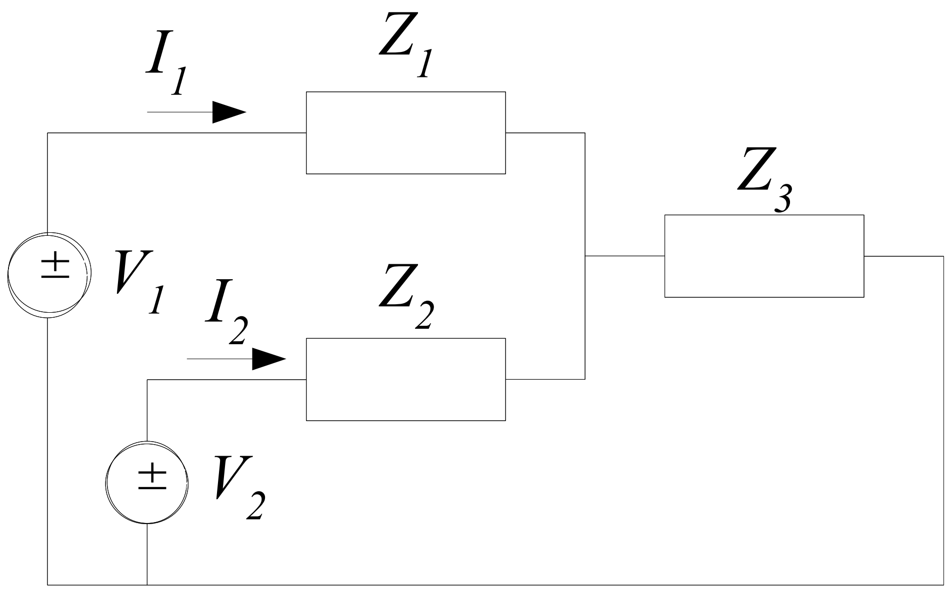
Differential Amplifier
Common & Differential Modes

- Common & Differential Modes
- BJT Differential Amplifier
- Diff. Amp Voltage Gain and Input Impedance
- Small Signal Analysis – Differential Mode
- Small Signal Analysis – Common Mode

Common and Differential Modes

Consider a circuit with the *two-port* configuration shown



Define (convention) the voltages:

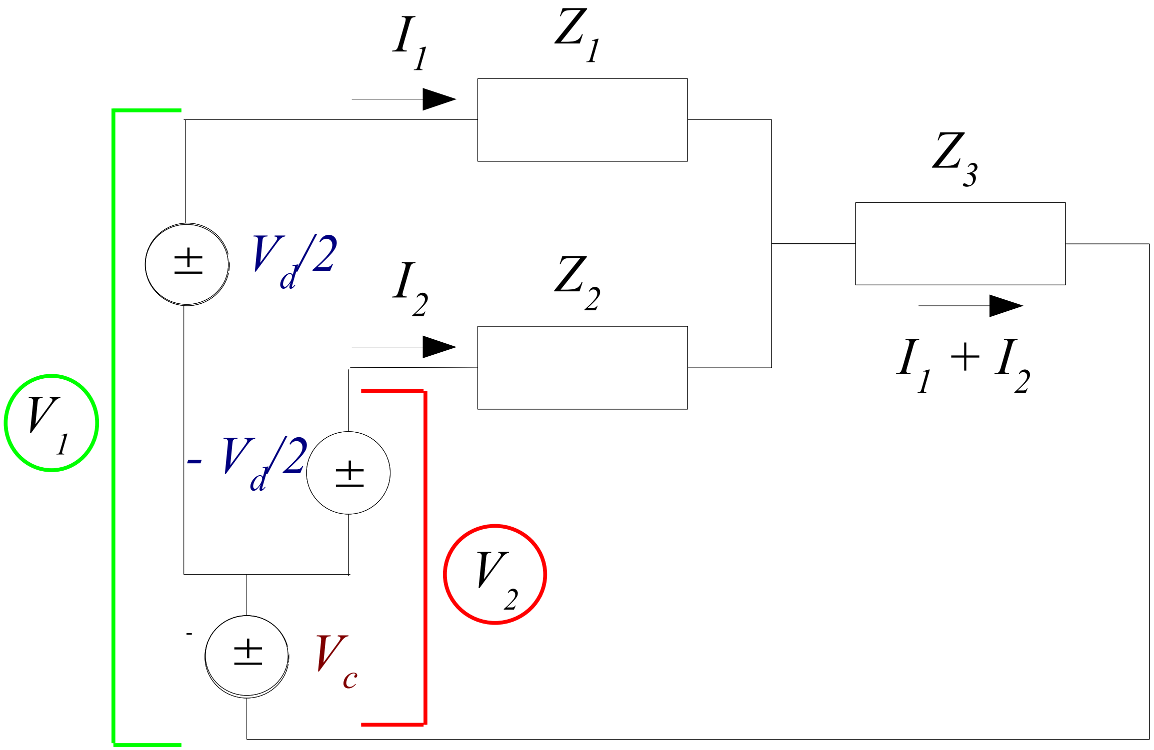
$$V_c = \frac{V_1 + V_2}{2} \qquad V_d = V_1 - V_2$$

