

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## Intro to Compression

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## Data Compression

- What is compression?
  - Encoding information using fewer bits than the original representation
- Why do we need compression?
  - Most digital data is not sampled/quantized/represented in the most compact form
    - It takes up more space on a hard drive/memory
    - It takes longer to transmit over a network
  - Why? Because data is represented so that it is easiest to use
- Two broad categories of compression algorithms:
  - **Lossless** – when data is un-compressed, data is its original form
    - No data is lost or distorted
  - **Lossy** – when data is un-compressed, data is in **approximate** form
    - Some of the original data is lost

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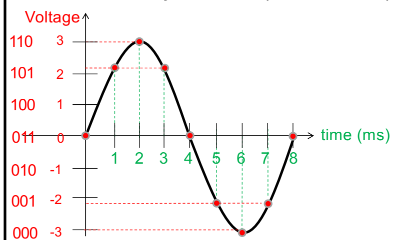
## Recall ADC Process?

- **Analog-to-Digital (ADC) Conversion**
  - **We have 7 discrete voltages, # of bits to represent 7 things?**
  - **3-bits! Why? $2^{3\text{-bits}} = 8$ (1 unused state)**




| Samples: | Binary Encoding: |
|---|---|
| { 0 ms, 0 Volts } → | 011 |
| { 1 ms, 2 Volts } → | 101 |
| { 2 ms, 3 Volts } → | 110 |
| { 3 ms, 2 Volts } → | 101 |
| { 4 ms, 0 Volts } → | 011 |
| { 5 ms, -2 Volts } → | 001 |
| { 6 ms, -3 Volts } → | 000 |
| { 7 ms, -2 Volts } → | 001 |
| { 8 ms, 0 Volts } → | 011 |

Encoding: mapping data from one form to another (not always conversion) 14

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## Example of Lossy Compression

- Sample Rate: **1000 samples/sec**, Resolution: **3-bits** per sample
- Our Sampled Signal: {0, 2.2V, 3V, 2.2V, 0, -2.2V, -3, -2.2V, 0}
- Our Quantized Signal: {0, 2V, 3, 2V, 0, -2, -3, -2, 0}
- Our 3-bit Digitized Data: {011, 101, 110, 101, 011, 001, 000, 001, 011}
  - space required to store/transmit: **27 bits**
- ADC related compression algorithm:
  - CS&Q (Coarser Sampling AND/OR Quantization)
    - Either reduce number of bits per sample AND/OR discard a sample completely
  - Example with our digitized data:
  - Our 3-bit Digitized Data: {011, 101, 110, 101, 011, 001, 000, 001, 011}
  - If we drop the sampling rate by a factor of 2, how impact number of bits needed?
  - ***Lossy*** because we **cannot** restore exact original

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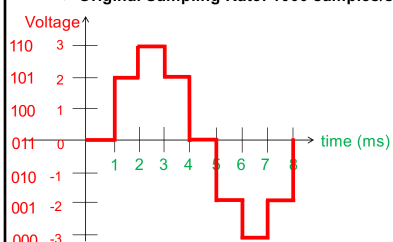
## De-Compression of Signal:

- **Decompression & DAC Process**
  - **Original digital signal: {011, 101, 110, 101, 011, 001, 000, 001, 011}**
  - **Original Sampling Rate: 1000 samples/sec**




Original Signal (recall ADC/DAC is approximation at best anyway!) 16

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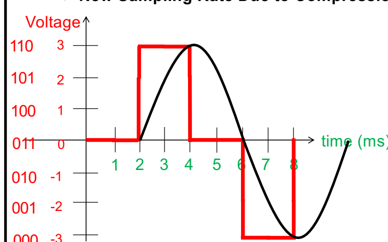
## De-Compression of Signal:

- **Decompression & DAC Process**
  - **Original compressed signal: {011, , 110, , 011, , 000, , 011}**
  - **New Sampling Rate Due to Compression: 500 samples/sec**




Effect of CS&Q compression
Lowered Sampling Rate
Added "noise" to signal
Listeners might not notice!

Lossy Compression:
One can achieve high Compression ratios

Frequently used for Audio:
*MP3 format uses lossy compression algorithm*

Lossy: Compression removed every other sample 17

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## Two forms of classification

Compression Algorithms → Lossy, Lossless

Compression Algorithms → Fixed Group Size, Variable Group Size

Examples of **Fixed** Group Size:
Take in 2 samples: (6-bits) always spit out: (3-bits)
Take in 8-bit ASCII character (group), spit out 7-bit ASCII character (group)

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## Probability-Based Lossless Compression

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## Information Content

- Does each character contain the same amount of "information"?

