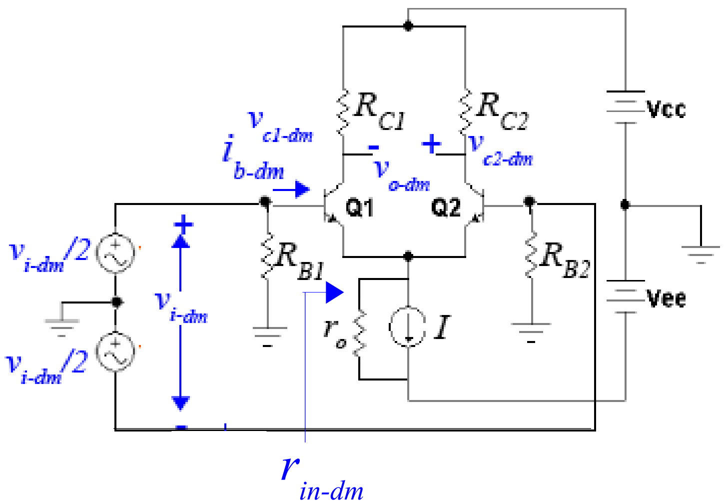
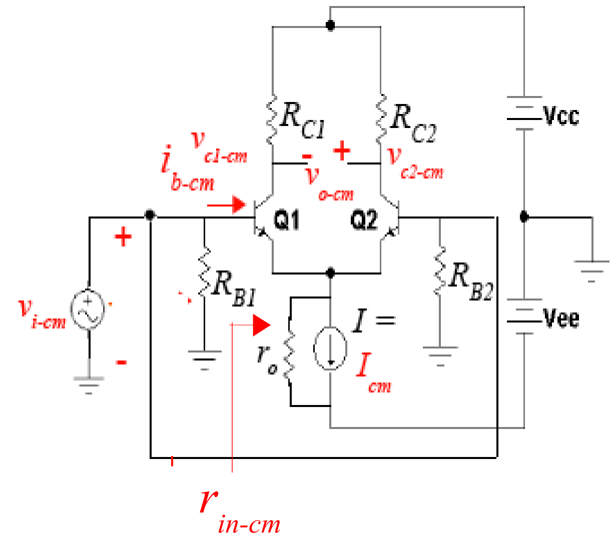
Differential Amplifier Cont.

- Differential amplifier with emitter resistors
- Single-ended output
- Common mode rejection ratio (CMRR)

Summary – Previous Lecture





Differential mode:

$$r_{in-dm}=2(\beta+1)r_e$$

Differential DM Voltage gain:

$$A_{v-dm}=\frac{v_{o-dm}}{v_{i-dm}}\approx\frac{R_C}{r_e}$$

Single-Ended DM Voltage gain:

$$A_{v-dm1}=-A_{v-dm2}=\frac{v_{c1-dm}}{v_{i-dm}}\approx-\frac{R_C}{2r_e}$$

Common mode:

$$r_{in-cm}=(\beta+1)\left(\frac{r_e}{2}+r_o\right)\approx(\beta+1)r_o$$

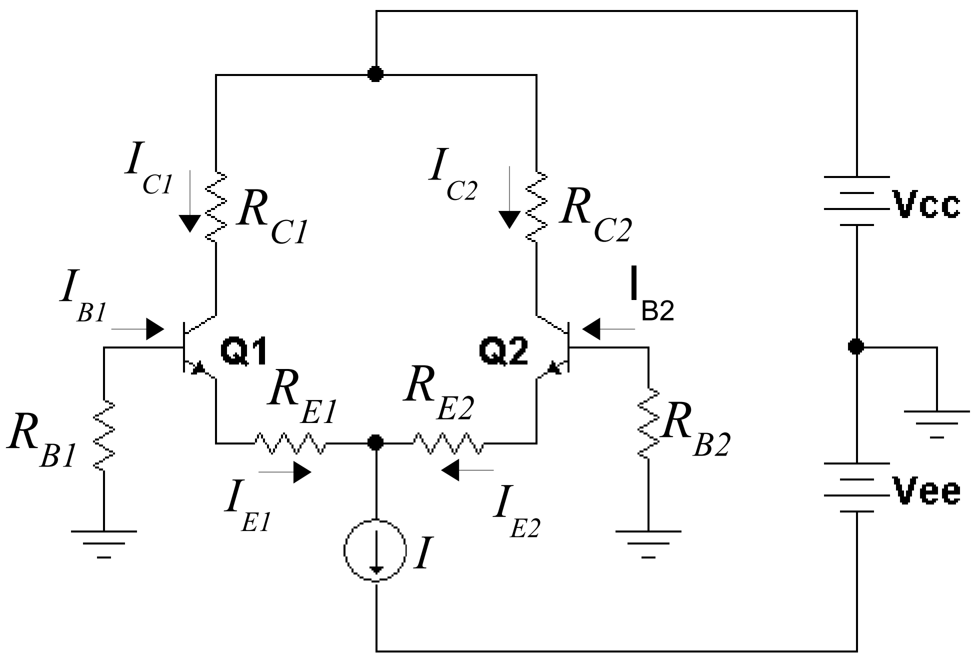
Differential CM Voltage gain:

$$A_{v-cm}=\frac{v_{o-cm}}{v_{i-cm}}=0$$

Single-Ended CM Voltage gain:

$$A_{v-cm1}=A_{v-cm2}=\frac{v_{c1-cm}}{v_{i-cm}}\approx-\frac{R_C}{2r_o}$$

Differential Amplifier With Emitter Resistors



Balance assumed:

$$R_{C1}=R_{C2}=R_C$$

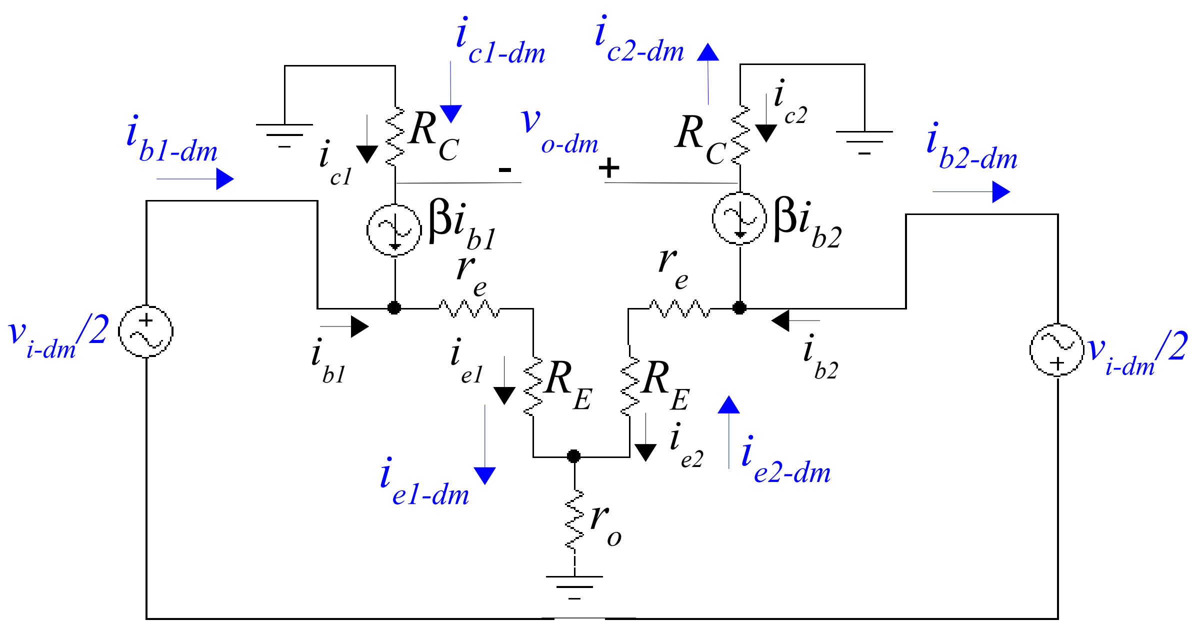
$$R_{E1}=R_{E2}=R_E$$

$$R_{B1}=R_{B2}=R_B$$

$$Q1=Q2\,(Matched\ Pair)$$

Differential Mode Analysis ($v_{i-cm} = 0$)

Balanced circuit



NOTE:

1. r_o for Q1 and Q2 are ignored.
2. Both R_B's are ignored.