V_c = *"common mode voltage"*

V_d = *"differential mode voltage"*

1. Let $V_1 = V_2 = V \Rightarrow V_c = V$ & $V_d = 0$
2. Let $V_1 = -V_2 = V \Rightarrow V_c = 0$ & $V_d = 2V$

Replace V_1, V_2 with CM and DM Sources



Simultaneous Equations:

$$V_1 + V_2 = 2V_c$$
$$V_1 - V_2 = V_d$$

Adding: $2V_1 = 2V_c + V_d$

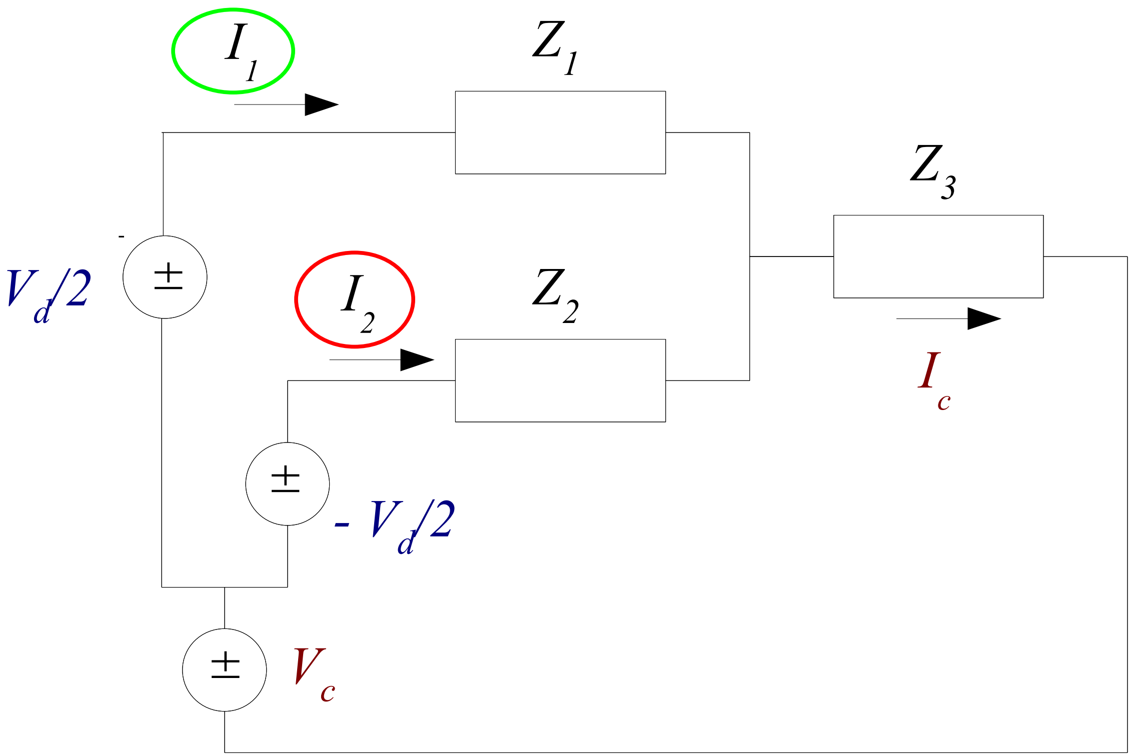
$$V_1 = V_c + V_d/2$$

Subtracting: $2V_2 = 2V_c - V_d$

$$V_2 = V_c - V_d/2$$

Definition $V_c = \frac{V_1 + V_2}{2}$ $\quad V_d = V_1 - V_2$

Common and Differential Mode Currents



Simultaneous Equations:

$$I_1 + I_2 = I_c$$

$$I_1 - I_2 = 2\,I_d$$

Adding:

$$2\,I_1 = I_c + 2\,I_d$$

$$I_1 = I_c/2 + I_d$$

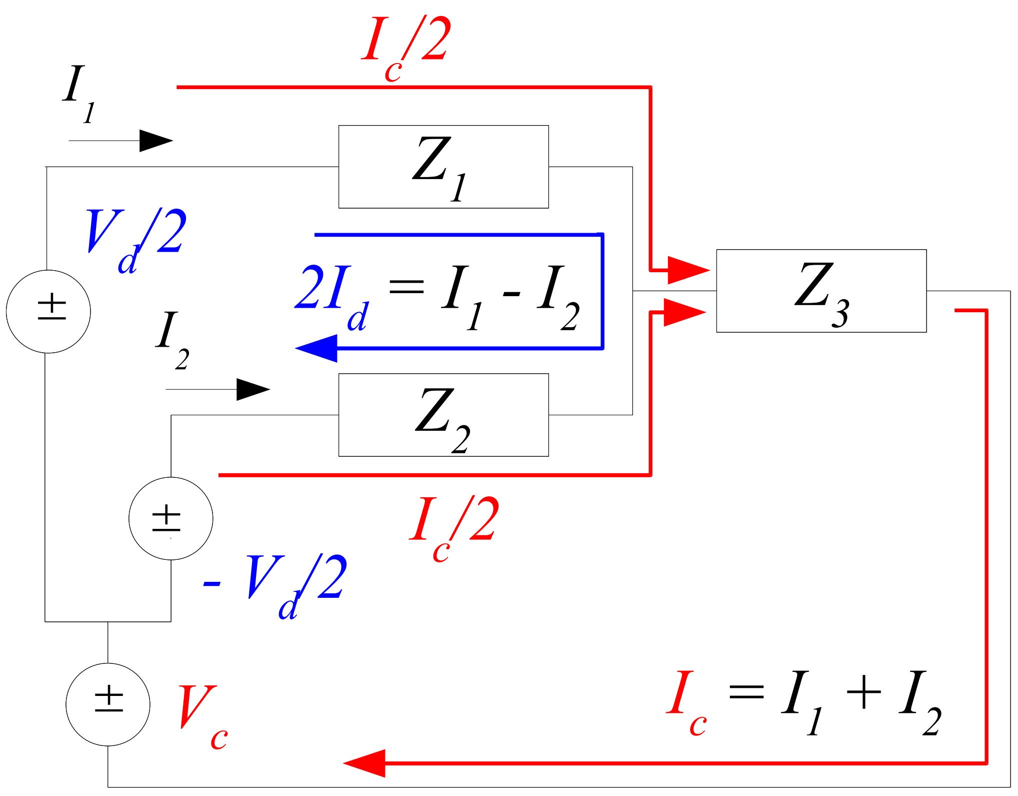
Subtracting:

$$2\,I_2 = I_c - 2\,I_d$$

$$I_2 = I_c/2 - I_d$$

Define (convention): $I_c = I_1 + I_2 \quad I_d = \dfrac{I_1 - I_2}{2}$

CM-DM Circuit Description



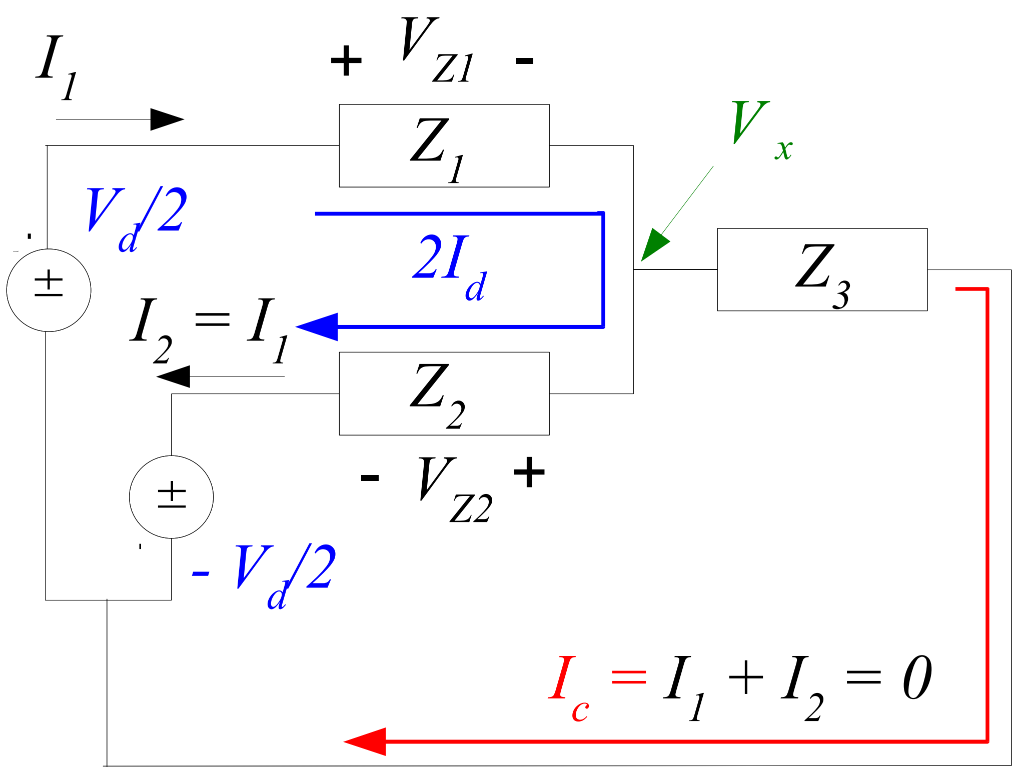
$$I_1 = I_c/2 + I_d \qquad I_2 = I_c/2 - I_d$$

1. Voltage sources redefined as common and differential mode quantities.

2. Differential mode currents take “blue” path.

3. Common mode current takes “red” path.

Analyze Circuits by Superposition Balanced Circuit Differential Mode ($V_C = 0$)



$$I_c = I_1 + I_2 = I_d - I_d = 0$$

$$I_c = I_1 + I_2 = \frac{\frac{V_d}{2} - V_x}{Z_1} + \frac{\frac{-V_d}{2} - V_x}{Z_2} = \frac{-2V_x}{Z} = 0$$

$$I_c = 0 \Rightarrow V_x = 0$$

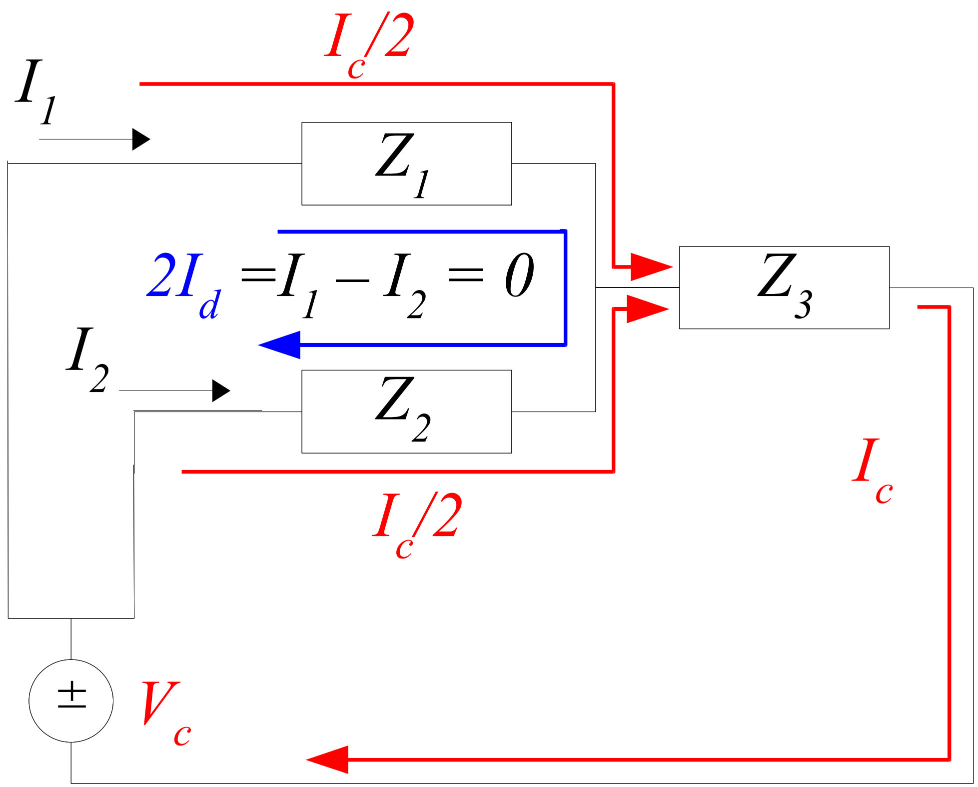
$$2I_d = \frac{\frac{V_d}{2} - (\frac{-V_d}{2})}{Z_1 + Z_2} = \frac{V_d}{2Z}$$

$$Z_{in-dm} = \frac{V_d}{2I_d} = 2Z$$

balanced circuit => $Z_1 = Z_2 = Z$

NOTE: If $Z_2 = Z_1 + \delta Z$, then $I_c \neq 0$ => $V_x \neq 0$.