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## Statistics

- How often does each character occur?
  - Capital letters versus non-capitals?
  - How many e's in preclass quote?
  - How many z's?
  - How many q's?

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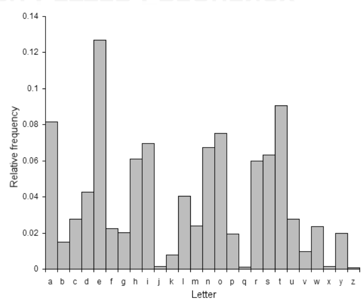
## English Letter Frequency




http://en.wikipedia.org/wiki/File:English-slf.png

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## Huffman Encoding

- Developed in 1950's (D.A. Huffman)
- Takes advantage of frequency of stream of bits occurrence in data
  - Can be done for ASCII (8-bits per character)
    - Characters do not occur with equal frequency.
    - How can we exploit statistics (frequency) to pick character encodings?
  - But can also be used for anything with symbols occurring frequently
    - E.g., Music (drum beats…frequently occurring data)
  - Example of **variable length** compression algorithm
    - Takes in fixed size group – spits out variable size replacement

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## How many Bits to represent all letters?

| Letter | Binary Encoding | Letter | Binary Encoding |
|---|---|---|---|
| A | 00000 | N | 01101 |
| B | 00001 | O | 01110 |
| C | 00010 | P | 01111 |
| D | 00011 | Q | 10000 |
| E | 00100 | R | 10001 |
| F | 00101 | S | 10010 |
| G | 00110 | T | 10011 |
| H | 00111 | U | 10100 |
| I | 01000 | V | 10101 |
| J | 01001 | W | 10110 |
| K | 01010 | X | 10111 |
| L | 01011 | Y | 11000 |
| M | 01100 | Z | 11001 |

Including upper and lower case?
…and numbers, how many bits?

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## ASCII Encoding (7-bit encoding)

| Letter | ASCII Code | Binary | Letter | ASCII Code | Binary |
|---|---|---|---|---|---|
| a | 097 | 01100001 | A | 065 | 01000001 |
| b | 098 | 01100010 | B | 066 | 01000010 |
| c | 099 | 01100011 | C | 067 | 01000011 |
| d | 100 | 01100100 | D | 068 | 01000100 |
| e | 101 | 01100101 | E | 069 | 01000101 |
| f | 102 | 01100110 | F | 070 | 01000110 |
| g | 103 | 01100111 | G | 071 | 01000111 |
| h | 104 | 01101000 | H | 072 | 01001000 |
| i | 105 | 01101001 | I | 073 | 01001001 |
| j | 106 | 01101010 | J | 074 | 01001010 |
| k | 107 | 01101011 | K | 075 | 01001011 |
| l | 108 | 01101100 | L | 076 | 01001100 |
| m | 109 | 01101101 | M | 077 | 01001101 |
| n | 110 | 01101110 | N | 078 | 01001110 |
| o | 111 | 01101111 | O | 079 | 01001111 |
| p | 112 | 01110000 | P | 080 | 01010000 |
| q | 113 | 01110001 | Q | 081 | 01010001 |
| r | 114 | 01110010 | R | 082 | 01010010 |
| s | 115 | 01110011 | S | 083 | 01010011 |
| t | 116 | 01110100 | T | 084 | 01010100 |
| u | 117 | 01110101 | U | 085 | 01010101 |
| v | 118 | 01110110 | V | 086 | 01010110 |
| w | 119 | 01110111 | W | 087 | 01010111 |
| x | 120 | 01111000 | X | 088 | 01011000 |
| y | 121 | 01111001 | Y | 089 | 01011001 |
| z | 122 | 01111010 | Z | 090 | 01011010 |

ASCII:
***American Standard Code for Information Interchange***
$2^7$=128 combinations

Standard encoding, developed in the 1960's

Didn't take into account international standards!

UNICODE
8-bit encoding
$2^8$=256 possibilities!

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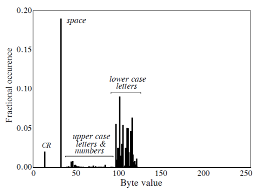
## Huffman Encoding – The Basics




- Example: more than 96% of file consists of 31 characters
- Idea: Assign frequently used characters fewer bits
  - 31 common characters get 5b codes 00000--11110
  - Rest get 13b: 11111+original 8b code
- How many bits do we need on average per original byte?



