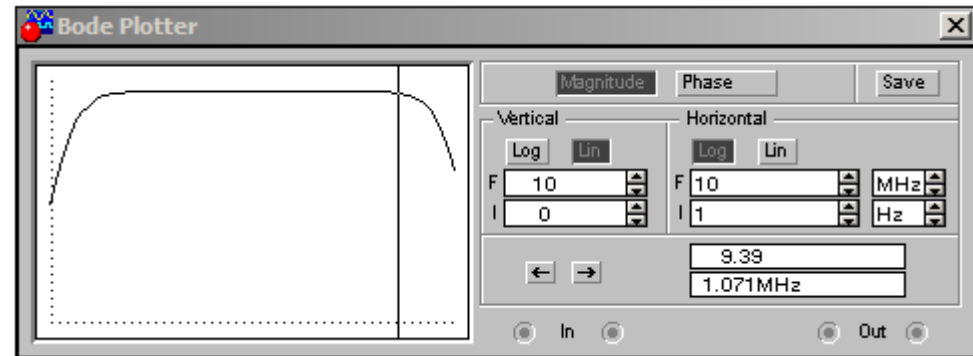
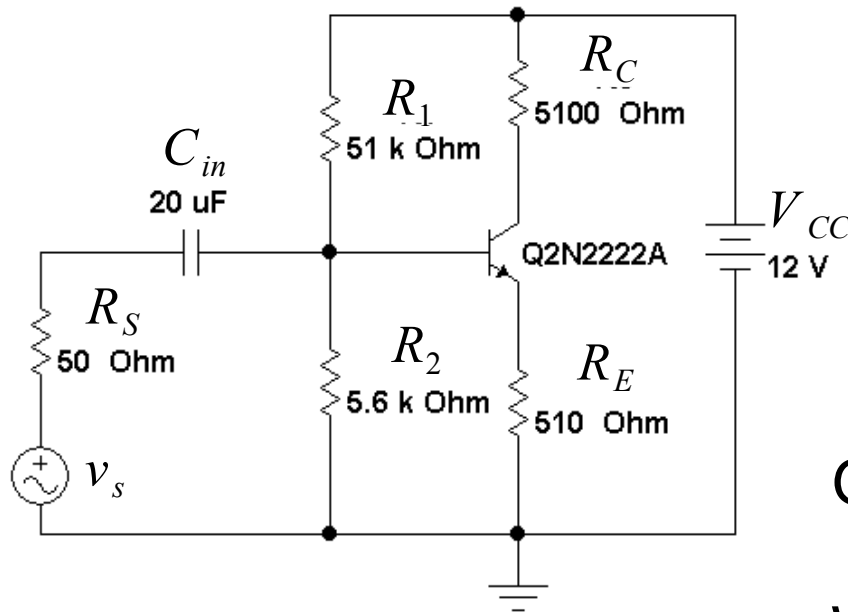




High Frequency BJT Model

Cascode BJT Amplifier

Gain of 10 Amplifier – Non-ideal Transistor

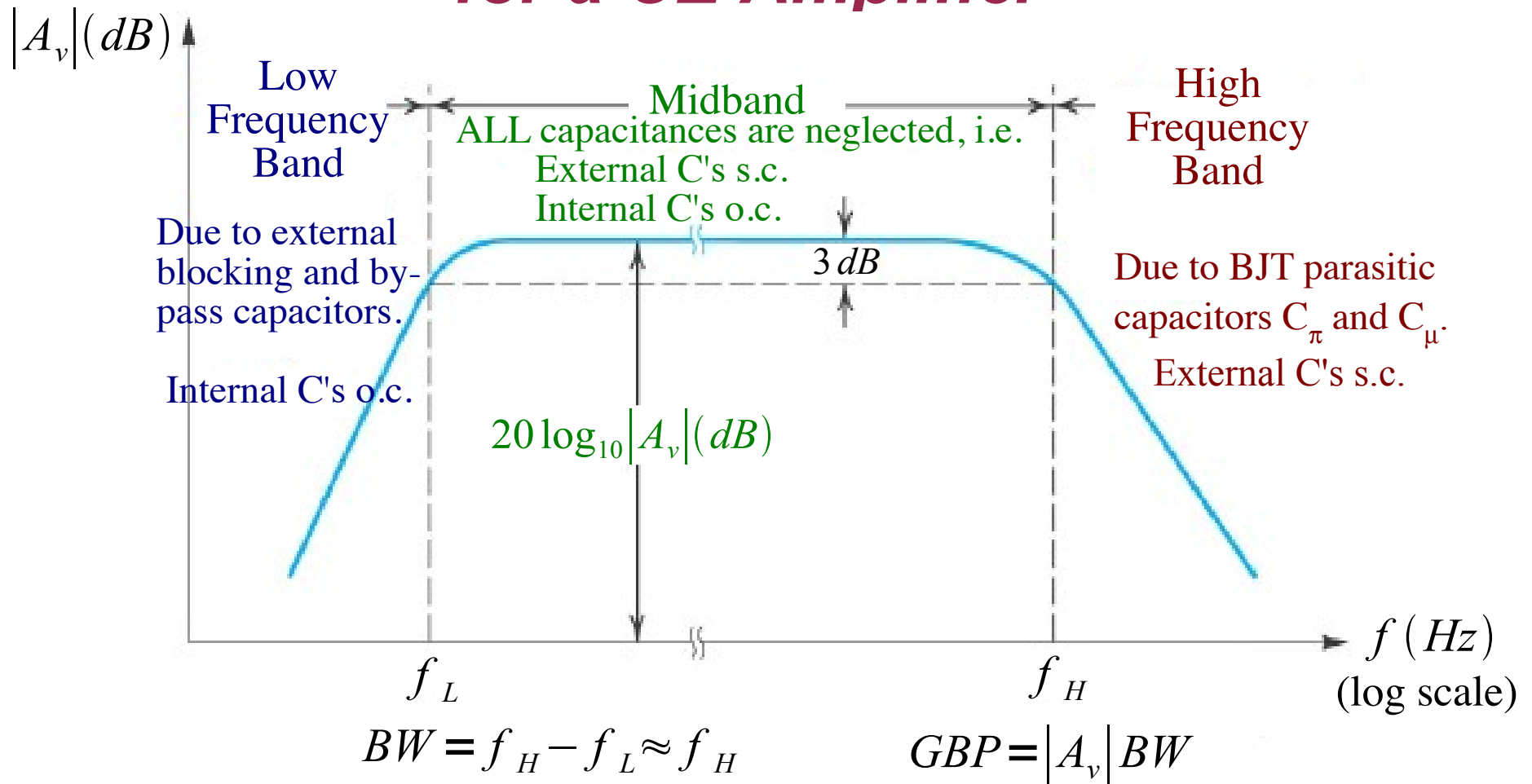


Gain starts dropping at $> 1\text{MHz}$.

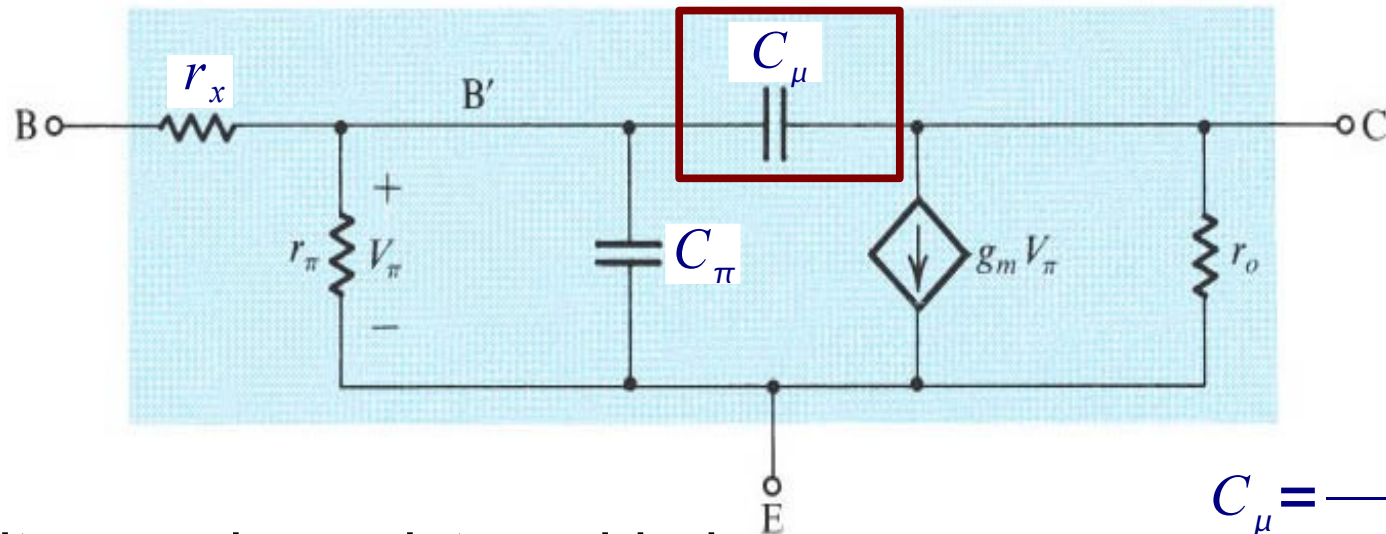
Why!

Because of internal transistor capacitances that we have ignored in our mid-band models.

Sketch of Typical Voltage Gain Response for a CE Amplifier



High Frequency Small-signal Model



SPICE	
CJC	$C_{\mu 0}$
CJE	C_{je0}
TF	τ_F
RB	r_x

$$C_{\mu} = \frac{C_{\mu 0}}{\left(1 + \frac{V_{CB}}{V_{0c}}\right)^m}$$

Two capacitors and a resistor added.

A base to emitter capacitor, C_{π}

A base to collector capacitor, C_{μ}

A resistor, r_x , representing the base terminal resistance ($r_x \ll r_{\pi}$)