Analyze Circuits by Superposition Balanced Circuit Common Mode ($V_d = 0$)



$$I_c = \frac{V_c}{Z_1 \| Z_2 + Z_3} = \frac{V_c}{\frac{Z}{2} + Z_3}$$

$$2I_d = I_1 - I_2 = \frac{I_c}{2} - \frac{I_c}{2} = 0 \Rightarrow I_d = 0$$

$$Z_{in-cm} = \frac{V_c}{I_c} = \frac{Z}{2} + Z_3$$

balanced circuit => $Z_1 = Z_2 = Z$

Summary

In a **balanced differential circuit** ($Z_1 = Z_2 = Z$)**:**

1. Differential mode voltages result in differential mode currents.
2. Common mode voltages result in common mode currents.

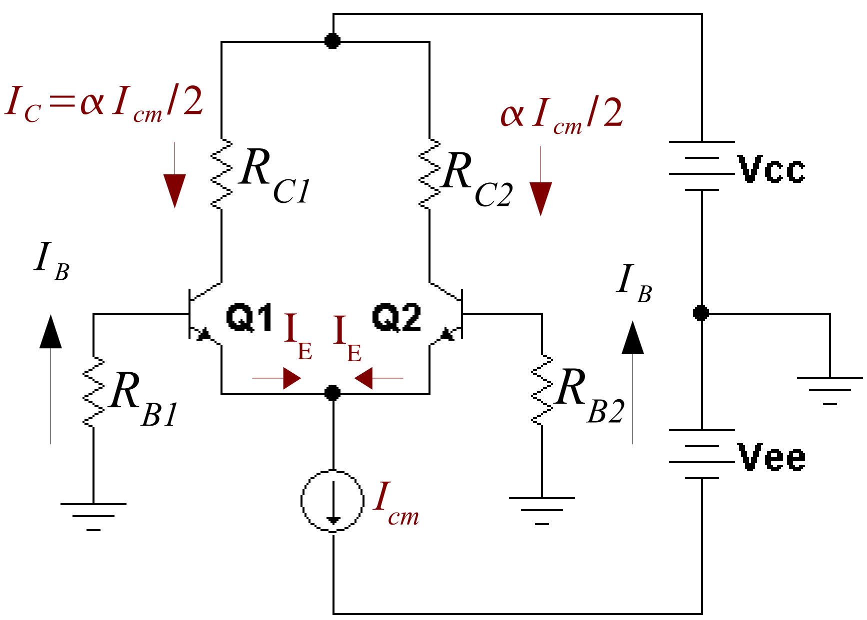
The *differential mode input impedance* of a balanced circuit is:

$$Z_{in-dm} = 2Z$$

The *common mode input impedance* of a balanced circuit is:

$$Z_{in-cm} = \frac{Z}{2} + Z_3$$

BJT Differential Amplifier – Bias View



$$I_E=\frac{I_{cm}}{2}$$

$$I_B=\frac{I_{cm}}{2(\beta+1)} \qquad I_C=\alpha\frac{I_{cm}}{2}\approx\frac{I_{cm}}{2}$$

$Q1=Q2$

$R_{C1}=R_{C2}=R_C$

$R_{B1}=R_{B2}=R_B$

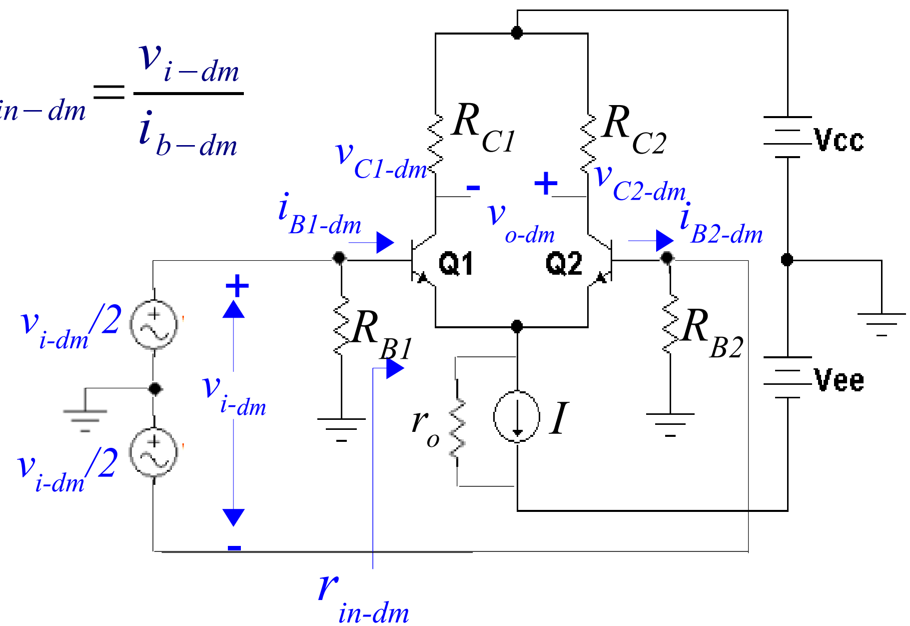
balanced circuit

Collector bias path inherently common mode.