$$r_{in-dm} = 2(\beta+1)(r_e+R_E)$$

Differential DM Voltage gain:

$$A_{v-dm} = \frac{v_{o-dm}}{v_{i-dm}} \approx \frac{R_C}{(r_e+R_E)}$$

Single-Ended DM Voltage gains:

$$A_{v-dm1} = \frac{v_{c1-dm}}{v_{i-dm}} = \frac{-\beta R_C}{2(\beta+1)(r_e+R_E)}$$

$$A_{v-dm2} = \frac{v_{c2-dm}}{v_{i-dm}} = \frac{\beta R_C}{2(\beta+1)(r_e+R_E)}$$

$$i_{e1-dm} = i_{e-dm} = i_{e1} = i_e \qquad i_{e2-dm} = i_{e-dm} = -i_{e2} = -i_e$$

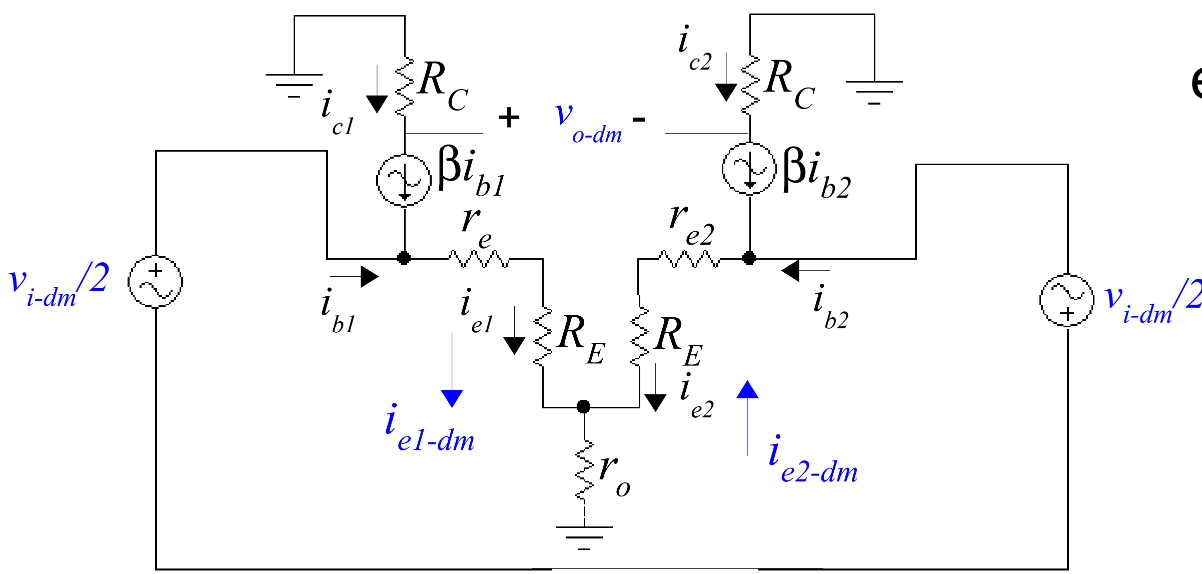
$$i_{b1-dm} = i_{b-dm} = i_{b1} = i_b \qquad i_{b2-dm} = i_{b-dm} = -i_{b2} = -i_b$$

$$i_{c1-dm} = i_{c-dm} = i_{c1} = i_c \qquad i_{c2-dm} = i_{c-dm} = -i_{c2} = -i_c$$

$$i_{c-dm} = \beta\, i_{b-dm}$$

Differential Mode Analysis - Continued

Balanced circuit



The collector-to-ground (single-ended) gain of the Q2 stage is:

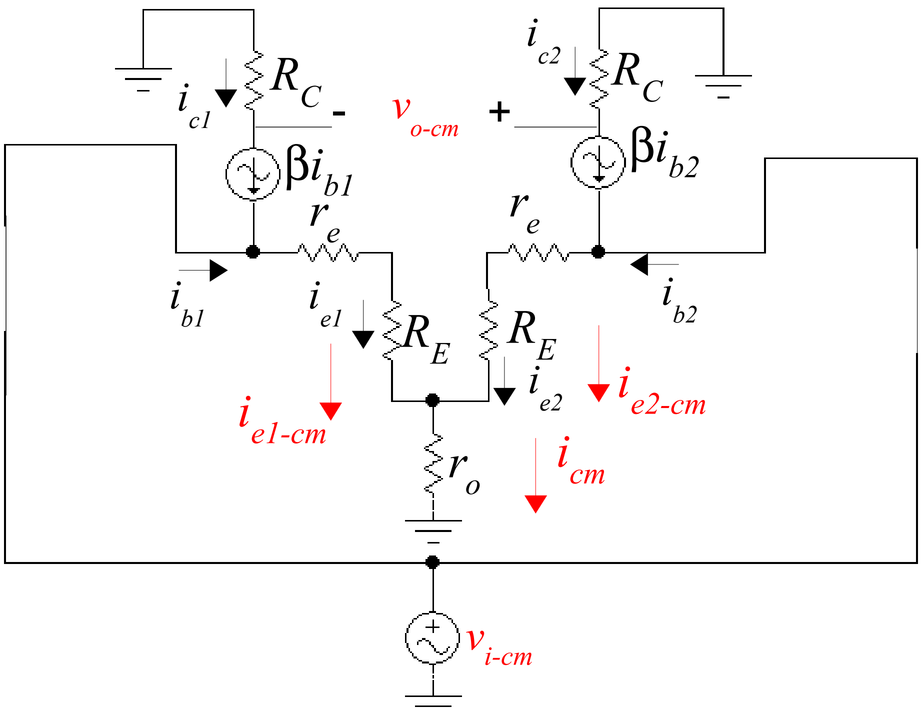
$$A_{v-dm2}=\frac{v_{c2-dm}}{v_{i-dm}}\approx\frac{R_C}{2(r_e+R_E)}$$

Experimentally (or MultiSim) the voltage at the base of Q1 is actually v_i, where in our analysis $v_i = v_{i-dm}/2$. E.g. the gain at Q1 output w.r.t v_i is:

$$A'_{v-dm2}=\frac{v_{c2-dm}}{v_i}=\frac{v_{c2-dm}}{v_{dm}/2}\approx\frac{R_C}{r_e+R_E}$$

Common Mode Analysis (v_{i-dm} = 0)

Balanced circuit



$$i_{cm} = i_{e1-cm} + i_{e2-cm} = 2 i_{e-cm}$$

NOTE:

1. r_o for Q1 and Q2 are ignored.
2. Both R_B's are ignored.

$$r_{in-cm} = (\beta+1)\left(\frac{r_e + R_E}{2} + r_o\right) \approx (\beta+1) r_o$$

Differential CM Voltage gain:

$$A_{v-cm} = \frac{v_{o-cm}}{v_{i-cm}} = 0$$

Single-Ended CM Voltage gains:

$$A_{v-cm1} = \frac{v_{c1-cm}}{v_{i-cm}} = -\frac{\beta R_C}{(\beta+1)(r_e + R_E + 2 r_0)} \approx -\frac{R_C}{2 r_o}$$

$$A_{v-cm2} = \frac{v_{c2-cm}}{v_{i-cm}} = A_{v-cm1}$$

Common Mode Rejection Ratio

Common mode rejection ratio is the magnitude ratio of the differential mode gain over the common mode gain expressed in dB:

Differential:

$A_{v-cm}=0$ (iff balanced)

Single-Ended:

$$A_{v-cm2}=A_{v-cm1}\approx-\frac{R_C}{2r_o}$$

Differential:

$$A_{v-dm}\approx\frac{-R_C}{r_e+R_E}$$

Single-Ended:

$$A_{v-dm2}=-A_{v-dm1}\approx\frac{R_C}{2(r_e+R_E)}$$

Differential: (iff balanced)

