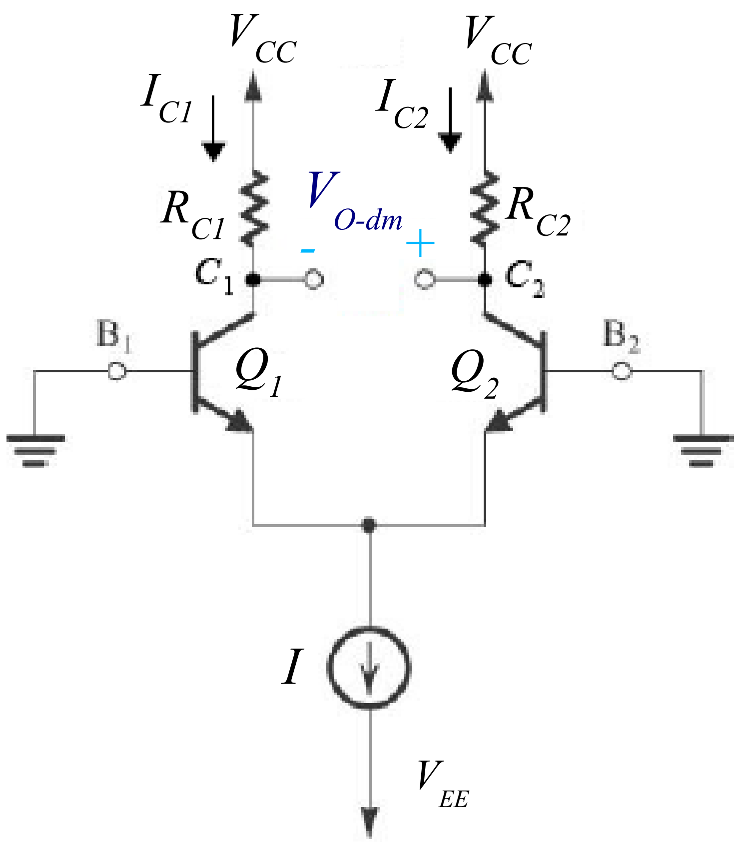
Differential Amplifier Offset

- Causes of dc voltage and current offset
- Modeling dc offset
  - R_C mismatch
  - I_S mismatch
  - β mismatch
  - dc offsets in differential amplifiers due to component mismatch are differential phenomena

transistor mismatch



1. Consider the emitter-current bias case where bases of matched Q1 and Q2 are connected to ground.
2. To further simplify, let there be no external base and emitter resistors.
3. Let the mismatch be in R_C where $R_{C1} \neq R_{C2}$.

$$V_{C1} = V_{CC} - R_{C1} I_{C1}$$

$$V_{C2} = V_{CC} - R_{C2} I_{C2}$$

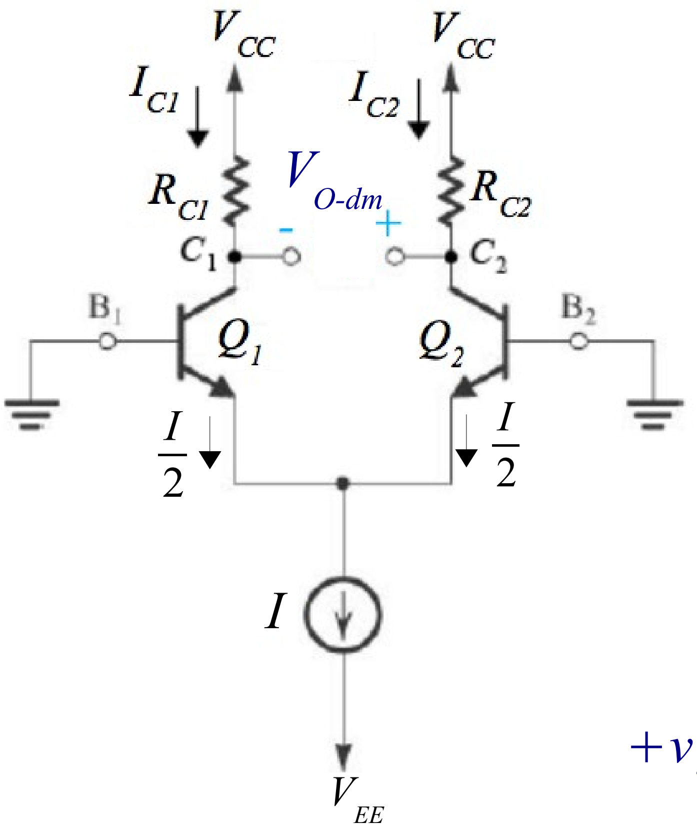
$$V_{O\text{-}dm} = V_{C2} - V_{C1} \quad (= 0 \text{ ideally})$$

