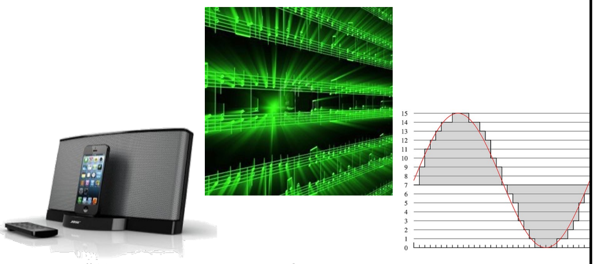


Lecture #10– Discrete Fourier Transform

ESE 1500 –
DIGITAL AUDIO BASICS

1

LECTURE TOPICS

- **Where are we on course map?**
- **Reminder: The Fourier Series**
  - can represent any signal in frequency domain
- **Vectors, Dot Products, Change of basis**
- **Part 2: The Discrete Fourier Transform (DFT)**
  - can translate any signal between time and frequency domain
  - change of basis
- **References**

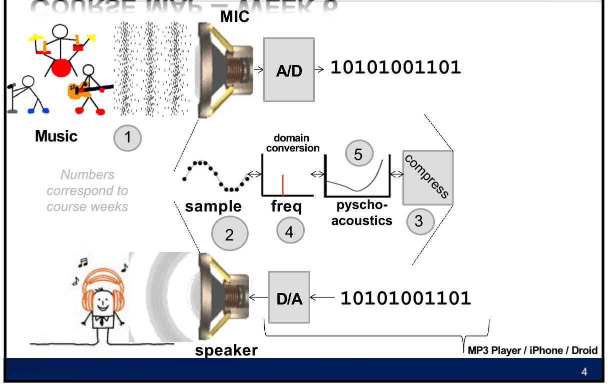
2

WARNING

- **Hardest (most tedious?) lecture**
- **Trying to balance big picture concept with operational math**
  - Big picture:
    - Can represent sound discretely in frequency domain
    - Can perform computation to convert between discrete time samples and discrete frequency amplitudes
  - Operational:
    - Conversion is dot product against frequency to extract how much of a frequency is present (to frequency)
    - …or dot product against frequency samples at time to reconstruct time value (to time)

3

COURSE MAP – WEEK 6



4

The frequency domain &

THE FOURIER SERIES

5

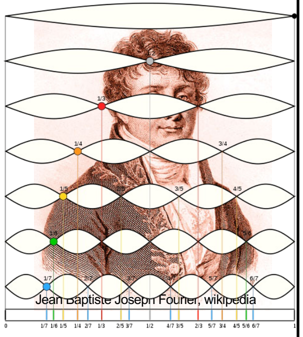
HISTORY…

- **Fourier series:**
  - Any *periodic* signal can be represented as a sum of simple periodic functions: *sin* and *cos*

sin(nt) and cos(nt)

where n = 1, 2, 3, …

These are called the ***harmonics*** of the signal



Jean Baptiste Joseph Fourier, wikipedia

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FOURIER SERIES – MORE FORMALLY

The Fourier Theorem states that any *periodic* function f(t) of period L can be cast in the form:

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi t}{L} + b_n \sin\frac{n\pi t}{L}\right)$$

The constants: a_0, a_n , and b_n are called the Fourier coefficients of *f(t)*

That's the fancy, general form for the examples we've seen like:
$f(\mathbf{t}) = \sin(2\times 2\pi\times t) + \frac{1}{2}\sin(2\pi\times t)$

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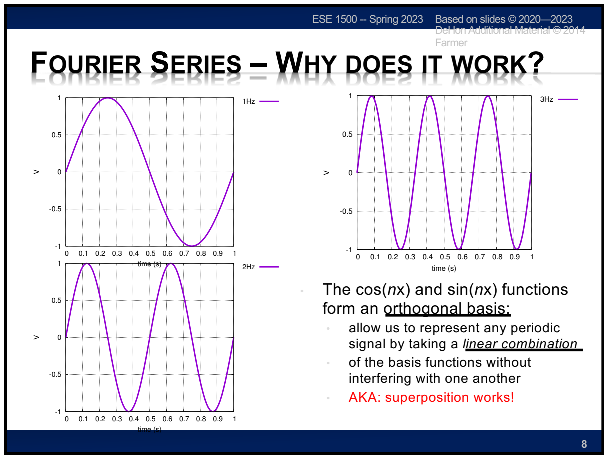

FOURIER SERIES – WHY DOES IT WORK?

- The cos(*nx*) and sin(*nx*) functions form an orthogonal basis:
  - allow us to represent any periodic signal by taking a *linear combination*
  - of the basis functions without interfering with one another
  - AKA: superposition works!

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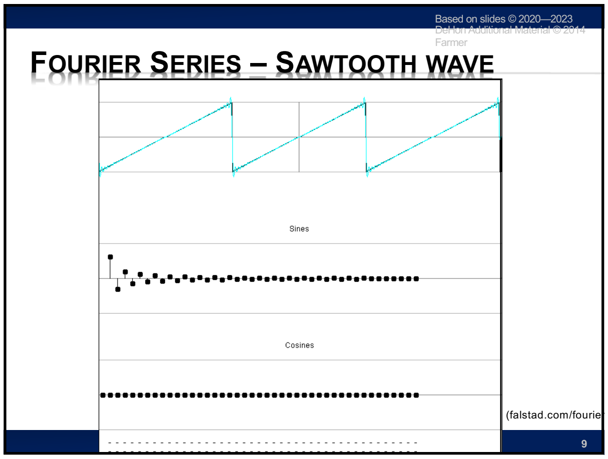

FOURIER SERIES – SAWTOOTH WAVE

(falstad.com/fourie

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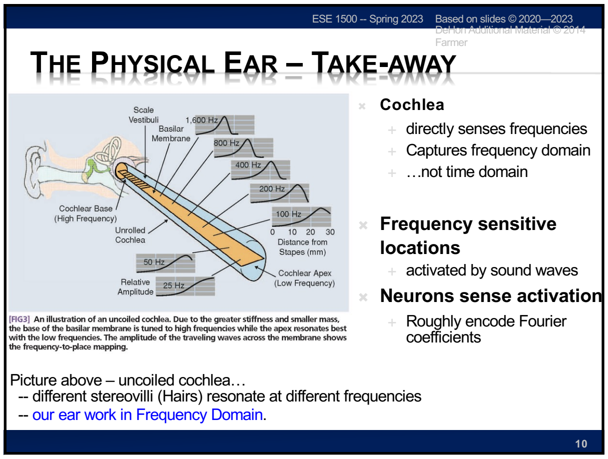

THE PHYSICAL EAR – TAKE-AWAY

[FIG3] **An illustration of an uncoiled cochlea. Due to the greater stiffness and smaller mass, the base of the basilar membrane is tuned to high frequencies while the apex resonates best with the low frequencies. The amplitude of the traveling waves across the membrane shows the frequency-to-place mapping.**

- **Cochlea**
  - directly senses frequencies
  - Captures frequency domain
  - …not time domain
- **Frequency sensitive locations**
  - activated by sound waves
- **Neurons sense activation**
  - Roughly encode Fourier coefficients

Picture above – uncoiled cochlea…
-- different stereovilli (Hairs) resonate at different frequencies
-- our ear work in Frequency Domain.