$$C_{\pi} = C_{de} + C_{je} \approx C_{de} + 2C_{je0}$$

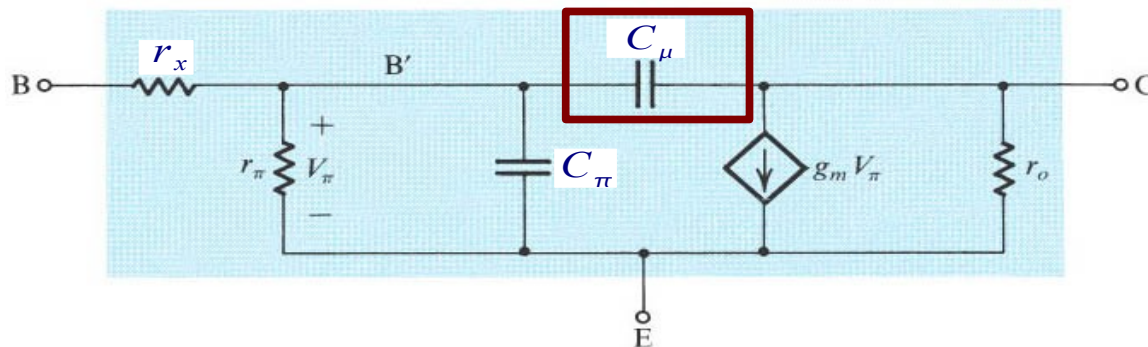
$$C_{de} = \tau_F g_m$$

τ_F = forward-base transit time

High Frequency Small-signal Model

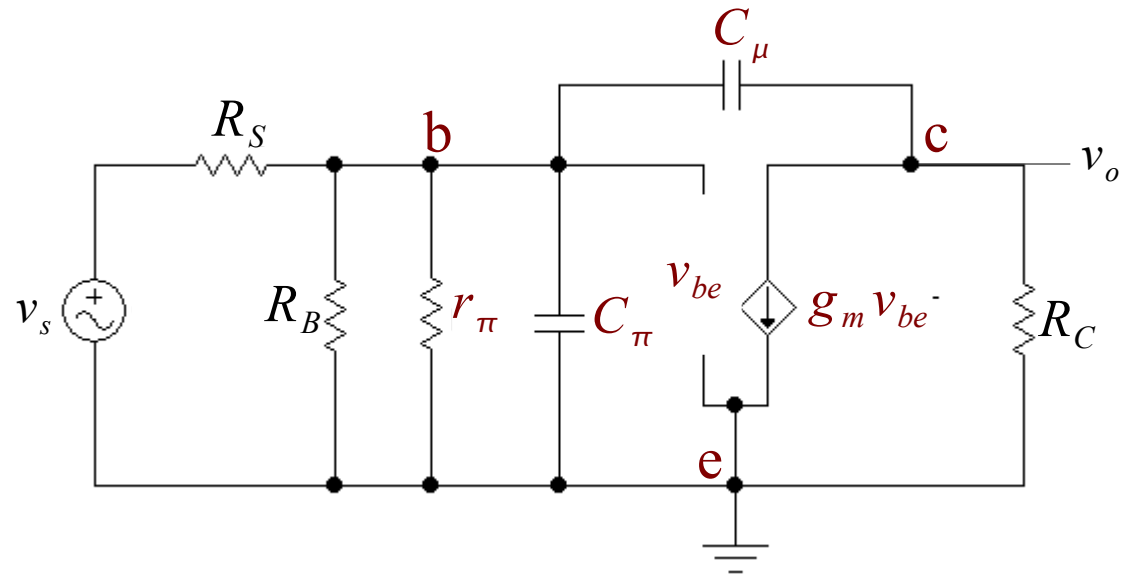
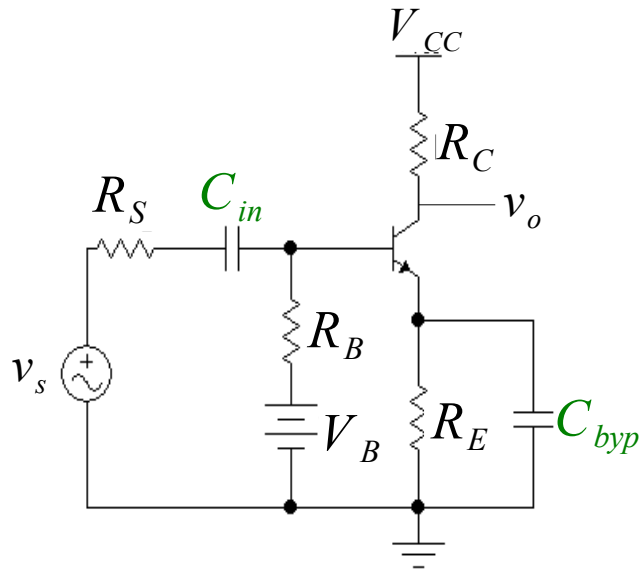
The transistor parasitic capacitances have a strong effect on circuit high frequency performance! They attenuate base signals, decreasing v_{be} since their reactance approaches zero (short circuit) at high frequencies.

As we will see later C_{μ} is the principal cause of this gain loss at high frequencies. At the base C_{μ} looks like a capacitor of value $k C_{\mu}$ connected between base and emitter, where $k > 1$ and may be $\gg 1$.



This phenomenon is called the *Miller Effect*.

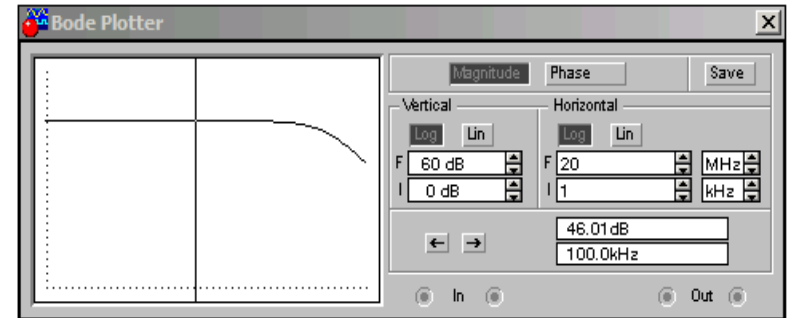
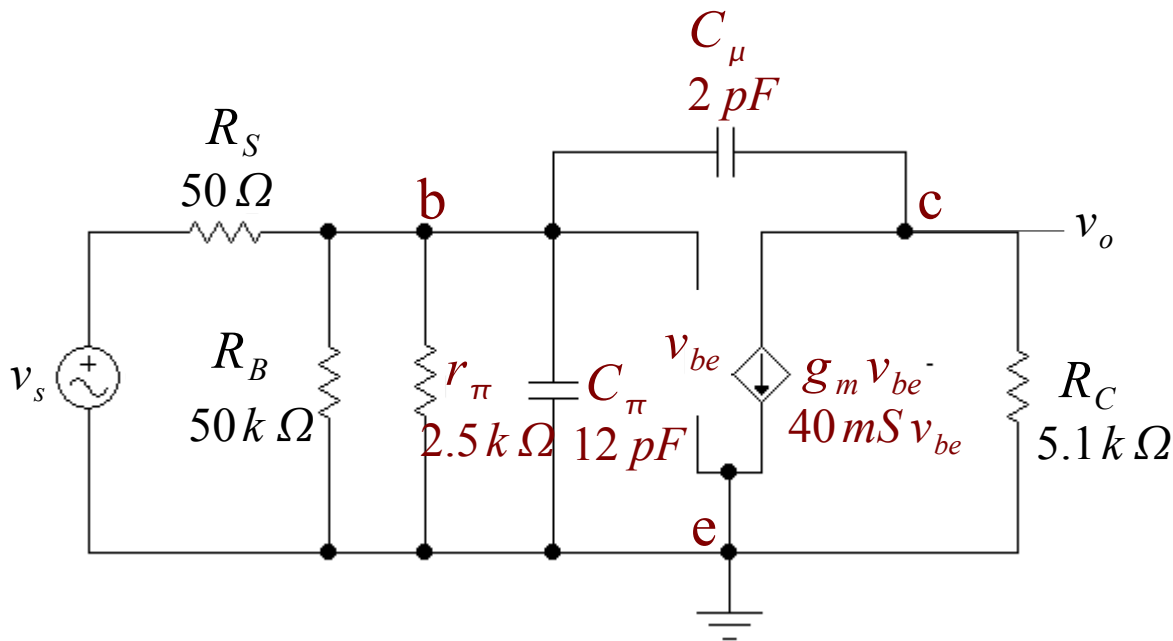
Prototype Common Emitter Circuit



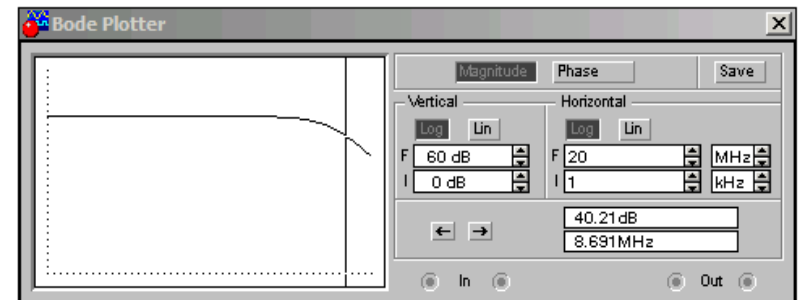
At high frequencies
“low frequency”
capacitors are “short circuits”

High frequency
small-signal ac model

Multisim Simulation

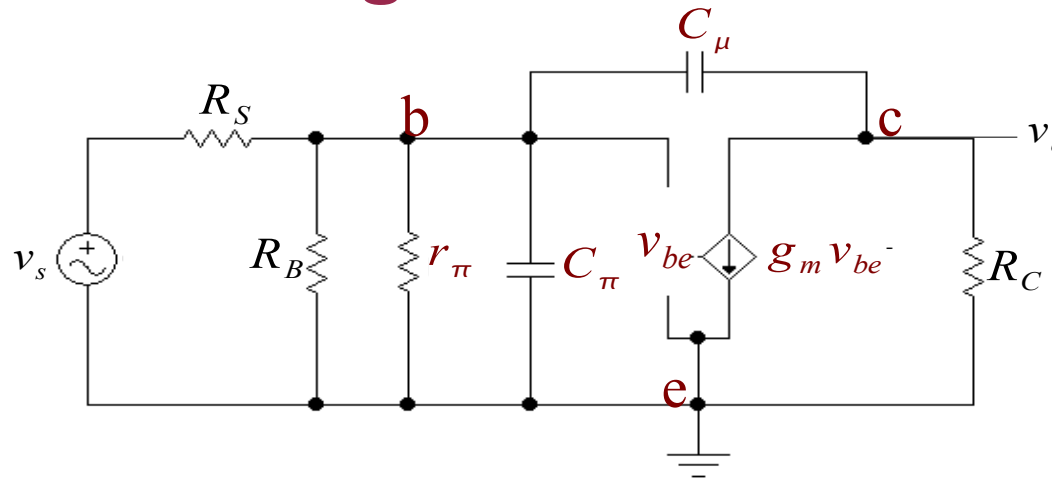


Mid-band gain



Half-gain point

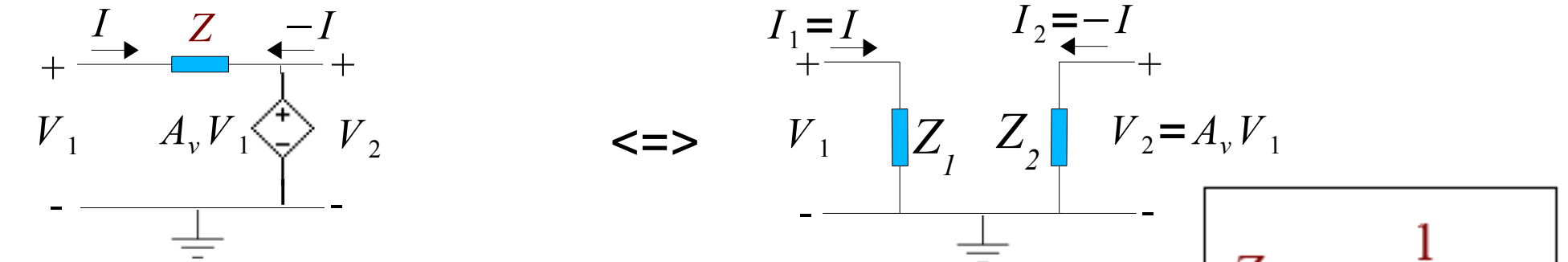
Introducing the Miller Effect



The feedback connection of C_μ between base and collector causes it to appear in the amplifier like a large capacitor $(1-K)C_\mu$ has been inserted between the base and emitter terminals. This phenomenon is called the “Miller effect” and the capacitive multiplier “ $1-K$ ” acting on C_μ equals the CE amplifier mid-band gain, i.e. $K = A_v = -g_m R_C$.

NOTE: CB and CC amplifiers do not suffer from the Miller effect, since in these amplifiers, one side of C_μ is connected directly to ground.

Miller's Theorem



$$I = \frac{V_1 - V_2}{Z} = \frac{V_1 - A_v V_1}{Z} = \frac{V_1}{Z} \Rightarrow Z_1 = \frac{V_1}{I_1} = \frac{V_1}{I} = \frac{Z}{1 - A_v}$$

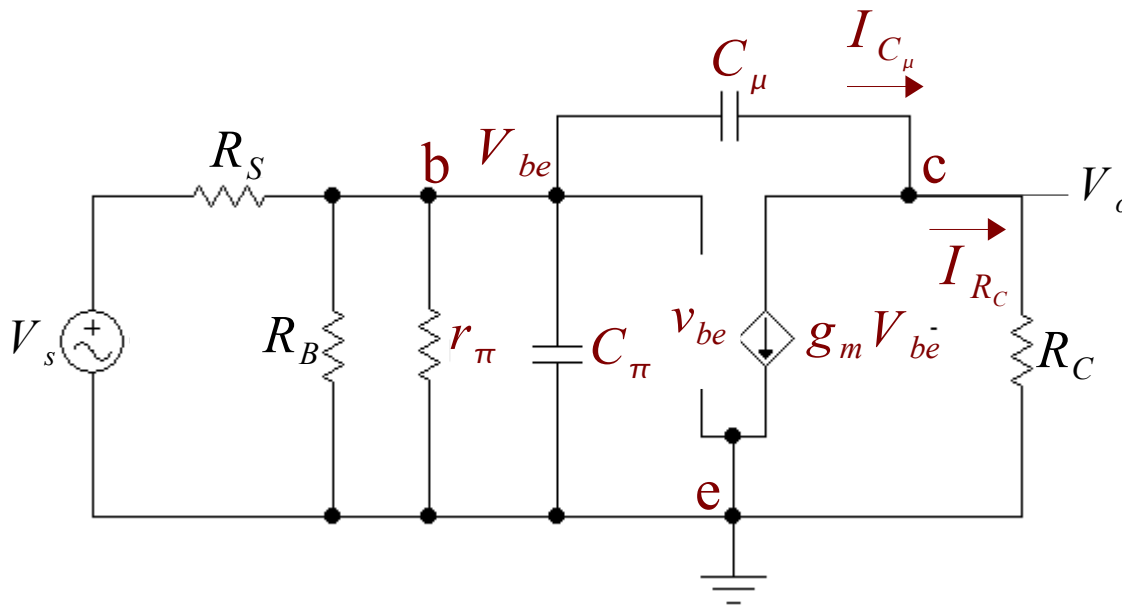
$$-I = \frac{V_2 - V_2}{Z} = \frac{V_2 - \frac{1}{A_v} V_2}{Z} \Rightarrow Z_2 = \frac{V_2}{I_2} = \frac{V_2}{-I} = \frac{Z}{1 - \frac{1}{A_v}} \approx Z \quad \text{if } A_v \gg 1$$

$$Z_2 = \frac{V_2}{I_2} = \frac{V_2}{-I} \approx \frac{1}{j2\pi f C_\mu}$$

$$Z = \frac{1}{j2\pi f C_\mu}$$

Ignored in practical circuits

Common Emitter Miller Effect Analysis



Note: The current through C_μ depends only on V_{be} !

