University of Pennsylvania
Department of Electrical and Systems Engineering
ESE 319 Microelectronic Circuits
Fall 2009

Midterm Exam #1 Solutions
October 16, 2009

This exam is a closed book exam. Students are allowed to use a calculator and a single page reference sheet (two sided). Please show all work, justify all approximations and give the units for all calculated parameters.

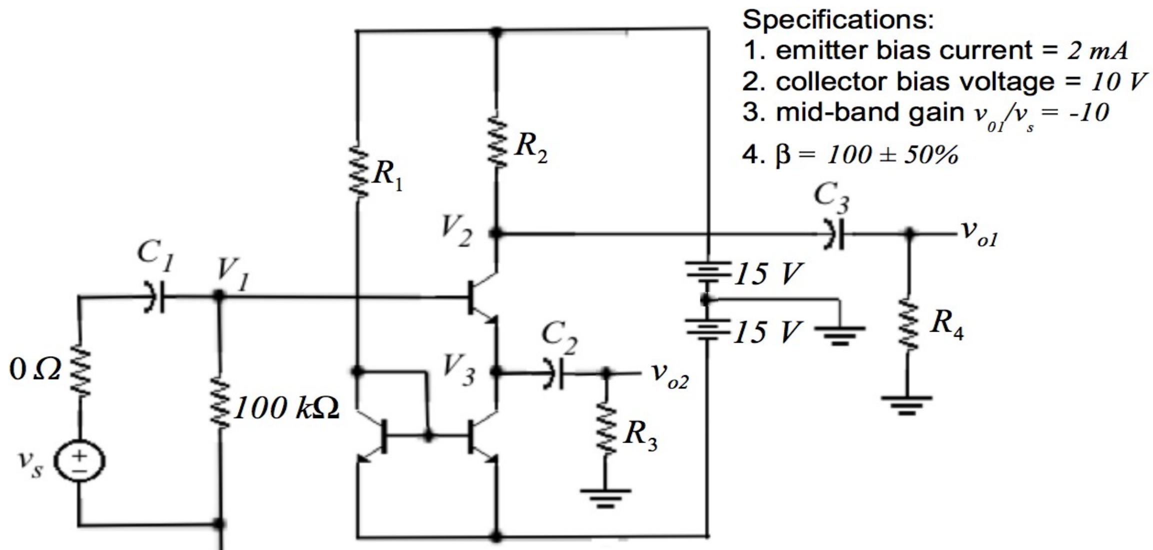


Figure 1

1. Consider the amplifier in Fig. 1. Please assume that the three transistors are forward-active with V_{BE}= 0.7 V and $r_o = \infty$, the ambient temperature is $25^o C$ and the current mirror transistors are perfectly matched. It is your task to design and evaluate this amplifier by developing your answers to a) through j) below. Be mindful that signs are important.

a) Determine the values of appropriate resistors to set the emitter bias current for the amplifier BJT at 2 mA and collector bias voltage at V_2 = 10 V, assume β = 100. (10pts)

$15\text{V}=I_{ref}\,R_1=V_{BE}-15\text{V}$ where $I_{ref} = I_E = 2\ mA$ => $R_1=\dfrac{30\text{V}-V_{BE}}{I_{ref}}=\dfrac{(30-0.7)V}{2\,mA}=14.65\,k\,\Omega$

$R_2=\dfrac{15\text{V}-V_2}{\dfrac{\beta}{\beta+1}I_E}=\dfrac{(15-10)V}{1.98\,mA}=2.53\,k\,\Omega$ or $R_2=\dfrac{(15-10)V}{2\,mA}=2.5\,k\,\Omega$

b) Determine values for the small-signal parameters g_m, r_π and r_e for the amplifier transistor (β = 100). (10pts)

$g_m=\dfrac{I_C}{V_T}=\dfrac{\dfrac{\beta}{\beta+1}I_E}{25\,mV}=\dfrac{1.98\,mA}{25\,mV}=79.2\,mS$ or $g_m=\dfrac{I_C}{V_T}=\dfrac{I_E}{25\,mV}=\dfrac{2\,mA}{25\,mV}=80\,mS$

$r_e=\dfrac{25\,mV}{I_E}=\dfrac{25\,mV}{2\,mA}=12.5\,\Omega$ and $r_\pi=\dfrac{g_m}{\beta}=r^e(\beta+1)=12.5\,\Omega(101)=1.26\,k\,\Omega$ or
$r_\pi=12.5\,\Omega(100)=1.25\,k\,\Omega$

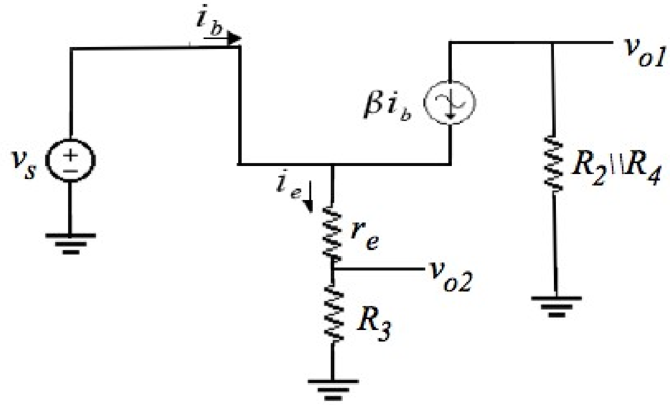
c) Explain the role for each of the capacitors C_1, C_2 and C_3. (10pts)

C_1 is a blocking capacitor to delete dc on v_S, otherwise C_1 is not needed. C_1 can usually be ignored.

