SOLUTIONS TO PROBLEM SET 5

1. Given: $n = 20,\ \bar{x}_{20} = 55.74,\ s_{20} = 1.03$

(a) The Normal Quantile plot shows that the normality assumption is OK, so t-test can be used.

(b) $\left.\begin{array}{l} H_0 : \mu = 55 \\ H_a : \mu \neq 55 \end{array}\right\} \Rightarrow$ readjust speedometer *only* if H_0 can be rejected

Reject $\Leftrightarrow |\bar{x}_n - 55| \geq t_{\alpha/2,n-1} \dfrac{s_n}{\sqrt{n}}$

$\alpha / 2 = .0125 \Rightarrow t_{.0125,19} = 2.433$ (by JMPIN)

$\Rightarrow$ Rejected $\Leftrightarrow |\bar{x}_{20} - 55| \geq .56$

So $\bar{x}_{20} - 55 = 55.74 - 55 = .74 > .56 \Rightarrow$ Reject H_0

$\Rightarrow$ speedometers need adjusting.

(c) $\bar{x}_{20} = \mu + t_{\alpha/2,n-1} \dfrac{s_n}{\sqrt{n}} \Rightarrow t_{\alpha/2,n-1} = \dfrac{\bar{x}_n - \mu}{s_n / \sqrt{n}}$

$$\Rightarrow \alpha / 2 \equiv T_{n-1}(-t_{\alpha/2,n-1}) = T_{n-1}\left(\frac{\mu - \bar{x}_n}{s_n / \sqrt{n}}\right)$$

$$= T_{19}\left(\frac{55 - 55.74}{1.03/\sqrt{20}}\right) = T_{19}(-3.21) = .00229 \text{ (by JMPIN)}$$

$\Rightarrow$ P-value $\equiv \alpha = .00229 \times 2 = .0046$

$\Rightarrow$ The odds of observing a speed this far from a mean of $\mu = 55$ is are less than 5 in 1000 (equivalently: H_0 could have been rejected at the $\alpha = .0046$ level).

2. Given: $n = 950,\ \hat{p}_n = \dfrac{35}{950} = .0368$

(a) $\left.\begin{array}{l} H_0 : p \geq .05 \\ H_a : p < .05 \end{array}\right\}$ Postpone inventory *only* if H_0 can be rejected.

$\Rightarrow$ Normal approximation OK since $950(.05) = 47.5 > 10$.

Reject $\Leftrightarrow \hat{p}_n \leq p - z_\alpha \sqrt{\dfrac{p(1-p)}{n}} = c$

$\alpha = .01 \Rightarrow z_\alpha = 2.33 \Rightarrow z_\alpha \sqrt{\dfrac{.05(.95)}{950}} = .0165$

So: $c = p - z_\alpha \sqrt{\dfrac{p(1-p)}{n}} = .05 - .0165 = .0335 < .0368 = \hat{p}_n$

$\Rightarrow$ can reject H_0 (so inventory should be postponed)

(b) $\text{Prob}\left(H_0 \text{ not rejected} \,\middle|\, p = .04\right) = \text{Prob}\left(\hat{p}_n > c \mid p = .04\right)$

$= \text{Prob}\left(\dfrac{\hat{p}_n - p}{\sqrt{\frac{p(1-p)}{n}}} > \dfrac{c - p}{\sqrt{\frac{p(1-p)}{n}}} \mid p = .04 \right)$

$\Rightarrow \text{Prob}\left(Z > \dfrac{.0335 - .04}{\sqrt{\frac{.04(.96)}{950}}} \right) = \text{Prob}\left(Z > \dfrac{-.0065}{.0064} \right) = \text{Prob}(Z > -1.016)$

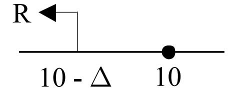
$= P(Z < 1.016) \approx .846$ (shows that $\alpha = .01$ is quite conservative)

(c) $\text{Prob}(\hat{p}_n \leq c \mid p = .06) = \text{Prob}\left(\dfrac{\hat{p}_n - .06}{\sqrt{\frac{.06(.94)}{950}}} \leq \dfrac{c - .06}{\sqrt{\frac{.06(.94)}{950}}} \right)$

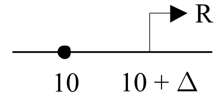
$= \text{Prob}\left(Z \leq \dfrac{-.0265}{.0077} \right) = \Phi(-3.44) = .0003$

$\Rightarrow$ Very *small* chance of mistakenly postponing inventory.