Determine effect of C_μ :

Using phasor notation:

$$I_{R_C} = -g_m V_{be} + I_{C_\mu}$$

or

$$V_o = (-g_m V_{be} + I_{C_\mu}) R_C$$

where

$$I_{C_\mu} = (V_{be} - V_o) s C_\mu$$

$$I_{C_\mu} = \frac{(V_{be} + g_m V_{be} R_C - I_{C_\mu} R_C)}{V_o} s C_\mu$$

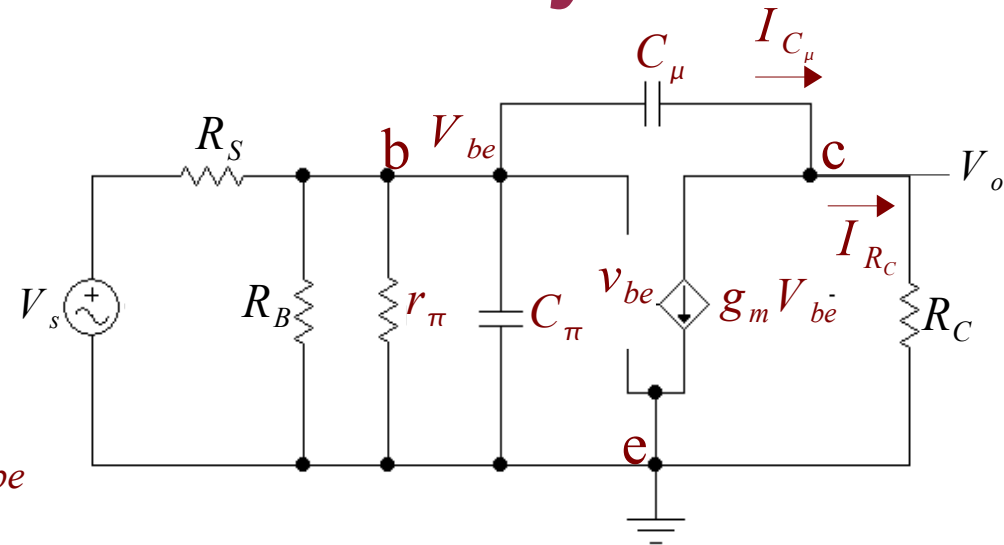
Common Emitter Miller Effect Analysis II

From slide 7:

$$I_{C_\mu} = (V_{be} + g_m V_{be} R_C - I_{C_\mu} R_C) s C_\mu$$

Collect terms for I_{C_μ} and V_{be} :

$$(1 + s R_C C_\mu) I_{C_\mu} = (1 + g_m R_C) s C_\mu V_{be}$$



$$I_{C_\mu} = \frac{(1 + g_m R_C) s C_\mu}{(1 + s R_C C_\mu)} V_{be} = \frac{s(1 + g_m R_C) C_\mu}{(1 + s R_C C_\mu)} V_{be} \approx s(1 + g_m R_C) C_\mu V_{be}$$

$\omega R_C C_\mu \ll 1$

Miller Capacitance C_{eq} : $C_{eq} = (1 - A_v) C_\mu = (1 + g_m R_C) C_\mu$

Common Emitter Miller Effect Analysis III

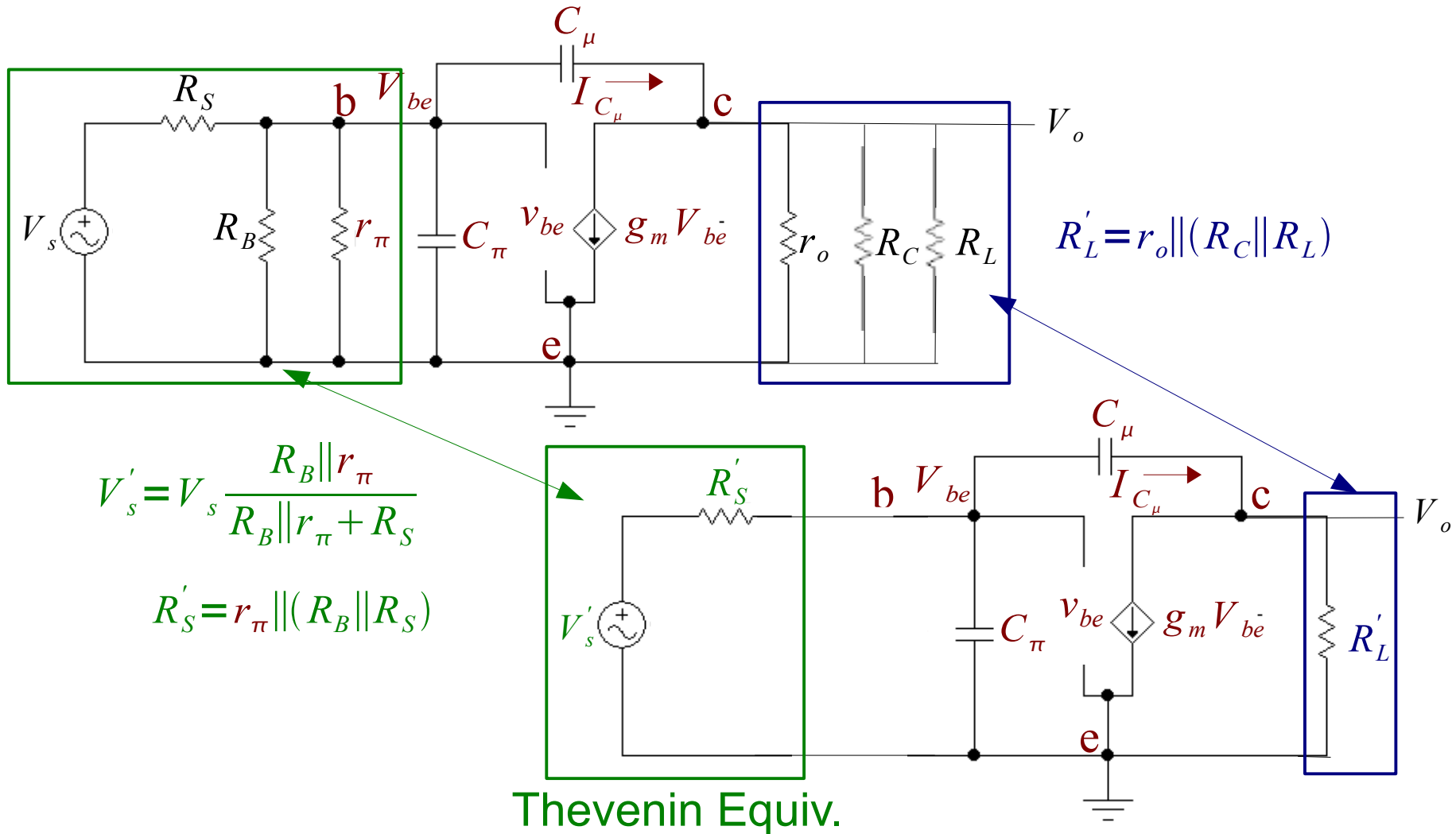
$$C_{eq} = (1 + g_m R_C) C_\mu$$

For our example circuit:

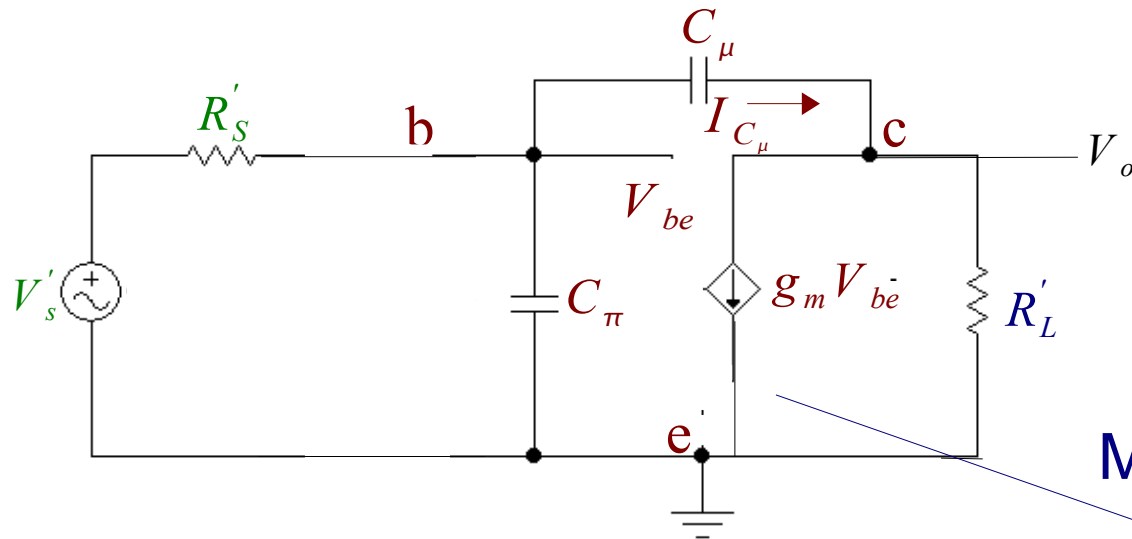
$$1 + g_m R_C = 1 + 0.040 \cdot 5100 = 205$$

