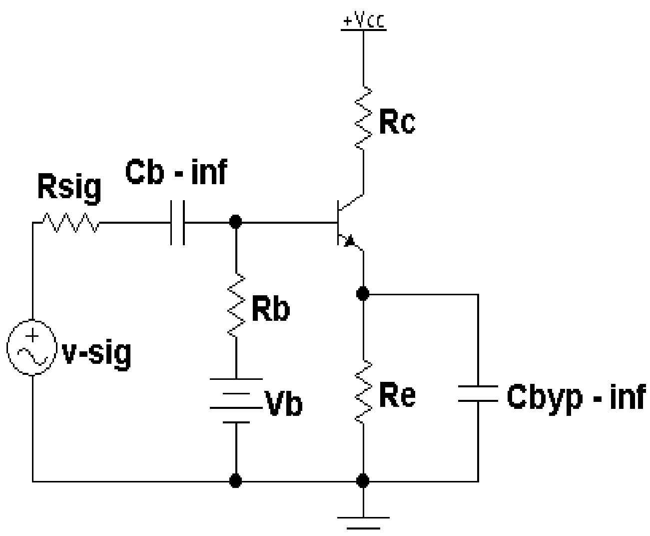
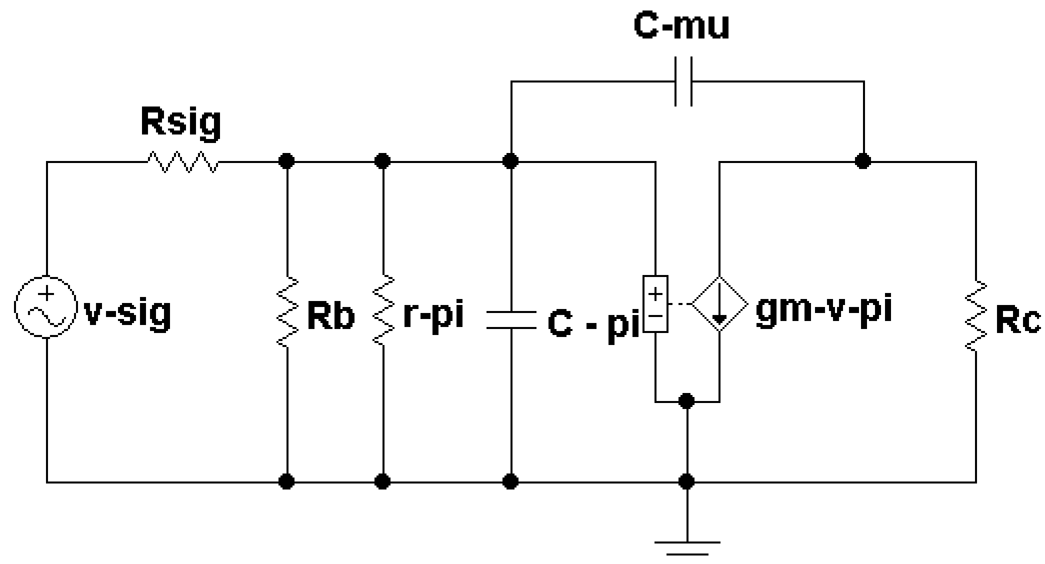
Miller Effect
Cascode BJT Amplifier

Prototype Common Emitter Circuit

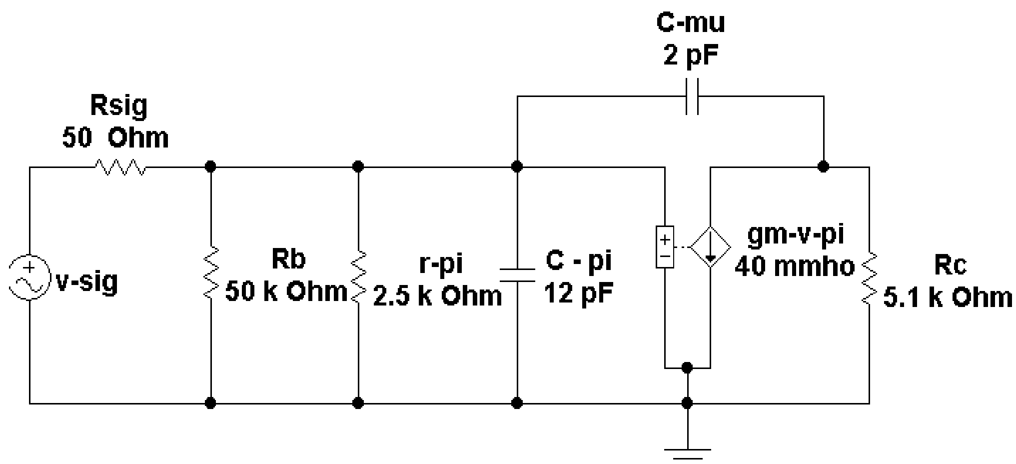


Ignore “low frequency” capacitors

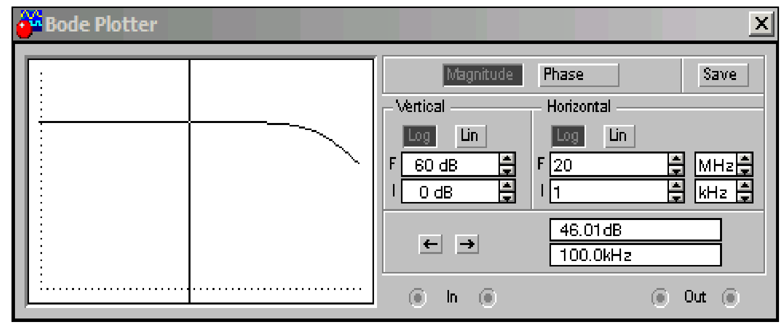


High frequency model

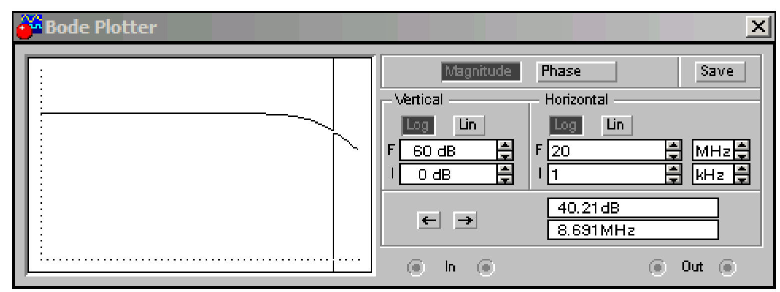
Multisim Simulation



$$|A_v| = g_m R_C = (40\,mS) * (5.1\,k\,\Omega) = 204\,(46.2\,dB)$$

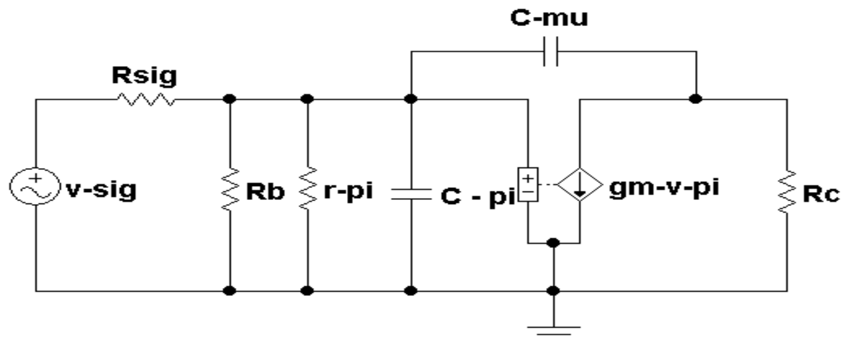


Mid-band gain



Half-gain point

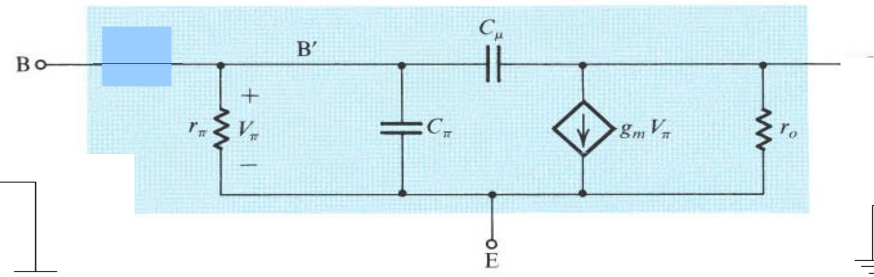
Introducing the Miller Effect

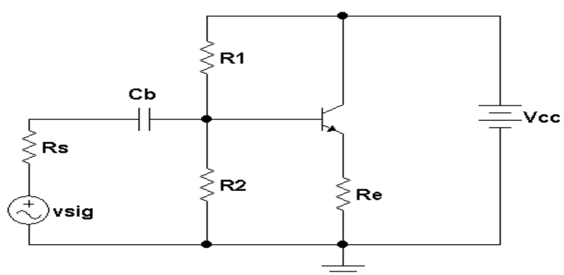


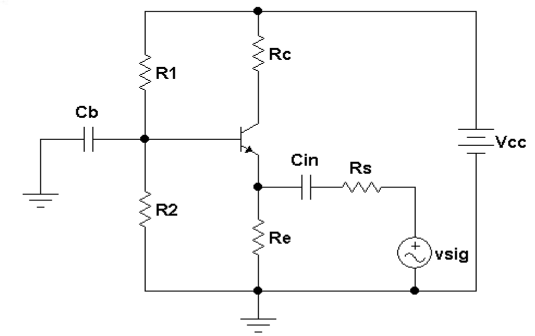
The feedback connection of C_μ between base and collector causes it to appear to the amplifier like a large capacitor $(1-K)C_\mu$ has been inserted between the base and emitter terminals. This phenomenon is called the "Miller effect" and the capacitive multiplier "$1-K$" acting on C_μ equals the common emitter amplifier mid-band gain, i.e. $K=-g_m R_C$.

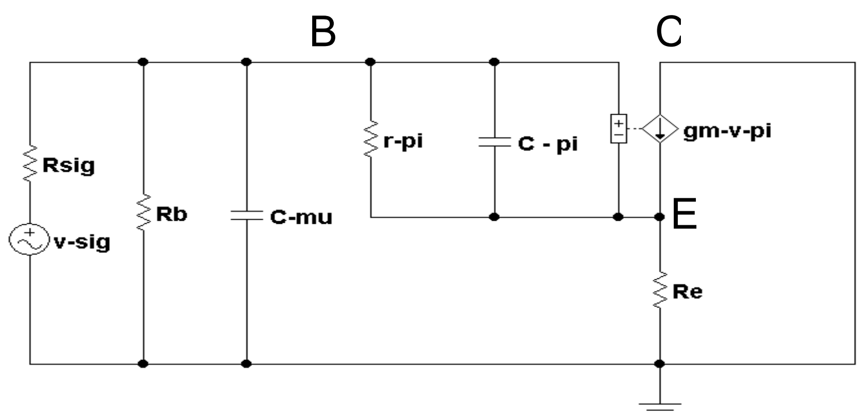
Common base and common collector amplifiers do not suffer from the Miller effect, since in these amplifiers, one side of C_μ is connected directly to ground.