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VECTOR BACKGROUND

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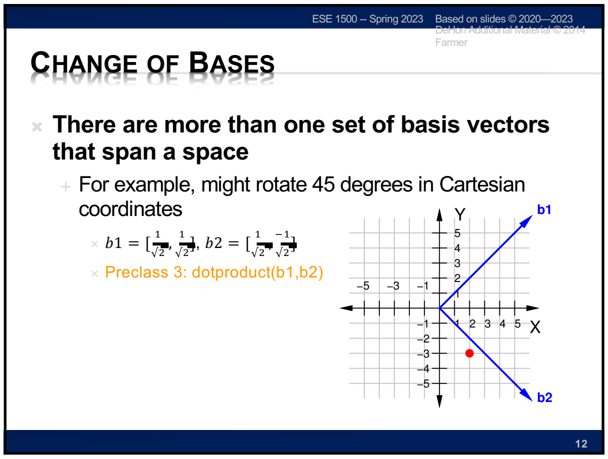

CHANGE OF BASES

- **There are more than one set of basis vectors that span a space**
  - For example, might rotate 45 degrees in Cartesian coordinates
    - $b1 = [\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}], b2 = [\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}]$
    - Preclass 3: dotproduct(b1,b2)

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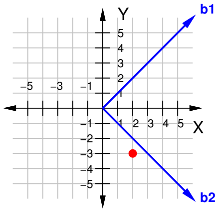
CHANGE OF BASES

- **Can change basis by performing dot product**
  - $b1 = [\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}], b2 = [\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}]$
  - Represent points as linear combination: a*b1+c*b2
  - a=dotproduct([x,y],b1); c=dotproduct([x,y],b2)
    - What are a and c for case shown?
      - Preclass 3
    - Check correct by seeing that a*b1+c*b2 is what we expect
    - $\frac{-1}{\sqrt{2}}$b1+ $\frac{5}{\sqrt{2}}$*b2
    - [-1/2,-1/2]+[5/2,-5/2]
    - [2,-3]



THE FOURIER TRANSFORM

PRECLASS 1

- **Compute Dot Products**

i	0	1	2	3	4	5	6	7	8	9	10
time	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Sample	0	0.95	0.59	-0.59	-0.95	0	0.95	0.59	-0.59	-0.95	0

i	0	1	2	3	4	5	6	7	8	9	10
B1	0.00	0.59	0.95	0.95	0.59	0.00	-0.59	-0.95	-0.95	-0.59	0.00
B2	0.00	0.95	0.59	-0.59	-0.95	0.00	0.95	0.59	-0.59	-0.95	0.00
B3	0.00	0.95	-0.59	-0.59	0.95	0.00	-0.95	0.59	0.59	-0.95	0.00
B4	0.00	0.59	-0.95	0.95	-0.59	0.00	0.59	-0.95	0.95	-0.59	0.00
B5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

DOT PRODUCT

- **Dot(A,B1) = $\sum_{i=0}^{10} A[i] * B[i]$**
- **=A[0]*B[0]+A[1]*B[1]+A[2]*B[2]+…+A[10]*B[10]**
- **=0*0 +0.95*0.59+0.59*0.95+…+0*0**
- **= 0**

PRECLASS 1

- **Compute Dot Products – what did we get?**

i	0	1	2	3	4	5	6	7	8	9	10
time	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Sample	0	0.95	0.59	-0.59	-0.95	0	0.95	0.59	-0.59	-0.95	0

i	0	1	2	3	4	5	6	7	8	9	10
B1	0.00	0.59	0.95	0.95	0.59	0.00	-0.59	-0.95	-0.95	-0.59	0.00
B2	0.00	0.95	0.59	-0.59	-0.95	0.00	0.95	0.59	-0.59	-0.95	0.00
B3	0.00	0.95	-0.59	-0.59	0.95	0.00	-0.95	0.59	0.59	-0.95	0.00
B4	0.00	0.59	-0.95	0.95	-0.59	0.00	0.59	-0.95	0.95	-0.59	0.00
B5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

PRECLASS 1

$$Sample[k] = \sin\left(\frac{2\pi n \times k}{T}\right)$$

- **Note B1 to B5 were sample values from 1, 2, 3, 4, 5 Hz sine waves**

i	0	1	2	3	4	5	6	7	8	9	10
time	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Sample	0	0.95	0.59	-0.59	-0.95	0	0.95	0.59	-0.59	-0.95	0

i	0	1	2	3	4	5	6	7	8	9	10
n=1	0.00	0.59	0.95	0.95	0.59	0.00	-0.59	-0.95	-0.95	-0.59	0.00
n=2	0.00	0.95	0.59	-0.59	-0.95	0.00	0.95	0.59	-0.59	-0.95	0.00
n=3	0.00	0.95	-0.59	-0.59	0.95	0.00	-0.95	0.59	0.59	-0.95	0.00
n=4	0.00	0.59	-0.95	0.95	-0.59	0.00	0.59	-0.95	0.95	-0.59	0.00
n=5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

OBSERVE

$$Sample[k] = \sin\left(\frac{2\pi n \times k}{T}\right)$$

- When we compute the dot-product with discrete frequency samples, the only non-zero was the frequency in the signal!

i	0	1	2	3	4	5	6	7	8	9	10
time	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Sample	0	0.95	0.59	-0.59	-0.95	0	0.95	0.59	-0.59	-0.95	0