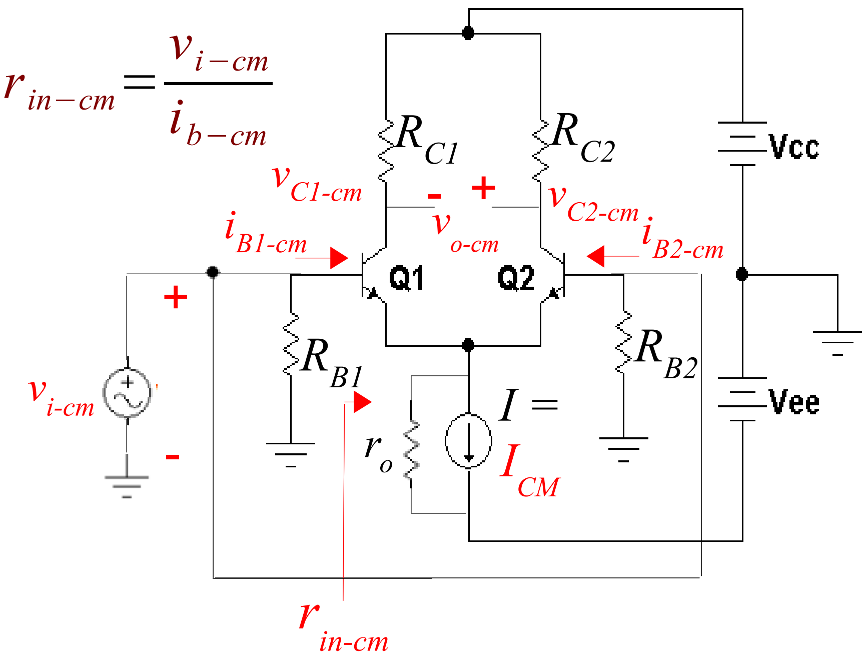
Choose I_{cm} and R_C for approximate ½ V_{CC} drop across R_C.

Voltage Gain & Input Impedance

$$r_{in-dm} = \frac{v_{i-dm}}{i_{b-dm}}$$



$$r_{in-cm} = \frac{v_{i-cm}}{i_{b-cm}}$$



Single-ended outputs: $v_{c1\text{-}dm}$, $v_{c2\text{-}dm}$

Differential output: $v_{o\text{-}dm} = v_{c2\text{-}dm} - v_{c1\text{-}dm}$

(diff) $$A_{v-dm} = \frac{v_{o-dm}}{v_{i-dm}}$$

(se) $$A_{v-dm1,2} = \frac{v_{c1,2-dm}}{v_{i-dm}}$$

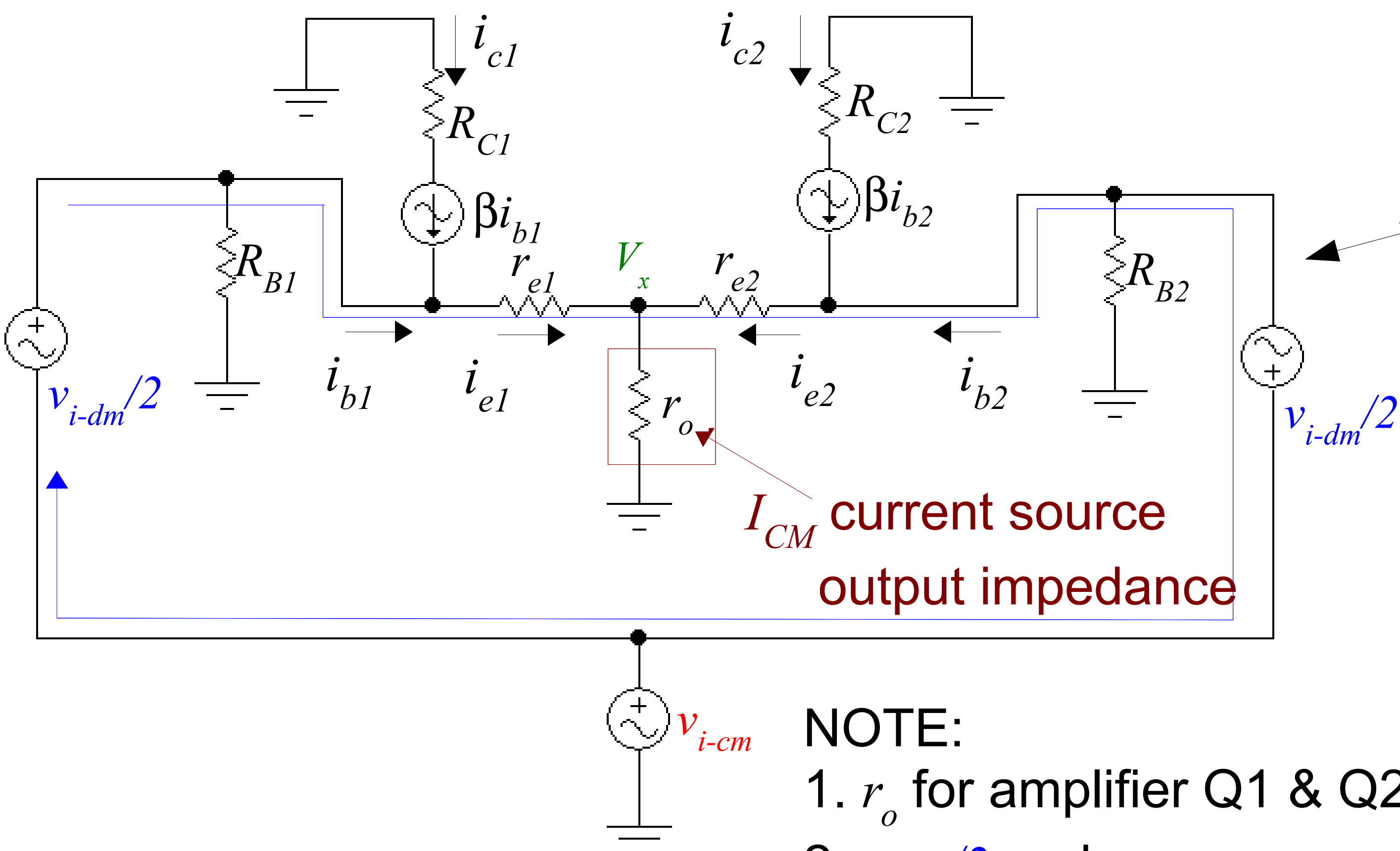
Single-ended outputs: $v_{c1\text{-}cm}$, $v_{c2\text{-}cm}$