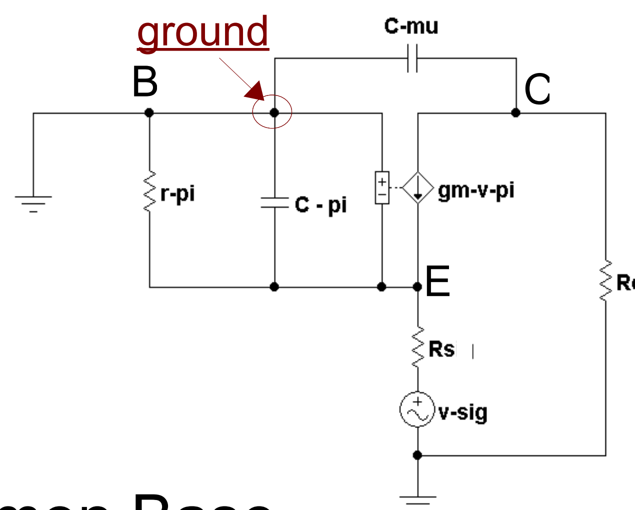
High Frequency CC and CB Models











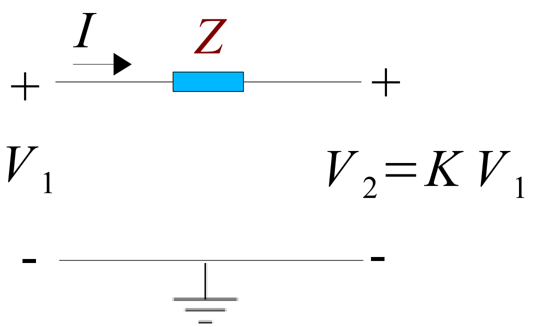
Common Collector

C_μ is in parallel with R_B.

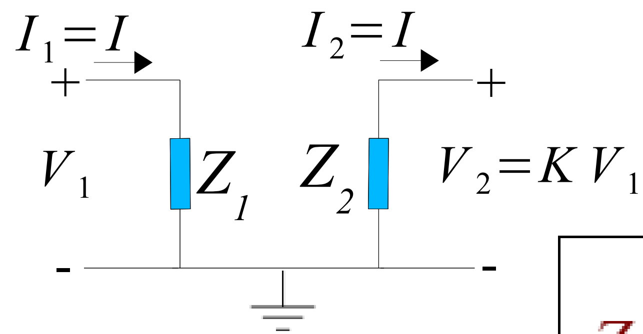
Common Base

C_μ is in parallel with R_C.

Miller's Theorem



<=>



$$I = \frac{V_1 - V_2}{Z} = \frac{V_1 - K V_1}{Z} = \frac{V_1}{\frac{Z}{1-K}}$$

=>

$$Z_1 = \frac{V_1}{I_1} = \frac{V_1}{I} = \frac{Z}{1-K}$$

$$Z = \frac{1}{j 2 \pi f C_\mu}$$

$$Z_1 \approx \frac{1}{j 2 \pi f C_\mu (1-K)}$$

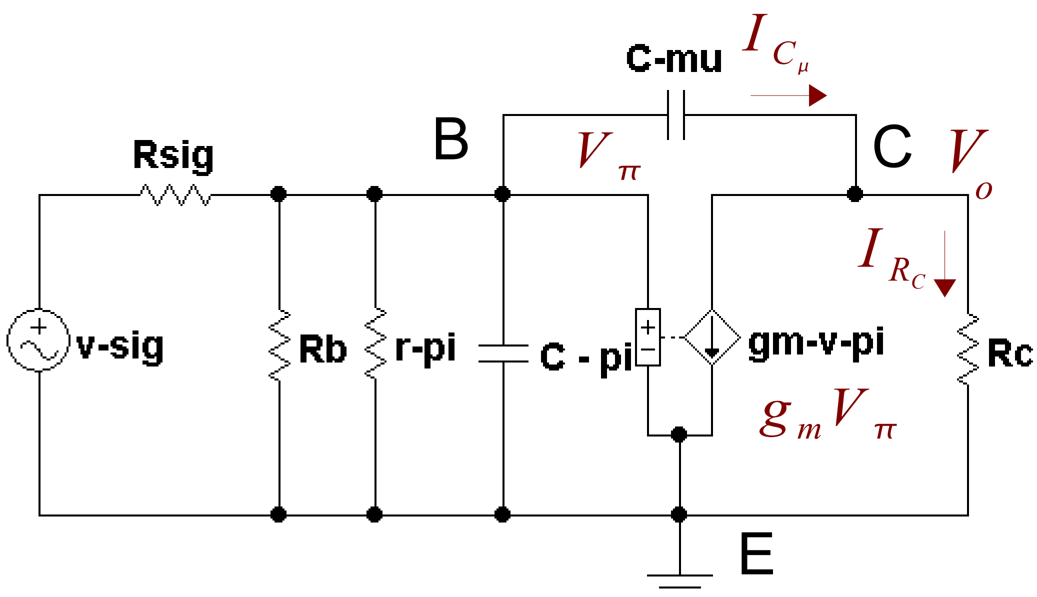
=>

$$Z_2 = \frac{V_2}{-I_2} = \frac{V_2}{-I} = -\frac{Z}{\frac{1}{K} - 1} = \frac{Z}{1 - \frac{1}{K}} \approx Z$$

if $K >> 1$

Let's examine Miller's Theorem as it applies to the HF model for the BJT CE amplifier.

Common Emitter Miller Effect Analysis



Determine effect of C_μ:

Using phasor notation:

$$I_{R_C} = -g_m V_\pi + I_{C_\mu}$$

or

$$V_o = \left(-g_m V_\pi + I_{C_\mu}\right) R_C$$

where

$$I_{C_\mu} = \left(V_\pi - V_o\right) s C_\mu$$

Note: The current through C_μ depends only on V_π!

$$I_{C_\mu} = \left(V_\pi + \underbrace{g_m V_\pi R_C - I_{C_\mu} R_C}_{V_o}\right) s C_\mu$$

Common Emitter Miller Effect Analysis II

From slide 7:

$$I_{C_\mu} = \left(V_\pi + g_m V_\pi R_C - I_{C_\mu} R_C \right) s C_\mu$$

Collect terms for I_{C_μ} and V_π:

$$\left(1 + s R_C C_\mu\right) I_{C_\mu} = \left(1 + g_m R_C\right) s C_\mu V_\pi$$

$$I_{C_\mu} = \frac{\left(1 + g_m R_C\right) s C_\mu}{\left(1 + s R_C C_\mu\right)} V_\pi = \frac{s \left(1 + g_m R_C\right) C_\mu}{\left(1 + s R_C C_\mu\right)} V_\pi$$

Miller Capacitance C_{eq}: $C_{eq} = (1 - K) C_\mu = (1 + g_m R_C)$

Common Emitter Miller Effect Analysis III

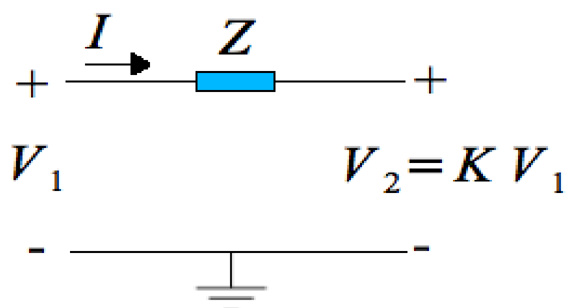
$$C_{eq} = \left(1 + g_m R_C\right) C_\mu$$

For our example circuit:

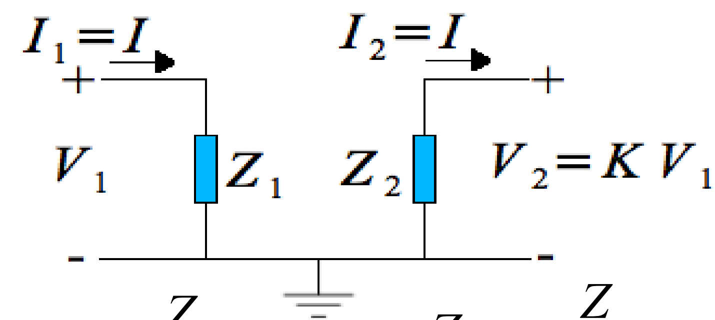
$$1 + g_m R_C = 1 + 0.040 \cdot 5100 = 205$$

$$\boxed{C_{eq} = (205) \cdot 2\ pF \approx 410\ pF}$$

Apply Miller's Theorem to BJT CE Amplifier



<=>
$K > 1$



$$Z_1 = \frac{Z}{1-K} \qquad Z_2 = \frac{Z}{1-\frac{1}{K}}$$

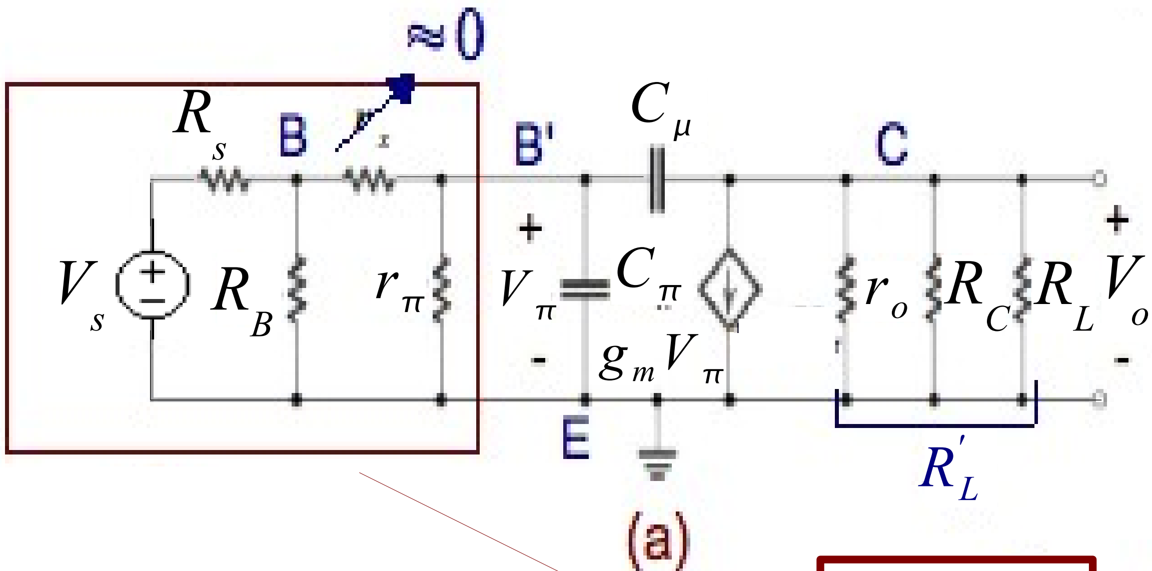
For the BJT CE Amplifier: $Z = \frac{1}{j\omega C_\mu}$ and $K = -g_m R_C$