$$CMRR=20\log_{10}\left(\frac{|A_{v-dm}|}{|A_{v-cm}|}\right)=\infty$$

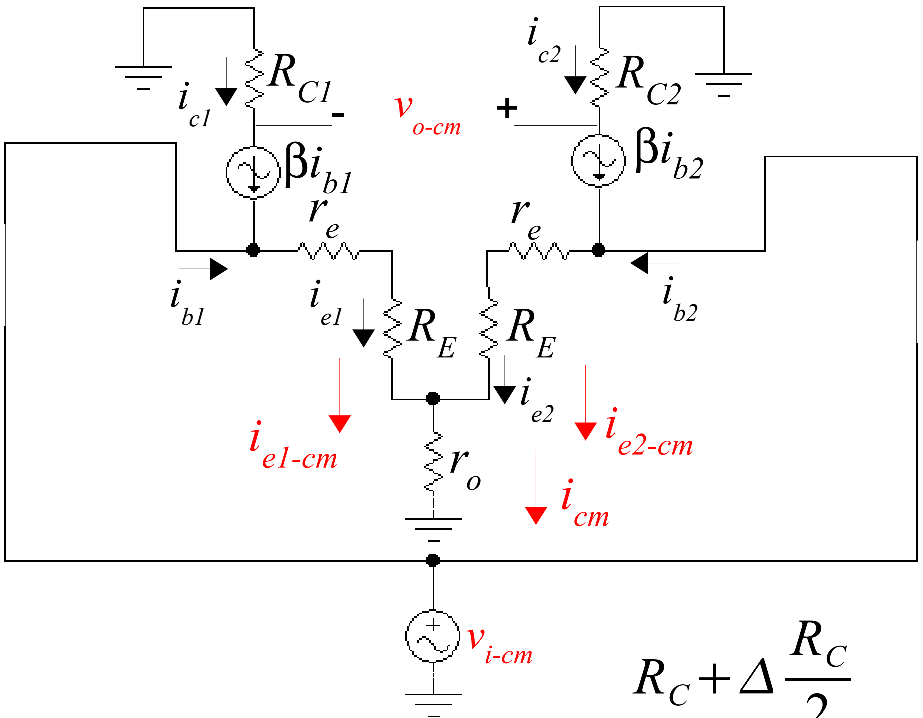
Single-Ended:

$$CMRR=20\log_{10}\left(\frac{|A_{v-dm1,2}|}{|A_{v-cm1,2}|}\right)$$

$$CMRR\approx20\log_{10}\left(\frac{R_C}{2(r_e+R_E)}*\frac{2r_o}{R_C}\right)$$

$$CMRR\approx20\log_{10}\left(\frac{r_o}{r_e+R_E}\right)$$

What if the Diff Amp isn't Perfectly Balanced?



Let $R_{C2} > R_{C1}$, s.t. $R_{C1} = R_C$ and

$$\boxed{R_{C2} = R_C + \Delta R_C}$$ where $\Delta R_C << R_C$

$$A_{v-cm1} = \frac{v_{c1-cm}}{v_{i-cm}} \approx -\frac{R_C}{(r_e + R_E + 2r_0)} \approx -\frac{R_C}{2r_o}$$

$$A_{v-cm2} = \frac{v_{c2-cm}}{v_{i-cm}} \approx -\frac{R_C + \Delta R_C}{2r_0}$$

$$= -\frac{R_C}{2r_o}\left(1 + \frac{\Delta R_C}{R_C}\right)$$

$$A_{v-dm} \approx \frac{R_C + \Delta\frac{R_C}{2}}{r_e + R_E} \approx \frac{R_C}{r_e + R_E}$$

Hence!

$$A_{v-cm2} \neq A_{v-cm1} \approx -\frac{R_C}{2r_o} \text{ and } A_{v-cm} \neq 0$$

$$\boxed{A_{v-cm} = A_{v-cm2} - A_{v-cm1} = -\frac{R_C}{2r_o}\frac{\Delta R_C}{R_C}}$$

Hence!

$$\boxed{CMRR = 20\log_{10}\left(\frac{|A_{v-dm}|}{|A_{v-cm}|}\right) = 20\log_{10}\left(\frac{2r_o}{r_e + R_E}\frac{1}{\Delta R_C / R_C}\right) \neq \infty}$$

Summary

Differential mode:

$$r_{in-dm}=\frac{v_{i-dm}}{i_{b1-dm}}=(\beta+1)2(r_e+R_E)$$

$$A_{v-dm2}=-A_{v-dm1}\approx\frac{R_C}{2(r_e+R_E)} \qquad A_{v-dm}=\frac{v_{o-dm}}{v_{i-dm}}=\frac{R_C}{r_e+R_E}$$

Common mode:

$$r_{in-cm}=\frac{v_{i-cm}}{2i_{b-cm}}=(\beta+1)(\frac{r_e+R_E}{2}+r_0)$$

$$A_{v-cm1}=A_{v-cm2}\approx-\frac{R_C}{2r_o}$$

$A_{v-cm}=0$ Balanced

$A_{v-cm}\approx\frac{R_C}{2r_o}\frac{\Delta R_C}{R_C}$ Unbalanced due to $R_{C1}\neq R_{C2}$

Single-ended (balanced)

$$CMRR\approx 20\log_{10}\left(\frac{r_o}{r_e+R_E}\right)$$

Differential

$$CMRR=20\log_{10}(2\frac{r_o}{r_E+R_E}\frac{1}{\Delta R_C/R_C})$$

$CMRR=\infty$ iff balanced