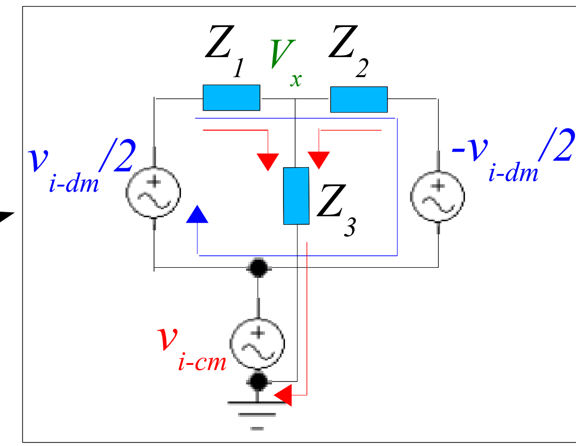
Differential output: $v_{o\text{-}cm} = v_{c2\text{-}cm} - v_{c1\text{-}cm}$

(diff) $$A_{v-cm} = \frac{v_{o-cm}}{v_{i-cm}}$$

(se) $$A_{v-cm1,2} = \frac{v_{c1,2-cm}}{v_{i-cm}}$$

BJT Differential Amplifier – Small-signal View





$$Z_1 = Z_2 = (\beta+1) r_e$$

$$Z_3 = (\beta+1) r_o$$

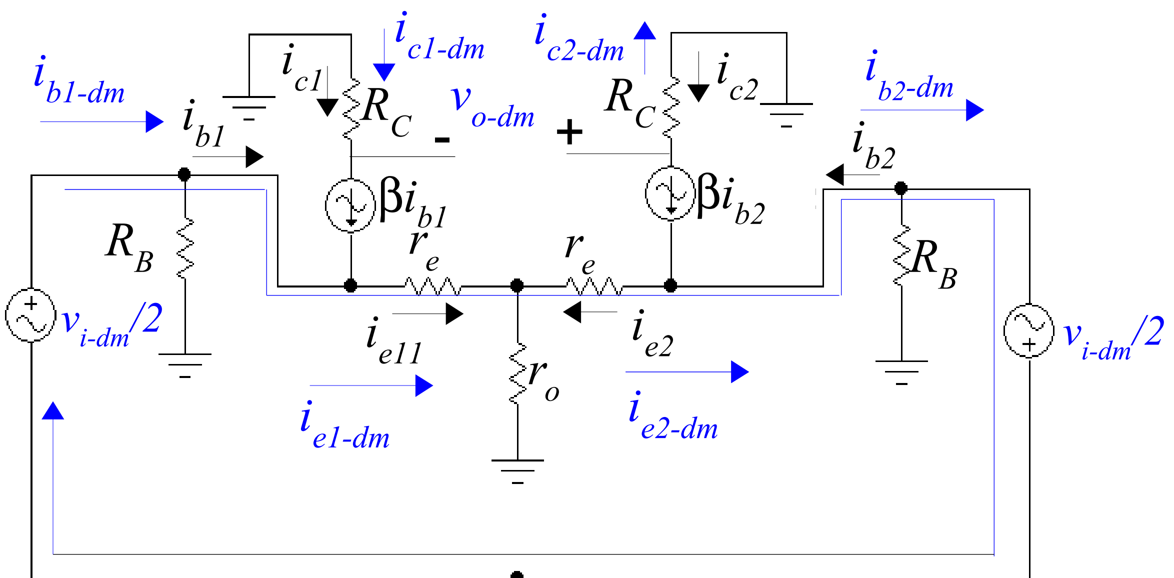
NOTE:

1. r_o for amplifier Q1 & Q2 is ignored.
2. $v_{i\text{-}dm}/2$ and $v_{i\text{-}cm}$ are ac small signals

Superposition Small-signal Analysis Differential Mode ($v_{i\text{-}cm} = 0$)

Balanced circuit => $R_{C1}=R_{C2}=R_C,\ R_{B1}=R_{B2}=R_B,\ r_{e1}=r_{e2}=r_e,\ \beta_1=\beta_2=\beta$

$\Rightarrow i_{e1}=i_{e2}=i_e,\ i_{b1}=i_{b2}=i_b,\ i_{c1}=i_{c2}=i_c$



$Z_1=Z_2=(\beta+1)r_e$

$i_{e1-dm}=i_{e-dm}=i_{e1}=i_e$

$i_{b1-dm}=i_{b-dm}=i_{b1}=i_b$

$i_{c1-dm}=i_{c-dm}=i_{c1}=i_c$

$Z_3=(\beta+1)r_o$

$i_{e2-dm}=i_{e-dm}=-i_{e2}=-i_e$

$i_{b2-dm}=i_{b-dm}=-i_{b2}=-i_b$

$i_{c2-dm}=i_{c-dm}=-i_{c2}=-i_c$

DM Path (ignoring both R_B):

