

Example of a second order circuit response

Consider the following function transfer function of a 2nd order (RLC) low-pass filter.

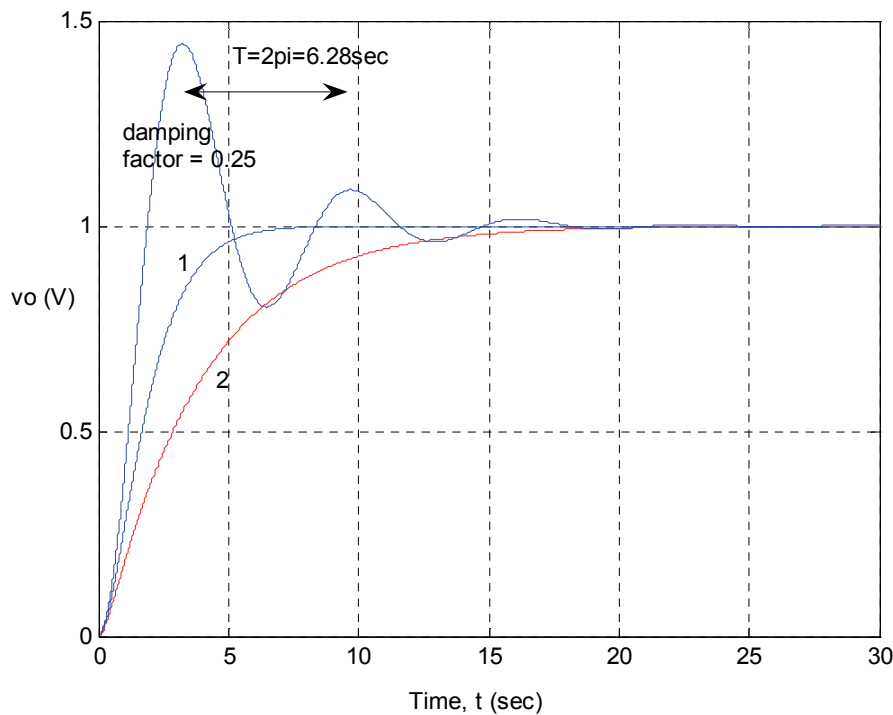
$$\frac{V_o(s)}{V_i(s)} = \frac{1}{(s^2 + 2\zeta s + 1)}$$

in which the undamped natural frequency $\omega_0=1$ rad/s

For a unit step function as the input ($V_i(s)=1/s$), the output can be written as,

$$V_o(s) = \frac{1}{s(s^2 + 2\zeta s + 1)}$$

The following plot shows the time response of the output for a unit step input, for the case of $\zeta=0.25$ (under-damped), $\zeta=1$ (critically damped) and $\zeta=2$ (over-damped).



The expressions of the output are equal to:

a) $v_o=1+1.03*\exp(-0.25*t).*\cos(0.97*t+165.54^\circ)$ (case $\zeta=0.25$)

b) $v_o=1+0.077*\exp(-3.732*t)-1.077*\exp(-0.268*t)$ (case $\zeta=2$)

c) $v_o=1-t.*\exp(-t)-\exp(-t)$ (case $\zeta=1$)