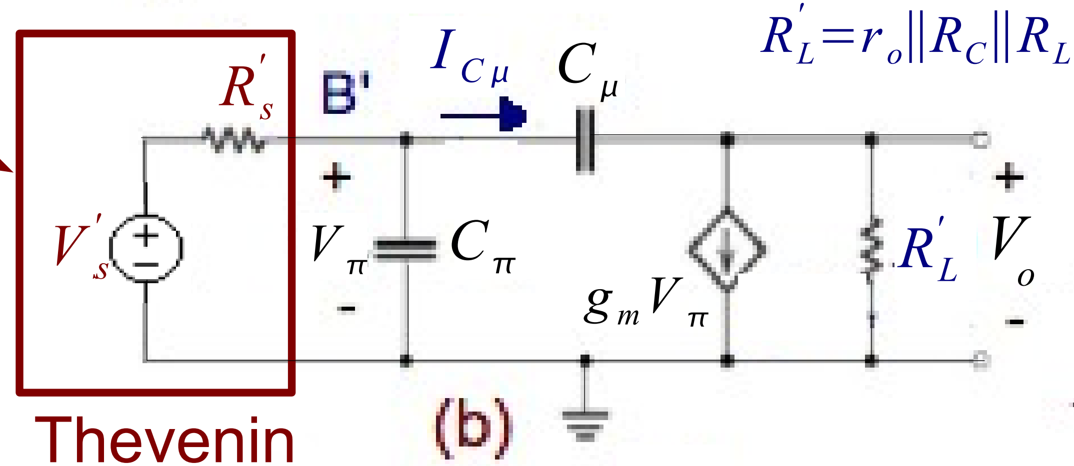
=> $Z_1 = \frac{1}{j\omega(1+g_m R_C)C_\mu}$ and $Z_2 = \frac{1}{j\omega\left(1+\frac{1}{g_m R_C}\right)C_\mu} \approx \frac{1}{j\omega C_\mu}$

$$C_{eq} = (1+g_m R_C)C_\mu$$

Miller's Theorem => important simplification to the HF BJT CE Model

Simplified HF Model



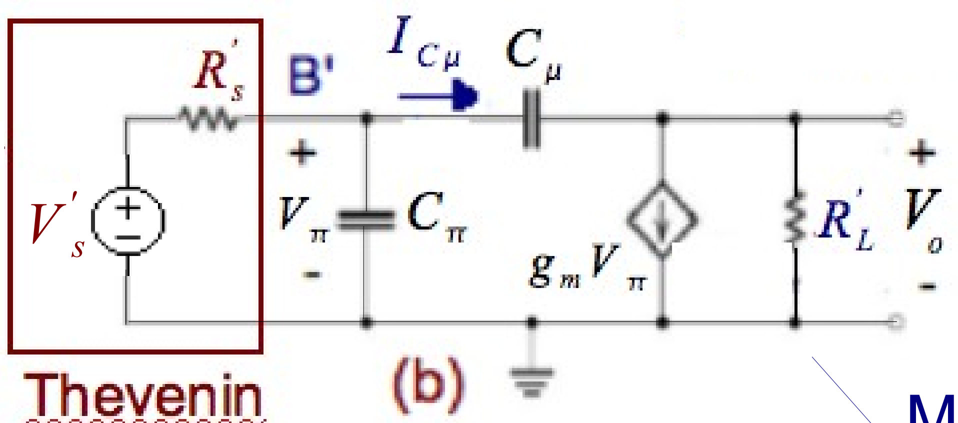


$$R_L^{'}=r_o\|R_C\|R_L$$

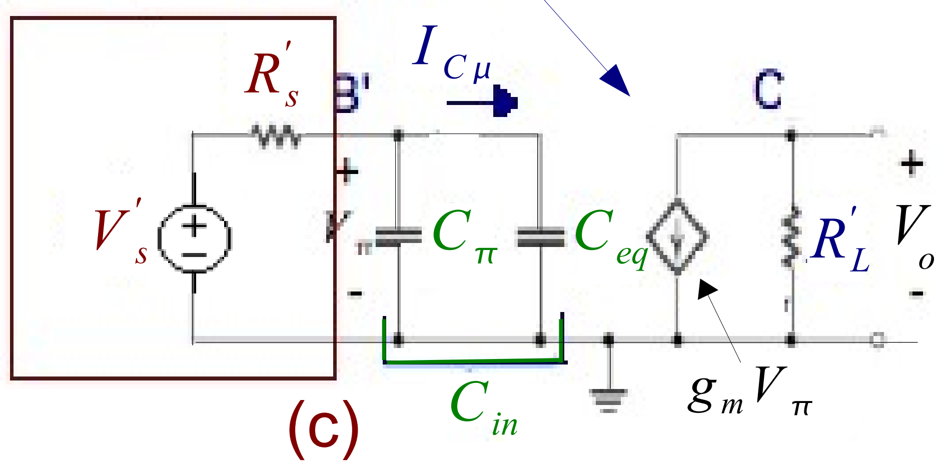
$$V_s^{'}=V_s\frac{R_B\|r_\pi}{R_B\|r_\pi+R_{sig}}$$

$$R_s^{'}=r_\pi\|(R_B\|R_s)$$

Simplified HF Model



Miller's Theorem

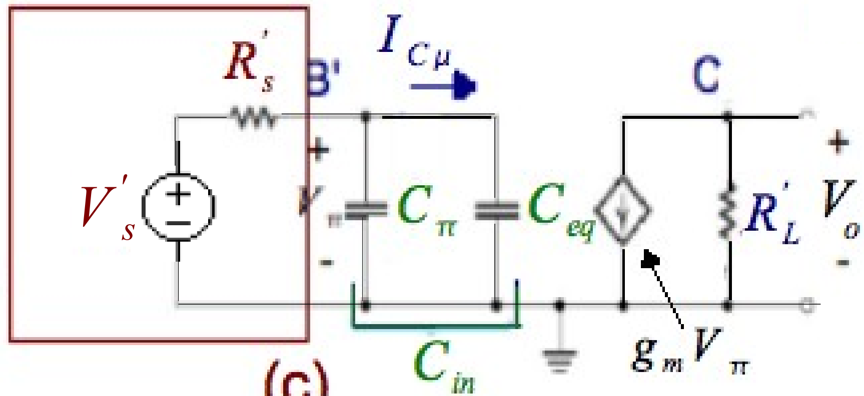


$$C_{in} = C_\pi + C_{eq}$$

$$= C_\pi + C_\mu (1 + g_m R_L')$$

Simplified HF Model

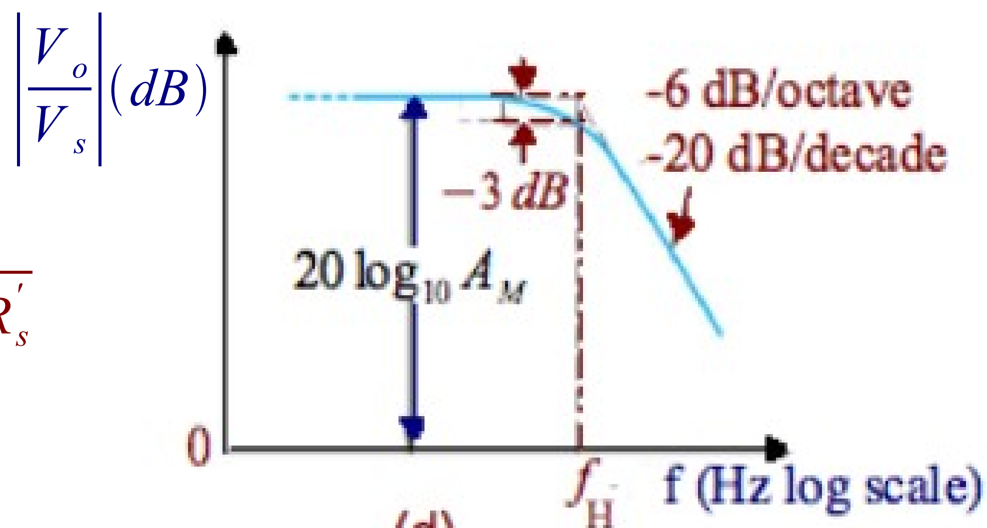
$$C_{in} = C_{\pi} + C_{eq}$$
$$= C_{\pi} + C_{\mu}(1 + g_m R_L')$$



(c)

$$A_v(f) = \frac{V_o}{V_s} \approx \frac{-g_m R_L'}{1 + j2\pi f C_{in} R_s'}$$

$$f_H = \frac{1}{2\pi C_{in} R_s'}$$



(d)

