Miller Effect
Cascode BJT Amplifier
Prototype Common Emitter Circuit

Ignore “low frequency” capacitors

High frequency model
\[ |A_v| = g_m R_C = (40 \text{ mS}) \times (5.1 \text{ k}\Omega) = 204 (46.2 \text{ dB}) \]
Introducing the Miller Effect

The feedback connection of $C_\mu$ between base and collector causes it to appear to the amplifier like a large capacitor $(1 - K)C_\mu$ has been inserted between the base and emitter terminals. This phenomenon is called the “Miller effect” and the capacitive multiplier “$1 - K$” acting on $C_\mu$ equals the common emitter amplifier mid-band gain, i.e. $K = -g_m R_C$.

Common base and common collector amplifiers do not suffer from the Miller effect, since in these amplifiers, one side of $C_\mu$ is connected directly to ground.
High Frequency CC and CB Models

Common Collector
$C_\mu$ is in parallel with $R_B$.

Common Base
$C_\mu$ is in parallel with $R_C$. 
Let's examine Miller's Theorem as it applies to the HF model for the BJT CE amplifier.
Common Emitter Miller Effect Analysis

Determine effect of $C_\mu$:

Using phasor notation:

$$I_{RC} = -g_m V_\pi + I_{C_\mu}$$

or

$$V_o = \left(-g_m V_\pi + I_{C_\mu}\right) R_C$$

where

$$I_{C_\mu} = \left(V_\pi - V_o\right) s C_\mu$$

$$I_{C_\mu} = \left(V_\pi + g_m V_\pi R_C - I_{C_\mu} R_C\right) s C_\mu$$

Note: The current through $C_\mu$ depends only on $V_\pi$!
Common Emitter Miller Effect Analysis II

From slide 7:

\[ I_{C_\mu} = \left( V_\pi + g_m V_\pi R_C - I_{C_\mu} R_C \right) s C_\mu \]

Collect terms for \( I_{C_\mu} \) and \( V_\pi \):

\[ \left( 1 + s R_C C_\mu \right) I_{C_\mu} = \left( 1 + g_m R_C \right) s C_\mu V_\pi \]

Miller Capacitance \( C_{eq} \):

\[ C_{eq} = (1 - K) C_\mu = (1 + g_m R_C) \]

\[ I_{C_\mu} = \frac{\left( 1 + g_m R_C \right) s C_\mu}{\left( 1 + s R_C C_\mu \right)} \quad V_\pi = \frac{s \left( 1 + g_m R_C \right) C_\mu}{\left( 1 + s R_C C_\mu \right)} \]
Common Emitter Miller Effect Analysis III

\[ C_{eq} = (1 + g_m R_C) C_\mu \]

For our example circuit:

\[ 1 + g_m R_C = 1 + 0.040 \cdot 5100 = 205 \]

\[ C_{eq} = (205) \cdot 2 \, pF \approx 410 \, pF \]
Apply Miller's Theorem to BJT CE Amplifier

For the BJT CE Amplifier: \( Z = \frac{1}{j\omega C_\mu} \) and \( K = -g_m R_C \)

\[
Z_1 = \frac{1}{j\omega (1 + g_m R_C) C_\mu}
\]

\[
Z_2 = \frac{1}{j\omega C_\mu} \approx \frac{1}{j\omega (1 + \frac{1}{g_m R_C}) C_\mu}
\]

Miller's Theorem => important simplification to the HF BJT CE Model
**Simplified HF Model**

(a) Circuit diagram showing the simplified high-frequency (HF) model of a circuit.

