1 Option pricing.

A Derivation of (3) and (4).
Because \( Y(t) \) is a Brownian Motion.

B Determination of drift and volatility.
The code follows
\[
\begin{align*}
Z &= \log(\text{close\_price}); \\
Y &= Z(2:end) - Z(1:end-1); \\
N &= \text{length}(Y); \\
h &= 1/365; \\
\mu_{\text{hat}} &= \frac{\text{sum}(Y)}{(N \cdot h)}; \\
\sigma_{\text{sqr\_hat}} &= \frac{\text{sum}((Y-\mu_{\text{hat}} \cdot h)^2)}{(N-1) \cdot h};
\end{align*}
\]
The results are:
\[
\begin{align*}
\mu_{\text{hat}} &= 0.6275 \\
\sigma_{\text{sqr\_hat}} &= 0.2174
\end{align*}
\]

Note that throughout this text, we assume the unit for time to be one year (not one day).

C Is Geometric Brownian motion a good model?
Code follows:
\[
\begin{align*}
x &= -0.1:0.01:0.1; \\
n_{\text{elements}} &= \text{histc}(Y,x); \\
\end{align*}
\]
\[
\begin{align*}
\text{figure}(1) \\
\text{bar}(x,n_{\text{elements}}/N/0.01,'r') \\
\text{hold on} \\
x_{\text{padded}} &= -0.1:0.001:0.1; \\
\text{plot}(x_{\text{padded}}, \text{normpdf}(x_{\text{padded}}, \mu_{\text{hat}} \cdot h, \sqrt{\sigma_{\text{sqr\_hat}} \cdot h}),'Linewidth',3) \\
\text{grid on} \\
\text{axis([-0.1 0.1 0 25])}
\end{align*}
\]
\[
\begin{align*}
\text{figure}(2) \\
c_{\text{elements}} &= \text{cums\_sum}(n_{\text{elements}})/N; \\
\text{bar}(x,c_{\text{elements}},'r') \\
\text{hold on} \\
x_{\text{padded}} &= -0.1:0.001:0.1; \\
\text{plot}(x_{\text{padded}}, \text{normcdf}(x_{\text{padded}}, \mu_{\text{hat}} \cdot h, \sqrt{\sigma_{\text{sqr\_hat}} \cdot h}),'Linewidth',3) \\
\text{grid on} \\
\text{axis([-0.1 0.1 0 1])}
\end{align*}
\]
The result is depicted in fig. 1. The comparison shows that the geometric exponential model is just barely acceptable.

D Expected return.
Referring to slide 27 of arbitrage_stock_and_option_pricing, the expected return is simply:
\[
\mathbb{E}\left[ \log\left( \frac{e^{-\alpha t}X(t)}{X(0)} \right) \right] = (\hat{\mu} + \hat{\sigma}^2/2 - \alpha)t
\]
For the parameters given, we have:

\[ E[(\text{discounted return})] = (0.6275 + 0.2174/2 - 0.0375) \times 1 = 0.6987 \]

\[ P \left( \log \left( \frac{e^{-\alpha t}X(t)}{X(0)} \right) \geq 0.05 \right) = F_{X(t)/X(0)}(0.05 + 0.0375) \]

knowing that \( X(t)/X(0) \sim N(\mu t, \sigma t) \) we get:

\[ P \{ (\text{discounted return}) \geq 0.0875 \} = 1 - \Phi \left( \frac{0.0875 - \hat{\mu} \times 1}{\sqrt{\hat{\sigma} \times 1}} \right) \]

Using Matlab's \text{normcdf} command, the number we obtain is:

\[ 1 - \text{normcdf}(0.0875, \mu_{hat}, \sqrt{\sigma_{sqr_{hat}}}) = 0.8766 \]

**E  Risk neutral measure.**

Referring to slide 26 of \textit{arbitrage_stock_and_option_pricing}, the risk neutral measure is a re-scaled geometric Brownian motion (GBM) whose drift is \( \alpha - \sigma^2/2 \) and whose variance is \( \sigma^2 \). So, the risk neutral measure is a geometric Brownian motion with drift \( -0.0712 \) and variance 0.2174.

**F  Expected return for risk neutral measure.**

Following the concept of risk-neutral measure, expected discounted rate of return in the alternative reality is zero, non-discounted rate of return would be \( \alpha \).

**G  Derive Black-Scholes formula.**

Refer to slide 40.

**H  Determine option price.**

The following code calculates the risk neural price of the buying option (based on Black-Scholes formula is slide 40.):

```matlab
alpha=0.0375;
X_0=close_price(1,1);
EX=X_0*exp(mu_hat+sigma_sqr_hat/2);
K=[0.8 1 1.2]*EX;
a=(log(K/X_0)-(alpha-sigma_sqr_hat/2))/(sqrt(sigma_sqr_hat));
b=a-sqrt(sigma_sqr_hat);
Q_a=1-normcdf(a,0,1);
Q_b=1-normcdf(b,0,1);
c=X_0*Q_b-exp(-alpha) * K.*Q_a;
```

The resulting prices are:

\[ c = \begin{bmatrix} 0.7190 & 0.2941 & 0.1243 \end{bmatrix} \]