SYSTEMS 302

LECTURE 18

- **TESTS OF HYPOTHESES**
  - **One-Sided Tests**
  - **P-Values for Tests**
- **TESTS WITH σ^2 UNKNOWN**
  - **Tests of Proportions**
  - **General *t*-Tests**
- **For next time:**
  - **Devore, Sections 8.3-8.4, 9.1**

SLOT MACHINE PROBLEM

Given a slot machine with mean profits $\mu_0 = .07$ and standard deviation $\sigma = 2.72$, it follows from the CLT that for a large number of plays, n, the average profit should be distributed as:

$$\bar{X}_n \sim N\left(\mu_0, \tfrac{\sigma^2}{n}\right)$$

To determine whether pilfering may have occurred on this machine, one can test whether the realized average profits are 'unusually low'. If we denote the true mean profit by μ then an appropriate hypothesis is given by:

$$\left.\begin{array}{l} H_0 : \mu \geq \mu_0 \\ H_a : \mu < \mu_0 \end{array}\right\} \text{ Machine 'suspect' if } H_0 \text{ rejected}$$

Type I Error: Falsely conclude that pilfering occurred

➡ **How can we test this hypothesis using $\bar{X}_n$?**

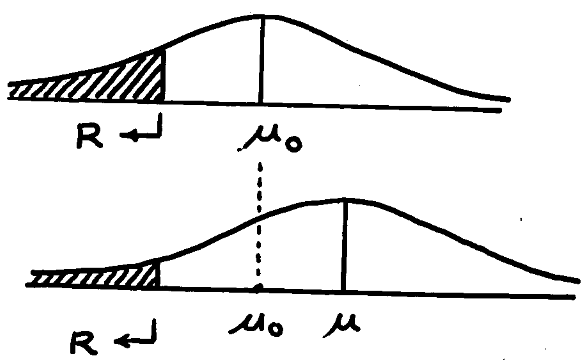
ONE-SIDED HYPOTHESES

Composite Null Hypothesis: $H_0 = \{\mu : \mu \geq \mu_0\}$

Rejection Regions: $R = \{x : x \leq x_R\}$

$$\alpha(\mu, R) = Pr(\bar{X}_n \in R \mid \mu \text{ True}) \,, \quad \mu \in H_0$$

$$\boxed{\alpha(R) = \max\{\alpha(\mu,R) : \mu \in H_0\} \equiv \alpha(\mu_0, R)}$$



$$\boxed{H_0 : \mu = \mu_0 \,, \quad H_a : \mu < \mu_0}$$

Hence $Pr(\bar{X}_n \leq \mu_0 - Z_\alpha \frac{\sigma}{\sqrt{n}}) = \alpha$

$$\Rightarrow \quad \boxed{R_\alpha = \{x : x \leq \mu_0 - Z_\alpha \frac{\sigma}{\sqrt{n}}\}}$$

SUMMARY OF HYPOTHESIS TESTS

TWO-SIDED

$$H_0 : \mu = \mu_0$$
$$H_a : \mu \neq \mu_0$$

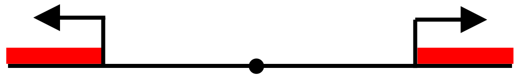
Choose α **-error**

Sampling Error $= z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

Rejection Region for H_0**:**

$$\left| \bar{X}_n - \mu_0 \right| \geq z_{a/2} \frac{\sigma}{\sqrt{n}}$$

$$c_{\alpha/2} = \mu_0 \pm z_{a/2} \frac{\sigma}{\sqrt{n}}$$



ONE-SIDED

$$H_0 : \mu = \mu_0$$
$$H_a : \mu < \mu_0$$

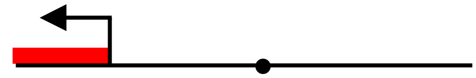
Choose α **-error**

Sampling Error $= z_{\alpha} \frac{\sigma}{\sqrt{n}}$

Rejection Region for H_0**:**

$$\bar{X}_n \leq \mu_0 - z_a \frac{\sigma}{\sqrt{n}}$$

$$c_{\alpha} = \mu_0 - z_a \frac{\sigma}{\sqrt{n}}$$



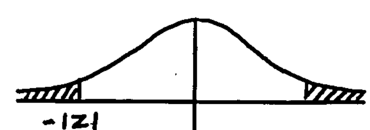
P-VALUES

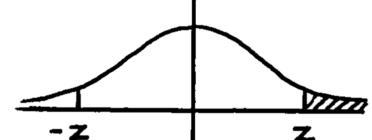
For any given sample $(x_1,..,x_n)$ to test a null hypothesis $H_0 : \mu = \mu_0$ with known variance σ^2, we denote the standardized sample mean by

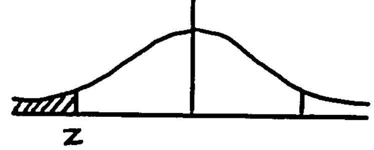
$$(1) \qquad z = \frac{\overline{x}_n - \mu_0}{\sigma / \sqrt{n}}$$

then the P-value for the test is given by

TEST	P-VALUE
Two-Tailed	$2 \cdot \Phi(-\lvert z \rvert)$
Upper-Tailed	$\Phi(-z)$
Lower-Tailed	$\Phi(z)$







P-VALUE INTUITION

Given observations $(x_1,..,x_n)$ with sample mean $\overline{x}_n$, the P-value for any test of the null hypothesis $H_0 : \mu = \mu_0$ is always obtained by answering the following question:

"If H_0 were true, then *how likely* would it be:

TEST	P-VALUE
Two-Tailed	**" to get a sample-mean value as far from μ_0 as $\overline{x}_n$?"**
Upper-Tailed	**" to get a sample-mean value as high as $\overline{x}_n$?"**
Lower-Tailed	**" to get a sample-mean value as low as $\overline{x}_n$?"**

POLITICAL POLL PROBLEM

A Gallup Poll conducted on Oct.29-31, 2008 questioned 2840 'likely voters' and found 55% in favor of Obama (over McCain) at that time. [Final vote was actually 53% of the popular vote.]

Gallup Concluded: "A sure victory for Obama"

Question: How can this conclusion be reached by hypothesis testing?

Hypothesis Formulation: Let p denote the true fraction of voters in favor of Obama.

$$\left.\begin{aligned} H_0 &: p \le .50 \\ H_a &: p > .50 \end{aligned}\right\} \text{ predict victory iff } H_0 \text{ is rejected}$$

Type I Error: Falsely predict victory for Obama.