(b) Mathematical representation:

\[
V'_{\text{sig}} = V_{\text{sig}} \frac{R_B || r_\pi}{R_B || r_\pi + R_{\text{sig}}}
\]

\[
R'_{\text{sig}} = r_o || R_C || R_L
\]

(c) Thevenin equivalent circuit:

\[
C_{\text{eq}} = C_{\pi} + C_{\text{eq}} = C_{\pi} + C_{\mu} (1 + g_m R'_L)
\]

(d) Frequency response graph:

\[
|V_o/V_{\text{sig}}| \text{ (dB)}
\]

-6 dB/octave

-20 dB/decade

20 log_{10} A_M

\[
f_H = \frac{1}{2\pi C_{\text{in}} R'_\text{sig}}
\]
The Cascode Amplifier

A two transistor amplifier used to obtain simultaneously:
1. Reasonably high input impedance.
2. Reasonable voltage gain.
3. Wide bandwidth.

None of the conventional single transistor designs will meet all of the criteria above. The cascode amplifier will meet all of these criteria. A cascode is a combination of a common emitter stage cascaded with a common base stage. (In “olden days” the cascode amplifier was a cascade of grounded cathode and grounded grid vacuum tube stages – hence the name “cascode,” which has persisted in modern terminology.)
The Cascode Circuit

Comments:
1. $R_1$, $R_2$, $R_3$, and $R_C$ set the bias levels for both Q1 and Q2.
2. Determine $R_E$ for the desired voltage gain.
3. $C_b$ and $C_{byp}$ are to act as “open circuits” at dc and act as “short circuits” at all operating frequencies of interest, i.e. $\omega > \omega_{min}$. 

$R_B = R_2 \parallel R_3$

$I_{in} = \frac{V_{eg1}}{I_{e1}} = \frac{V_{eg2}}{I_{e2}} = \text{low}$

ac equivalent circuit
**Cascode Mid-Band Small Signal Model**

1. Show reduction inMiller effect
2. Evaluate small-signal voltage gain

**OBSERVATIONS**

a. The emitter current of the CB stage is the collector current of the CE stage. (This also holds for the dc bias current.)

\[ i_{e1} = i_{c2} \]

b. The base current of the CB stage is:

\[ i_{b1} = \frac{i_{e1}}{\beta + 1} = \frac{i_{c2}}{\beta + 1} \]

c. Hence, both stages have about same collector current \( i_{c1} \approx i_{c2} \) and same \( g_m \).

\[ g_{m1} = g_{m2} = g_m \]
**Cascode Small Signal Analysis cont.**

The input resistance $R_{\text{in}l}$ to the CB stage is the small-signal “$R_C$” for the CE stage

$$i_{bl} = \frac{i_{e1}}{\beta + 1} = \frac{i_{c2}}{\beta + 1}$$

The CE output voltage, the voltage drop from Q2 collector to ground, is:

$$v_{cg2} = v_{eg1} = -r_{\pi1}i_{bl} = -\frac{r_{\pi1}}{\beta + 1}i_{c2} = -\frac{r_{\pi1}}{\beta + 1}i_{e1}$$

Therefore, the CB Stage input resistance is:

$$R_{\text{in}l} = \frac{v_{eg1}}{-i_{e1}} = \frac{r_{\pi1}}{\beta + 1} = r_{el}$$

$$A_{v,CE-Stage} = \frac{v_{cg2}}{v_{sig}} \approx -\frac{R_{\text{in}l}}{R_E} = -\frac{r_{el}}{R_E} < 1 \Rightarrow C_{eq} = \left(1 + \frac{r_{el}}{R_E}\right)C_{\mu} < 2C_{\mu}$$
**Cascode Small Signal Analysis - cont.**

Now, find the CE collector current in terms of the input voltage $V_{\text{sig}}$: Recall $i_{c1} \approx i_{c2}$

$$i_{c2} \approx \frac{V_{\text{sig}}}{R_E}$$

Now, find the CE collector current in terms of the input voltage $V_{\text{sig}}$:

$$i_{b2} \approx \frac{\beta \, V_{\text{sig}}}{R_{\text{sig}} \| R_B + r_{\pi 2} + (\beta + 1) \, R_E} \approx \frac{\beta \, V_{\text{sig}}}{(\beta + 1) \, R_E}$$

for bias insensitivity: $(\beta + 1) \, R_E \gg R_{\text{sig}} \| R_B + r_{\pi 2}$

$$i_{c2} = \beta \, i_{b2} \approx \frac{\beta \, V_{\text{sig}}}{R_{\text{sig}} \| R_B + r_{\pi 2} + (\beta + 1) \, R_E} \approx \frac{\beta \, V_{\text{sig}}}{(\beta + 1) \, R_E}$$

