Week 12: Gaussian processes
Arbitrages
Solutions

A Single booker arbitrage. We can complete this proof by absurd. Suppose there exists an arbitrage. Then, we must be able to achieve a net gain strictly larger than zero regardless of the outcome. Explicitly, the inequalities

$$6.6x - c > 0, \quad 8.1y - c > 0, \quad \text{and} \quad 1.2z - c > 0$$

must hold simultaneously. Observe that this is equivalent to writing

$$x > \frac{c}{6.6}, \quad y > \frac{c}{8.1}, \quad \text{and} \quad z > \frac{c}{1.2}.$$

However, if we add these equations we obtain

$$c = x + y + z > \left(\frac{1}{6.6} + \frac{1}{8.1} + \frac{1}{1.2} \right) c \approx 1.1083c, \tag{1}$$

which is a contradiction. Thus, no arbitrage exist using the odds of Booker 1.

B Many bookers arbitrage. Actually, there is no arbitrage in this either. The best combination, i.e., the one that makes the factor in front of (1) as large as possible, would be to bet on Brazil with Booker 1, Other with Booker 2, and Spain with Booker 3. These are the best odds we could get. Still, the argument of Part A would lead to the inequality

$$c = x + y + z > \left(\frac{1}{6.6} + \frac{1}{8.4} + \frac{1}{1.3} \right) c \approx 1.0398c,$$

which is again a contradiction.