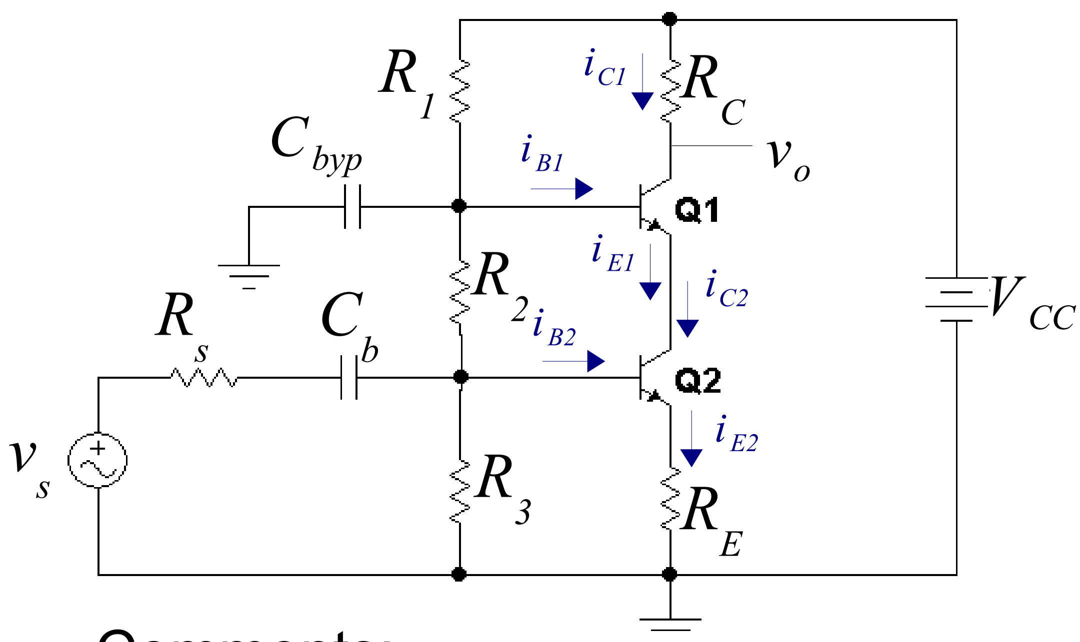
The Cascode Amplifier

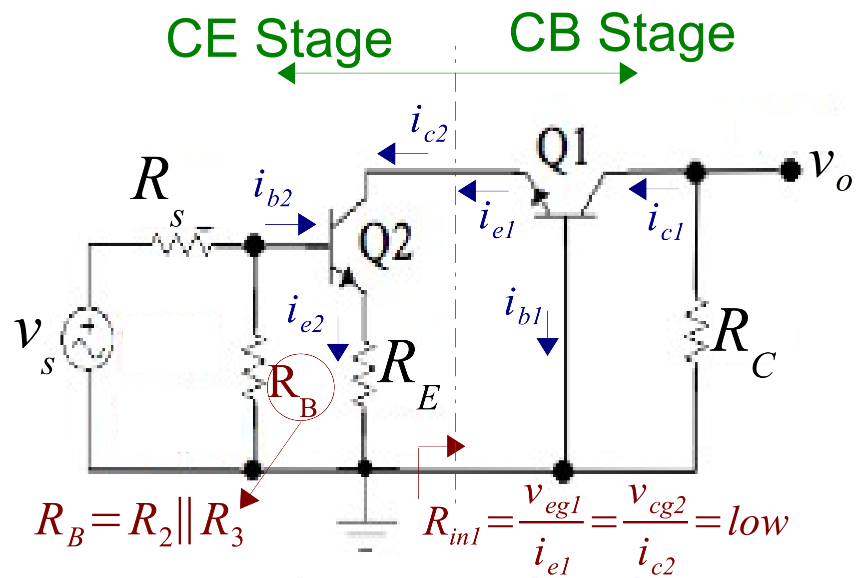
A two transistor amplifier used to obtain simultaneously:

1. Reasonably high input impedance.
2. Reasonable voltage gain.
3. Wide bandwidth.

None of the conventional single transistor designs will meet all of the criteria above. The cascode amplifier will meet all of these criteria. a cascode is a combination of a common emitter stage cascaded with a common base stage. (In “olden days” the cascode amplifier was a cascade of *grounded cathode* and *grounded grid* vacuum tube stages – hence the name “cascode,” which has persisted in modern terminology.

The Cascode Circuit



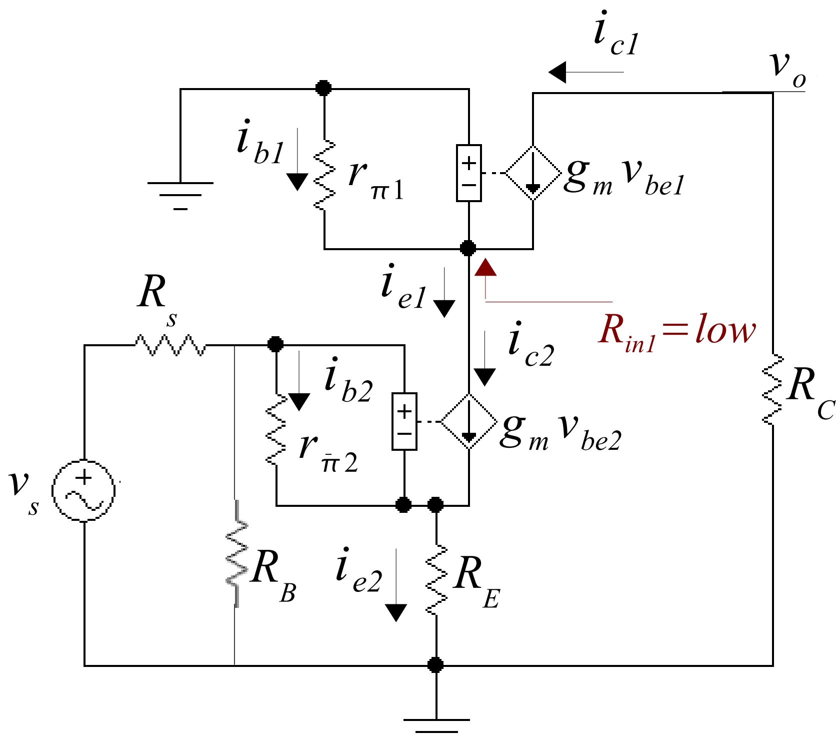


ac equivalent circuit

Comments:

1. R_1, R_2, R_3, and R_C set the bias levels for both Q1 and Q2.
2. Determine R_E for the desired voltage gain.
3. C_b and C_{byp} are to act as “open circuits” at dc and act as “short circuits” at all operating frequencies of interest, i.e. $f > f_{min}$.

Cascode Mid-Band Small Signal Model



$g_{m1} = g_{m2} = g_m$

$r_{e1} = r_{e2} = r_e$

$r_{\pi 1} = r_{\pi 2} = r_\pi$

1. Show reduction in Miller effect
2. Evaluate small-signal voltage gain

OBSERVATIONS

a. The emitter current of the CB stage is the collector current of the CE stage. (This also holds for the dc bias current.)

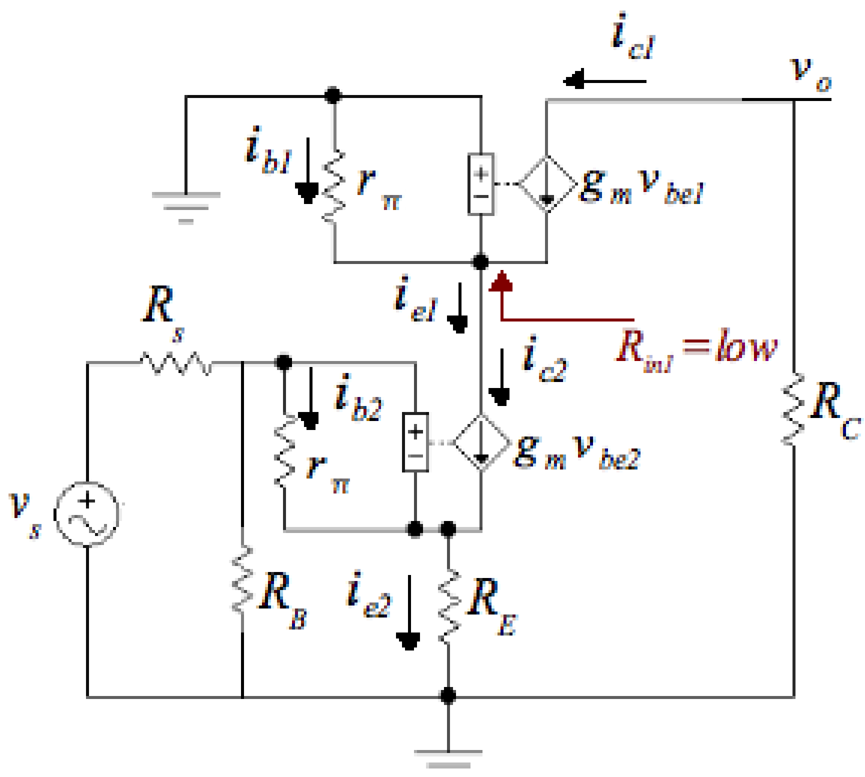
$$i_{e1} = i_{c2}$$

b. The base current of the CB stage is:

$$i_{b1} = \frac{i_{e1}}{\beta + 1} = \frac{i_{c2}}{\beta + 1}$$

c. Hence, both stages have about same collector current $i_{c1} \approx i_{c2}$ and same g_m, r_e, r_π.

Cascode Small Signal Analysis cont.



The input resistance R_{in1} to the CB stage is the small-signal "R_C" for the CE stage

$$i_{b1}=\frac{i_{e1}}{\beta+1}=\frac{i_{c2}}{\beta+1}$$

The CE output voltage, the voltage drop from Q2 collector to ground, is:

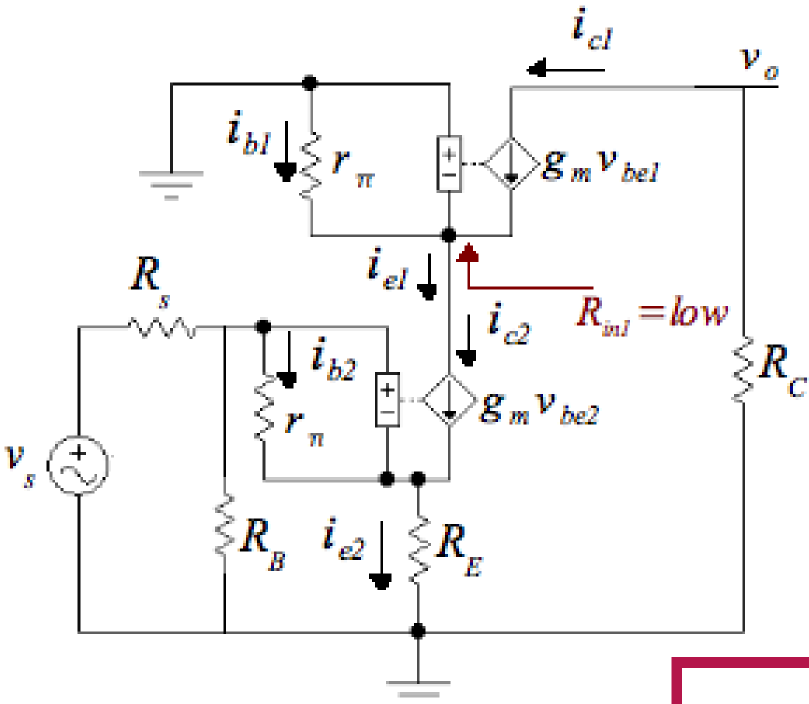
$$v_{cg2}=v_{eg1}=-r_{\pi 1}i_{b1}=-\frac{r_{\pi 1}}{\beta+1}i_{c2}=-\frac{r_{\pi 1}}{\beta+1}i_{e1}$$

Therefore, the CB Stage input resistance is:

$$R_{in1}=\frac{v_{eg1}}{-i_{e1}}=\frac{r_{\pi 1}}{\beta+1}=r_{e1}$$

$$A_{vCE-Stage}=\frac{v_{cg2}}{v_{sig}}\approx-\frac{R_{in1}}{R_E}=-\frac{r_e}{R_E}<1 \Rightarrow C_{eq}=(1+\frac{r_e}{R_E})C_\mu<2C_\mu$$

Cascode Small Signal Analysis - cont.