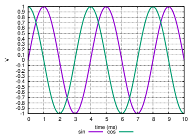
i	0	1	2	3	4	5	6	7	8	9	10
n=1	0.00	0.59	0.95	0.95	0.59	0.00	-0.59	-0.95	-0.95	-0.59	0.00
n=2	0.00	0.95	0.59	-0.59	-0.95	0.00	0.95	0.59	-0.59	-0.95	0.00
n=3	0.00	0.95	-0.59	-0.59	0.95	0.00	-0.95	0.59	0.59	-0.95	0.00
n=4	0.00	0.59	-0.95	0.95	-0.59	0.00	0.59	-0.95	0.95	-0.59	0.00
n=5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

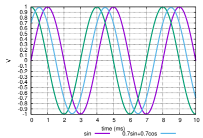
OBSERVE

- **Can identify frequencies with dot product**
  - Identifying projection onto each basis vector in Fourier Series
- **Works because frequency sine waves are orthogonal**
- **Performing a change of basis**
  - From time-sample basis
  - To Fourier (sine, cosine) basis

COSINES

- **For simplicity – preclass 1 demonstrated with sine**
  - Show sine of different frequencies orthogonal
- **Also true for cosines**
  - Cosines of different frequencies orthogonal
  - Cosines orthogonal to sines
- **Sine/cosine are just phase shift**





TIME AND FREQUENCY BASES

- **Time Sample basis**
  - Also a multi-dimensional space
  - Dimension = # time samples
  - Vector [t_0,t_1,t_2,t_3, ….]
- **Frequency basis**
  - Multi-dimensional
  - Dimensions = Coefficients of sine and cosine components
  - $f(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n2\pi t}{L} + b_n \sin\frac{n2\pi t}{L}\right)$
  - Vector [a_0,b_1,a_1,b_2,a_2,…]

Part 2

DISCRETE FOURIER TRANSFORM

DISCRETE FOURIER TRANSFORMS

- **Fourier Transforms are nice,**
  - but we want to store and process our signals with computers
- **We extend Fourier Transforms into Discrete Fourier Transforms, or DFT**
  - We know our music signal is now discrete: $x(t) \rightarrow f[k]$
  - The signal contains **N** samples: $0 \leq k \leq \boldsymbol{N} - 1$
  - Treat as **single period** for Fourier
    - Remember Fourier Transform works on periodic signals

WARNING

- **Don't get lost in mathematical notation**
- **Math is not hard**
  - …but compact (dense) with many variables
- **k – sample – correspond to a time point**
- **n -- frequency component**
- **(note on p2 of preclass so can refer back to)**

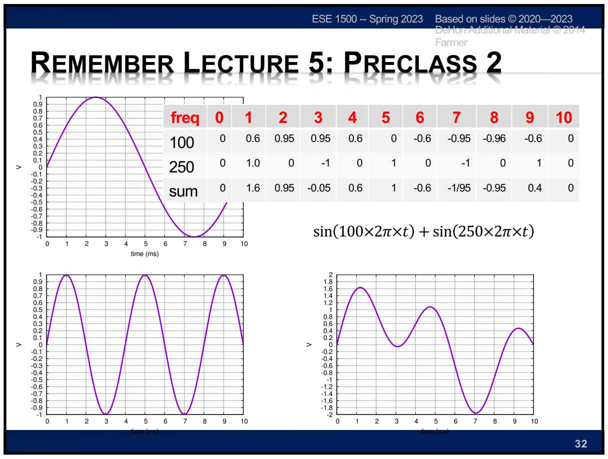
DFT – DISCRETE FOURIER TRANSFORM

- **Represent any sequence of time samples as**

$$f[k] = a_0 + \sum_{n=1}^{N/2}\left(a_n \cos\frac{n2\pi k}{N} + b_n \sin\frac{n2\pi k}{N}\right)$$

Replaced t/L with k/N (and finite sum – Nyquist no freq. over half sample rate)

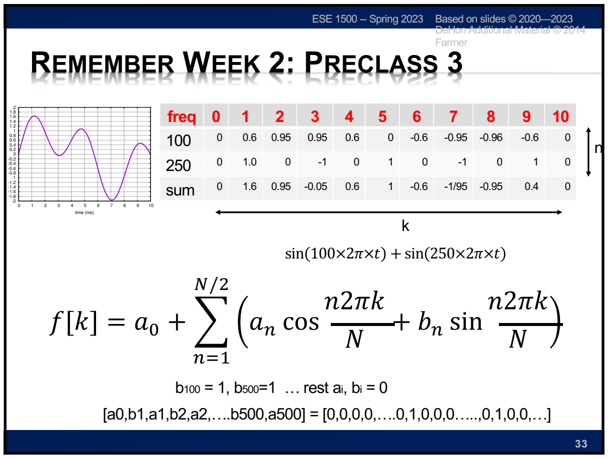
$$f(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi t}{L} + b_n \sin\frac{n\pi t}{L}\right)$$



REMEMBER LECTURE 5: PRECLASS 2

freq	0	1	2	3	4	5	6	7	8	9	10
100	0	0.6	0.95	0.95	0.6	0	-0.6	-0.95	-0.96	-0.6	0
250	0	1.0	0	-1	0	1	0	-1	0	1	0
sum	0	1.6	0.95	-0.05	0.6	1	-0.6	-1/95	-0.95	0.4	0

$\sin(100\times 2\pi\times t) + \sin(250\times 2\pi\times t)$



REMEMBER WEEK 2: PRECLASS 3

freq	0	1	2	3	4	5	6	7	8	9	10
100	0	0.6	0.95	0.95	0.6	0	-0.6	-0.95	-0.96	-0.6	0
250	0	1.0	0	-1	0	1	0	-1	0	1	0
sum	0	1.6	0.95	-0.05	0.6	1	-0.6	-1/95	-0.95	0.4	0

$\sin(100\times 2\pi\times t) + \sin(250\times 2\pi\times t)$

$$f[k] = a_0 + \sum_{n=1}^{N/2}\left(a_n \cos\frac{n2\pi k}{N} + b_n \sin\frac{n2\pi k}{N}\right)$$

$b_{100} = 1$, $b_{500}=1$ … rest a_i, $b_i = 0$

[a0,b1,a1,b2,a2,….b500,a500] = [0,0,0,0,….0,1,0,0,0…..,0,1,0,0,…]

DFT – DISCRETE FOURIER TRANSFORM

- **Represent any sequence of time samples as**

$$f[k] = a_0 + \sum_{n=1}^{N/2}\left(a_n \cos\frac{n2\pi k}{N} + b_n \sin\frac{n2\pi k}{N}\right)$$