**OBSERVATIONS:**

1. Voltage gain $A_v$ is about the same as a stand-alone CE Amplifier.
2. HF cutoff is much higher than a CE Amplifier due to the reduced $C_{eq}$.

$$A_v = \frac{V_{\text{out}}}{V_{\text{sig}}} = -\frac{R_C}{R_E}$$
Approximate Cascode HF Voltage Gain

\[
A_v(\omega) = \frac{V_{out}(\omega)}{V_{sig}} \approx \frac{-\frac{R_C}{R_E}}{1 + j \omega C_{in} R'_{sig}}
\]

where

\[
C_{in} = C_{\pi} + C_{eq} = C_{\pi} + (1 + \frac{I_{ce,1}}{R_E}) C_{\mu} < C_{\pi} + 2 C_{\mu}
\]

\[
R'_{sig} = R_{sig} \parallel R_B \approx R_{sig}
\]
Cascode Biasing

1. Choose $I_{E1}$ – make it relatively large to reduce $R_{in1} = r_e = V_T / I_{E1}$ to push out HF break frequencies.

2. Choose $R_C$ for suitable voltage swing $V_{CL}$ and $R_E$ for desired gain.

3. Choose bias resistor string such that its current $I_1$ is about 0.1 of the collector current $I_{C1}$.

4. Given $R_E$, $I_{E2}$ and $V_{BE2} = 0.7 V$ calculate $R_3$. 
Cascode Biasing - cont.

Since the CE-Stage gain is very small:

a. The collector swing of Q2 will be small.
b. The Q2 collector bias $V_{C2G} = V_{B1G} - 0.7\, V$.

5. Set $V_{B1G} - V_{B2G} \approx 1\, V \Rightarrow V_{CE2} \approx 1\, V$

This will limit $V_{CB2} = V_{CE2} - V_{BE2} = 0.3\, V$ which will keep Q2 forward active.

6. Next determine $R_2$. Its drop $V_{R2} = 1\, V$ with the known current.

$$R_2 = \frac{V_{B1G} - V_{B2G}}{I_1}$$

7. Then calculate $R_1$. $R_1 = \frac{V_{CC} - V_{B1G}}{I_1}$
**Cascode Bias Example**

**Cascode circuit**

**Typical Bias Conditions**

\[
\begin{align*}
I_{E2} & \approx I_{C2} = I_{E1} \approx I_{C1} \Rightarrow I_{C1} & \approx I_{E2} \\
V_{CE1} & = I_{C}R_{C} - 1 - I_{C}R_{E} \\
V_{CE2} & = 1 \\
V_{CC} & = 12 \text{ V} \\
V_{out} & \approx I_{C2} = I_{E2} \\
\end{align*}
\]
Cascode Bias Example cont.

1. Choose $I_{E1}$ – make it a bit high to lower $r_{e1}$ or $r_{\pi}$. Try $I_{E1} = 5 \ mA \Rightarrow r_{e1} = 0.025 \ V / I_E = 5 \Omega$.