C_2 is a bypass capacitor to separate the dc operating point design from the ac mid-band gain design. With C_2 the resistor R_3 only participates in the ac behavior of the amplifier.

C_3 is a blocking capacitor to prevent the dc component of the collector voltage v_C from being passed to R_4.

d) Draw a suitable small-signal model for the amplifier in Fig. 1 that is valid for mid-band frequencies. (10pts)



e) Let $C_1 = C_2 = C_3 = \infty$, $R_4 = \infty$ and $\beta = 100$. Use the model drawn in part d) to determine the value(s) for appropriate resistor(s) to realize $A_{vI} = -10$, without disrupting the dc bias set in part a). (10pts)

$$v_s = (\beta+1) i_b (r_e + R_3) \quad \Rightarrow \quad i_b = \frac{v_s}{(\beta+1)(r_e+R_3)}$$

$$v_{oI} = -\beta i_b R_2 \quad \Rightarrow \quad v_{oI} = \frac{-\beta R_2 v_s}{(\beta+1)(r_e+R_3)} \quad \Rightarrow \quad A_{vI} = \frac{v_{oI}}{v_s} = -\frac{\beta}{\beta+1}\frac{R_2}{r_e+R_3} \approx \frac{-R_2}{r_e+R_3} = -10$$

Using $r_e = 12.5\,\Omega$ and leaving $R_2 = 2.53$ kΩ, $R_3 = \frac{R_2 - 10 r_e}{10} = \frac{2.53\,k\,\Omega - 125\,\Omega}{10} = 240.5\,\Omega$

f) If the design in part e) were redone to maximize the voltage gain magnitude $|A_{vI}|$, what is the .maximum value for $|A_{vI}|$ that can be achieved without disrupting the dc bias set in part a) and what would be the resistor value(s). (10pts)

To maximize $|A_{vI}|$ one minimize R_3, i.e. set $R_3 = 0\Omega$, then $|A_{vI}| = \frac{R_2}{r_e} = \frac{2.53\,k\,\Omega}{12.5\,\Omega} = 202.4$

g) Let R_4 be finite, what is the role of R_4 in the amplifier circuit? It is desired to specify to the user of the amplifier the range of R_4 such that $|A_{vI}|$ does not deviate from 10 by more than 10%. What is the desired specification for the range of R_4. (10pts)

R_4 is the load resistor and for ac signals R_4 is in parallel with R_2; hence

$$|A_{vI}| = \left|\frac{v_{oI}}{v_s}\right| = \frac{\beta}{\beta+1}\frac{R_2 \| R_4}{r_e+R_3} \approx \frac{R_2 \| R_4}{r_e+R_3} = \frac{R_2\left(\frac{R_4}{R_2+R_4}\right)}{r_e+R_3}$$ = 10 ± 10% (implies $11 \le |A_{vI}| \ge 9$)

Since $\dfrac{R_4}{R_2+R_4} \le 1$ => $\dfrac{R_2\left(\dfrac{R_4}{R_2+R_4}\right)}{r_e+R_3} = 10\dfrac{R_4}{R_2+R_4} \ge 9$ => $10\,R_4 \ge 9\,R_4 + 9\,R_2$ =>

$R_4 \ge 9\,R_2 = 9(2.53\,k\,\Omega) = 22.8\,k\,\Omega$

h) Let $C_1 = C_2 = \infty$, $C_3 = 0$ and $\beta = 100$. Use then model drawn in part d) to determine the voltage gain $A_{v2} = v_{o2}/v_s$ at output v_{o2}. (10pts)

$$v_s = (\beta+1)i_b(r_e+R_3) \quad \Rightarrow \quad i_b = \frac{v_s}{(\beta+1)(r_e+R_3)}$$

$$v_{o2} = R_3(\beta+1)i_b = \frac{R_3(\beta+1)v_s}{(\beta+1)(r_e+R_3)} = \frac{R_3 v_s}{r_e+R_3} \quad \Rightarrow \quad A_{v2} = \frac{v_{o2}}{v_s} = \frac{R_3}{r_e+R_3} = 0.95 \approx 1$$

i) Determine the maximum and minimum values for dc collector current and collector bias voltage V_2. (10pts)

$I_C = \alpha I_E = \dfrac{\beta}{\beta+1} I_E$ and let $\beta_1 = 50$ and $\beta_2 = 150$

Let $\alpha_1 = \dfrac{50}{51} = 0.980$ and $\alpha_2 = \dfrac{150}{151} = 0.993$ => min $I_C = \alpha_1 I_E = 0.980 * 2$ mA = 1.96 0 mA

Let max $I_C = \alpha_2 I_E = 0.993 * 2$ mA = 1.986 mA

max V_{R2} = max $I_C * R_2$ = 1.986 mA * 2.53 kΩ = 5.025 V = > min V_2 = 15 V – 5.025 V = 9.975 V

min V_{R2} = min $I_C * R_2$ = 1.960 mA * 2.53 kΩ = 4.960 V = > max V_2 = 15 V – 4.960 V = 10.040 V

j) Determine the maximum and minimum values for dc base current and base bias voltage V_1. (10pts)

V_1 = - 100 kΩ I_B

max $I_{B1} = \dfrac{I_E}{(\beta_1+1)} = \dfrac{2\,mA}{51} = 39.2\,\mu A$ => min V_1 = - 100 kΩ I_{B1} = - 3.92 V

min $I_{B2} = \dfrac{I_E}{(\beta_2+1)} = \dfrac{2\,mA}{151} = 13.2\,\mu A$ => max V_1 = - 100 kΩ I_{B2} = - 1.32 V