3. (a) $H_1 : \mu \geq 10$ (Reject $\Leftrightarrow \bar{x}_n \leq 10 - \Delta$)



$H_2 : \mu \leq 10$ (Reject $\Leftrightarrow \bar{x}_n \geq 10 + \Delta$)



$\Rightarrow$ second procedure allows much higher emission levels before the company is fined.

(b) H_1 assumes the firm is *in violation*, and a fine is levied only if data shows otherwise.

H_2 assumes the firm is *not in violation*, and fine is levied only if data shows otherwise.

$\Rightarrow$ H_2 is more compatible with our system of justice: one is *presumed innocent* until proven guilty.

4. Given: $n_1 = 93,\ \bar{x}_1 = 7.85,\ s_1 = 1.42$

$n_2 = 112,\ \bar{x}_2 = 10.35,\ s_2 = 1.50$

(a) $\left.\begin{array}{l} H_0 : \mu_2 - \mu_1 \le 2 \\ H_a : \mu_2 - \mu_1 > 2 \end{array}\right\}$ Conclude that μ_2 does exceed μ_1 by at least two kips iff H_0 can be rejected

$$\text{Reject } H_0 \Leftrightarrow \bar{x}_2 - \bar{x}_1 > 2 + z_d \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$\Rightarrow$ *Normal approximation* acceptable here because $\min(n_1, n_2) > 40$

$$\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{(1.42)^2}{93} + \frac{(1.50)^2}{112}} = .204$$

$$\Rightarrow 2 + z_{.01}(.204) = 2 + (2.33)(.204) = 2.475 < 10.35 - 7.85\ (= 2.50)$$

$\Rightarrow$ reject H_0 at the $\alpha = .01$ level (conclude: $\mu_2 - \mu_1 > 2$)

P-value: $\bar{x}_2 - \bar{x}_1 = 2 + z_\alpha(.204) \Rightarrow z_\alpha = \dfrac{\bar{x}_2 - \bar{x}_1 - 2}{.204}$

$$\Rightarrow\ z_\alpha = \frac{.50}{.204} = 2.45\ \Rightarrow\ \alpha = \Phi(-2.45) = .0072$$

$\Rightarrow$ Almost *no chance* that $\mu_2 - \mu_1 \le 2$

(b) Because $\min(n_1, n_2) > 40$ we can use the approximation:

$$\text{Prob}\left(\left| \frac{\bar{X}_2 - \bar{X}_1 - (\mu_2 - \mu_1)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \right| \le z_{\alpha/2} \right) = 1-\alpha$$

$$\Rightarrow\ \text{Prob}\left(\bar{X}_2 - \bar{X}_1 - z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \le \mu_2 - \mu_1 \le \bar{X}_2 - \bar{X}_1 + z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \right) = 1-\alpha$$

$$\Rightarrow\ \left[\bar{x}_2 - \bar{x}_1 \pm z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \right] = 100(1-\alpha)\%\ CI \quad \text{for } \mu_2 - \mu_1$$

So: $(10.35 - 7.85) \pm (1.96)(.204) = 2.5 \pm .4$

$\Rightarrow$ $[2.1, 2.9]$ = 95% *CI* for $\mu_2 - \mu_1$

$\Rightarrow$ Can be 95% confident that the true mean difference is between 2.1 and 2.9 kips.

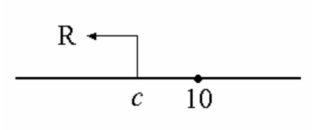
5. Let $L = X_1 - X_2 =$ *weight loss*, and let $\mu = E(L)$, $n = 20$,

$\bar{l}_n = \bar{x}_1 - \bar{x}_2 = 9.45$ and $s_n = 1.468$ (from JMPIN)

$\rightarrow$ use *paired t-test* here because X_1 and X_2 are *not independent*

(a) $\left. \begin{array}{l} H_0 : \mu \ge 10 \\ H_a : \mu < 10 \end{array} \right\}$ Conclude 'false advertising' only if H_0 is rejected.