2. Set desired gain magnitude. For example if $|A_v| = 10$, then $R_C/R_E = 10$.

3. Since the CE stage gain is very small, $V_{CE2}$ can be small. Use $V_{CE2} = V_{B1G} - V_{B2G} = 1 \ V$. 

$$V_{CC} - I_C R_E - 1.7$$

$$V_{CE1} = I_C R_C - 1 - I_C R_E$$

$$V_{CC} = 12 \ V$$

$$1.0$$

$$V_{CE2} = 1$$

$I_C R_E + 0.7$
Cascode Bias Example cont.

\[ V_{CC} = 12 \quad I_C = 5 \text{ mA} \]

\[ |A_v| = \frac{R_C}{R_E} = 10 \]

Determine \( R_C \) for a 5 V drop across \( R_C \).

\[ R_C = \frac{5 \text{ V}}{5 \cdot 10^{-3} \text{ A}} = 1000 \Omega \]

\[ R_E = \frac{R_C}{|A_v|} = \frac{R_C}{10} = 100 \Omega \]
Cascode Bias Example cont.

\[ V_{CC} = 12 \quad R_C = 1 \, k\Omega \quad I_C = 5 \, mA \quad R_E = 100 \, \Omega \]

Make current through the string of bias resistors \( I_1 = 1 \, mA \).

\[
R_1 + R_2 + R_3 = \frac{V_{CC}}{I_1} = \frac{12}{1 \cdot 10^{-3}} = 12 \, k\Omega
\]

We now calculate the bias voltages:

\[
V_{CC} - I_C R_E - 1.7 \, V = 12 \, V - 0.5 \, V - 1.7 \, V = 9.8 \, V
\]

\[
V_{B1B2} = 1.0 \, V
\]

\[
V_{B2G} = I_C R_E + 0.7 = 5 \cdot 10^{-3} \cdot 100 + 0.7 = 1.2 \, V
\]
Cascode Bias Example cont.

\[ V_{B2G} = 10^{-3} \quad R_3 = 1.2 \text{ V} \]
\[ R_3 = 1.2 \text{ k}\Omega \]
\[ V_{B1G} - V_{B2G} = 1 \cdot 10^{-3} \quad R_2 = 1.0 \text{ V} \]
\[ R_2 = 1 \text{ k}\Omega \]
\[ R_1 = 12000 - 1200 - 1000 = 9.8 \text{ k}\Omega \]
\[ R_1 = 10 \text{ k}\Omega \]

\[ V_{CC} = 12 \quad R_C = 1 \text{ k}\Omega \quad V_{B2G} = 1.2 \text{ V} \]
\[ I_C = 5 \text{ mA} \quad R_E = 100 \text{ } \Omega \quad V_{B1G} - V_{B2G} = 1.0 \text{ V} \]
Multisim Results – Bias Example

Check $I_{CI}$:

$$I_{CI} = \frac{V_{CC} - (7.329 + 0.971 + 0.339)}{R_C} = \frac{12 V - 8.639 V}{1000 \, \Omega} = 3.36 \, mA$$

$I_{CI}$ about 3.4 mA. That's a little low.

Increase $R_3$ to 1.5 k Ohms and re-simulate.
Improved Biasing

That's better! Now measure the gain at a mid-band frequency with some “large” coupling capacitors, say 10 μF inserted.
Gain $|A_v|$ of about 8.75 at 100 kHz - OK for rough calculations. Some attenuation from low CB input impedance ($R_B = R_2||R_3$) and some from 5 Ohm $r_e$. 

2008 Kenneth R. Laker (based on P. V. Lopresti 2006) update 15Oct08 KRL
Bode Plot for the Amplifier

Low frequency break point with 10 µF. capacitors

High frequency break point – internal capacitances only
Scope Plot – Near 5 V Swing on Output
Determine Bypass Capacitors

Low Frequency $f \leq f_{\text{min}}$

From CE stage determine $C_b$

$$C_b \geq \frac{10}{2 \pi \frac{f_{\text{min}}}{R_B} r_{bg}} \approx \frac{10}{2 \pi f_{\text{min}} R_B \left(\beta + 1\right) R_E} F$$

$$R_B = R_2 \parallel R_3$$

From CB stage determine $C_{\text{byp}}$

$$C_{\text{byp}} \geq \frac{10}{2 \pi f_{\text{min}} \left(\beta + 1\right) R_E + r_{\pi 1}} F$$

where $R_S \rightarrow R_E$