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## Calculation

- Bits = #5b-characters * 5 + #13b-character * 13
- Bits=#bytes*0.96*5 + #bytes*0.04*13
- Bits/original-byte = 0.96*5+0.04*13
- Bits/original-byte = 5.32



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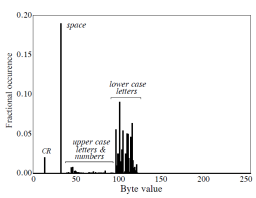
## Huffman Encoding – More Advanced




Example Encoding Table

| letter | probability | Huffman code |
|---|---|---|
| A | .154 | 1 |
| B | .110 | 01 |
| C | .072 | 0010 |
| D | .063 | 0011 |
| E | .059 | 0001 |
| F | .015 | 000010 |
| G | .011 | 000011 |

- Huffman goes further: Assign MOST used characters least # of bits:
  - Most frequent: A= 1, least frequent: G=00011, etc.
  - **Example:** original data stream: C E G A D F B E A…

Huffman encoded: 0010 0001 000011 1 0011 000010 01 0001 1 …



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## Preclass Encoding

| symbol | encode | occur |
|---|---|---|
| (space) | 00 | 15 |
| A | 1110 | |
| B | 100100 | |
| C | 100101 | |
| D | 10110 | |
| E | 110 | 11 |
| F | 011010 | |
| G | 011011 | |
| H | 011000 | |
| I | 0111 | |

| symbol | encode | occur |
|---|---|---|
| L | 0100 | |
| M | 1111 | |
| N | 1010 | |
| O | 10011 | |
| R | 0101 | |
| T | 10111 | |
| V | 10000 | |
| Y | 011001 | |
| , | 10001 | |



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## Preclass Encoding

| symbol | encode | occur |
|---|---|---|
| (space) | 00 | 15 |
| A | 1110 | 6 |
| B | 100100 | |
| C | 100101 | |
| D | 10110 | |
| E | 110 | 11 |
| F | 011010 | |
| G | 011011 | |
| H | 011000 | |
| I | 0111 | 4 |

| symbol | encode | occur |
|---|---|---|
| L | 0100 | 4 |
| M | 1111 | 6 |
| N | 1010 | 5 |
| O | 10011 | |
| R | 0101 | 4 |
| T | 10111 | |
| V | 10000 | |
| Y | 011001 | |
| , | 10001 | |



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## Preclass Encoding

| symbol | encode | occur |
|---|---|---|
| (space) | 00 | 15 |
| A | 1110 | 6 |
| B | 100100 | |
| C | 100101 | |
| D | 10110 | 3 |
| E | 110 | 11 |
| F | 011010 | |
| G | 011011 | |
| H | 011000 | |
| I | 0111 | 4 |

| symbol | encode | occur |
|---|---|---|
| L | 0100 | 4 |
| M | 1111 | 6 |
| N | 1010 | 5 |
| O | 10011 | 2 |
| R | 0101 | 4 |
| T | 10111 | 3 |
| V | 10000 | 2 |
| Y | 011001 | |
| , | 10001 | 2 |



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## PRECLASS ENCODING

| symbol | encode | occur |
|---|---|---|
| (space) | 00 | 15 |
| A | 1110 | 6 |
| B | 100100 | 1 |
| C | 100101 | 1 |
| D | 10110 | 3 |
| E | 110 | 11 |
| F | 011010 | 1 |
| G | 011011 | 1 |
| H | 011000 | 1 |
| I | 0111 | 4 |

| symbol | encode | occur |
|---|---|---|
| L | 0100 | 4 |
| M | 1111 | 6 |
| N | 1010 | 5 |
| O | 10011 | 2 |
| R | 0101 | 4 |
| T | 10111 | 3 |
| V | 10000 | 2 |
| Y | 011001 | 1 |
| , | 10001 | 2 |

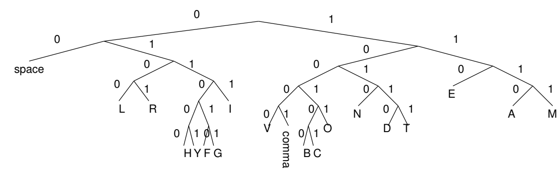
See how variable length encoding saved bits?
Questions?