Since Q1=Q2 and I is fixed, currents are matched i.e.

$$I_{C1} = I_{C2} = \frac{\beta}{\beta+1} \frac{I}{2} = \alpha \frac{I}{2}$$

$$V_{O-dm} = -R_{C2} I_{C2} - (-R_{C1} I_{C1}) = R_{C1} I_{C1} - R_{C2} I_{C2}$$

if $V_{O-dm} = 0 \Rightarrow R_{C1} I_{C1} = R_{C2} I_{C2}$



Let: $R_{C1} \neq R_{C2}$ s.t. $R_{C2} > R_{C1}$

Split the mismatch between both collector resistors, and decompose into common and differential components:

$$R_C = \frac{R_{C2} + R_{C1}}{2} \quad \text{(common)}$$

$$\Delta R_C = R_{C2} - R_{C1} \quad \text{(differential)}$$

Then, it can be shown that:

$+v_{i-dm}/2$

v_{i-cm}

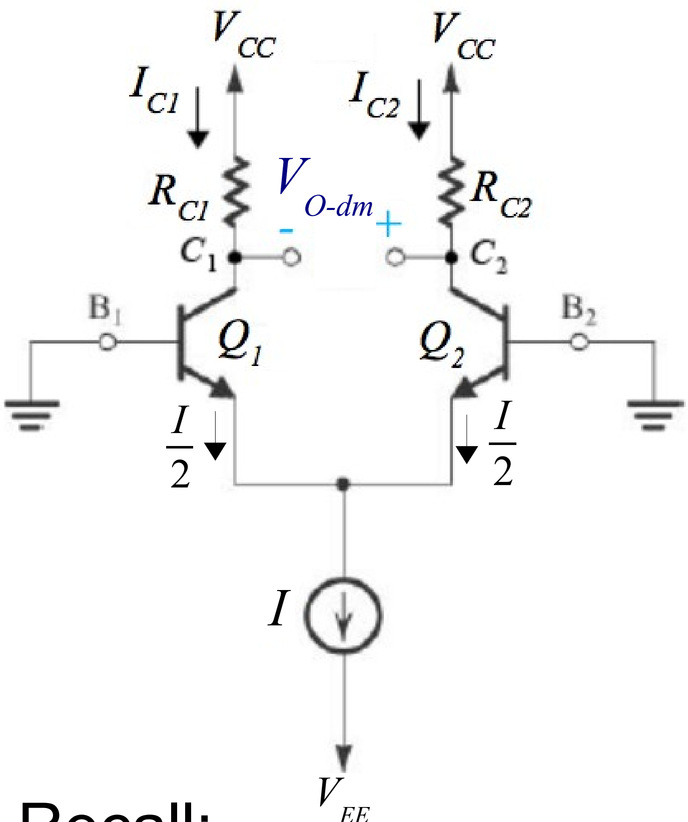
$$R_{C2} = R_C + \boxed{\frac{\Delta R_C}{2}} = R_C \left(1 + \frac{1}{2} \frac{\Delta R_C}{R_C}\right)$$

$-v_{i-dm}/2$

$$R_{C1} = R_C \boxed{- \frac{\Delta R_C}{2}} = R_C \left(1 - \frac{1}{2} \frac{\Delta R_C}{R_C}\right)$$

Assume matched currents:

$$I_{C1} = I_{C2} = \alpha \frac{I}{2}$$



Hence:

$$V_{O-dm}=\left(R_C-\frac{\Delta R_C}{2}\right)I_{C1}-\left(R_C+\frac{\Delta R_C}{2}\right)I_{C2}$$

$$V_{O-dm}=\left(R_C-\frac{\Delta R_C}{2}\right)\alpha\frac{I}{2}-\left(R_C+\frac{\Delta R_C}{2}\right)\alpha\frac{I}{2}$$

or:

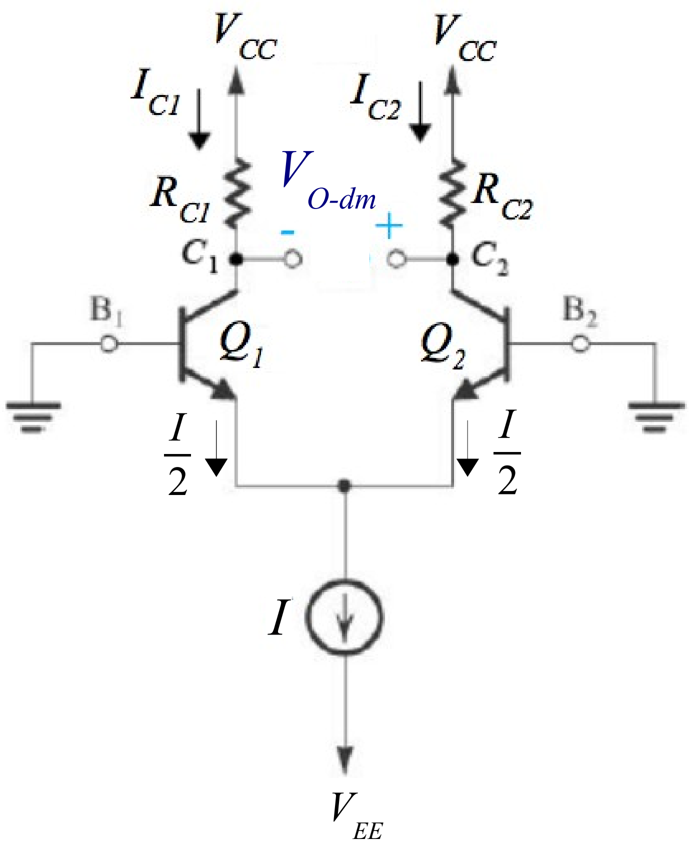
random variable (rv)

$$V_{O-dm}=-\alpha\frac{I}{2}\boxed{\Delta R_C}$$ output offset voltage

Recall:

$$V_{O-dm}=R_{C2}I_{C2}-R_{C1}I_{C1}$$

$$I_{C1}=I_{C2}=\alpha\frac{I}{2} \qquad R_{C1}\neq R_{C2}$$



Collector-collector voltage due to resistor mismatch:

$$V_{O-dm} = -\alpha \frac{I}{2} \Delta R_C$$

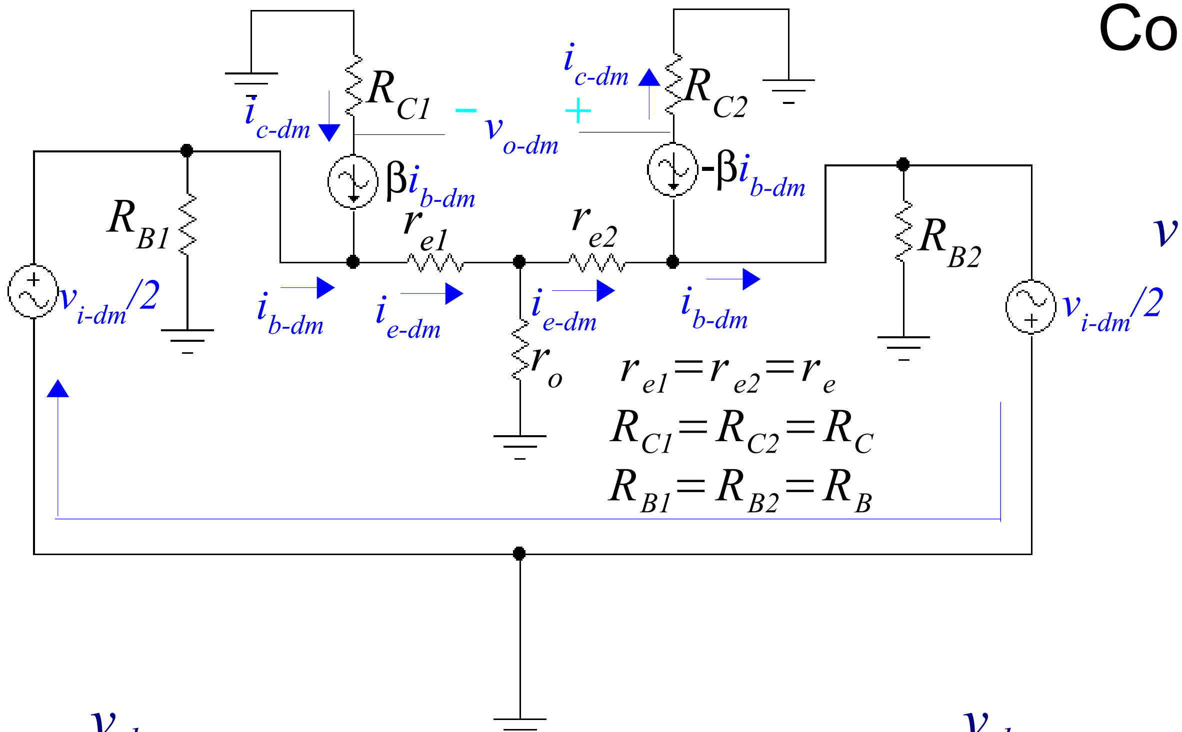
Define the ***input offset voltage*** as that input voltage that will cancel $V_{O\text{-}dm}$. If the amplifier differential gain is $A_{v\text{-}dm}$:

$$V_{OS} \equiv \frac{V_{O-dm}}{A_{v-dm}}$$

V_{OS} is highly variable, rv & can be + or -

Input offset voltage is the output offset voltage referred to the input due to mismatch ΔR_C.