$$C_{eq} = (205) \cdot 2 \text{ pF} \approx 410 \text{ pF}$$

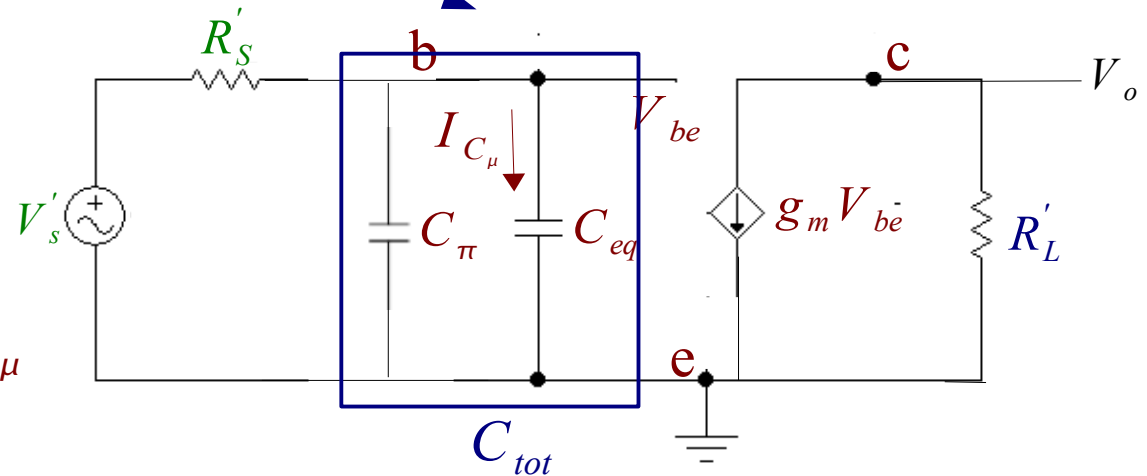
Simplified HF Model



Simplified HF Model



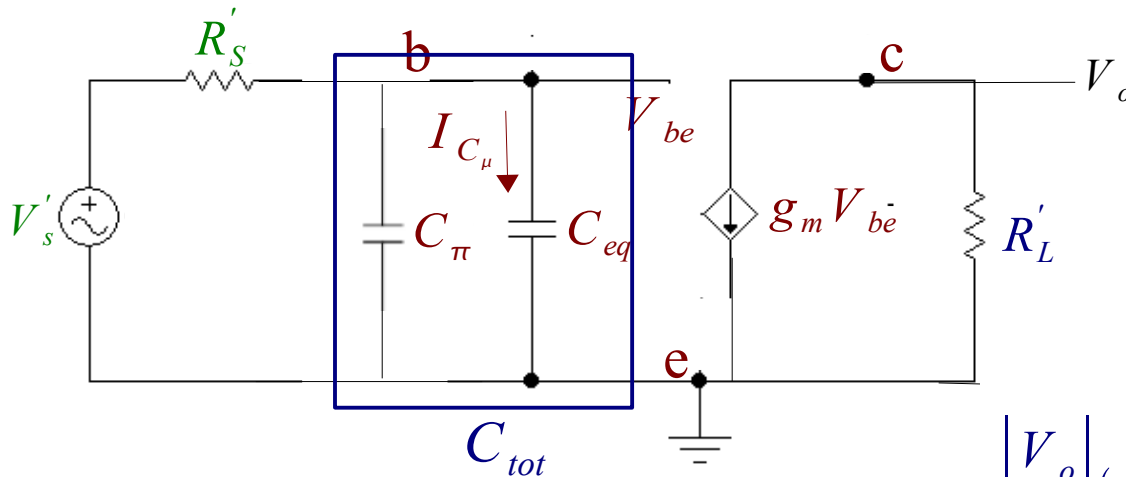
Miller's Theorem



$$C_{eq} = (1 + g_m R_C) C_\mu$$

$$C_{tot} = C_\pi + C_{eq}$$

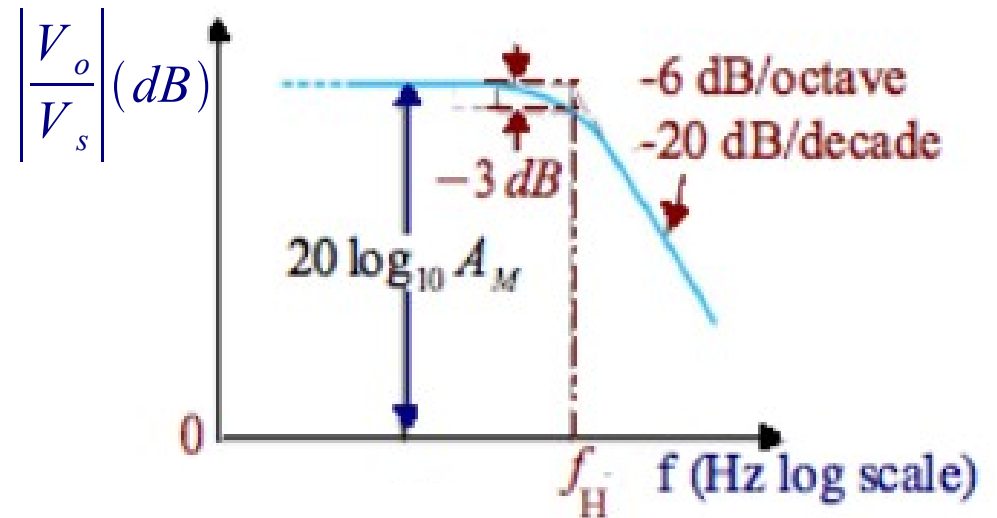
Simplified HF Model



$$C_{tot} = C_{\pi} + (1 + g_m R_C) C_{\mu}$$

$$A_v(f) = \frac{V_o}{V_s} \approx \frac{-g_m R'_L}{1 + j2\pi f C_{tot} R'_S} = \frac{-g_m R'_L}{1 + j \frac{f}{f_H}}$$

$$A_M = g_m R'_L \quad \text{and} \quad f_H = \frac{1}{2\pi C_{tot} R'_S}$$

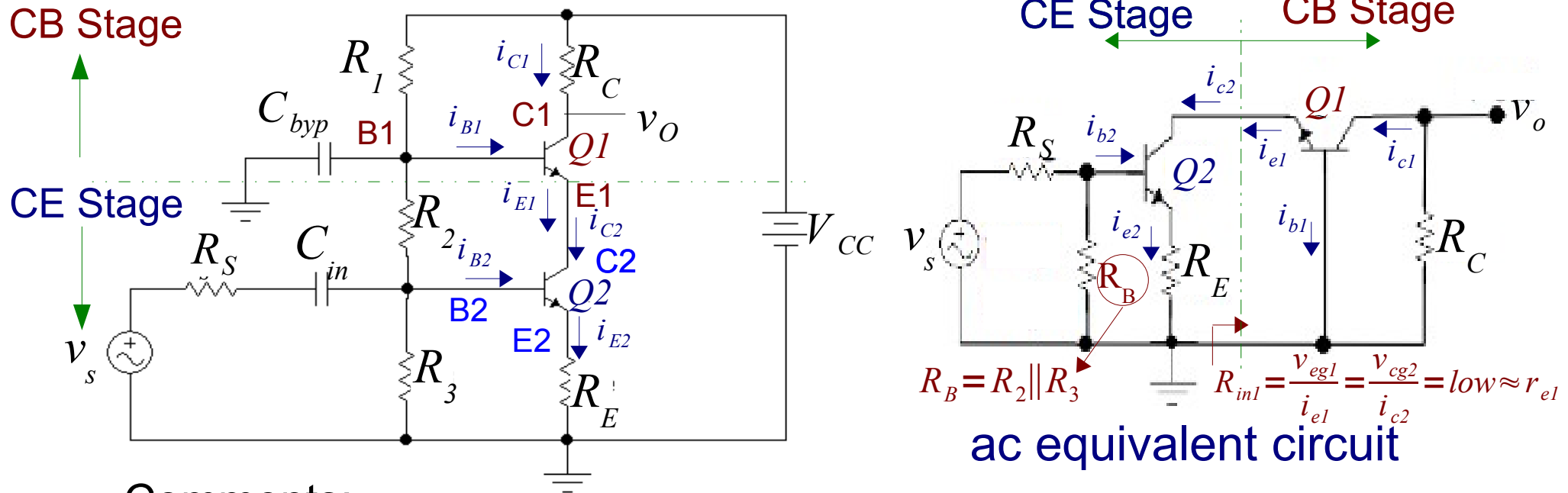


The Cascode Amplifier

- A two transistor amplifier used to obtain simultaneously:
 1. Reasonably high input impedance.
 2. Reasonable voltage gain.
 3. Wide bandwidth.
- None of the conventional single transistor designs will satisfy all of the criteria above.
- The cascode amplifier will satisfy all of these criteria.
- A cascode is a **CE Stage** cascaded with a **CB Stage**.

(Historical Note: the cascode amplifier was a cascade of *grounded cathode* and *grounded grid* vacuum tube stages – hence the name “cascode,” which has remained in modern terminology.)

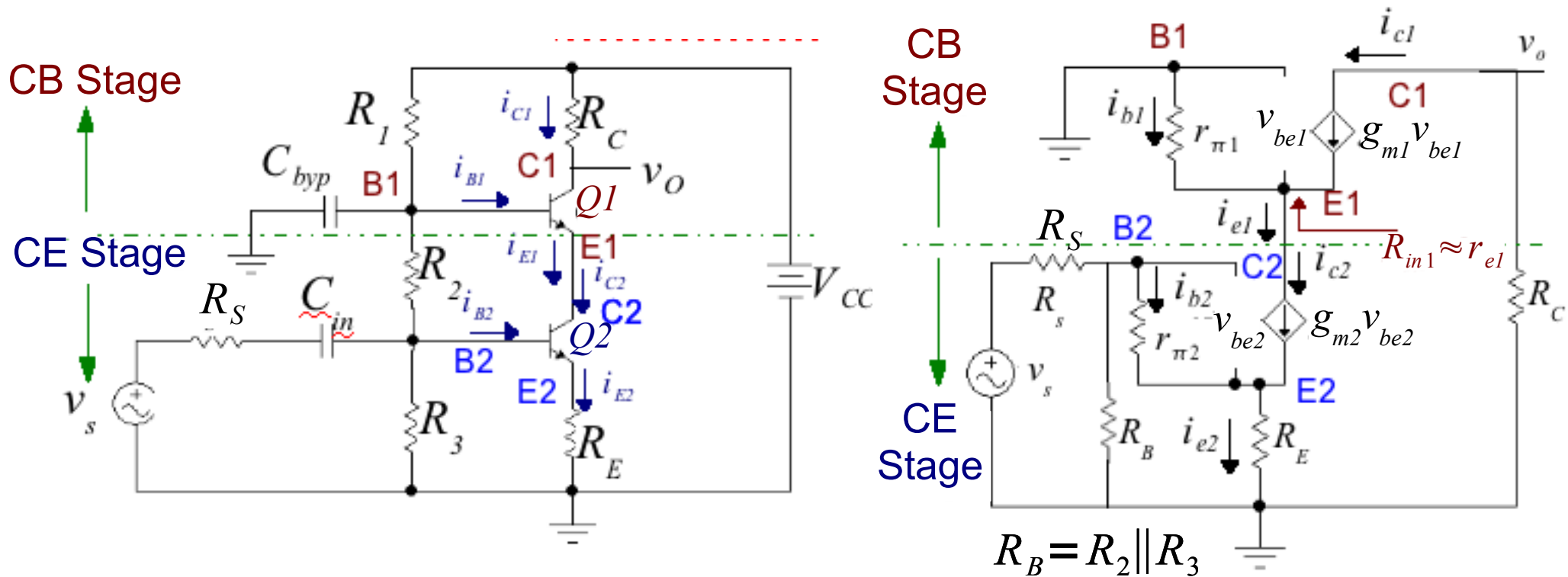
The Cascode Amplifier



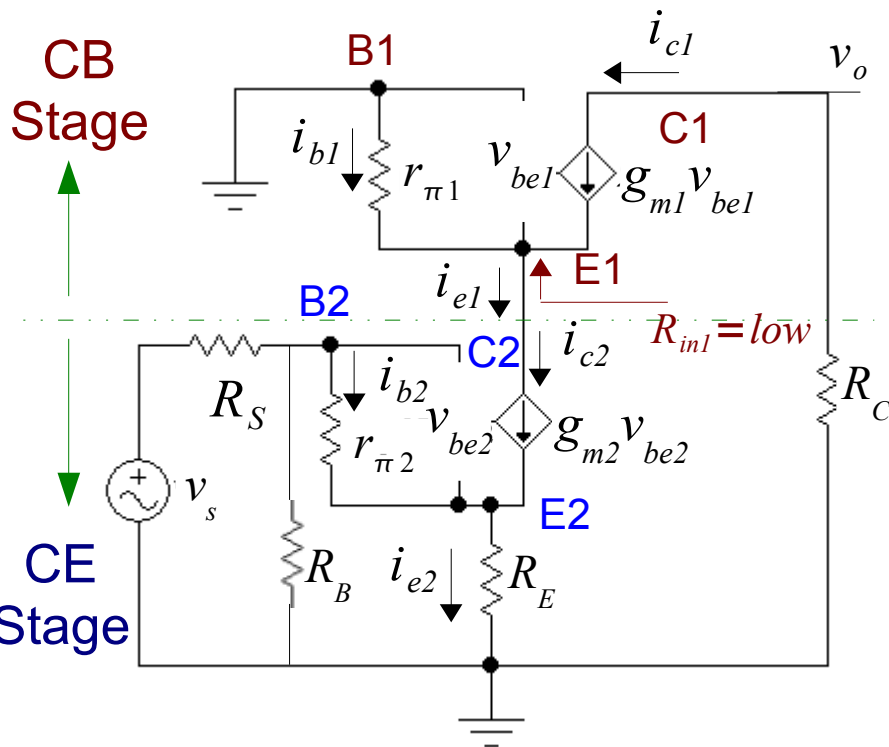
Comments:

1. R_1 , R_2 , R_3 , and R_C set the bias levels for both Q1 and Q2.
2. Determine R_E for the desired voltage gain.
3. C_{in} and C_{byp} are to act as “open circuits” at dc and act as “short circuits” at all operating frequencies $f > f_{min}$.

Cascode Mid-Band Small Signal Model



Cascode Small Signal Analysis



1. Show reduction in Miller effect
2. Evaluate small-signal voltage gain

OBSERVATIONS

a. The emitter current of the **CB Stage** is the collector current of the **CE Stage**. (This also holds for the dc bias current.)

$$i_{e1} = i_{c2}$$

b. The base current of the **CB Stage** is:

$$i_{b1} = \frac{i_{e1}}{\beta + 1} = \frac{i_{c2}}{\beta + 1}$$

c. Hence, both stages have about same collector current $i_{c1} \approx i_{c2}$ and same g_m, r_e, r_{π} .

$$g_{m1} = g_{m2} = g_m$$

$$r_{e1} = r_{e2} = r_e$$

$$r_{\pi 1} = r_{\pi 2} = r_{\pi}$$

Cascode Small Signal Analysis cont.

