Differential Amplifier Cont.

- Differential amplifier with emitter resistors
- Single-ended output
- Common mode rejection ratio (CMRR)
Summary – Previous Lecture

**Differential mode:**

\[ r_{in-dm} = 2(\beta + 1)r_e \]

**Differential DM Voltage gain:**

\[ A_{v-dm} = \frac{v_{o-dm}}{v_{i-dm}} \approx \frac{R_C}{r_e} \]

**Single-Ended DM Voltage gain:**

\[ A_{v-dm} = -A_{v-dm2} = \frac{v_{c1-dm}}{v_{i-dm}} \approx -\frac{R_C}{2r_e} \]

**Common mode:**

\[ r_{in-cm} = (\beta + 1)\left(\frac{r_e}{2} + r_o\right) \approx (\beta + 1)r_o \]

**Differential CM Voltage gain:**

\[ A_{v-cm} = \frac{v_{o-cm}}{v_{i-cm}} = 0 \]

**Single-Ended CM Voltage gain:**

\[ A_{v-cm1} = A_{v-cm2} = \frac{v_{c1-cm}}{v_{i-cm}} \approx -\frac{R_C}{2r_o} \]
**Differential Amplifier With Emitter Resistors**

Balance assumed:

\[ R_{C1} = R_{C2} = R_C \]
\[ R_{E1} = R_{E2} = R_E \]
\[ R_{B1} = R_{B2} = R_B \]

\[ Q1 = Q2 (Matched Pair) \]
Differential Mode Analysis ($v_{i-cm} = 0$)

NOTE:
1. $r_o$ for Q1 and Q2 are ignored.
2. Both $R_B$'s are ignored.

**Balanced circuit**

\[
\begin{align*}
    v_{i-dm}/2 & = i_{c1-dm} = i_{e1-dm} = i_{e1} = i_e \\
    i_{b1-dm} & = i_{b-dm} = i_{b1} = i_b \\
    i_{c1-dm} & = i_{c-dm} = i_{c1} = i_c \\
    i_{b2-dm} & = i_{b-dm} = -i_{b2} = -i_b \\
    i_{c2-dm} & = i_{c-dm} = -i_{c2} = -i_c \\
    \beta i_{b2} & = i_{c1-dm} = i_{e1-dm} = -i_e = -i_{e2} = -i_{e2-dm} \\
\end{align*}
\]

\[i_{e-dm} = \beta i_{b-dm}\]

**Differential DM Voltage gain:**

\[A_{v-dm} = \frac{v_{o-dm}}{v_{i-dm}} \approx \frac{R_C}{(r_e + R_E)}\]

**Single-Ended DM Voltage gains:**

\[A_{v-dm1} = \frac{v_{c1-dm}}{v_{i-dm}} = \frac{-\beta R_C}{2(\beta + 1)(r_e + R_E)}\]

\[A_{v-dm2} = \frac{v_{c2-dm}}{v_{i-dm}} = \frac{\beta R_C}{2(\beta + 1)(r_e + R_E)}\]
Differential Mode Analysis - Continued

The collector-to-ground (single-ended) gain of the Q2 stage is:

\[ A_{v-\text{dm}} = \frac{v_{c2-\text{dm}}}{v_{i-\text{dm}}} \approx \frac{R_C}{2(r_e + R_E)} \]

Experimentally (or MultiSim) the voltage at the base of Q1 is actually \( v_i \), where in our analysis \( v_i = v_{i-\text{dm}}/2 \). E.g. the gain at Q1 output w.r.t \( v_i \) is:

\[ A'_{v-\text{dm}} = \frac{v_{c2-\text{dm}}}{v_i} = \frac{v_{c2-\text{dm}}}{v_{\text{dm}}/2} \approx \frac{R_C}{r_e + R_E} \]
Common Mode Analysis ($v_{i-dm} = 0$)

Balanced circuit

\[ i_{cm} = i_{e1-cm} + i_{e2-cm} = 2i_{e-cm} \]

NOTE:
1. $r_o$ for Q1 and Q2 are ignored.
2. Both $R_B$'s are ignored.