- **Compute a_n, b_n by dot product – preclass 1!**
  - $a_n = \left(\frac{2}{N}\right)\sum_{k=0}^{k=N}\left(Sample[k]\times\cos\left(\frac{n2\pi k}{N}\right)\right)$
  - $b_n = \left(\frac{2}{N}\right)\sum_{k=0}^{k=N}\left(Sample[k]\times\sin\left(\frac{n2\pi k}{N}\right)\right)$

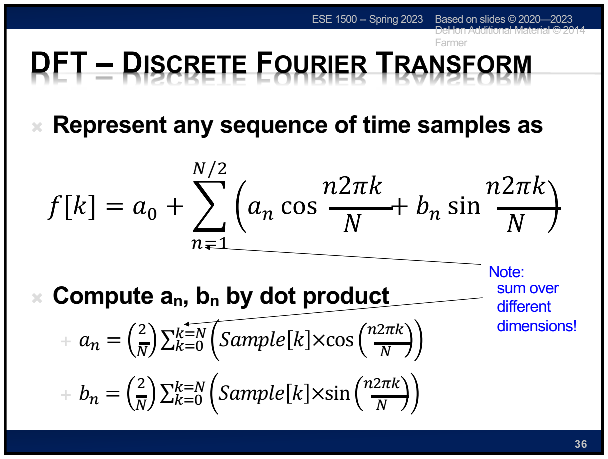
DFT – DISCRETE FOURIER TRANSFORM

- **Compute a_n, b_n by dot product – preclass 1!**
  - $a_n = \left(\frac{2}{N}\right)\sum_{k=0}^{k=N}\left(Sample[k]\times\cos\left(\frac{n2\pi k}{N}\right)\right)$
  - $b_n = \left(\frac{2}{N}\right)\sum_{k=0}^{k=N}\left(Sample[k]\times\sin\left(\frac{n2\pi k}{N}\right)\right)$

k

n

i	0	1	2	3	4	5	6	7	8	9	10
n=1	0.00	0.59	0.95	0.95	0.59	0.00	-0.59	-0.95	-0.95	-0.59	0.00
n=2	0.00	0.95	0.59	-0.59	-0.95	0.00	0.95	0.59	-0.59	-0.95	0.00
n=3	0.00	0.95	-0.59	-0.59	0.95	0.00	-0.95	0.59	0.59	-0.95	0.00
n=4	0.00	0.59	-0.95	0.95	-0.59	0.00	0.59	-0.95	0.95	-0.59	0.00
n=5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00



DFT – DISCRETE FOURIER TRANSFORM

- **Represent any sequence of time samples as**

$$f[k] = a_0 + \sum_{n=1}^{N/2}\left(a_n \cos\frac{n2\pi k}{N} + b_n \sin\frac{n2\pi k}{N}\right)$$

- **Compute a_n, b_n by dot product**
  - $a_n = \left(\frac{2}{N}\right)\sum_{k=0}^{k=N}\left(Sample[k]\times\cos\left(\frac{n2\pi k}{N}\right)\right)$
  - $b_n = \left(\frac{2}{N}\right)\sum_{k=0}^{k=N}\left(Sample[k]\times\sin\left(\frac{n2\pi k}{N}\right)\right)$

Note:
sum over
different
dimensions!

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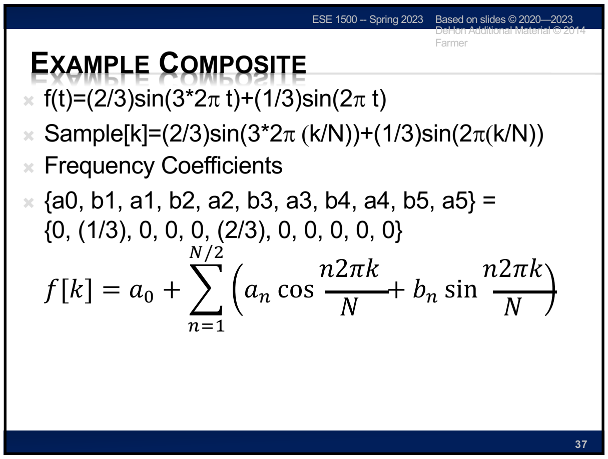

EXAMPLE COMPOSITE

- f(t)=(2/3)sin(3*2π t)+(1/3)sin(2π t)
- Sample[k]=(2/3)sin(3*2π (k/N))+(1/3)sin(2π(k/N))
- Frequency Coefficients
- {a0, b1, a1, b2, a2, b3, a3, b4, a4, b5, a5} = {0, (1/3), 0, 0, 0, (2/3), 0, 0, 0, 0, 0}

$$f[k] = a_0 + \sum_{n=1}^{N/2}\left(a_n \cos\frac{n2\pi k}{N} + b_n \sin\frac{n2\pi k}{N}\right)$$

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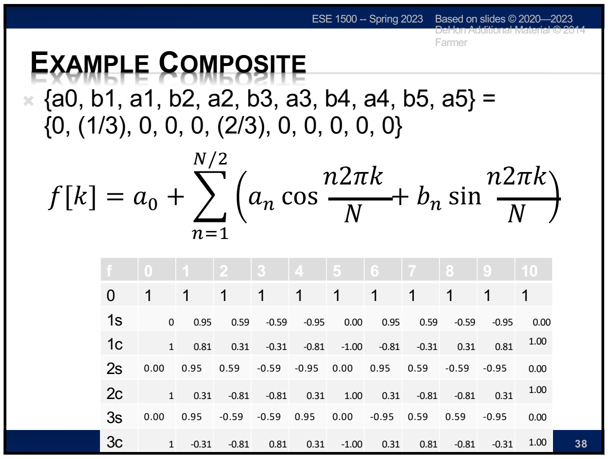