The input resistance R_{in1} to the CB Stage is the small-signal “ r_{el} ” for the CB Stage, i.e.

$$i_{b1} = \frac{i_{e1}}{\beta + 1} = \frac{i_{c2}}{\beta + 1}$$

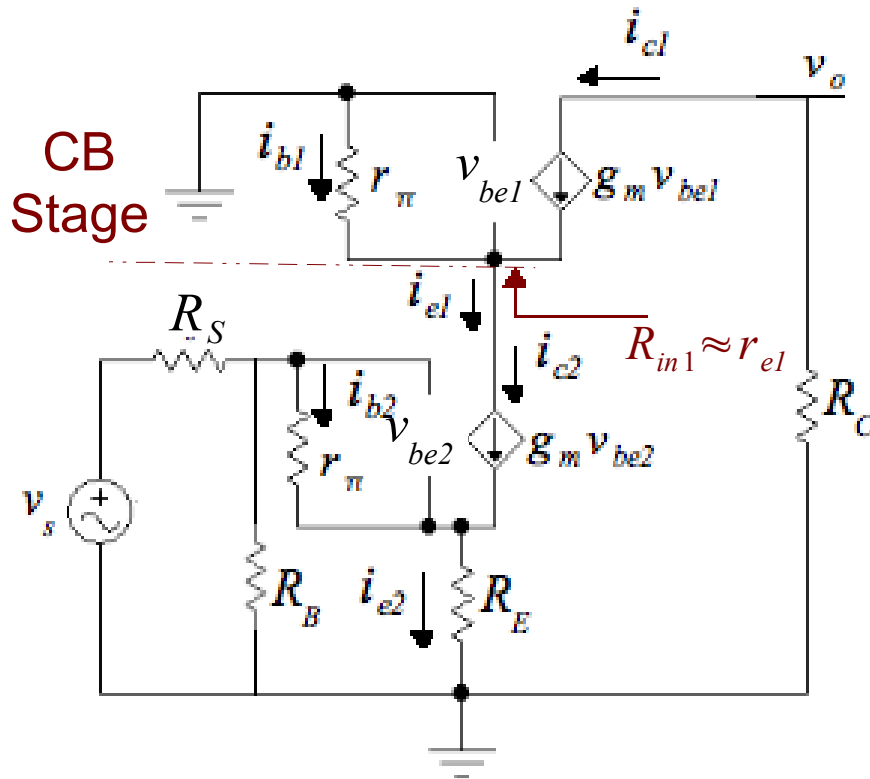
The CE output voltage, the voltage drop from Q2 collector to ground, is:

$$v_{c2} = v_{e1} = -r_{\pi} i_{b1} = -\frac{r_{\pi}}{\beta + 1} i_{c2} = -\frac{r_{\pi}}{\beta + 1} i_{e1}$$

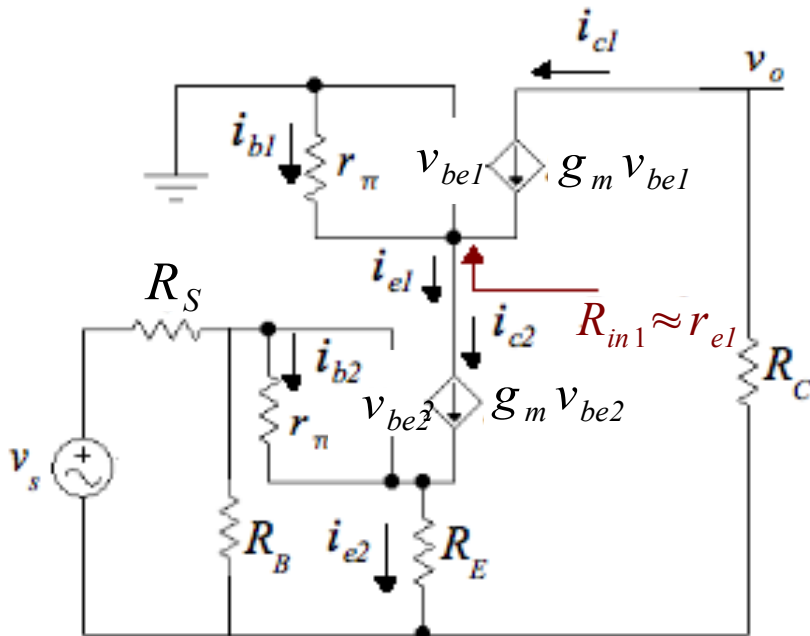
Therefore, the CB Stage input resistance is:

$$R_{in1} = \frac{v_{e1}}{-i_{e1}} = \frac{r_{\pi}}{\beta + 1} = r_{e1}$$

$$A_{vCE-Stage} = \frac{v_{c2}}{v_s} \approx -\frac{R_{in1}}{R_E} = -\frac{r_e}{R_E} < 1 \Rightarrow C_{eq} = \left(1 + \frac{r_e}{R_E}\right) C_{\mu} < 2 C_{\mu}$$



Cascode Small Signal Analysis - cont.



Now, find the CE collector current in terms of the input voltage v_s : **Recall** $i_{c1} \approx i_{c2}$

$$i_{b2} \approx \frac{v_s}{R_S \parallel R_B + r_{\pi} + (\beta + 1) R_E}$$

$$i_{c2} = \beta i_{b2} \approx \frac{\beta v_s}{R_S \parallel R_B + r_{\pi} + (\beta + 1) R_E} \approx \frac{\beta v_s}{(\beta + 1) R_E}$$

for bias insensitivity: $(\beta + 1) R_E \gg R_S \parallel R_B + r_{\pi}$

$$i_{c1} \approx i_{e1} = i_{c2} \approx i_{e2}$$

$$i_{c2} \approx \frac{v_s}{R_E}$$

$$v_o = -i_{c2} R_C$$

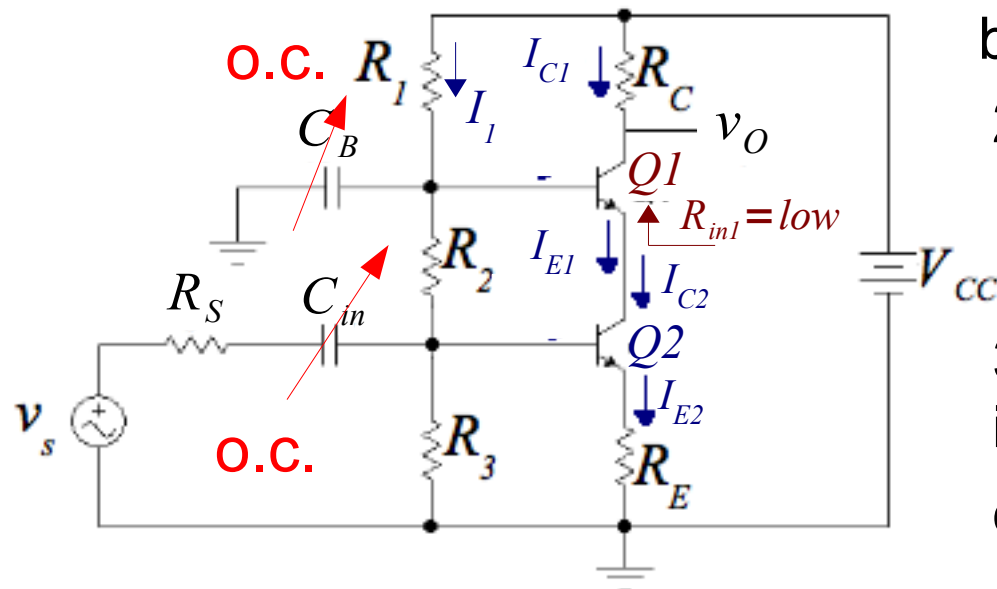
\Rightarrow

$$A_v = \frac{v_o}{v_s} = \frac{-R_C}{R_E}$$

OBSERVATIONS:

1. Voltage gain A_v is about the same as a stand-alone CE Amplifier.
2. HF cutoff is much higher than a CE Amplifier due to the reduced C_{eq} .

Cascode Biasing



1. Choose I_{E1} – make it relatively large to reduce $R_{in1} = r_e = V_T / I_{E1}$ to push out HF break frequencies.

2. Choose R_C for suitable voltage swing V_{C1} and R_E for desired gain.

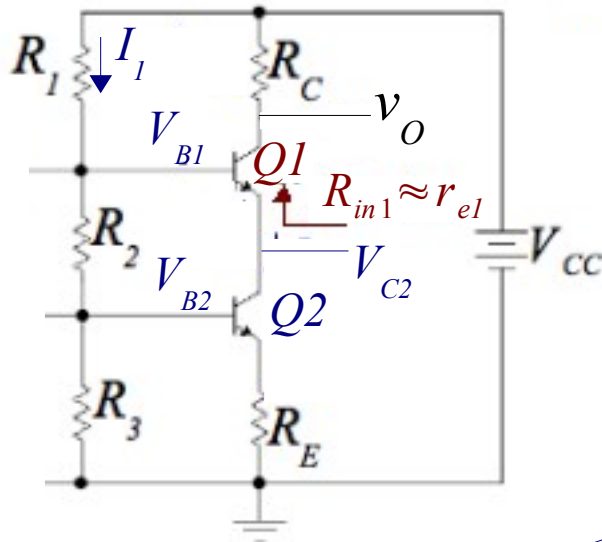
3. Choose bias resistor string such that its current I_1 is about 0.1 of the collector current I_{C1} .

4. Given R_E , I_{E2} and $V_{BE2} = 0.7 V$ calc. R_3 .

5. Need to also determine R_1 & R_2 .

$$\alpha_2 I_{E2} = I_{C2} = I_{E1} = \frac{1}{\alpha_1} I_{C1} \Rightarrow I_{C1} \approx I_{E2}$$

Cascode Biasing - cont.



Since the CE-Stage gain is very small:
a. The collector swing of Q2 will be small.
b. The Q2 collector bias $V_{C2} = V_{B1} - 0.7 V$.

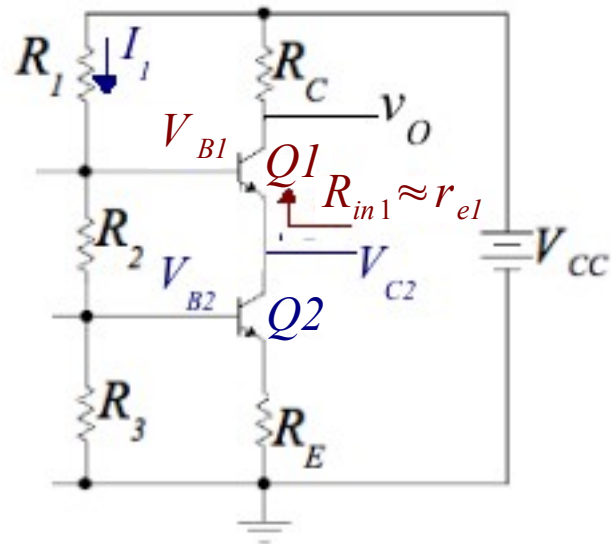
6. Set $V_{B1} - V_{B2} = 1 V \Rightarrow V_{CE2} = 1 V$

This will limit V_{CB2} $V_{CB2} = V_{CE2} - V_{BE2} = 0.3 V$
which will keep Q2 forward active.

$$\begin{aligned} V_{CE2} &= V_{C2} - V_{Re} = V_{C2} - (V_{B2} - 0.7 V) \\ &= V_{B1} - 0.7 V - V_{B2} + 0.7 V \\ &= V_{B1} - V_{B2} \end{aligned}$$

7. Next determine R_2 . Its drop $V_{R2} = 1 V$
with the known current. $R_2 = \frac{V_{B1} - V_{B2}}{I_1}$

Cascode Biasing - cont.