## PRECLASS ENCODING

- Prefix decodable on tree

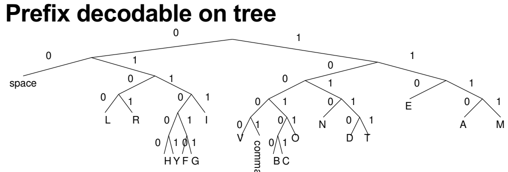







## PRECLASS ENCODING

- Prefix decodable on tree



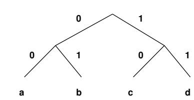

- Cannot have 4 length 2 codes and longer codes
- Each length 2 code consumes 25% of code space
- Each length n code consumes $2^{-n}$ of space
- Having some short codes means some things get longer encodings



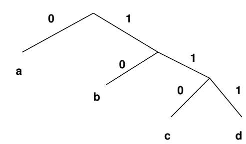


## PREFIX DECODABLE

- Consider small 4 symbol case
  - Uniform 2b each




  - Can give one symbol 1b code: say 0
  - But then must code remaining 3 cases start with 1
    - 3 cases left – need at least 2 more bits for some to differentiate

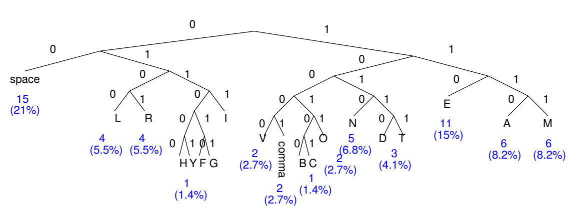







## PRECLASS ENCODING

- Probability, encoding, length








## INTERLUDE

## Interlude

- **SNL – 5 minute University**
  - Father Guido Sarducci
- https://www.youtube.com/watch?v=kO8x8eoU3L4
- **What form of compression here?**

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## For Computer Engineering?

- **Make the common case fast**
- **Make the frequent case small**

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## Part 2

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## Common Case

- **Big idea in optimization engineering**
  - Make the common case inexpensive
- **Shows up throughout computer systems**
  - Computer architecture
    - Caching, instruction selection, branch prediction, …
  - Networking and communication, data storage
    - Compression, error-correction/retransmission
  - Algorithms and software optimization
  - User Interfaces
    - Where things live on menus, shortcuts, …
    - How you organize your apps on screens

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## Entropy

Is there a lower bound for compression?

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## How low can we go with compression?

What is the least # of bits required to encode information?

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## Claude Shannon

- Father of Information Theory, brilliant mathematician
- While at AT&T Bell Labs, landmark paper in 1948
- Determined exactly how low we can go with compression!

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## Shannon's Entropy

- What is entropy?
  - Chaos/Disorganization/Randomness/Uncertainty
- Shannon's Famous Entropy Formula:

$$H = -\sum p(x)\log_2 p(x)$$

Shannon's Entropy *(measured in bits)*

Negative Sum Of:

Probability of each outcome X $\log_2$ of (probability of each outcome)

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## Estimating Entropy of English Language

- Example: 32 Characters
- If we assume all characters are equally probable:
  - $\boldsymbol{p(each\ character) = \frac{1}{32}}$
- Information Entropy per character:

$$H = -\sum p(x)\log_2 p(x)$$

$$H = -32\left(\frac{1}{32}\right)\log\left(\frac{1}{32}\right) = -\log\left(\frac{1}{32}\right) = +5\ bits$$

Same thing we got when we said we needed $\log_2$(unique_things) bits

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## Estimating Entropy of English Language

- 27 Characters (26 letters + space)
- If we assume all characters are equally probable:
  - $\boldsymbol{p(each\ character) = \frac{1}{27}}$
- Information Entropy per character:

$$H = -\sum p(x)\log_2 p(x)$$

$$H = -27\left(\frac{1}{27}\right)\log\left(\frac{1}{27}\right) = -\log\left(\frac{1}{27}\right) = +4.75\ bits$$

Same thing we got when we said we needed $\log_2$(unique_things) bits

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## Preclass 5c

- Computed total bits as sum of bits

$$TotalBits = \sum_{1}^{73} bits[quote[i]]$$

- Per character
  - Divide by total characters
  - Group by same symbols
  - $p_i$ = #occurrences/total_characters

$$\text{Average Bits} = \sum_i p_i \times \text{bits}(i)$$

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## Shannon Entropy

- Essentially says
  - Should be able to encode symbol with log(1/p) bits

$$\text{Average Bits} = \sum_i p_i \times \text{bits}(i)$$

$$H = -\sum_i p_i \times \log_2(p_i)$$

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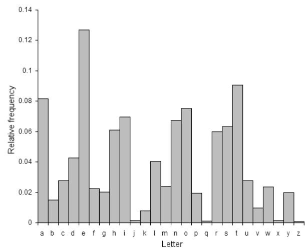
## RECALL:

Not all letters are equally probable in English Language




http://en.wikipedia.org/wiki/File:English-slf.png

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## SHANNON ENTROPY ENGLISH LETTERS

$$H = -\sum_i p_i \times \log_2(p_i)$$

| letter | p | $-\log_2(p)$ | $-p*\log_2(p)$ |
|---|---|---|---|
| a | 8.17% | 3.61 | 0.30 |
| b | 1.49% | 6.07 | 0.09 |
| c | 2.78% | 5.17 | 0.14 |
| d | 4.25% | 4.56 | 0.19 |
| e | 12.70% | 2.98 | 0.38 |
| f | 2.23% | 5.49 | 0.12 |
| | | | |
| z | 0.07% | 10.40 | 0.01 |
| sum | 100.00% | | 4.18 |



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## SHANNON ENTROPY PRECLASS QUOTE

$$H = -\sum_i p_i \times \log_2(p_i)$$

| Symbol | Bits | Occur | P | $-\log_2(p)$ | $-p*\log_2(p)$ | p*bits |
|---|---|---|---|---|---|---|
| (space) | 2 | 15 | 0.21 | 2.28 | 0.47 | 0.41 |
| A | 4 | 6 | 0.08 | 3.60 | 0.30 | 0.33 |
| B | 6 | 1 | 0.01 | 6.19 | 0.08 | 0.08 |
| C | 6 | 1 | 0.01 | 6.19 | 0.08 | 0.08 |
| D | 5 | 3 | 0.04 | 4.60 | 0.19 | 0.21 |
| E | 3 | 11 | 0.15 | 2.73 | 0.41 | 0.45 |
| | | | | | | |
| , | 5 | 2 | 0.03 | 5.19 | 0.14 | 0.14 |
| | | | | sum | 3.74 | 3.77 |



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## ENCODING TARGET

- Right bits target is:
  - Bits(i) = $-\log_2(p_i)$
  - $2^{-\text{Bits}(i)} = p_i$
- Symbol should take up fraction of encoding space matching probability of occurrence



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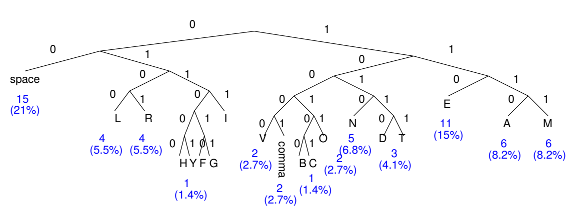
## PRECLASS ENCODING

- **Probability, encoding, length**



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## SUMMING IT UP: SHANNON & COMPRESSION

- **Shannon's Entropy represents a lower limit for lossless data compression**
  - It tells us the minimum number of bits that can be used to encode a message without loss (according to a particular model)
- **Shannon's Source Coding Theorem:**
  - A lossless data compression algorithm cannot compress messages to have (on average) more than 1 bit of Shannon's Entropy per bit of encoded message



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## Learn More

- **ESE 3010– Probability**
  - Central to understanding probabilities
    - What cases are common and how common they are
- **ESE 6740 – Information Theory**
- **Most all computer engineering courses**
  - Deal with common-case optimizations
  - CIS2400, CIS4710, CIS3800, ESE4070, ….

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## Big Ideas

- **Lossless Compression**
  - Exploit non-uniform statistics of data
  - Given short encoding to most common items
- **Common Case**
  - Make the common case inexpensive
- **Shannon's Entropy**
  - Gives us a formal tool to define lower bound for compressibility of data

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## Monday in Lab

- **Implement Compression!**
  - Implement Huffman Compression
  - **Note:** longer prelab with MATLAB intro; plan accordingly
    - Budget a few hours
- **Remember**
  - Feedback

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## References


- **S. Smith, "The Scientists and Engineer's Guide to Digital Signal Processing," 1997.**
- **Shannon's Entropy (excellent video) http://www.youtube.com/watch?v=JnJq3Py0dyM**
  - Used heavily in the creation of entropy slides


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