INTERPRETATION OF CATEGORICAL VARIABLES

Consider a model with *two* categorical variables $X = (0,1,2)$ and $Z = (0,1,2)$. [The same argument holds for *any* number of categorical variables.] Construct dummy variables (x_1, x_2) for category values $X = (1,2)$ and similarly two dummies (z_1, z_2) for $Z = (1,2)$, and consider the regression model:

(1) $$y = \beta_0 + \alpha_1 x_1 + \alpha_2 x_2 + \theta_1 z_1 + \theta_2 z_2 + \varepsilon$$

To interpret the meaning of these coefficients, consider the following cases:

(2) $$(X = 0, Z = 0) \Rightarrow y = \beta_0 + \varepsilon$$

(3) $$(X = 0, Z = 1) \Rightarrow y = \beta_0 + \theta_1 + \varepsilon$$

(4) $$(X = 0, Z = 2) \Rightarrow y = \beta_0 + \theta_2 + \varepsilon$$

So in the presence of $X = 0$, it is clear that θ_1 reflects the differential effect of $Z = 1$ versus $Z = 0$ on y, since $\theta_1 = (\beta_0 + \theta_1) - \beta_0$. Similarly, θ_2 represents the differential effect of $Z = 2$ versus $Z = 0$ on y in the presence of $X = 0$.

Now suppose that $X = 1$ and observe that

(5) $$(X = 1, Z = 0) \Rightarrow y = \beta_0 + \alpha_1 + \varepsilon$$

(6) $$(X = 1, Z = 1) \Rightarrow y = \beta_0 + \alpha_1 + \theta_1 + \varepsilon$$

(7) $$(X = 1, Z = 2) \Rightarrow y = \beta_0 + \alpha_1 + \theta_2 + \varepsilon$$

So here again we see that θ_1 reflects the differential effect of $Z = 1$ versus $Z = 0$ on y in the presence of $X = 1$, since now $\theta_1 = (\beta_0 + \alpha_1 + \theta_1) - (\beta_0 + \alpha_1)$. Similarly, θ_2 continues to represent the differential effect of $Z = 2$ versus $Z = 0$ on y in the presence of $X = 1$.

Finally, by similar evaluations for the case $X = 2$, one may verify that the same interpretation of θ_1 and θ_2 continue to hold.

Conclusion: *For any given value of X, the differential effects of* $Z = 1$ *and* $Z = 2$ *versus the base value,* $Z = 0$, *on y are given by the coefficients* θ_1 *and* θ_2. *By symmetry, exactly the same interpretation of* α_1 *and* α_2 *hold for X given Z. Such coefficients can thus be used to test for significant differences in the effects of any pair of categorical values on y by simply choosing one of these values to be the "base" value.*