EXAMPLE COMPOSITE

- {a0, b1, a1, b2, a2, b3, a3, b4, a4, b5, a5} = {0, (1/3), 0, 0, 0, (2/3), 0, 0, 0, 0, 0}

$$f[k] = a_0 + \sum_{n=1}^{N/2}\left(a_n \cos\frac{n2\pi k}{N} + b_n \sin\frac{n2\pi k}{N}\right)$$

f	0	1	2	3	4	5	6	7	8	9	10
0	1	1	1	1	1	1	1	1	1	1	1
1s	0	0.95	0.59	-0.59	-0.95	0.00	0.95	0.59	-0.59	-0.95	0.00
1c	1	0.81	0.31	-0.31	-0.81	-1.00	-0.81	-0.31	0.31	0.81	1.00
2s	0.00	0.95	0.59	-0.59	-0.95	0.00	0.95	0.59	-0.59	-0.95	0.00
2c	1	0.31	-0.81	-0.81	0.31	1.00	0.31	-0.81	-0.81	0.31	1.00
3s	0.00	0.95	-0.59	-0.59	0.95	0.00	-0.95	0.59	0.59	-0.95	0.00
3c	1	-0.31	-0.81	0.81	0.31	-1.00	0.31	0.81	-0.81	-0.31	1.00

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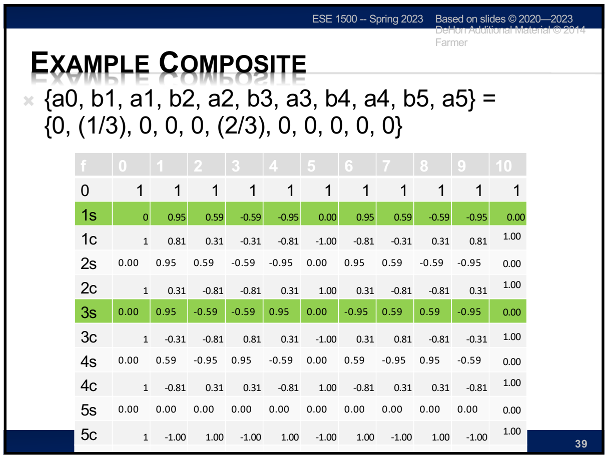

EXAMPLE COMPOSITE

- {a0, b1, a1, b2, a2, b3, a3, b4, a4, b5, a5} = {0, (1/3), 0, 0, 0, (2/3), 0, 0, 0, 0, 0}

f	0	1	2	3	4	5	6	7	8	9	10
0	1	1	1	1	1	1	1	1	1	1	1
1s	0	0.95	0.59	-0.59	-0.95	0.00	0.95	0.59	-0.59	-0.95	0.00
1c	1	0.81	0.31	-0.31	-0.81	-1.00	-0.81	-0.31	0.31	0.81	1.00
2s	0.00	0.95	0.59	-0.59	-0.95	0.00	0.95	0.59	-0.59	-0.95	0.00
2c	1	0.31	-0.81	-0.81	0.31	1.00	0.31	-0.81	-0.81	0.31	1.00
3s	0.00	0.95	-0.59	-0.59	0.95	0.00	-0.95	0.59	0.59	-0.95	0.00
3c	1	-0.31	-0.81	0.81	0.31	-1.00	0.31	0.81	-0.81	-0.31	1.00
4s	0.00	0.59	-0.95	0.95	-0.59	0.00	0.59	-0.95	0.95	-0.59	0.00
4c	1	-0.81	0.31	0.31	-0.81	1.00	-0.81	0.31	0.31	-0.81	1.00
5s	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5c	1	-1.00	1.00	-1.00	1.00	-1.00	1.00	-1.00	1.00	-1.00	1.00

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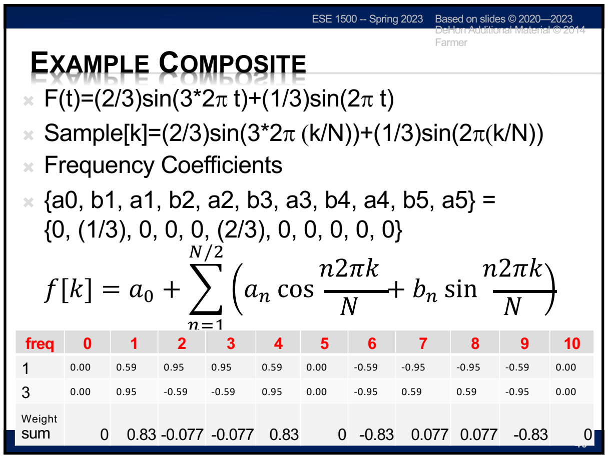

EXAMPLE COMPOSITE

- F(t)=(2/3)sin(3*2π t)+(1/3)sin(2π t)
- Sample[k]=(2/3)sin(3*2π (k/N))+(1/3)sin(2π(k/N))
- Frequency Coefficients
- {a0, b1, a1, b2, a2, b3, a3, b4, a4, b5, a5} = {0, (1/3), 0, 0, 0, (2/3), 0, 0, 0, 0, 0}

$$f[k] = a_0 + \sum_{n=1}^{N/2}\left(a_n \cos\frac{n2\pi k}{N} + b_n \sin\frac{n2\pi k}{N}\right)$$

freq	0	1	2	3	4	5	6	7	8	9	10
1	0.00	0.59	0.95	0.95	0.59	0.00	-0.59	-0.95	-0.95	-0.59	0.00
3	0.00	0.95	-0.59	-0.59	0.95	0.00	-0.95	0.59	0.59	-0.95	0.00
Weight sum	0	0.83	-0.077	-0.077	0.83	0	-0.83	0.077	0.077	-0.83	0

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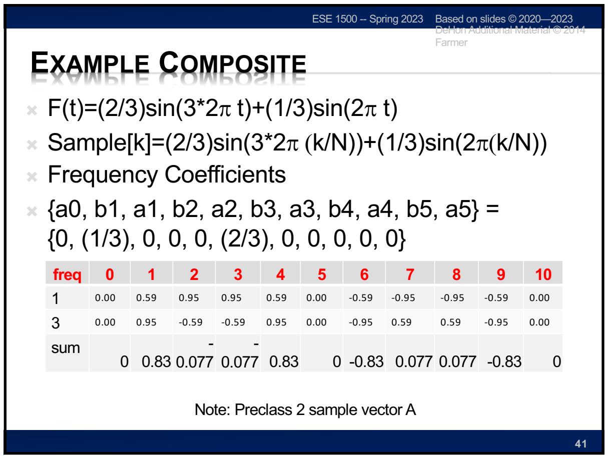