$$\frac{v_{i-dm}}{2}=i_{e-dm}r_e+i_{e-dm}r_e-\frac{v_{i-dm}}{2}$$

$$v_{i-dm}=2r_e i_{e-dm}=2(\beta+1)r_e i_{b-dm}$$

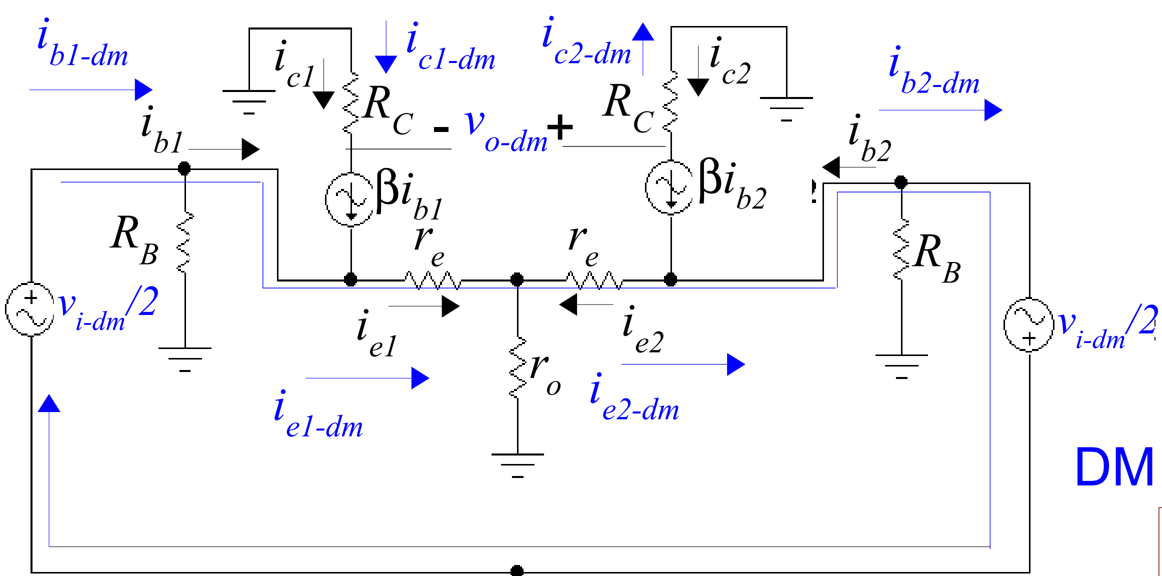
$$=r_{in-dm}i_{b-dm}$$

$$\boxed{r_{in-dm}=2(\beta+1)r_e}$$

Recall: $Z_{in-dm}=2Z$

Superposition Small-signal Analysis Differential Mode ($v_{i\text{-}cm} = 0$) cont.

Balanced circuit



$$i_{e1-dm} = i_{e-dm} = i_{e1} = i_e \qquad i_{e2-dm} = i_{e-dm} = -i_{e2} = -i_e$$

$$i_{b1-dm} = i_{b-dm} = i_{b1} = i_b \qquad i_{b2-dm} = i_{b-dm} = -i_{b2} = -i_b$$

$$i_{c1-dm} = i_{c-dm} = i_{c1} = i_c \qquad i_{c2-dm} = i_{c-dm} = -i_{c2} = -i_c$$

$$i_{c-dm} = \beta i_{b-dm}$$

$$v_{o-dm} = v_{c2-dm} - v_{c1-dm}$$

$$v_{o-dm} = \underbrace{-R_C\beta(-i_{b-dm})}_{v_{c2-dm}} - \underbrace{(-R_C\beta i_{b-dm})}_{v_{c1-dm}}$$

$$i_{b-dm} = \frac{v_{i-dm}}{2(\beta+1)r_e}$$

$$v_{o-dm} = \cancel{2}\frac{R_C\beta}{\cancel{2}(\beta+1)r_e}v_{i-dm}$$

DM Differential voltage gain:

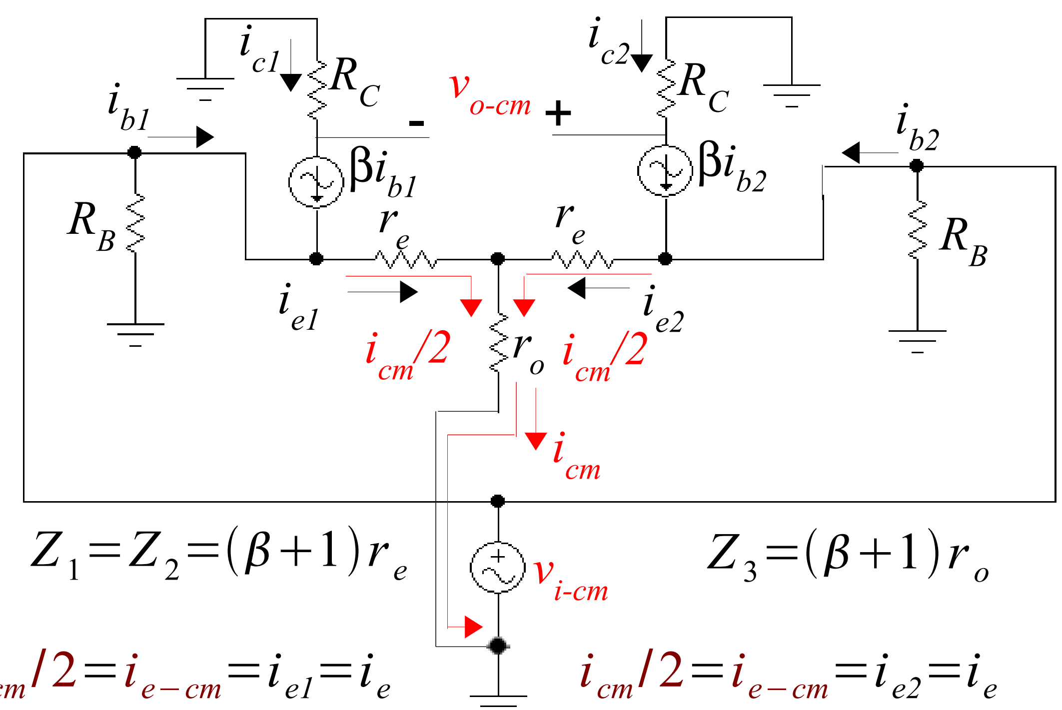
$$A_{v-dm} = \frac{v_{o-dm}}{v_{i-dm}} = \frac{R_C\beta}{(\beta+1)r_e} \approx \frac{R_C}{r_e}$$

DM Single-ended voltage gains:

$$A_{v-dm2} = -A_{v-dm1} = \frac{v_{c2-dm}}{v_{i-dm}} = \frac{\beta R_C}{2(\beta+1)r_e} \approx \frac{R_C}{2r_e}$$

Superposition Small-signal Analysis Common Mode ($v_{i-dm} = 0$)