Now, find the CE collector current in terms of the input voltage v_s: Recall $i_{c1} \approx i_{c2}$

$$i_{b2} \approx \frac{v_s}{R_s \| R_B + r_{\pi 2} + (\beta+1) R_E}$$

$$i_{c2} = \beta i_{b2} \approx \frac{\beta v_s}{R_s \| R_B + r_{\pi 2} + (\beta+1) R_E} \approx \frac{\beta v_s}{(\beta+1) R_E}$$

for bias insensitivity: $(\beta+1) R_E \gg R_s \| R_B + r_\pi$

$$i_{c2} \approx \frac{v_s}{R_E}$$

$$v_o = -i_{c2} R_C$$

=> $$A_v = \frac{v_{out}}{v_{sig}} = -\frac{R_C}{R_E}$$

OBSERVATIONS:

1. Voltage gain A_v is about the same as a stand-along CE Amplifier.
2. HF cutoff is much higher then a CE Amplifier due to the reduced C_{eq}.

Approximate Cascode HF Voltage Gain

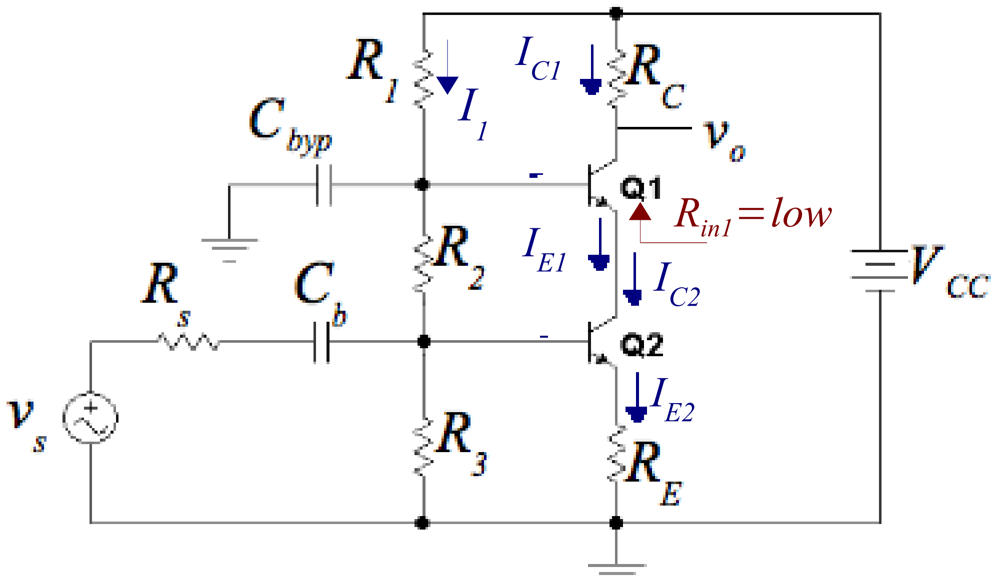
$$A_v(f) = \frac{V_o(f)}{V_s(f)} \approx \frac{-\frac{R_C}{R_E}}{1 + j2\pi f C_{in} R_s'}$$

where

$$C_{in} = C_\pi + C_{eq} = C_\pi + \left(1 + \frac{r_e}{R_E}\right) C_\mu < C_\pi + 2C_\mu$$

$$R_s' = R_s \| R_B \approx R_s$$

Cascode Biasing



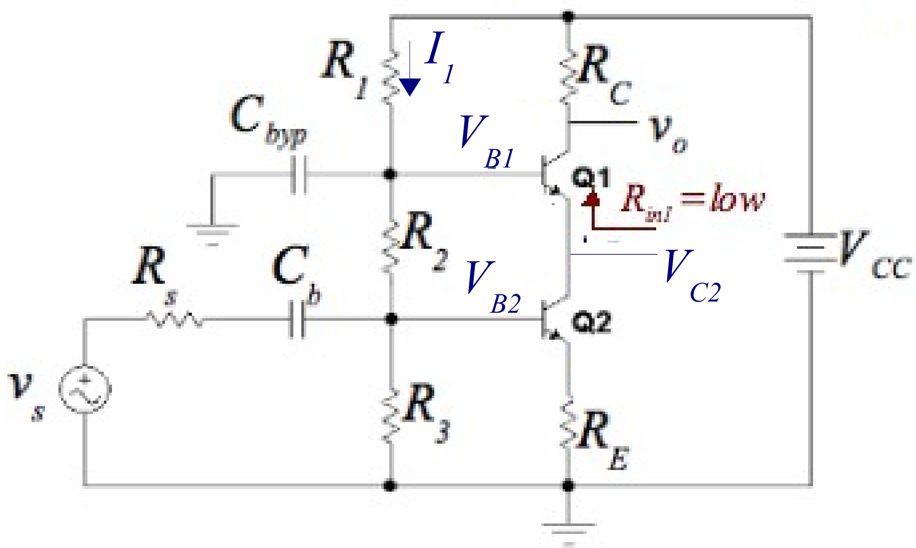
1. Choose I_{E1} – make it relatively large to reduce $R_{in1}=r_{e1}=V_T/I_{E1}$ to push out HF break frequencies.

2. Choose R_C for suitable voltage swing V_{C1G} and R_E for desired gain.

3. Choose bias resistor string such that its current I_1 is about 0.1 of the collector current I_{C1}.

4. Given R_E, I_{E2} *and* $V_{BE2} = 0.7\ V$ calculate R_3.

Cascode Biasing - cont.



$$V_{CE2} = V_{C2} - V_{Re} = V_{C2} - (V_{B2} - 0.7\,V)$$
$$= V_{B1} - V_{BE1} - V_{B2} + V_{BE2}$$
$$\approx V_{B1} - 0.7\,V - V_{B2} + 0.7\,V$$
$$= V_{B1} - V_{B2}$$

Since the CE-Stage gain is very small:

a. The collector swing of Q2 will be small.

b. The Q2 collector bias $V_{C2} = V_{B1} - 0.7\ V$.

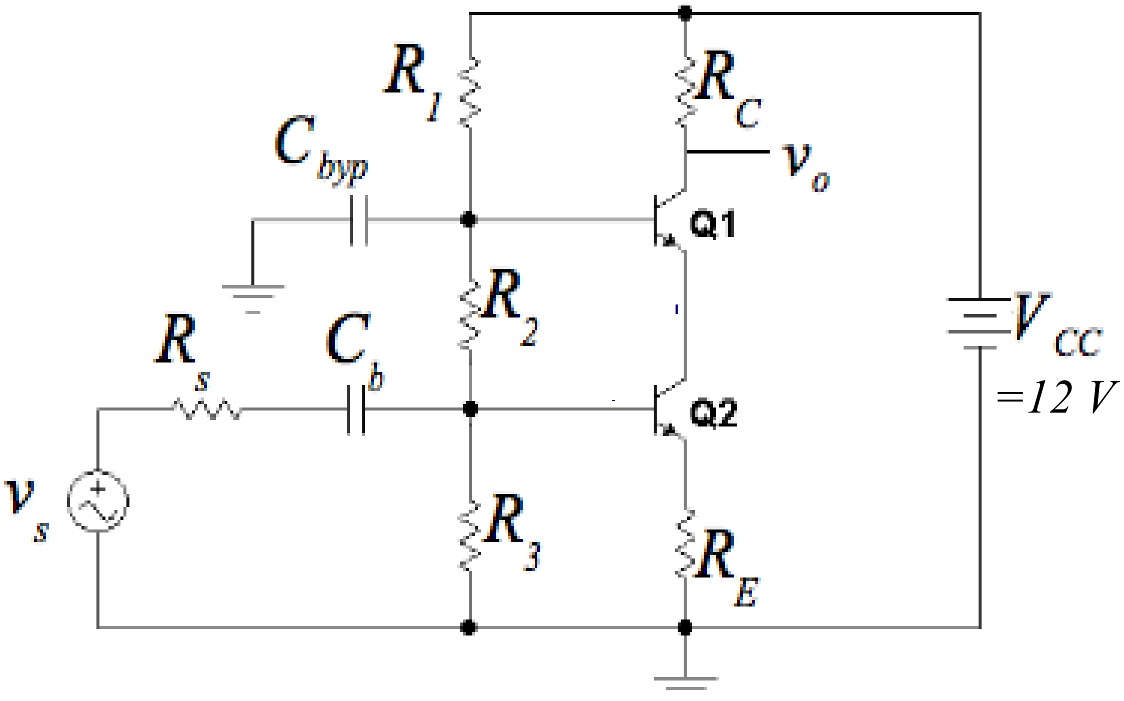
5. Set $V_{B1} - V_{B2} \approx 1\,V \Rightarrow V_{CE2} \approx 1\,V$

This will limit V_{CB2} $V_{CB2} = V_{CE2} - V_{BE2} = 0.3\,V$ which will keep Q2 forward active.