EXAMPLE COMPOSITE

- F(t)=(2/3)sin(3*2π t)+(1/3)sin(2π t)
- Sample[k]=(2/3)sin(3*2π (k/N))+(1/3)sin(2π(k/N))
- Frequency Coefficients
- {a0, b1, a1, b2, a2, b3, a3, b4, a4, b5, a5} = {0, (1/3), 0, 0, 0, (2/3), 0, 0, 0, 0, 0}

freq	0	1	2	3	4	5	6	7	8	9	10
1	0.00	0.59	0.95	0.95	0.59	0.00	-0.59	-0.95	-0.95	-0.59	0.00
3	0.00	0.95	-0.59	-0.59	0.95	0.00	-0.95	0.59	0.59	-0.95	0.00
sum	0	0.83	-0.077	-0.077	0.83	0	-0.83	0.077	0.077	-0.83	0

Note: Preclass 2 sample vector A

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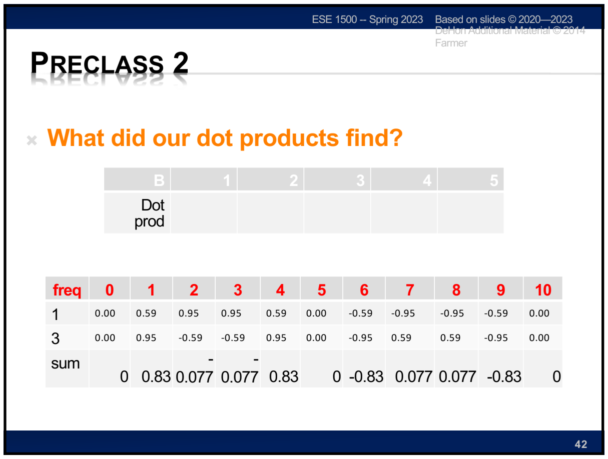

PRECLASS 2

- What did our dot products find?

B	1	2	3	4	5
Dot prod					

freq	0	1	2	3	4	5	6	7	8	9	10
1	0.00	0.59	0.95	0.95	0.59	0.00	-0.59	-0.95	-0.95	-0.59	0.00
3	0.00	0.95	-0.59	-0.59	0.95	0.00	-0.95	0.59	0.59	-0.95	0.00
sum	0	0.83	- 0.077	- 0.077	0.83	0	-0.83	0.077	0.077	-0.83	0

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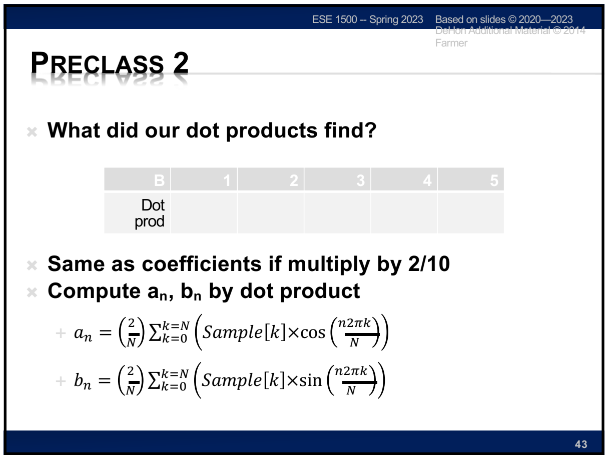

PRECLASS 2

- What did our dot products find?

B	1	2	3	4	5
Dot prod					

- Same as coefficients if multiply by 2/10
- Compute a_n, b_n by dot product
  - $a_n = \left(\frac{2}{N}\right)\sum_{k=0}^{k=N}\left(Sample[k]\times\cos\left(\frac{n2\pi k}{N}\right)\right)$
  - $b_n = \left(\frac{2}{N}\right)\sum_{k=0}^{k=N}\left(Sample[k]\times\sin\left(\frac{n2\pi k}{N}\right)\right)$

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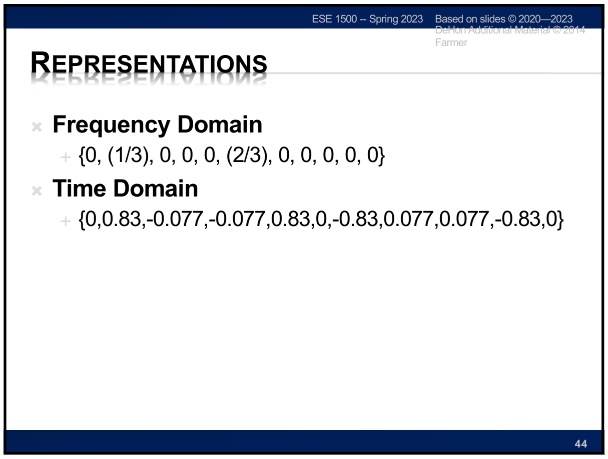

REPRESENTATIONS

- Frequency Domain
  - {0, (1/3), 0, 0, 0, (2/3), 0, 0, 0, 0, 0}
- Time Domain
  - {0,0.83,-0.077,-0.077,0.83,0,-0.83,0.077,0.077,-0.83,0}

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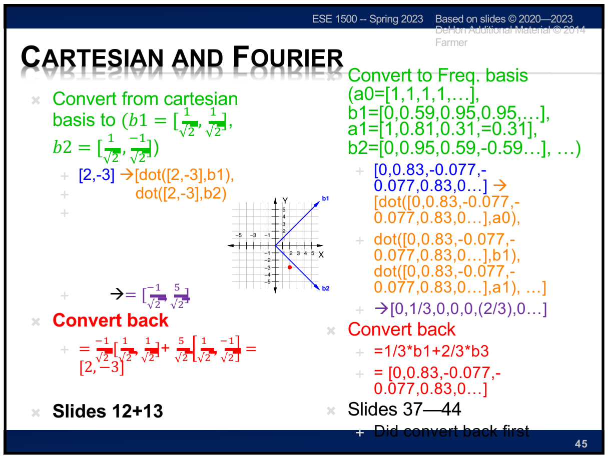

CARTESIAN AND FOURIER

- Convert from cartesian basis to ($b1 = [\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}]$, $b2 = [\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}]$)
  - [2,-3] →[dot([2,-3],b1),
  - dot([2,-3],b2)
  -
  - →$= [\frac{-1}{\sqrt{2}}, \frac{5}{\sqrt{2}}]$
- Convert back
  - $= \frac{-1}{\sqrt{2}}[\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}] + \frac{5}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right] = [2, -3]$
- Slides 12+13

Convert to Freq. basis (a0=[1,1,1,1,…], b1=[0,0.59,0.95,0.95,…], a1=[1,0.81,0.31,=0.31], b2=[0,0.95,0.59,-0.59…], …)