Differential Mode Small-signal Analysis



$$r_{e1}=r_{e2}=r_e$$
$$R_{C1}=R_{C2}=R_C$$
$$R_{B1}=R_{B2}=R_B$$

Collector-collector voltage:

$$v_{o-dm}=v_{c2g-dm}-v_{c1g-dm}$$

$$v_{o-dm}=R_{C2}\beta i_{b-dm}-(-R_{C1}\beta(-i_{b-dm}))$$

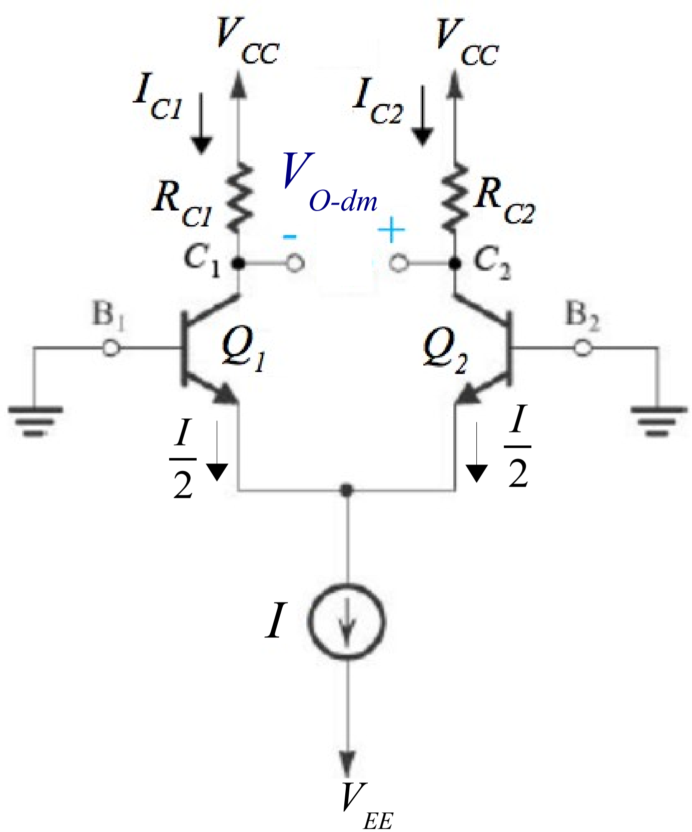
$$\frac{v_{o-dm}}{v_{dm}}=\frac{2\beta R_C}{2(\beta+1)r_e}$$

$$r_e=\frac{V_T}{I_E}=\frac{\beta}{\beta+1}\frac{V_T}{I_C}=\frac{\beta}{\beta+1}\frac{1}{g_m}$$

$$\boxed{A_{v-dm}=\frac{I_C}{V_T}R_C=g_m R_C}$$

$$\frac{v_{dm}}{2}=r_e(\beta+1)i_{b-dm}+r_e(\beta+1)i_{b-dm}-\frac{v_{dm}}{2}$$

$$i_{b-dm}=\frac{v_{dm}}{2(\beta+1)r_e}$$



$$V_{OS} = \frac{\alpha \frac{I}{2} \Delta R_C}{g_m R_C}$$

$$g_m = \frac{I_C}{V_T} = \alpha \frac{I}{2} \frac{1}{V_T}$$

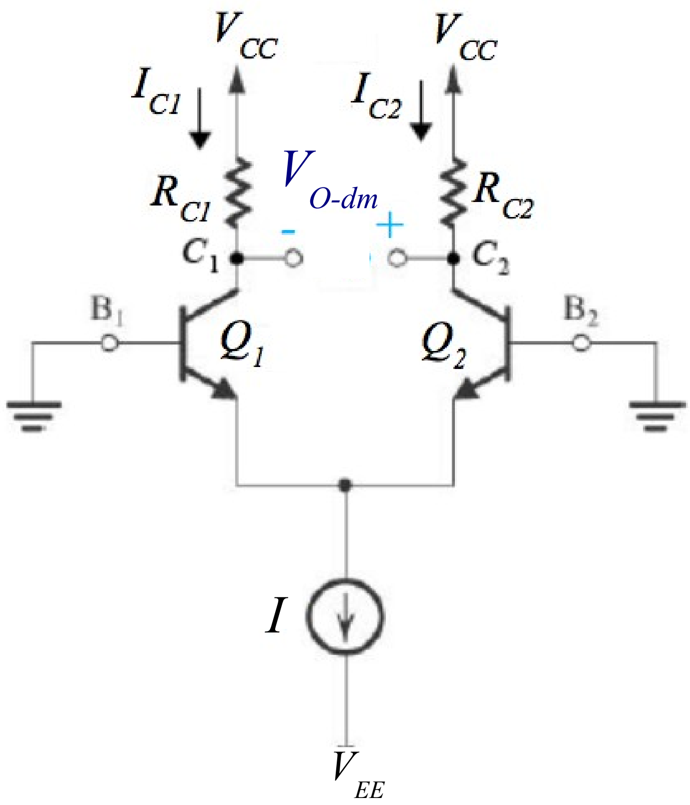
Input referred offset due to ΔR_C mismatch:

$$\boxed{V_{OS(\Delta R_C)} = V_T \boxed{\frac{\Delta R_C}{R_C}}}$$

random variable

$$V_{OS} = \frac{|V_{O-dm}|}{|A_{v-dm}|}$$ where $A_{v-dm} = g_m R_C$

Offset Voltage From Transistor Mismatch



Perfect balance requires:

$$V_{O-dm}=0 \Rightarrow R_{C1} I_{C1}=R_{C2} I_{C2}$$

Previous case considered $R_{C1} \neq R_{C2}$

& Q1 = Q2 => $I_{C1} = I_{C2}$.

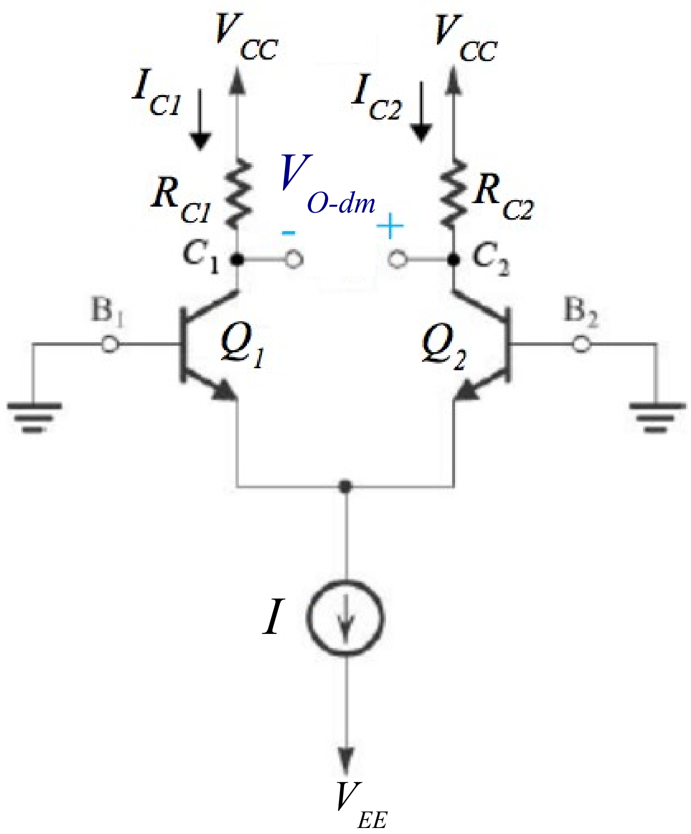
Consider now Q1 ≠ Q2 => $I_{C1} \neq I_{C2}$.

ASSUME: 1. $V_{BE1} = V_{BE2}$

2. $V_{T1}=V_{T2}$

Only difference is in the saturation currents of the transistors, i.e.

$$\boxed{I_{S1} \neq I_{S2}}$$



Again using common and differential mode concepts:

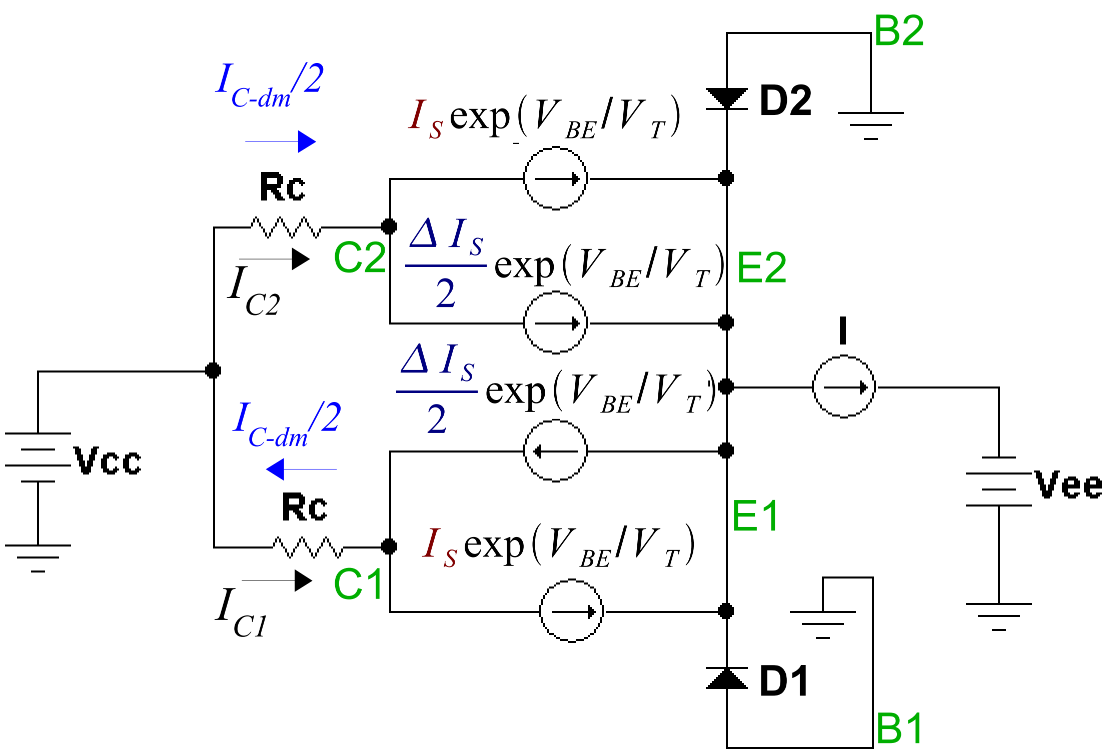
$$I_S = \frac{I_{S2} + I_{S1}}{2}$$

$$\Delta I_S = I_{S2} - I_{S1}$$

The two transistor saturation currents are:

$$\overbrace{I_{S2} = I_S}^{v_{i-cm}} + \underbrace{\boxed{\frac{\Delta I_S}{2}}}_{+v_{i-dm}/2}$$

$$I_{S1} = I_S - \underbrace{\boxed{\frac{\Delta I_S}{2}}}_{-v_{i-dm}/2}$$



Large signal Model:

$$I_{C1}=\left(I_S-\frac{\Delta I_S}{2}\right)e^{\frac{V_{BE}}{V_T}}$$

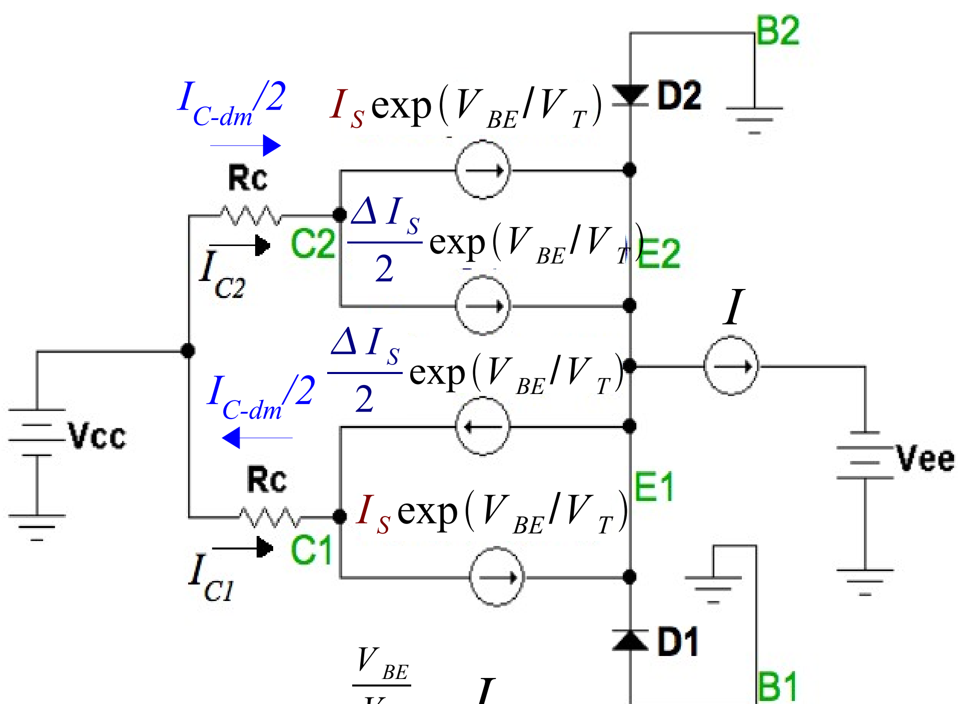
$$I_{C1}=I_S e^{\frac{V_{BE}}{V_T}}-\frac{\Delta I_S}{2}e^{\frac{V_{BE}}{V_T}}$$

also

$$I_{C2}=I_S e^{\frac{V_{BE}}{V_T}}+\frac{\Delta I_S}{2}e^{\frac{V_{BE}}{V_T}}$$

The parallel current sources are illustrated in the schematic.