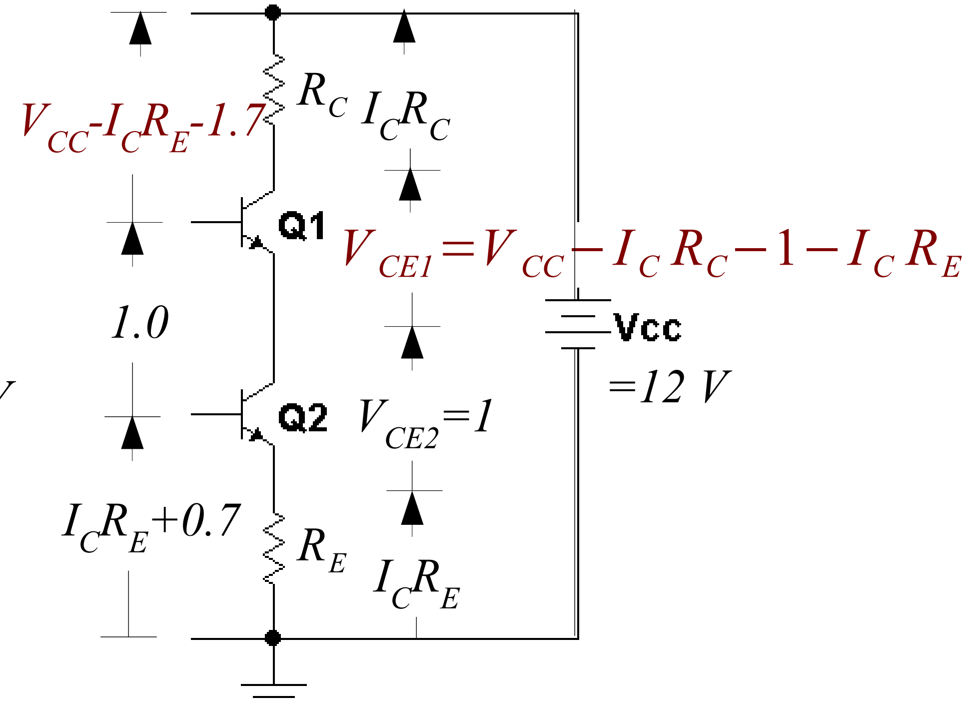
6. Next determine R_2. Its drop $V_{R2} = 1\ V$ with the known current. $R_2 = \frac{V_{B1} - V_{B2}}{I_1}$

7. Then calculate R_1. $R_1 = \frac{V_{CC} - V_{B1}}{I_1}$

Cascode Bias Example



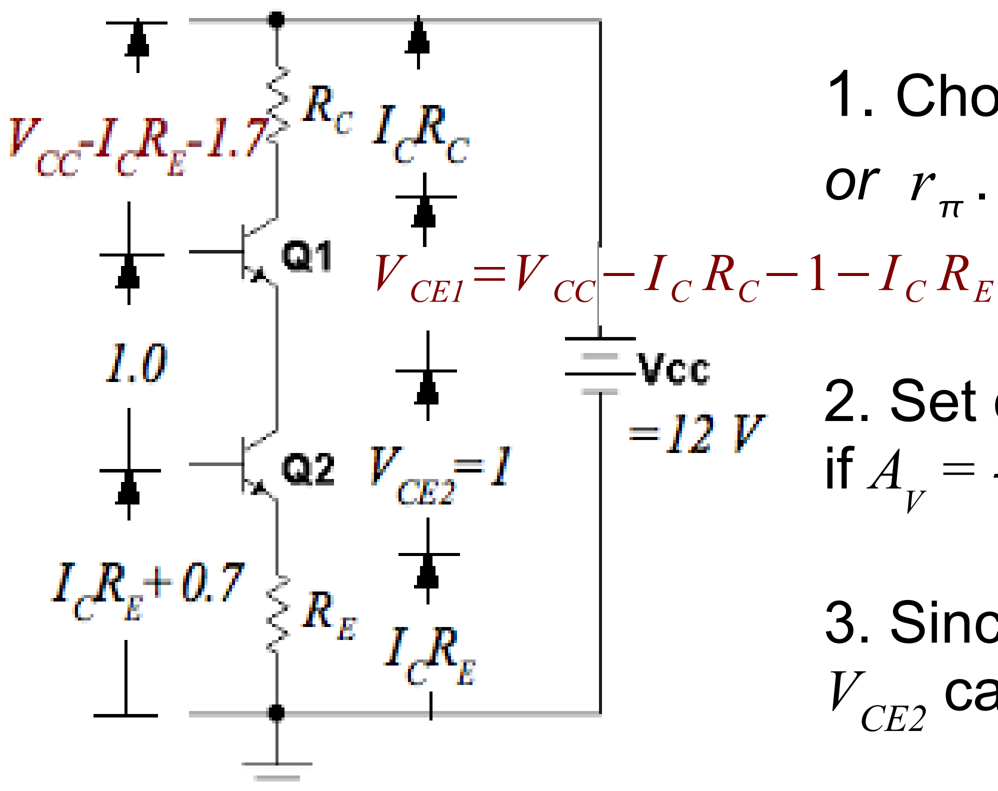
Cascode circuit



$$I_{E2}\approx I_{C2}=I_{E1}\approx I_{C1}\Rightarrow I_{C1}\approx I_{E2}$$

Typical Bias Conditions

Cascode Bias Example cont.

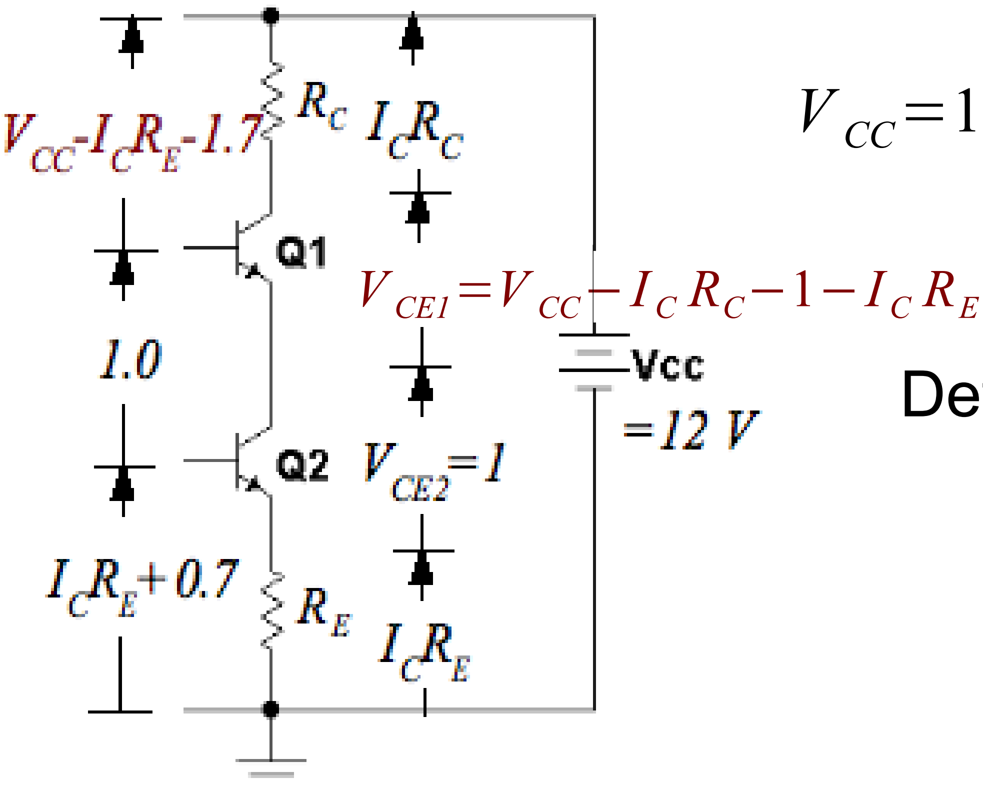


1. Choose I_{E1} – make it a bit high to lower r_e or r_π. Try $I_{E1} = 5\ mA$ => $r_e = 0.025\,V / I_E = 5\,\Omega$.

2. Set desired gain magnitude. For example if $A_V = -10$, then $R_C/R_E = 10$.

3. Since the CE stage gain is very small, V_{CE2} can be small. Use $V_{CE2} = V_{B1} - V_{B2} = 1\ V$.

Cascode Bias Example cont.



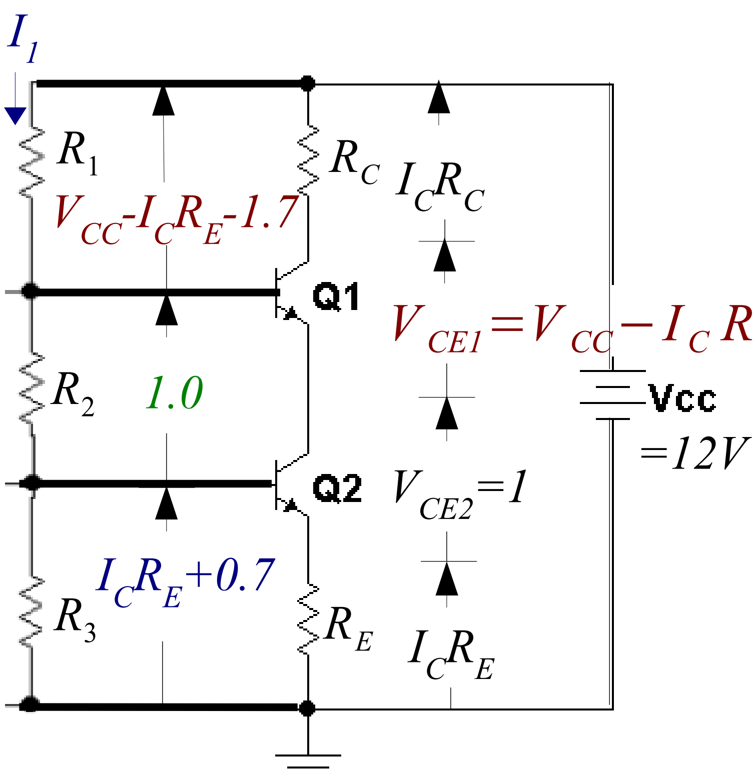
$$V_{CC}=12 \qquad I_C=5\,mA. \qquad |A_v|=\frac{R_C}{R_E}=10$$

Determine R_C for a $5\ V$ drop across R_C.

$$R_C=\frac{5\,V}{5\cdot 10^{-3}\,A}=1000\,\Omega$$

$$R_E=\frac{R_C}{|A_v|}=\frac{R_C}{10}=100\,\Omega$$

Cascode Bias Example cont.



$$V_{CC}=12 \quad R_C=1\,k\,\Omega \quad I_C=5\,mA. \quad R_E=100\,\Omega$$

Make current through the string of bias resistors $I_1 = 1\ mA$.