- [0,0.83,-0.077,-0.077,0.83,0…] → [dot([0,0.83,-0.077,-0.077,0.83,0…],a0),
- dot([0,0.83,-0.077,-0.077,0.83,0…],b1), dot([0,0.83,-0.077,-0.077,0.83,0…],a1), …]
- →[0,1/3,0,0,0,(2/3),0…]

- Convert back
  - =1/3*b1+2/3*b3
  - = [0,0.83,-0.077,-0.077,0.83,0…]
- Slides 37—44
  - Did convert back first

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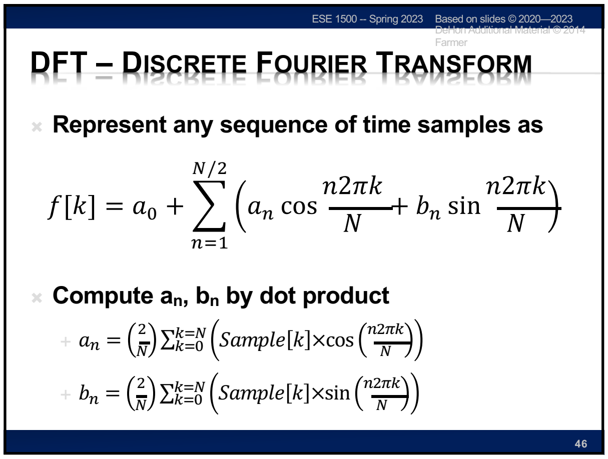

DFT – DISCRETE FOURIER TRANSFORM

- Represent any sequence of time samples as

$$f[k] = a_0 + \sum_{n=1}^{N/2}\left(a_n \cos\frac{n2\pi k}{N} + b_n \sin\frac{n2\pi k}{N}\right)$$

- Compute a_n, b_n by dot product
  - $a_n = \left(\frac{2}{N}\right)\sum_{k=0}^{k=N}\left(Sample[k]\times\cos\left(\frac{n2\pi k}{N}\right)\right)$
  - $b_n = \left(\frac{2}{N}\right)\sum_{k=0}^{k=N}\left(Sample[k]\times\sin\left(\frac{n2\pi k}{N}\right)\right)$

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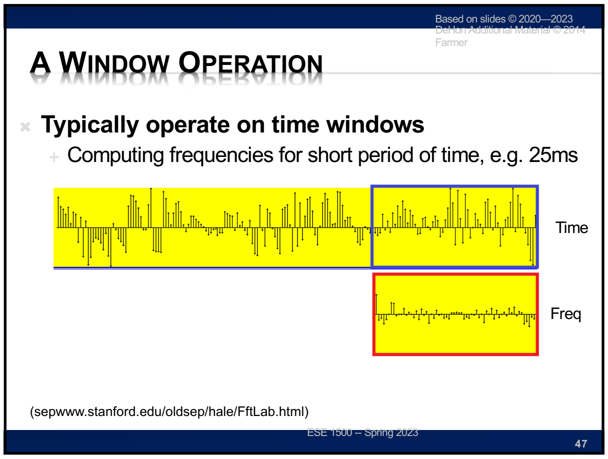

A WINDOW OPERATION

- Typically operate on time windows
  - Computing frequencies for short period of time, e.g. 25ms

(sepwww.stanford.edu/oldsep/hale/FftLab.html)

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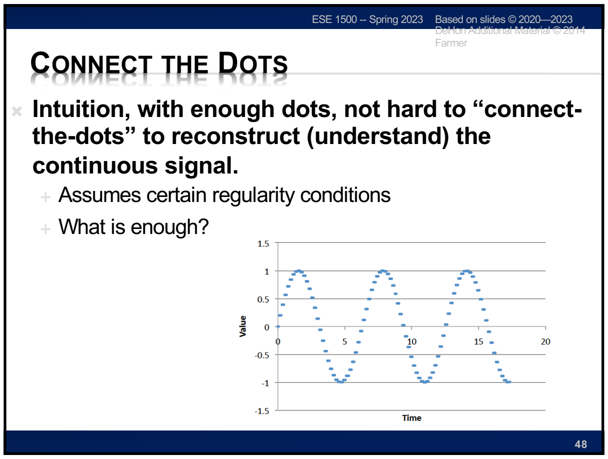

CONNECT THE DOTS

- **Intuition, with enough dots, not hard to "connect-the-dots" to reconstruct (understand) the continuous signal.**
  - Assumes certain regularity conditions
  - What is enough?

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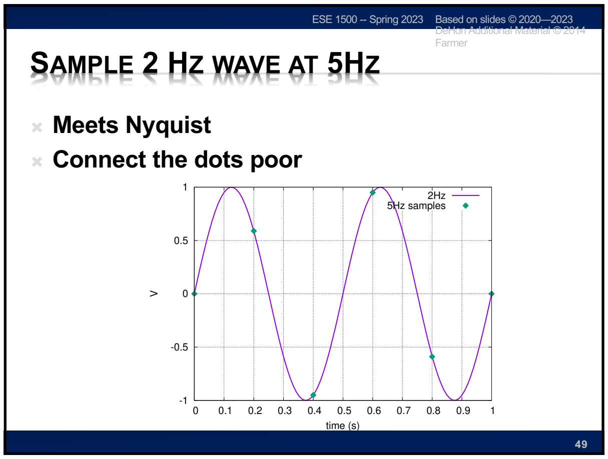

SAMPLE 2 HZ WAVE AT 5HZ

- **Meets Nyquist**
- **Connect the dots poor**

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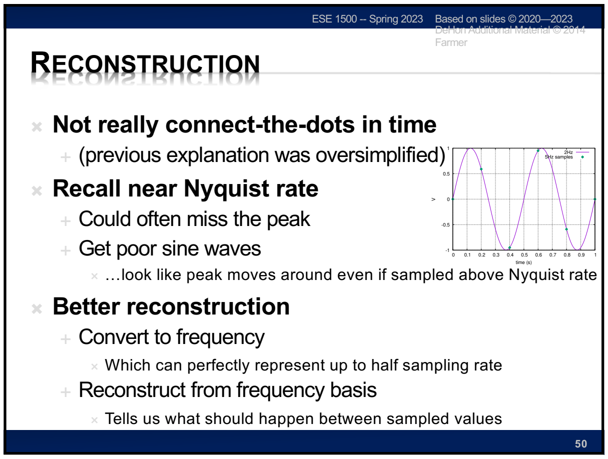