$$R_2 = \frac{V_{B1} - V_{B2}}{I_1} = \frac{1V}{I_1}$$

8. Then calculate R_3 . $R_3 = \frac{V_{B2}}{I_1}$

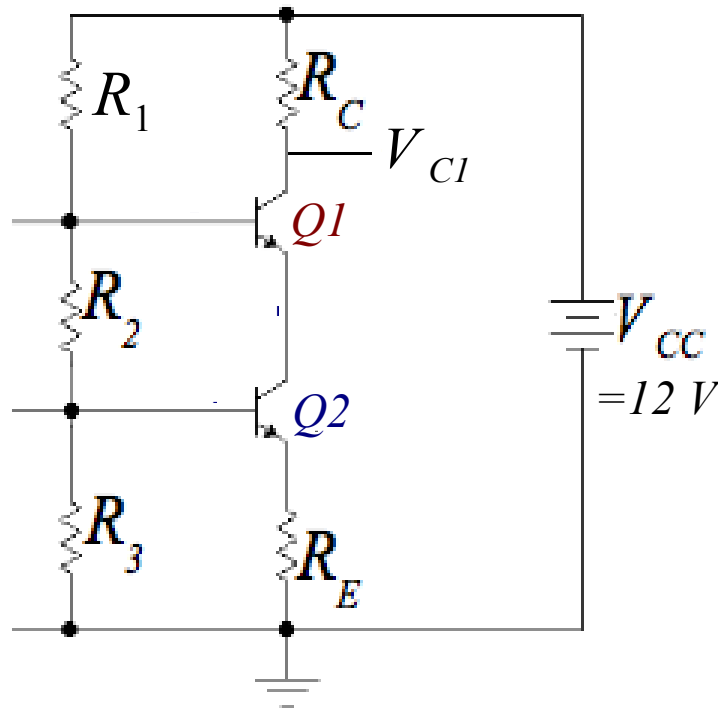
where $V_{B2} = 0.7V + I_E R_E$

Note: $R_1 + R_2 + R_3 = \frac{V_{CC}}{I_1}$

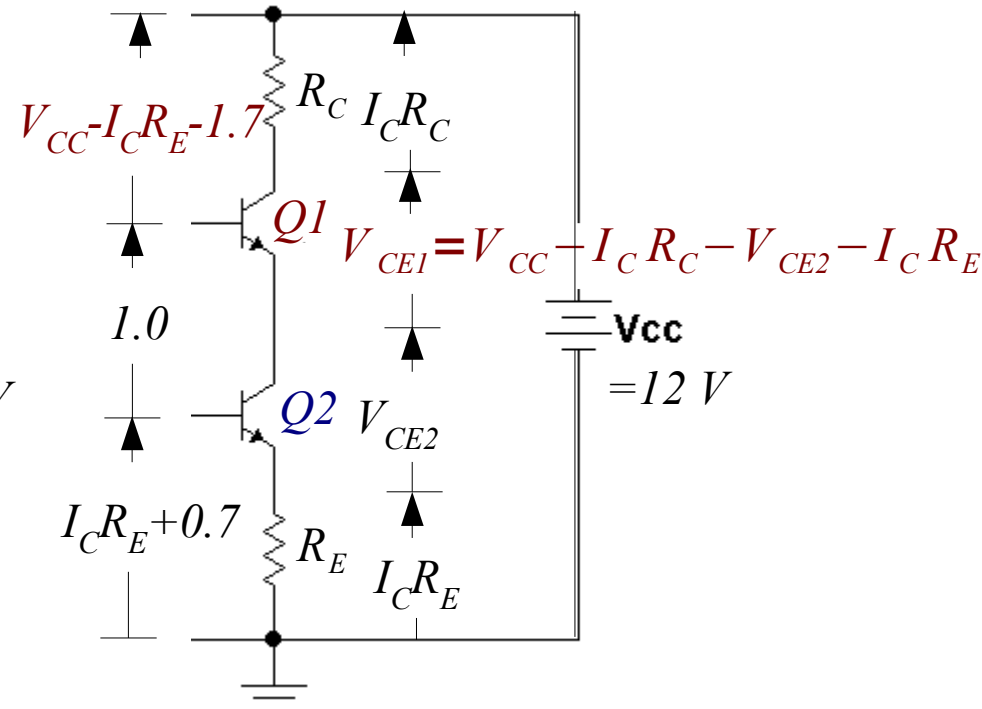
9. Then calculate R_1 .

$$R_1 = \frac{V_{CC}}{0.1 I_C} - R_2 - R_3$$

Cascode Bias Example



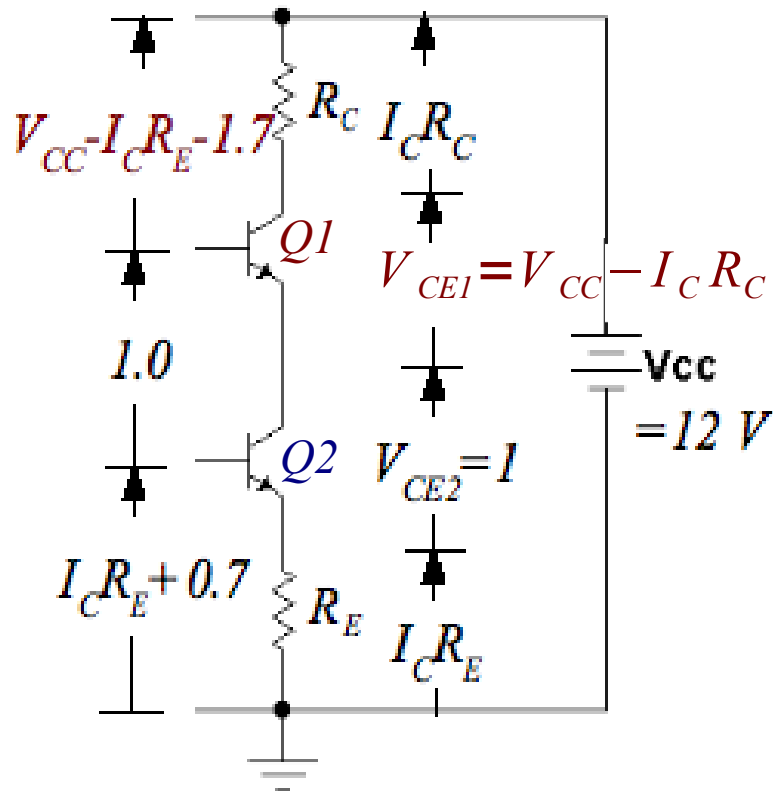
Cascode Amp



$$I_{E2} \approx I_{C2} = I_{E1} \approx I_{C1} \Rightarrow I_{C1} \approx I_{E2}$$

Typical Bias Conditions

Cascode Bias Example cont.

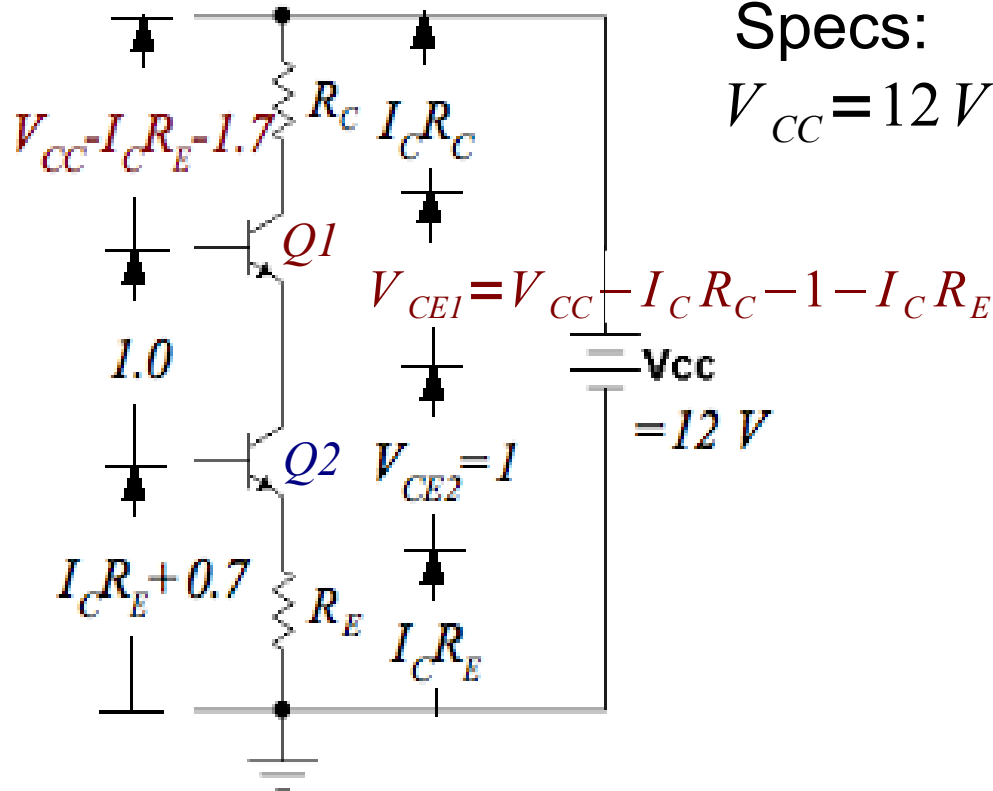


1. Choose I_{E1} – make it a bit high to lower r_e .
 Try $I_{E1} = 5 \text{ mA} \Rightarrow r_e = 0.025 \text{ V} / I_E = 5 \Omega$.

2. Set desired gain magnitude. For example if $A_V = -10$, then $R_C / R_E = 10$.

3. Since the CE stage gain is very small, V_{CE2} can be small, i.e. $V_{CE2} = V_{B1} - V_{B2} = 1 \text{ V}$.

Cascode Bias Example cont.



Specs:

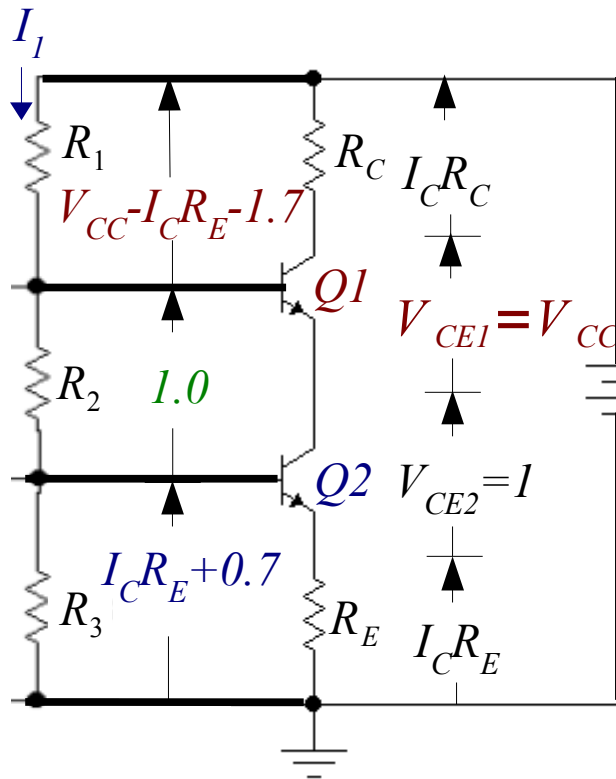
$$V_{CC} = 12 V \quad V_{CE1} = 7 V \quad I_C = 5 mA \quad |A_v| = \frac{R_C}{R_E} = 10$$

Determine R_C for $V_{CE1} = 7 V$.

$$R_C = \frac{V_{CC} - V_{CE1}}{I_C} = \frac{5 V}{5 \cdot 10^{-3} A} = 1000 \Omega$$

$$R_E = \frac{R_C}{|A_v|} = \frac{R_C}{10} = 100 \Omega$$

Cascode Bias Example cont.



$$V_{CC} = 12 \quad R_C = 1 \text{ k}\Omega \quad I_C = 5 \text{ mA} \quad R_E = 100 \Omega$$

Make current through the string of bias resistors $I_1 = 0.1 I_C = 0.5 \text{ mA}$.

$$R_1 + R_2 + R_3 = \frac{V_{CC}}{I_1} = \frac{12}{5 \cdot 10^{-4}} = 24 \text{ k}\Omega$$