Reject $H_0 \Leftrightarrow \bar{l}_n \le 10 - t_{\alpha,n-1}\dfrac{s_n}{\sqrt{n}} = c$



$t_{\alpha,n-1} = t_{.05,19} = 1.73$

$$\Rightarrow c = 10 - (1.73)\frac{1.468}{\sqrt{20}} = 10 - .568 = 9.43 < 9.45 = \overline{l}_n$$

$\Rightarrow H_0$ cannot be rejected (so one cannot conclude 'false advertising')

(b) P-value: $\overline{l}_n = 10 - t_{\alpha,n-1}\frac{s_n}{\sqrt{n}}$

$$\Rightarrow t_{\alpha,n-1} = \frac{10 - \overline{l}_n}{s_n/\sqrt{n}} = \frac{10 - 9.45}{1.468/\sqrt{20}} = 1.68$$

$\Rightarrow \alpha = T_{19}(-1.68) = .055$ (by JMPIN)

$\Rightarrow$ Diet product just 'barely passes' the test, and would be guilty of false advertising by any less conservative test, say $\alpha = .06$.

6. Given:

$n_1 = 25,\ \overline{x}_1 = 37.3,\ s_1 = 1.2$

$n_2 = 25,\ \overline{x}_2 = 31.5,\ s_2 = 1.5$

(a) $\left.\begin{array}{l} H_0 : \mu_1 - \mu_2 \le 5 \\ H_a : \mu_1 - \mu_2 > 5 \end{array}\right\}$ Accept new system iff H_0 can be rejected

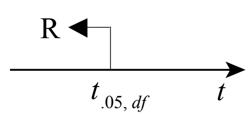
$$T = \frac{\overline{X}_1 - \overline{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \sim T_{df}$$

where: $df = \dfrac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}} = 45.79 \approx 46$

so $t_{.05,46} = 1.68$ and under H_0,

$$t = \frac{\overline{x}_1 - \overline{x}_2 - 5}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = 2.08$$

Reject $H_0 \Leftrightarrow t \ge t_{.05,df}$



So, $t = 2.08 > 1.68 = t_{.05,df} \Rightarrow$ Reject H_0

$\Rightarrow$ Adopt new fuel injection system

(b) $\text{Prob}(T \le t_{\alpha,df}) = 1-\alpha$

$\Rightarrow 1-\alpha = T_{df}(t)$, $\quad$ [T_{df} = cumulative distribution function]

= t Distribution(t, df) [JMPIN Notation]

= t Distribution$(2.08, 46)$

= .978

$\Rightarrow$ Honda can be almost 98% confident that the new system adds at least 5 *mpg*.

7. (a) Both *WB* and *W* are significant at the $\alpha = .05$ level (with *W* very significant). But the *P*-value for $\hat{\beta}_{HP}$ is .875, indicating that this variable is highly *insignificant*, and should be removed.

(b) Running the regression, we obtain

$$\hat{\beta}_W = -.0104,\ s_{\hat{\beta}_W} = .001065$$

Also, $\alpha = .01 \Rightarrow \alpha/2 = .005$ and $n-k-1 = 93-2-1 = 90$,
so that $\left| t_{\alpha/2,n-k-1} \right| = \left| t_{.005,90} \right| = 2.6316$

$$\Rightarrow CI = [\hat{\beta}_W \pm |t| \cdot s_{\hat{\beta}_W}] = [-.0104 \pm (2.6316)(.001065)] = [-.0104 \pm .0028]$$

$\Rightarrow$ one can be 99% confident that $\beta_W \in [-.0132, -.0076]$

(c) The same regression yields a Root Mean Square Error of $s_n = 2.9514$.

So by creating a new row with *WB* = 105 and *W* = 3000, and using 'Save Prediction Formula' and 'Save Std Err Pred Formula', we also obtain:

$$\widehat{MPG} = 23.369,\ s_{\widehat{MPG}} = .3496$$

Finally since $\alpha = .01 \Rightarrow \alpha/2 = .005$, we see that $\left| t_{\alpha/2,n-k-1} \right| = \left| t_{.005,90} \right| = 2.63$, that the desired 90% *PI* is given by

$$\left[\widehat{MPG} \pm |t_{.005,90}| \sqrt{s^2_{\widehat{MPG}} + s_n^{\ 2}}\right]$$

$$= \left[23.369 \ \pm 2.63\sqrt{(.3496)^2 + (2.9514)^2}\right]$$

$$= [23.369 \pm 7.816]$$

$\Rightarrow$ One can be 99% confident that the *MPG* for a car with (*WB*, *W*) = (105, 3000) would be in the range: [15.553, 31.185].