Differential CM Voltage gain:

\[ A_{v-cm} = \frac{v_{o-cm}}{v_{i-cm}} = 0 \]

Single-Ended CM Voltage gains:

\[ A_{v-cm1} = \frac{v_{c1-cm}}{v_{i-cm}} = -\frac{\beta R_C}{(\beta+1)(r_e + R_E + 2r_o)} \approx -\frac{R_C}{2r_o} \]

\[ A_{v-cm2} = \frac{v_{c2-cm}}{v_{i-cm}} = A_{v-cm1} \]
Common Mode Rejection Ratio

Common mode rejection ratio is the magnitude ratio of the differential mode gain over the common mode gain expressed in dB:

Differential:

\[ A_{v_{-cm}} = 0 \text{ (iff balanced)} \]

Single-Ended:

\[ A_{v_{-cm2}} = A_{v_{-cm1}} \approx -\frac{R_C}{2r_o} \]

Differential:

\[ A_{v_{-dm}} \approx \frac{-R_C}{r_e + R_E} \]

Single-Ended:

\[ A_{v_{-dm2}} = -A_{v_{-dm1}} \approx \frac{R_C}{2(r_e + R_E)} \]

Differential:

\[ CMRR = 20 \log_{10} \left( \left| \frac{A_{v_{-dm}}}{A_{v_{-cm}}} \right| \right) = \infty \]

Single-Ended:

\[ CMRR \approx 20 \log_{10} \left( \left| \frac{A_{v_{-dm1,2}}}{A_{v_{-cm1,2}}} \right| \right) \]

\[ CMRR \approx 20 \log_{10} \left( \frac{R_C}{2(r_e + R_E)} \cdot \frac{2r_o}{R_C} \right) \]

Single-Ended:

\[ CMRR \approx 20 \log_{10} \left( \frac{r_o}{r_e + R_E} \right) \]
Let \( R_{C2} > R_{C1} \), s.t. \( R_{C1} = R_C \) and
\[
R_{C2} = R_C + \Delta R_C
\]
where \( \Delta R_C \ll R_C \)

\[
A_{v-cm1} = \frac{v_{c1-cm}}{v_{i-cm}} \approx -\frac{R_C}{r_e + R_E + 2r_0} \approx -\frac{R_C}{2r_o}
\]

\[
A_{v-cm2} = \frac{v_{c2-cm}}{v_{i-cm}} \approx -\frac{R_C + \Delta R_C}{2r_0} = -\frac{R_C}{2r_o} \left( 1 + \frac{\Delta R_C}{R_C} \right)
\]

\[
A_{v-cm2} \neq A_{v-cm1} \approx -\frac{R_C}{2r_o} \quad \text{and} \quad A_{v-cm} \neq 0
\]

\[
A_{v-cm} = A_{v-cm2} - A_{v-cm1} = -\frac{R_C}{2r_o} \frac{\Delta R_C}{R_C}
\]

**Hence!**

\[
CMRR = 20 \log_{10} \left( \frac{A_{v-dm}}{A_{v-cm}} \right) = 20 \log_{10} \left( \frac{2r_o}{r_e + R_E} \frac{1}{\Delta R_C / R_C} \right) \neq \infty
\]
Summary

Differential mode:

\[ r_{in-dm} = \frac{v_{i-dm}}{i_{b1-dm}} = (\beta + 1) \left( r_e + R_E \right) \]

\[ A_{v-dm2} = -A_{v-dm1} \approx \frac{R_C}{2(r_e + R_E)} \quad A_{v-dm} = \frac{v_{o-dm}}{v_{i-dm}} = \frac{R_C}{r_e + R_E} \]

Common mode:

\[ r_{in-cm} = \frac{v_{i-cm}}{2i_{b-cm}} = (\beta + 1) \left( \frac{r_e + R_E}{2} + r_0 \right) \]

\[ A_{v-cm1} = A_{v-cm2} \approx -\frac{R_C}{2r_o} \quad A_{v-cm} = 0 \quad \text{Balanced} \quad A_{v-cm} \approx \frac{R_C}{2r_o} \frac{\Delta R_C}{R_C} \]

Single-ended (balanced)

\[ \text{CMRR} \approx 20 \log_{10} \left( \frac{r_o}{r_e + R_E} \right) \]

Differential

\[ \text{CMRR} = 20 \log_{10} \left( 2 \frac{r_o}{r_e + R_E} \frac{1}{\Delta R_C / R_C} \right) \]

Balanced

\[ \text{CMRR} = \infty \quad \text{iff balanced} \]

Unbalanced due to \( R_{C1} \neq R_{C2} \)