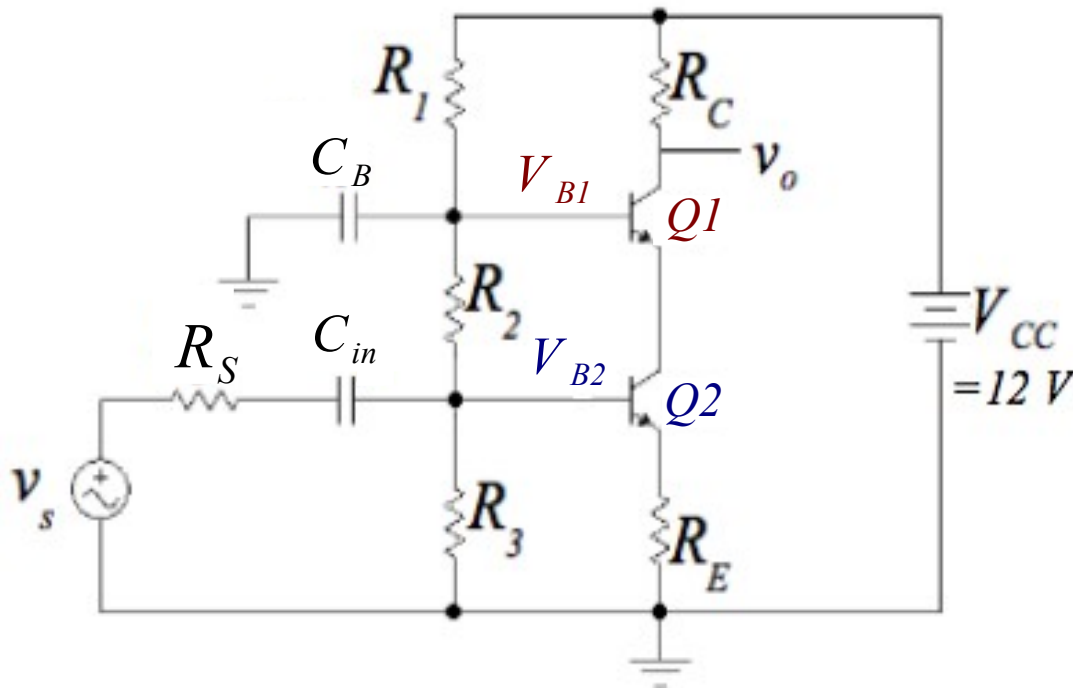
Calculate the bias voltages (base side of **Q1**, **Q2**):

$$V_{CC} - I_C R_E - 1.7 \text{ V} = 12 \text{ V} - 0.5 \text{ V} - 1.7 \text{ V} = 9.8 \text{ V}$$

$$V_{B1} - V_{B2} = 1 \text{ V}$$

$$V_{B2} = I_C R_E + 0.7 = 5 \cdot 10^{-3} \cdot 100 + 0.7 = 1.2 \text{ V}$$

Cascode Bias Example cont.



$$V_{B2} = 5 \cdot 10^{-4} R_3 = 1.2 V$$

$$R_3 = 2.4 k \Omega$$

$$V_{B1} - V_{B2} = 5 \cdot 10^{-4} R_2 = 1.0 V$$

$$R_2 = 2 k \Omega$$

$$\text{Recall: } R_1 + R_2 + R_3 = 24 k \Omega$$

$$R_1 = 24000 - 2400 - 2000 = 19.6 k \Omega$$

$$V_{CC} = 12, \quad R_C = 1 k \Omega, \quad V_{B2} = 1.2 V,$$

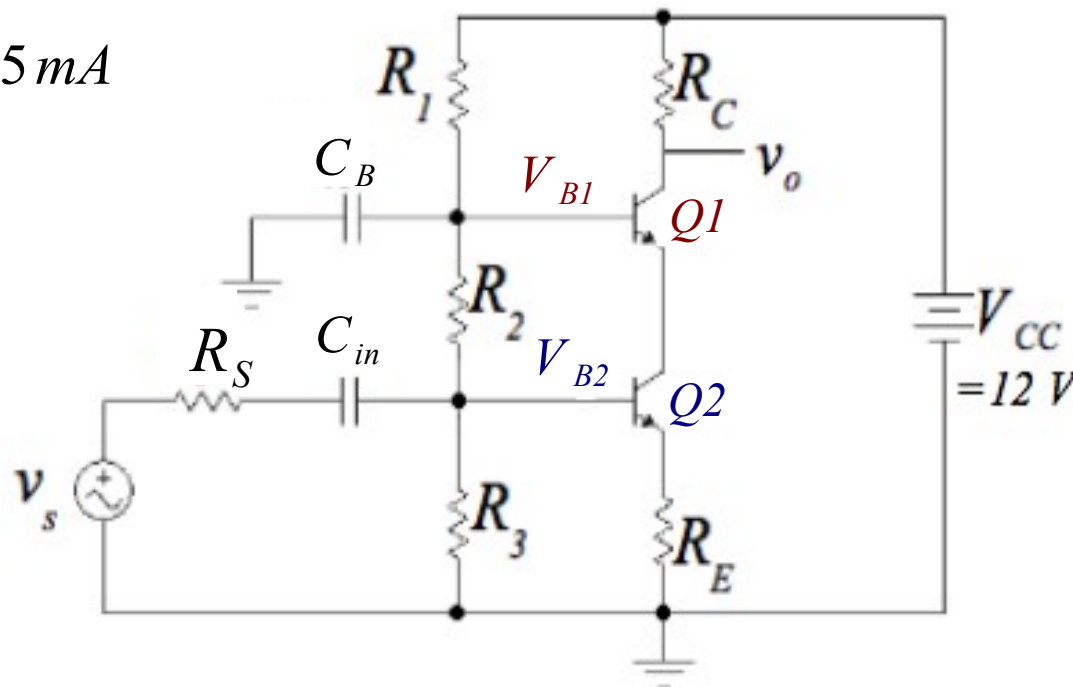
$$I_C = 5 mA, \quad R_E = 100 \Omega, \quad V_{B1} - V_{B2} = 1.0 V$$

Completed Design

$$r_e = 5 \Omega \Rightarrow I_C = 5 \text{ mA}$$

$$V_{CE} = 7 \text{ V}$$

$$|A_v| = \frac{R_C}{R_E} = 10$$



$$R_1 = 19.6 \text{ k}\Omega$$

$$R_C = 1 \text{ k}\Omega$$

$$R_2 = 2 \text{ k}\Omega$$

$$R_E = 100 \Omega$$

$$R_3 = 2.4 \text{ k}\Omega$$

$$f_H = \frac{1}{2\pi C_{tot} R'_S}$$

$$C_{tot} = C_\pi + \left(1 + \frac{r_e}{R_E}\right) C_\mu$$

$$= C_\pi + 1.05 C_\mu$$

$$\text{If } C_\pi = 12 \text{ pF}$$

$$C_\mu = 2 \text{ pF}$$

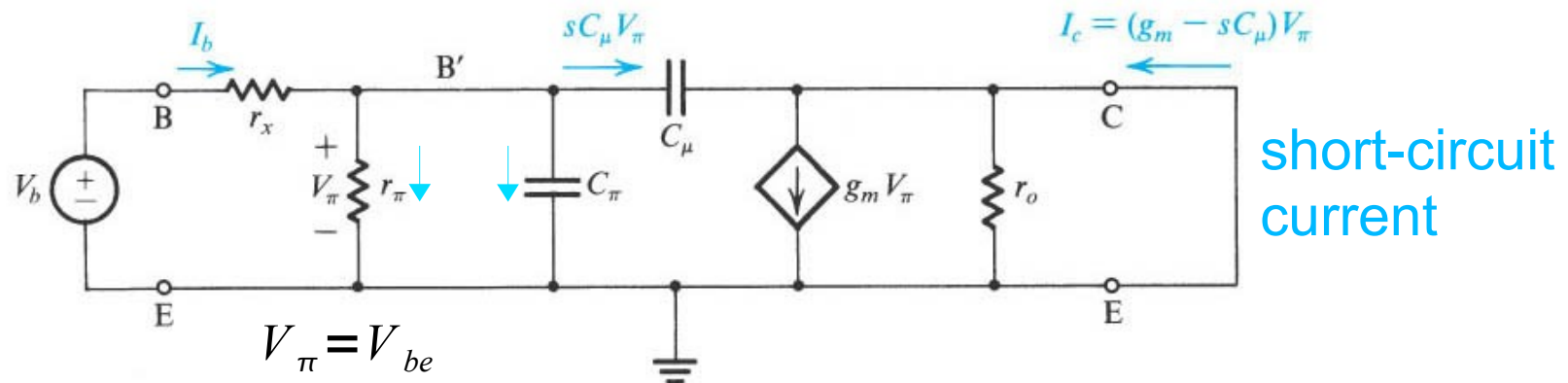
$$C_{tot} = 14.1 \text{ pF}$$

$$f_{Hcasc} = 225.8 \text{ MHz}$$

For CE with $|A_v| = 10$

$$f_{HCE} = 94 \text{ MHz}$$

Frequency-dependent “beta” h_{fe}



The relationship $i_c = \beta i_b$ does not apply at high frequencies $f > f_H$!

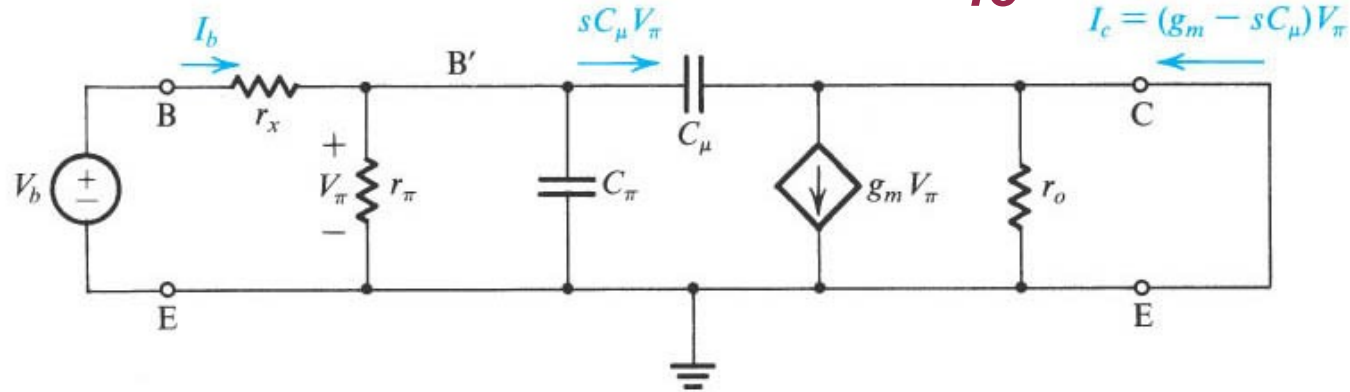
Using the relationship $-i_c = f(V_\pi)$ – find the new relationship

between i_b and i_c . For i_b (using *phasor notation* (I_x & V_x) for *frequency domain analysis*):

@ node B':
$$I_b = \left(\frac{1}{r_\pi} + sC_\pi + sC_\mu \right) V_\pi \quad \text{where } r_x \approx 0 \quad (\text{ignore } r_x)$$

NOTE: $s = \sigma + j\omega$, in sinusoidal steady-state $s = j\omega$.

Frequency-dependent h_{fe} or “beta”



$$I_b = \left(\frac{1}{r_\pi} + sC_\pi + sC_\mu \right) V_\pi \quad @ \text{ node C: } I_c = (g_m - sC_\mu) V_\pi \quad (\text{ignore } r_o)$$

Leads to a new relationship between the I_b and I_c :

$$h_{fe} = \frac{I_c}{I_b} = \frac{g_m - sC_\mu}{\frac{1}{r_\pi} + sC_\pi + sC_\mu}$$

Frequency Response of h_{fe}

$$h_{fe} = \frac{g_m - sC_\mu}{\frac{1}{r_\pi} + sC_\pi + sC_\mu}$$

multiply N&D by r_π and set $s = j\omega$

$$h_{fe} = \frac{(g_m - j\omega C_\mu)r_\pi}{1 + j\omega(C_\pi + C_\mu)r_\pi}$$

factor N to isolate g_m

$$h_{fe} = \frac{(1 - j\omega \frac{C_\mu}{g_m}) \underbrace{g_m r_\pi}_\beta}{1 + j\omega(C_\pi + C_\mu)r_\pi}$$

$$g_m = \frac{I_C}{V_T} \quad r_\pi = \beta \frac{V_T}{I_C}$$

For small $\omega = \omega_{low}$: $\omega_{low} \frac{C_\mu}{g_m} \ll 1 < \frac{1}{10}$

and: $\omega_{low}(C_\pi + C_\mu)r_\pi \ll 1 < \frac{1}{10}$

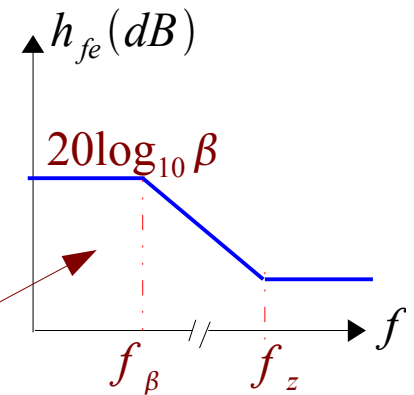
Note: $\omega_{low}(C_\pi + C_\mu)r_\pi = \omega_{low}(C_\pi + C_\mu) \frac{\beta}{g_m} \gg \omega_{low} \frac{C_\mu}{g_m}$

We have:

$$h_{fe} = g_m r_\pi = \beta$$

Frequency Response of h_{fe} cont.

$$h_{fe} = \frac{(1 - j\omega \frac{C_\mu}{g_m}) g_m r_\pi}{1 + j\omega (C_\pi + C_\mu) r_\pi} = \frac{\left(1 - j \frac{\omega}{\omega_z}\right) g_m r_\pi}{\left(1 + j \frac{\omega}{\omega_\beta}\right)} = \frac{\left(1 - j \frac{f}{f_z}\right)}{\left(1 + j \frac{f}{f_\beta}\right)} \beta$$



$$(C_\pi + C_\mu) r_\pi = (C_\pi + C_\mu) \frac{\beta}{g_m} \gg \frac{C_\mu}{g_m} \Rightarrow f_z \gg f_\beta$$

Hence, the lower break frequency or $-3dB$ frequency is f_β

$$f_\beta = \frac{1}{2\pi (C_\pi + C_\mu) r_\pi} = \frac{g_m}{2\pi (C_\pi + C_\mu) \beta} \quad \text{the upper:} \quad f_z = \frac{1}{2\pi C_\mu / g_m} = \frac{g_m}{2\pi C_\mu}$$

where $f_z > 10 f_\beta$

Frequency Response of h_{fe} cont.