$$R_1+R_2+R_3=\frac{V_{CC}}{I_1}=\frac{12}{1\cdot 10^{-3}}=12\,k\,\Omega$$

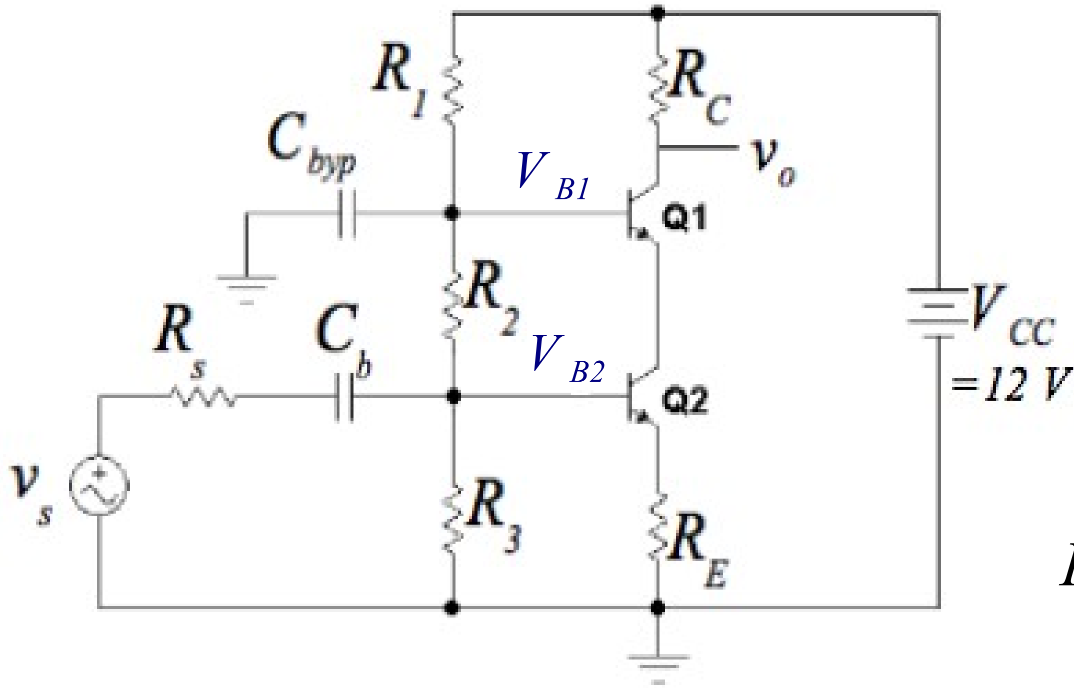
We now calculate the bias voltages:

$$V_{CC}-I_C R_E-1.7\,V=12\,V-0.5\,V-1.7\,V=9.8\,V$$

$$V_{B1B2}=V_{B1}-V_{B2}=1.0\,V$$

$$V_{B2}=I_C R_E+0.7=5\cdot 10^{-3}\cdot 100+0.7=1.2\,V$$

Cascode Bias Example cont.



$$V_{B2}=10^{-3}R_3=1.2\,V$$

$$R_3=1.2\,k\,\Omega$$

$$V_{B1}-V_{B2}=1\cdot10^{-3}R_2=1.0\,V$$

$$R_2=1\,k\,\Omega$$

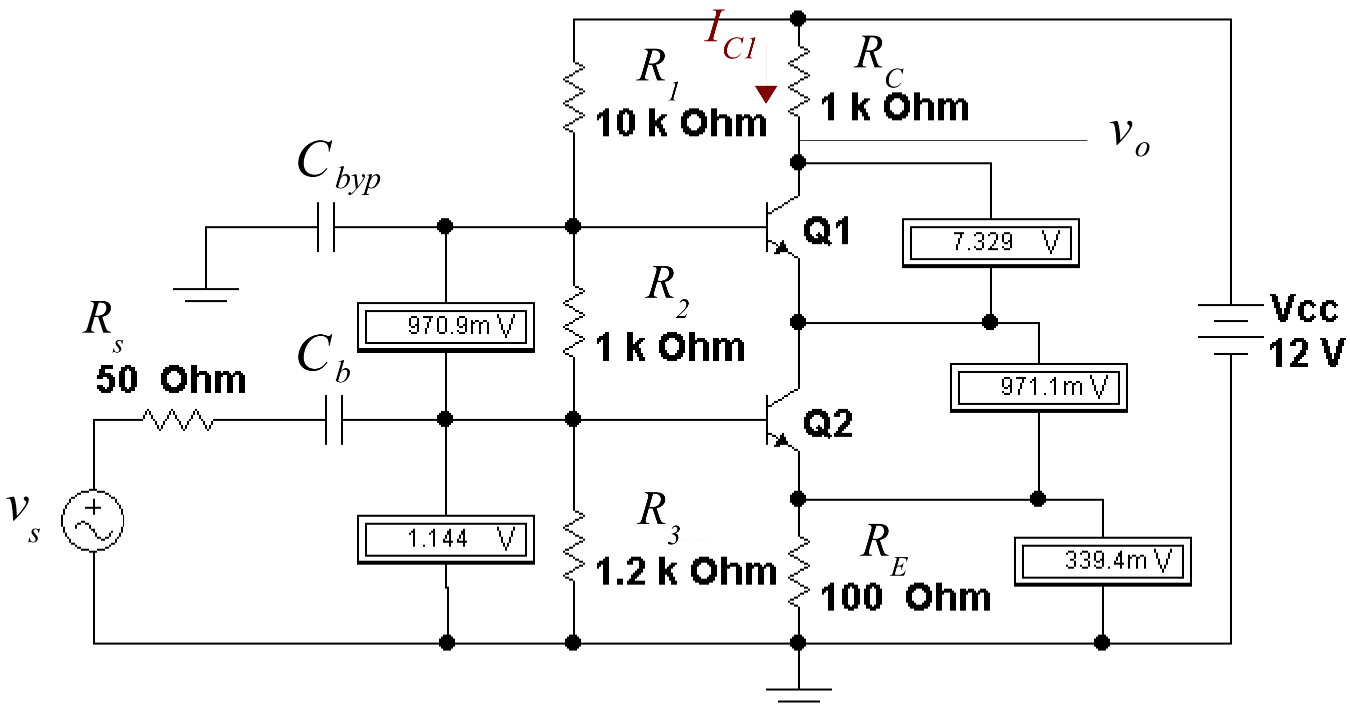
$$R_1=12000-1200-1000=9.8\,k\,\Omega$$

$$R_1=10\,k\,\Omega$$

$V_{CC}=12$ $R_C=1\,k\,\Omega$ $V_{B2}=1.2\,V$

$I_C=5\,mA.$ $R_E=100\,\Omega$ $V_{B1}-V_{B2}=1.0\,V$

Multisim Results – Bias Example

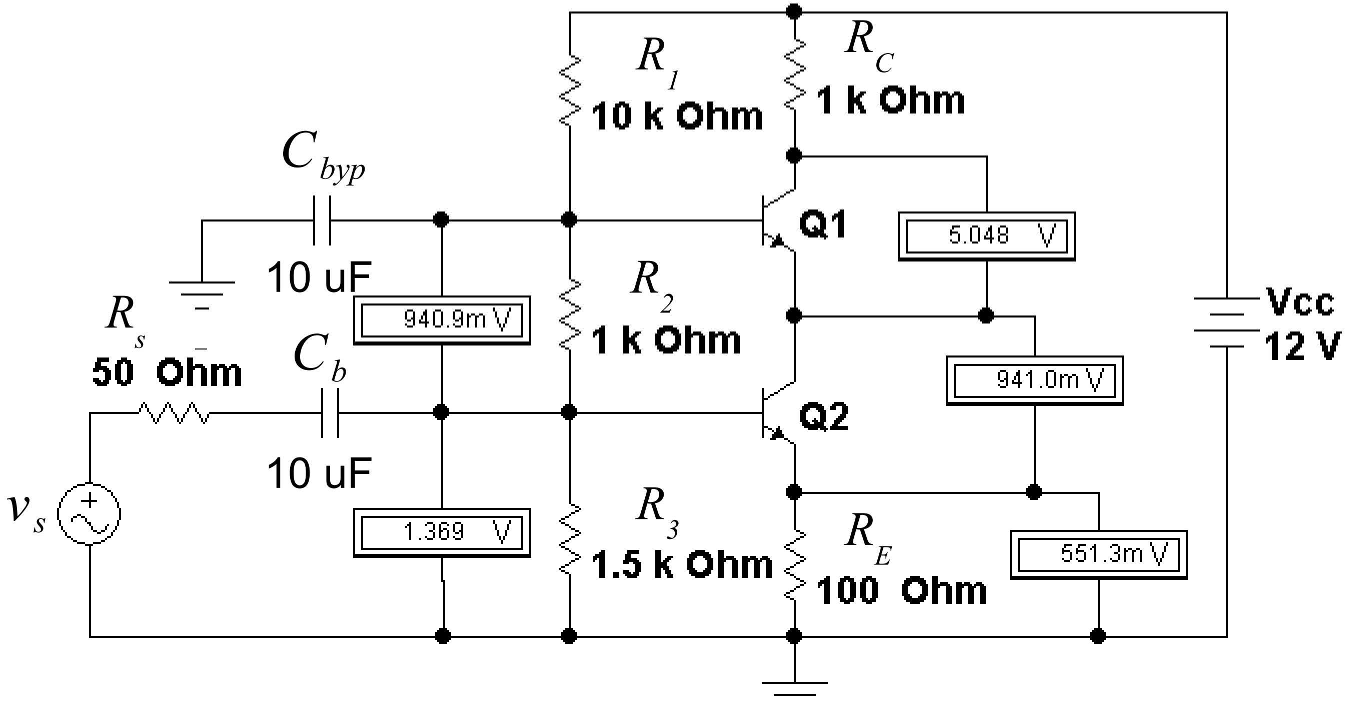


Check I_{C1}: $I_{C1}=\frac{V_{CC}-(7.329+0.971+0.339)}{R_C}=\frac{12\,V-8.639\,V}{1000\,\Omega}=3.36\,mA$

I_{C1} about $3.4\ mA$. That's a little low.

Increase R_3 to $1.5\ k\ Ohms$ and re-simulate.