Note: differential I_C components cause current flow in opposite directions through the R_C's resulting in an offset voltage. The common mode I_C components cause no offset voltage.



$$I_C = I_S e^{\frac{V_{BE}}{V_T}} = \frac{I}{2}$$

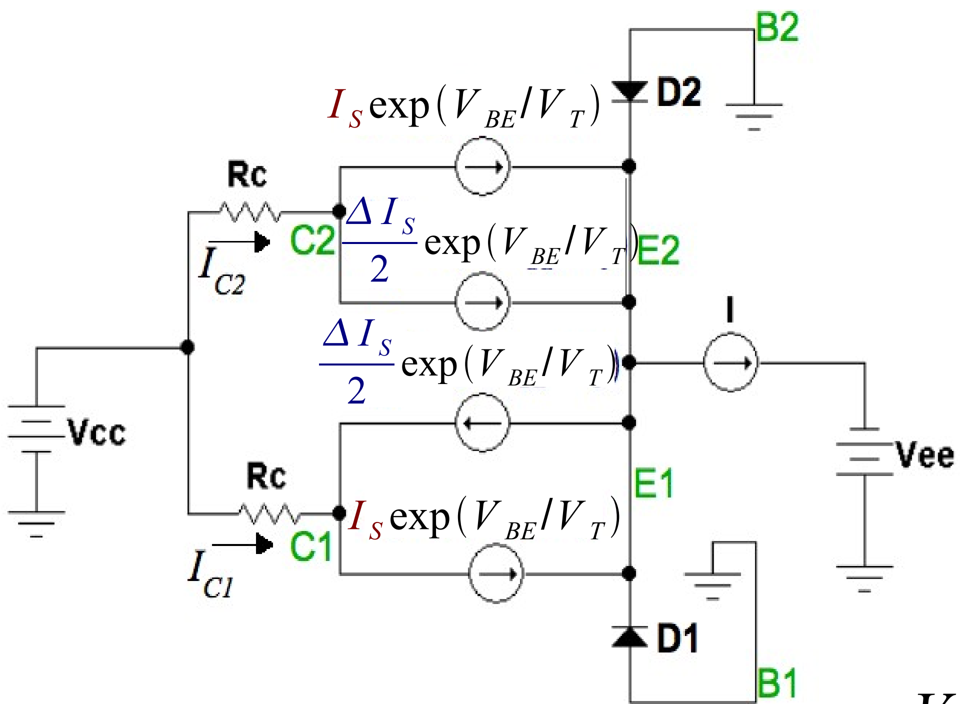
$$I_{C2} = I_S e^{\frac{V_{BE}}{V_T}} + \frac{\Delta I_S}{2} e^{\frac{V_{BE}}{V_T}} = I_S e^{\frac{V_{BE}}{V_T}} \left(1 + \frac{\Delta I_S}{2 I_S}\right)$$

$$= \alpha \frac{I}{2} \left(1 + \frac{\Delta I_S}{2 I_S}\right)$$

$$I_{C1} = I_S e^{\frac{V_{BE}}{V_T}} - \frac{\Delta I_S}{2} e^{\frac{V_{BE}}{V_T}} = I_S e^{\frac{V_{BE}}{V_T}} \left(1 - \frac{\Delta I_S}{2 I_S}\right)$$

$$= \alpha \frac{I}{2} \left(1 - \frac{\Delta I_S}{2 I_S}\right)$$

$$V_{O-dm} = R_C I_{C1} - R_C I_{C2} = -2 \alpha \frac{I}{2} \frac{\Delta I_S}{2 I_S} R_C = -\frac{\alpha I}{2} \frac{\Delta I_S}{I_S} R_C$$



$$V_{O-dm} = -\frac{\alpha I}{2}\frac{\Delta I_S}{I_S}R_C$$

$$V_{O-dm} = -\frac{\alpha I}{2}\frac{V_T}{V_T}\frac{\Delta I_S}{I_S}R_C$$

$$V_{O-dm} = -g_m V_T \frac{\Delta I_S}{I_S}R_C$$

$$V_{OS(\Delta I_S)} = \frac{V_{O-dm}}{A_{v-dm}} = \frac{V_{O-dm}}{g_m R_C} = V_T \frac{\Delta I_S}{I_S}$$

Recall:

$$g_m = \frac{I_C}{V_T} = \frac{\alpha I}{2}\frac{1}{V_T}$$

$$\boxed{V_{OS(\Delta I_S)} = V_T \frac{\Delta I_S}{I_S}}$$

random variable

Offset Voltage Summary

We considered two sources of offset voltage:

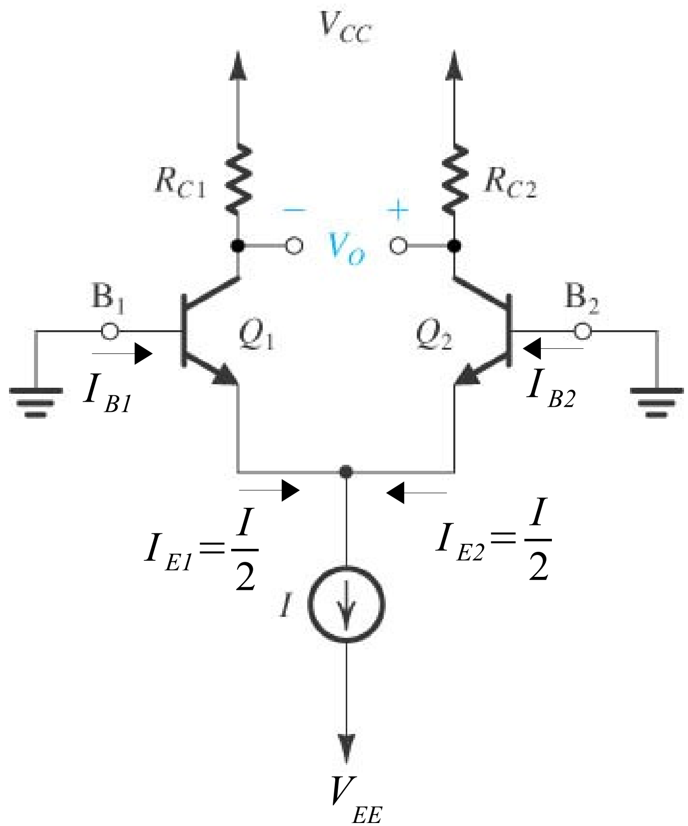
- Unbalanced collector resistors
- Unbalanced saturation currents
  - Mismatched transistor geometries

We ignored base or emitter circuit unbalance

Since the relationship between resistor and current unbalances are statistically random and assumed independent, we combine their effect as an rms quantity:

$$\boxed{V_{OS(rms)} = \sqrt{(V_{OS-\Delta R_C})^2 + (V_{OS-\Delta I_S})^2} = V_T \sqrt{\left(\frac{\Delta R_C}{R_C}\right)^2 + \left(\frac{\Delta I_S}{I_S}\right)^2}}$$

Average & Offset Base (input) Bias Currents



Consider the case where the base currents differ $I_{B1} \neq I_{B2}$.

Since I_{B1} & I_{B2} are related to bias current I, their mismatch is due to $\beta_1 \neq \beta_2$.

Let's use differential- common mode models for beta mismatch:

v_{i-cm} $\qquad$ $+v_{i-dm}/2$

$$\beta_1 = \beta + \boxed{\frac{\Delta\beta}{2}}$$

$-v_{i-dm}/2$

$$\beta_2 = \beta \boxed{-\frac{\Delta\beta}{2}}$$

Using this notation, the two base currents are:

$$I_{B1}=\frac{I}{2}\frac{1}{\beta_1+1}=\frac{I}{2}\frac{1}{\beta+\frac{\Delta\beta}{2}+1}=\frac{I}{2}\frac{1}{\beta+1}\frac{1}{(1+\frac{\Delta\beta}{2(\beta+1)})}$$

$$I_{B2}=\frac{I}{2}\frac{1}{\beta_2+1}=\frac{I}{2}\frac{1}{\beta-\frac{\Delta\beta}{2}+1}=\frac{I}{2}\frac{1}{\beta+1}\frac{1}{(1-\frac{\Delta\beta}{2(\beta+1)})}$$

nonlinear $f(\Delta\beta)$

For $x < 1$ we can expand the fraction as the series:

$$\frac{1}{1+x}=1-x+x^2-x^3+\ldots$$

where $$x=\pm\frac{\Delta\beta}{2(\beta+1)}\approx\frac{1}{2}\frac{\Delta\beta}{\beta}$$

Using the expansion and approximating $\beta+1\approx\beta$:

$$I_{B1}\approx\frac{I}{2}\frac{1}{\beta+1}\left(1-\frac{1}{2}\frac{\Delta\beta}{\beta}\right) \qquad I_{B2}\approx\frac{I}{2}\frac{1}{\beta+1}\left(1+\frac{1}{2}\frac{\Delta\beta}{\beta}\right)$$

$$\boxed{I_{OS}\equiv\left|I_{B1}-I_{B2}\right|=\frac{I}{\beta+1}\frac{\Delta\beta}{\beta}}$$ (input offset current)

Since the input bias current I_B is defined as: $I_B=\frac{I_{B1}+I_{B2}}{2}=\frac{I}{2(\beta+1)}$

The base or input offset current can also be written as:

$$\boxed{I_{OS(\Delta\beta)}=I_B\left(\frac{\Delta\beta}{\beta}\right)}$$