Using Bode plot concepts, for the range where: $f < f_{\beta}$

$$h_{fe} = g_m r_{\pi} = \beta$$

For the range where: $f_{\beta} < f < f_z$ s.t. $|1 - j f / f_z| \approx 1$

We consider the frequency-dependent numerator term to be 1 and focus on the response of the denominator:

$$h_{fe} = \frac{g_m r_{\pi}}{\left(1 + j \frac{f}{f_{\beta}}\right)} = \frac{\beta}{\left(1 + j \frac{f}{f_{\beta}}\right)}$$

Frequency Response of h_{fe} cont.

Neglecting numerator term:

$$h_{fe} = \frac{g_m r_\pi}{\left(1 + j \frac{f}{f_\beta}\right)} = \frac{\beta}{\left(1 + j \frac{f}{f_\beta}\right)}$$

And for $f/f_\beta \gg 1$ (but $< f/f_z$):

$$|h_{fe}| \approx \frac{\beta}{\left(\frac{f}{f_\beta}\right)} = \beta \frac{f_\beta}{f}$$

Unity gain bandwidth: $|h_{fe}| = 1 \Rightarrow \beta \frac{f_\beta}{f} \quad |_{f=f_T} = 1 \Rightarrow f_T = \beta f_\beta$

$$f_T = \frac{\omega_T}{2\pi} = \beta f_\beta$$

BJT unity-gain frequency or GBP

Frequency Response of h_{fe} cont.

$$\beta = 100 \quad r_{\pi} = 2500 \, \Omega \quad C_{\pi} = 12 \, \text{pF} \quad C_{\mu} = 2 \, \text{pF} \quad g_m = 40 \cdot 10^{-3} \, \text{S}$$

$$\omega_{\beta} = \frac{1}{(C_{\pi} + C_{\mu}) r_{\pi}} = \frac{10^{12} \cdot 10^{-3}}{(12 + 2) \cdot 2.5} = 28.57 \cdot 10^6 \, \text{rps}$$

$$f_{\beta} = \frac{\omega_{\beta}}{2\pi} = \frac{28.57}{6.28} 10^6 \, \text{Hz} = 4.55 \, \text{MHz} \quad f_T = \beta f_{\beta} = 455 \, \text{MHz}$$

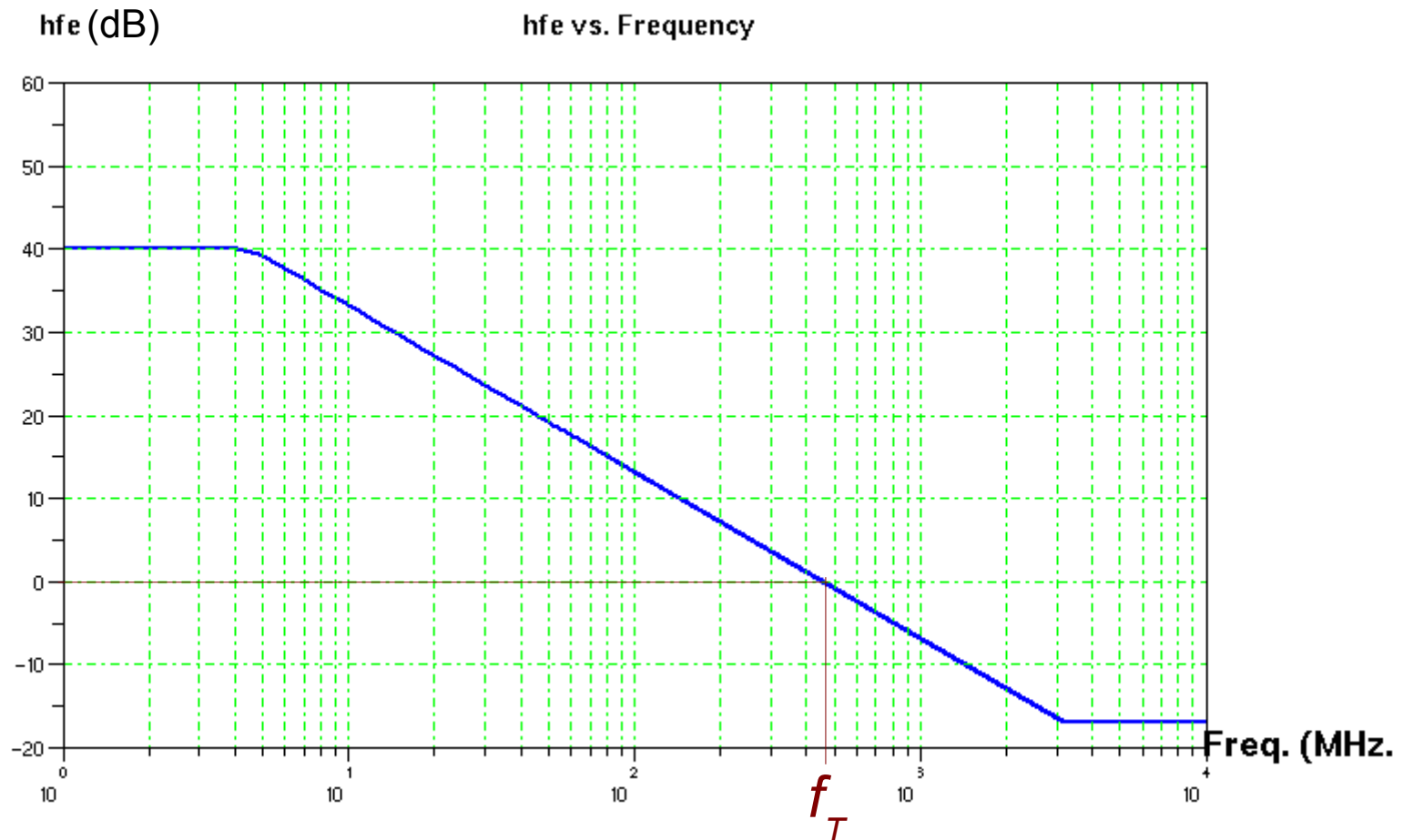
$$\omega_z = \frac{g_m}{C_{\mu}} = \frac{40 \cdot 10^{-3} \cdot 10^{12}}{2} \, \text{Hz} = 20 \cdot 10^9 \, \text{rps}$$

$$f_z = \frac{\omega_z}{2\pi} = 3.18 \cdot 10^9 \, \text{Hz} = 3180 \, \text{MHz}$$

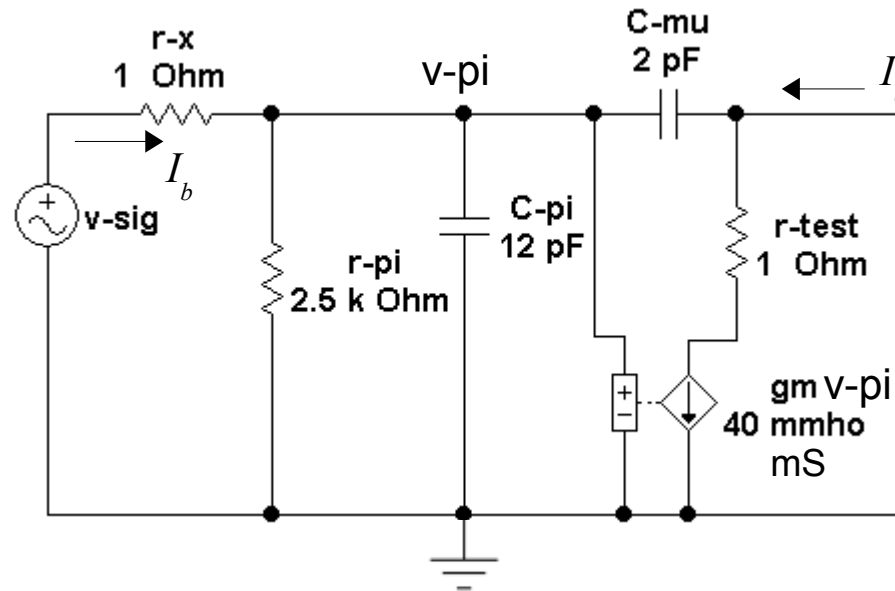
Scilab f_T Plot

```
//fT Bode Plot
Beta=100;
KdB= 20*log10(Beta);
fz=3180;
fp=4.55;
f= 1:1:10000;
term1=KdB*sign(f); //Constant array of len(f)
term2=max(0,20*log10(f/fz)); //Zero for f < fz;
term3=min(0,-20*log10(f/fp)); //Zero for f < fp;
BodePlot=term1+term2+term3;
plot(f,BodePlot);
```

h_{fe} Bode Plot



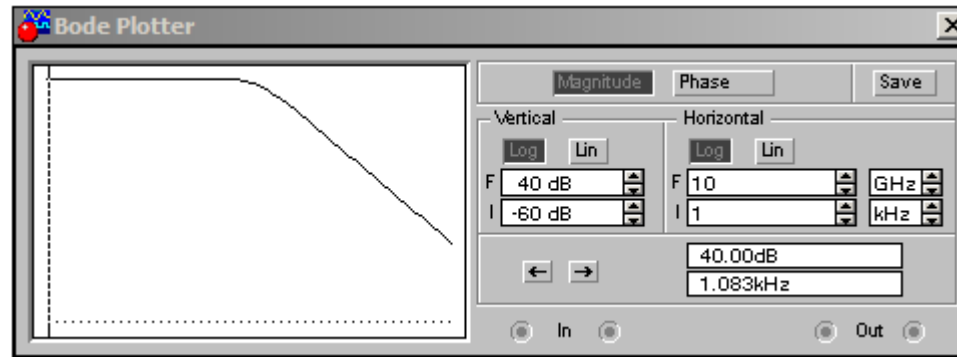
Multisim Simulation



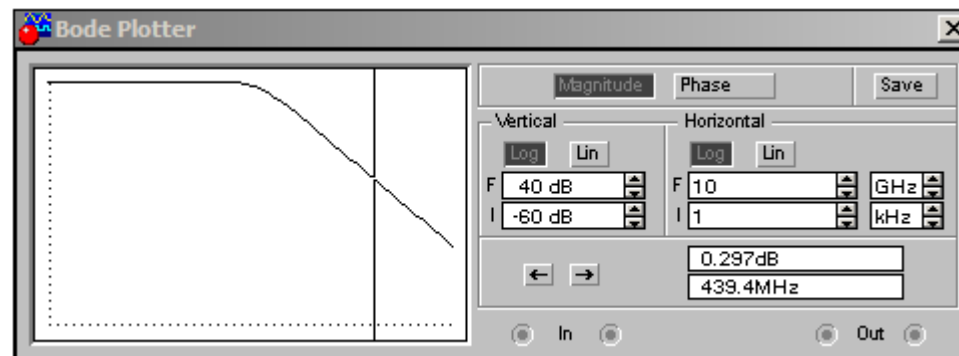
Insert 1 ohm resistors – we want to measure a current ratio.

$$h_{fe} = \frac{I_c}{I_b} = \frac{g_m - s C_\mu}{\frac{1}{r_\pi} + s(C_\pi + C_\mu)}$$

Simulation Results



Low frequency $|h_{fe}|$



Unity Gain frequency about **440 MHz**

Theory:

$$f_T = \beta f_\beta = 455 \text{ MHz}$$