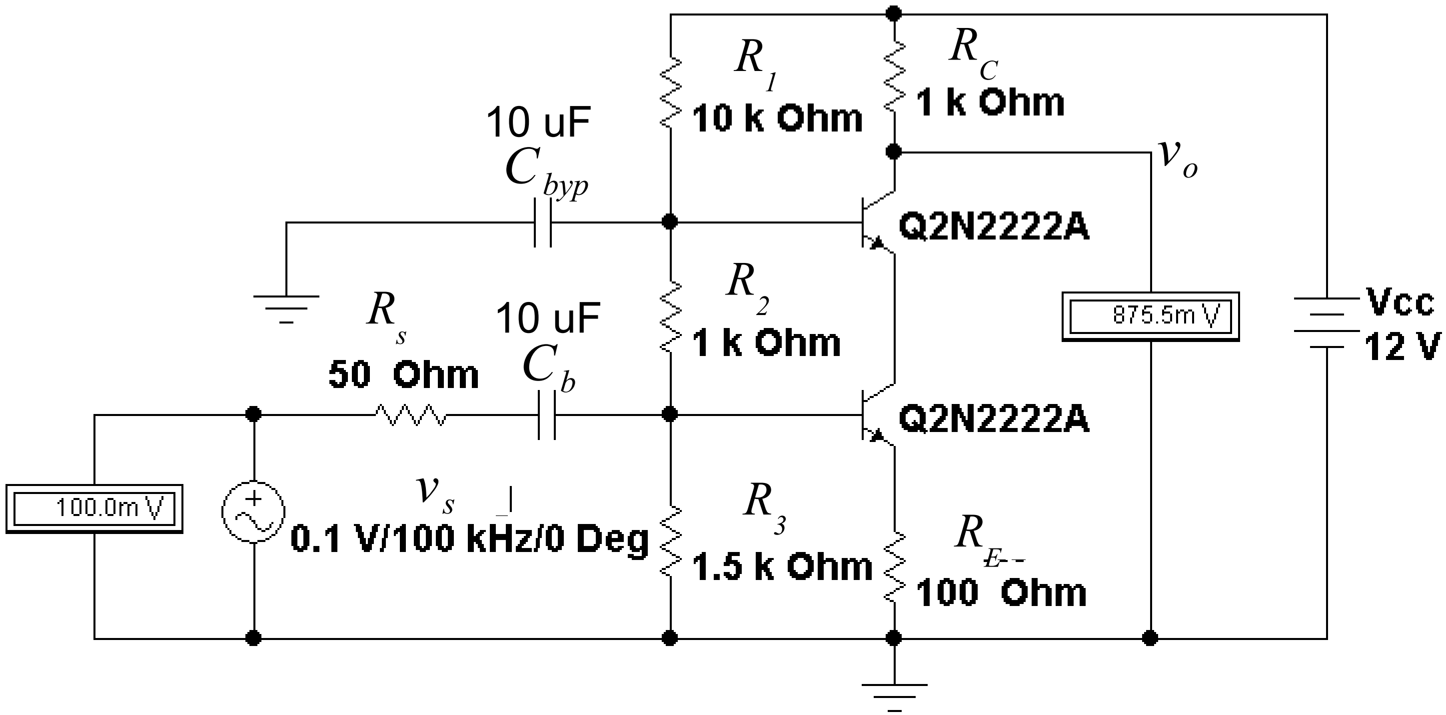
Improved Biasing



Check I_{C1}: $I_{C1}=\frac{V_{CC}-(5.048+0.941+0.551)}{R_C}=\frac{12\,V-6.540\,V}{1000\,\Omega}=5.46\,mA$

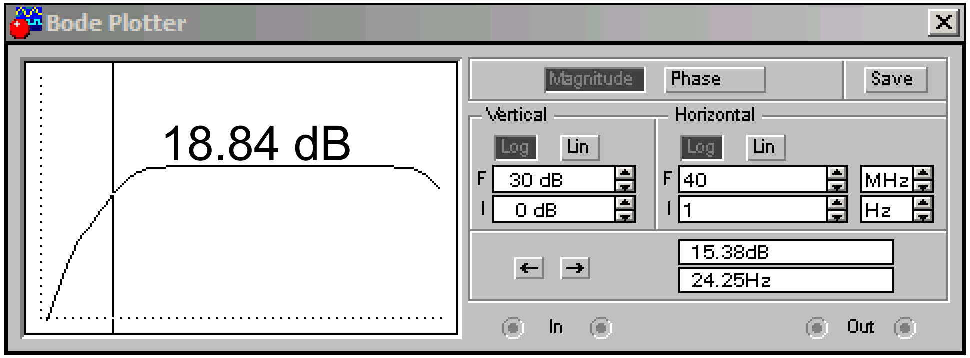
That's better! Now measure the gain at a mid-band frequency with some “large” coupling capacitors, say $10\,\mu F$ inserted.

Single Frequency Gain

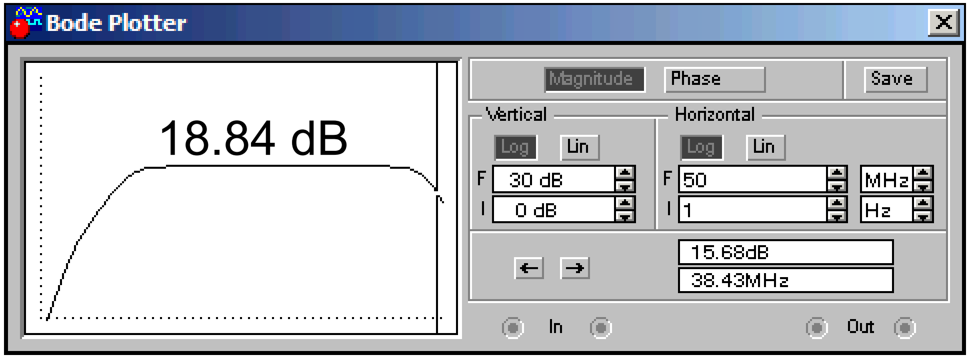


Gain $|A_v|$ of about 8.75 at $100\ kHz$ - OK for rough calculations. Some attenuation from low CB input impedance ($R_B = R_2||R_3$)and some from $5\Omega\ r_e$.

Bode Plot for the Amplifier

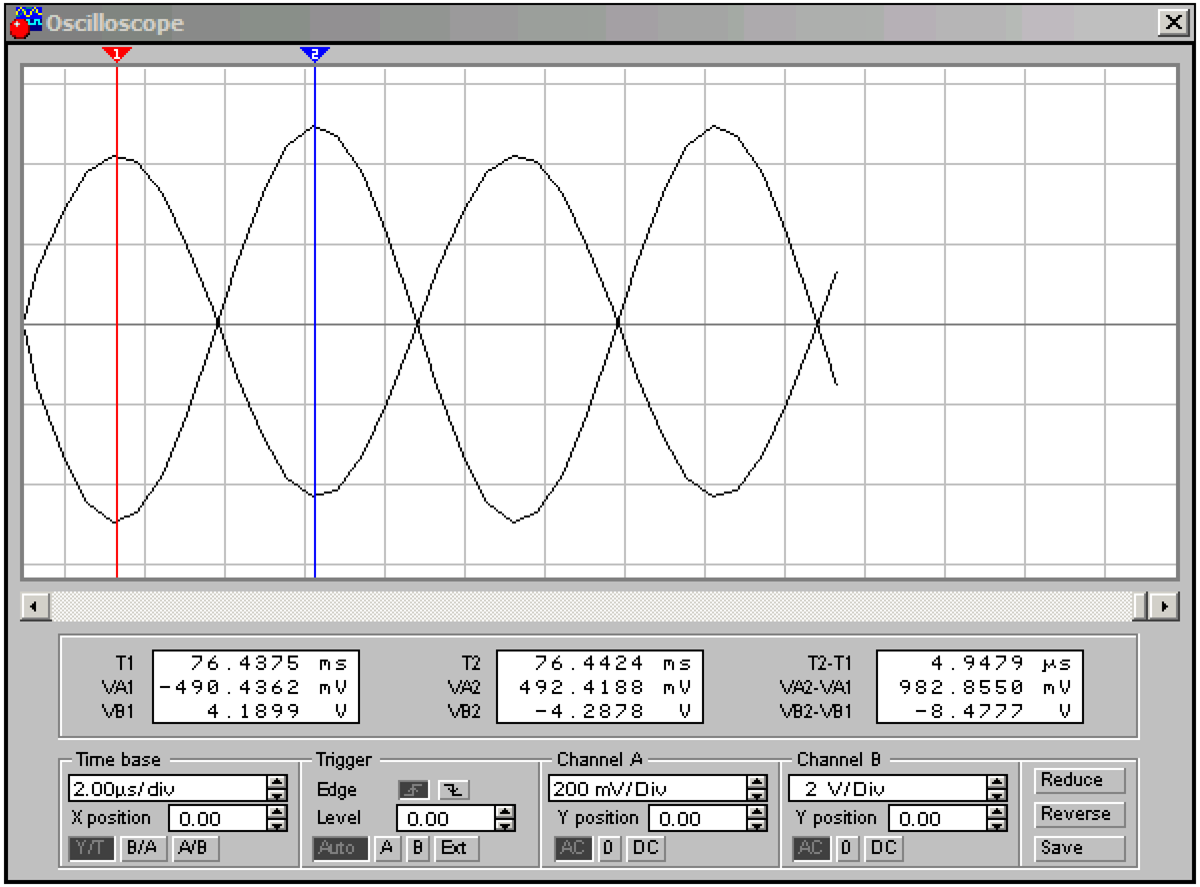


Low frequency break point with 10 µF. capacitors

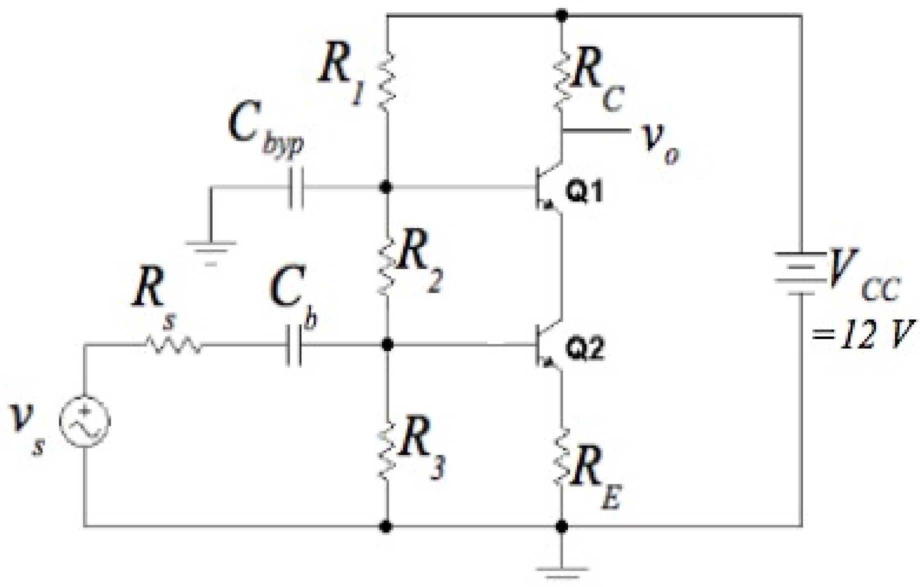


High frequency break point – internal capacitances only

Scope Plot – Near 5 V Swing on Output



Determine Bypass Capacitors



Low Frequency $f \leq f_{min}$

From CE stage determine C_b

$$C_b \geq \frac{10}{2\pi f_{min} R_B \| r_{bg}} \approx \frac{10}{2\pi f_{min} R_B \| (\beta+1) R_E} F$$

$$R_B = R_2 \| R_3$$

From CB stage determine C_{byp}

$$C_{byp} \geq \frac{10}{2\pi f_{min} \left((\beta+1) R_E + r_\pi \right)} F$$

where R_S -> R_E