Balanced circuit



$Z_1 = Z_2 = (\beta+1) r_e$

$Z_3 = (\beta+1) r_o$

$i_{cm}/2 = i_{e-cm} = i_{e1} = i_e$

$i_{b1-cm} = i_{b-cm} = i_{b1} = i_b$

$i_{c1-cm} = i_{c-cm} = i_{c1} = i_c$

$i_{cm}/2 = i_{e-cm} = i_{e2} = i_e$

$i_{b2-cm} = i_{b-cm} = i_{b2} = i_b$

$i_{c2-cm} = i_{c-cm} = i_{c2} = i_c$

CM path: both r_e are in parallel

$$v_{i-cm} = [r_e \| r_e + r_o] i_{cm}$$

$$i_{cm} = \frac{v_{i-cm}}{\frac{r_e}{2} + r_o} \approx \frac{v_{i-cm}}{r_o}$$

$$i_{cm} = 2 i_{e-cm} = (\beta+1) 2 i_{b-cm}$$

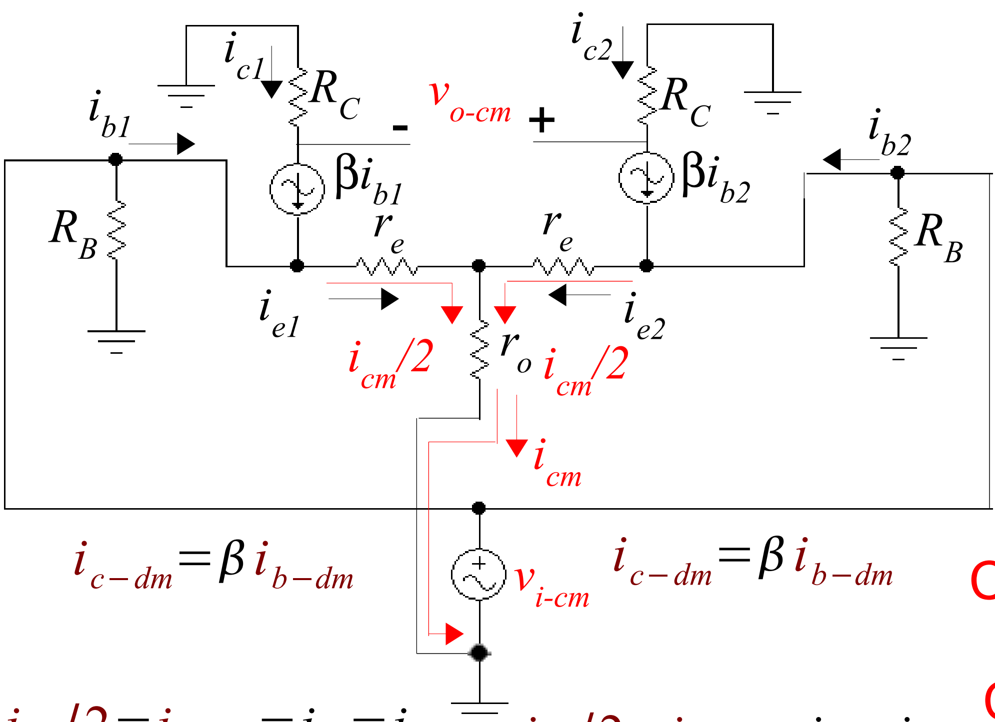
$$v_{i-cm} = 2 i_{b-cm} (\beta+1) \left(\frac{r_e}{2} + r_o \right)$$

$$r_{in-cm} = \frac{v_{i-cm}}{2 i_{b-cm}} = (\beta+1)\left(\frac{r_e}{2} + r_o\right) \approx (\beta+1) r_o$$

Recall: $Z_{in-cm} = \frac{Z}{2} + Z_3$

Superposition Small-signal Analysis Common Mode ($v_{i\text{-}dm} = 0$)

Balanced circuit



$$v_{o-cm} = v_{c2-cm} - v_{c1-cm}$$

$$v_{o-cm} = -R_C \beta i_{b-cm} - (-R_C \beta i_{b-cm}) = 0$$

$$i_{b-cm} = \frac{v_{i-cm}}{(\beta+1)(r_e + 2r_o)} \approx \frac{v_{i-cm}}{(\beta+1)2r_o}$$

$$v_{c2-cm} = v_{c1-cm} = -R_C \beta i_{b-cm} = \frac{-R_C \beta}{(\beta+1)2r_o} v_{i-cm}$$

$$i_{c-dm} = \beta i_{b-dm} \qquad i_{c-dm} = \beta i_{b-dm}$$

CM Differential voltage gain: $A_{v-cm} = \frac{v_{o-cm}}{v_{i-cm}} = 0$

CM Single-ended voltage gains:

$$A_{v-cm2} = A_{v-cm1} = \frac{v_{c1-cm}}{v_{i-cm}} = -\frac{R_C \beta}{(\beta+1)2r_o} \approx -\frac{R_C}{2r_o}$$

$$i_{cm}/2 = i_{e-cm} = i_{e1} = i_e \qquad i_{cm}/2 = i_{e-cm} = i_{e2} = i_e$$

$$i_{b1-cm} = i_{b-cm} = i_{b1} = i_b \qquad i_{b2-cm} = i_{b-cm} = i_{b2} = i_b$$

$$i_{c1-cm} = i_{c-cm} = i_{c1} = i_c \qquad i_{c2-cm} = i_{c-cm} = i_{c2} = i_c$$

Summary

Differential mode:

$$r_{in-dm} = 2(\beta+1)r_e$$

Differential DM Voltage gain:

$$A_{v-dm} = \frac{v_{o-dm}}{v_{i-dm}} \approx \frac{R_C}{r_e}$$

Single-Ended DM Voltage gain:

$$A_{v-dm1} = -A_{v-dm2} = \frac{v_{c1-dm}}{v_{i-dm}} \approx -\frac{R_C}{2r_e}$$

Common mode:

$$r_{in-cm} = (\beta+1)\left(\frac{r_e}{2} + r_o\right) \approx (\beta+1)r_o$$

Differential CM Voltage gain:

$$A_{v-cm} = \frac{v_{o-cm}}{v_{i-cm}} = 0$$

Single-Ended CM Voltage gain:

$$A_{v-cm1} = A_{v-cm2} = \frac{v_{c1-cm}}{v_{i-cm}} \approx -\frac{R_C}{2r_o}$$