RECONSTRUCTION

- **Not really connect-the-dots in time**
  - (previous explanation was oversimplified)
- **Recall near Nyquist rate**
  - Could often miss the peak
  - Get poor sine waves
    - …look like peak moves around even if sampled above Nyquist rate
- **Better reconstruction**
  - Convert to frequency
    - Which can perfectly represent up to half sampling rate
  - Reconstruct from frequency basis
    - Tells us what should happen between sampled values

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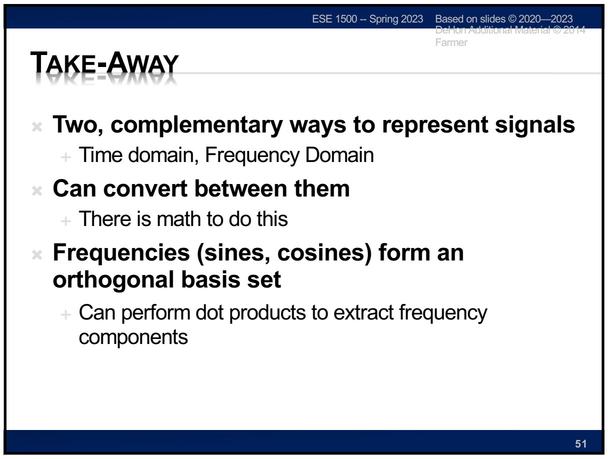

TAKE-AWAY

- **Two, complementary ways to represent signals**
  - Time domain, Frequency Domain
- **Can convert between them**
  - There is math to do this
- **Frequencies (sines, cosines) form an orthogonal basis set**
  - Can perform dot products to extract frequency components

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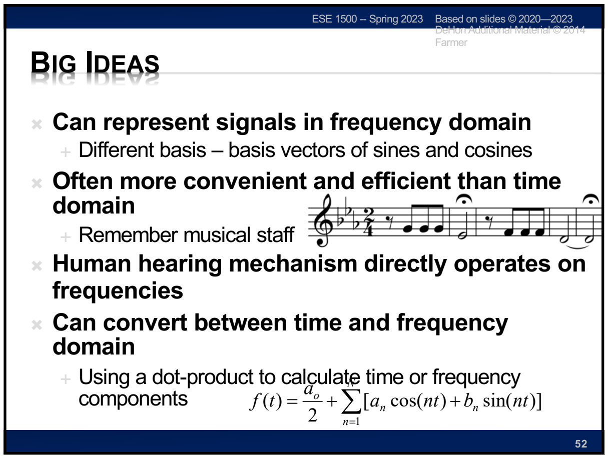

BIG IDEAS

- **Can represent signals in frequency domain**
  - Different basis – basis vectors of sines and cosines
- **Often more convenient and efficient than time domain**
  - Remember musical staff
- **Human hearing mechanism directly operates on frequencies**
- **Can convert between time and frequency domain**
  - Using a dot-product to calculate time or frequency components

$$f(t) = \frac{a_o}{2} + \sum_{n=1} [a_n \cos(nt) + b_n \sin(nt)]$$

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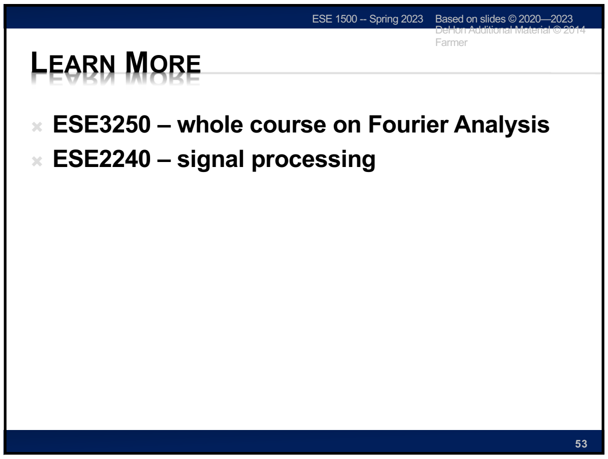

LEARN MORE

- **ESE3250 – whole course on Fourier Analysis**
- **ESE2240 – signal processing**

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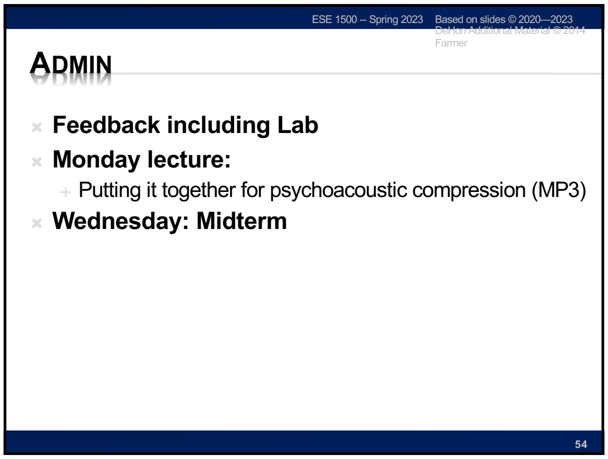
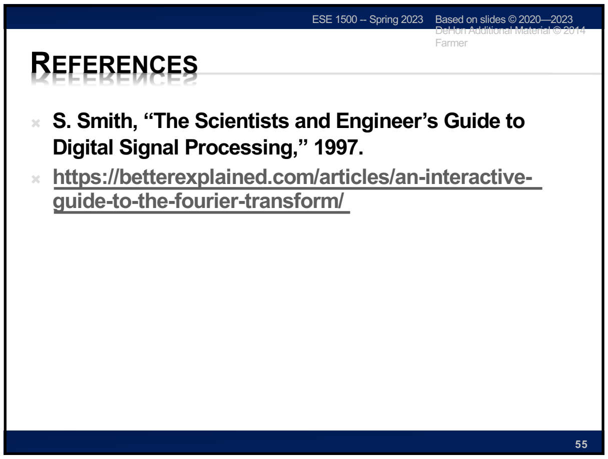


ADMIN

- **Feedback including Lab**
- **Monday lecture:**
  - Putting it together for psychoacoustic compression (MP3)
- **Wednesday: Midterm**



REFERENCES

- **S. Smith, “The Scientists and Engineer’s Guide to Digital Signal Processing,” 1997.**
- https://betterexplained.com/articles/an-interactive-guide-to-the-fourier-transform/