Laplace Transforms with MATLAB

a. Calculate the Laplace Transform using Matlab

Calculating the Laplace **F**(s) transform of a function f(t) is quite simple in Matlab. First you need to specify that the variable t and s are symbolic ones. This is done with the command

```
>> syms t s
```

Next you define the function f(t). The actual command to calculate the transform is

```
>> F=laplace(f,t,s)
```

To make the expression more readable one can use the commands, `simplify` and `pretty`.

here is an example for the function f(t),

$$f(t) = -1.25 + 3.5te^{-2t} + 1.25e^{-2t}$$

```
>> syms t s
>> f=-1.25+3.5*t*exp(-2*t)+1.25*exp(-2*t);
>> F=laplace(f,t,s)

F =

-5/4/s+7/2/(s+2)^2+5/4/(s+2)

>> simplify(F)

ans =

(s-5)/s/(s+2)^2

>> pretty(ans)
                                  s - 5
                                ----------
                                         2
                                 s (s + 2)
```

which corresponds to F(s),

$$F(s) = \frac{(s-5)}{s(s+2)^2}$$

Alternatively, one can write the function f(t) directly as part of the laplace command:

```
>>F2=laplace(-1.25+3.5*t*exp(-2*t)+1.25*exp(-2*t))
```

b. Inverse Laplace Transform

The command one uses now is `ilaplace`. One also needs to define the symbols t and s. Lets calculate the inverse of the previous function F(s),

$$F(s) = \frac{(s-5)}{s(s+2)^2}$$

```
>> syms t s
>> F=(s-5)/(s*(s+2)^2);
>> ilaplace(F)
ans =
-5/4+(7/2*t+5/4)*exp(-2*t)
>> simplify(ans)
ans =
-5/4+7/2*t*exp(-2*t)+5/4*exp(-2*t)
>> pretty(ans)
        - 5/4 + 7/2 t exp(-2 t) + 5/4 exp(-2 t)
```

Which corresponds to f(t)

$$f(t) = -1.25 + 3.5te^{-2t} + 1.25e^{-2t}$$

Alternatively one can write

```
>> ilaplace((s-5)/(s*(s+2)^2))
```

Here is another example.

$$F(s) = \frac{10(s+2)}{s(s^2+4s+5)}$$

```
>> F=10*(s+2)/(s*(s^2+4*s+5));
>> ilaplace(F)
ans =
-4*exp(-2*t)*cos(t)+2*exp(-2*t)*sin(t)+4
```

Which gives f(t),

$$f(t) = [4 - 4e^{-2t}\cos(t) + 2e^{-2t}\sin(t)]u(t)$$

making use of the trigonometric relationship,

$$x\sin(\alpha) + y\cos(\alpha) = R\sin(\alpha + \beta)$$

and

$$x\cos(\alpha) - y\sin(\alpha) = R\cos(\alpha + \beta)$$

with

$$R = \sqrt{x^2 + y^2}$$

$$\beta = \tan^{-1}(y/x)$$

One can also write that $\text{f(t)} = [4 + 4.47e^{-2t}\cos(t - 153.4^o)]u(t)$

Matlab often gives the inverse Laplace Transform in terms of sinh*x* and cosh*x*. Using the following definition one can rewrite the hyperbolic expression as a function of exponentials:

$$\sinh(x) = \frac{e^{x} + e^{-x}}{2}$$
$$\cosh(s) = \frac{e^{x} - e^{-x}}{2}$$

Also, you may find the "Heaviside(t) function which corresponds to the unit step function u(t): thus the function H(t) = heaviside(t) =0 for t<0 and H(t) = heaviside(t)=1 for t>0.

As an example, suppose that Matlab gives you the following result for the inverse Laplace transform:

```
2 heaviside(t-10) exp(-5/2t+25) sinh(1/2t-5)
```

This can be re-written, using the definition of the sinh(x) function:

```
2u(t)
```

$$f(t) = 2u(t-10).e^{-2.5(t-10)}[\frac{e^{0.5t-5} - e^{-2.5t+5}}{2}] = u(t-10).e^{-2.5t+25+0.5t-5} - e^{-2.5t+25-0.5t+5}$$
$$= u(t-10)[e^{-2t+20} - e^{-3t+30}]$$
$$= [e^{-2(t-10)} - e^{-3(t-10)}]u(t-10)$$

This last expression is closer to what your hand calculations will give you for the invers Laplace Transform.