Feedback Basics

- Stability
- Feedback concept
- Feedback in emitter follower
- One-pole feedback and root locus
- Frequency dependent feedback and root locus
- Gain and phase margins
- Conditions for closed loop stability
- Frequency compensation

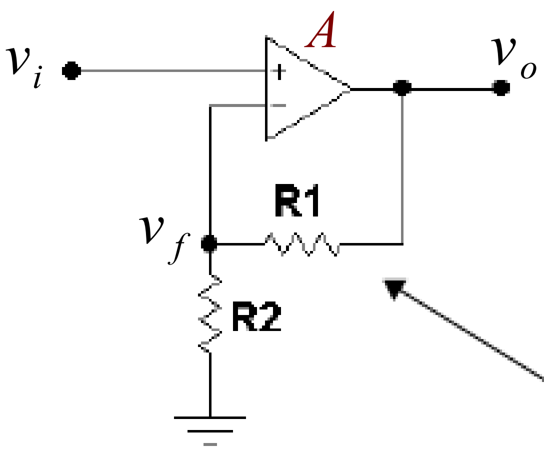
Circuit/System Stability

1. Simply put, a linear system such as an electronic circuit is "stable" if it has no poles (denominator roots or zeros) in the right-half of the complex s-plane.

2. Right-half plane poles correspond to growing exponential terms in the solution of the system/circuit differential equation.

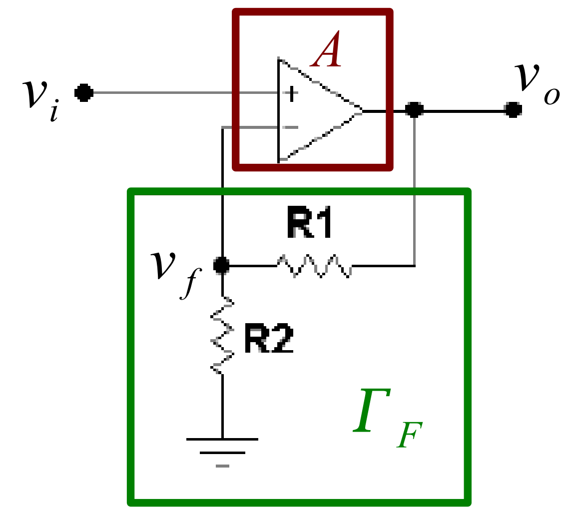
3. To determine stability, we need to check for RH plane poles. This requires either factoring the denominator polynomial or employing a number of other techniques developed to assist in the pencil and paper design of linear systems.

Operational Amplifier From Another View



$$v_o = A\left(v_i - \Gamma_F v_o\right)$$

$$\left(1 + A\,\Gamma_F\right) v_o = A\, v_i$$

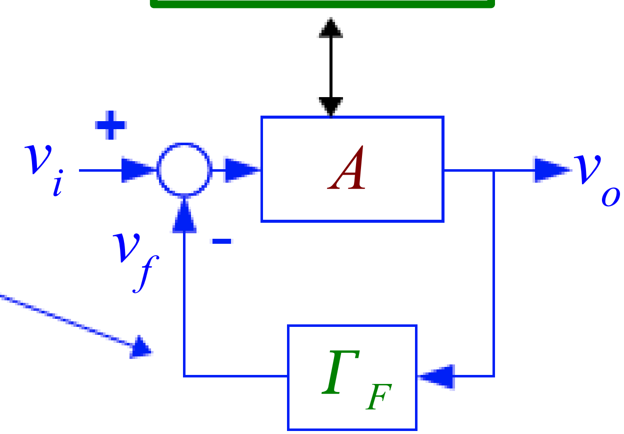


$$v_0 = A\left(v_i - v_f\right)$$

$$v_f = \frac{R_2}{R_1 + R_2} v_o = \Gamma_F v_o$$

$$v_o = \frac{A}{1 + A\,\Gamma_F} v_i$$

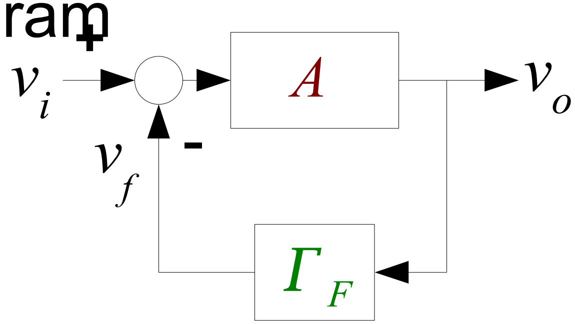
ideally $A = \infty \Rightarrow v_i = v_f$



classic feedback structure

Feedback Block Diagram Point of View

Fundamental block diagram



Fundamental feedback equation

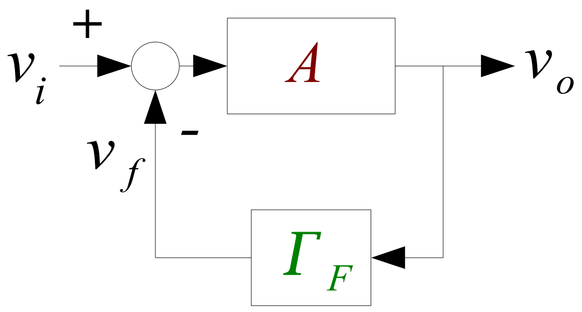
$$v_o = \frac{A}{1 + A\Gamma_F} v_i$$

We can split the ideal op amp A and resistive divider Γ_F into 2 separate blocks because they *do not interact with*, or "load," each other.

1. The op amp output is an ideal voltage source.

2. The op amp input draws no current.

Comments on the Feedback Equation



$$A_{cl} = \frac{v_o}{v_i} = \frac{A}{1 + A\Gamma_F}$$

$$A = \frac{v_o}{v_i}\Big|_{v_f = 0}$$

$$\Gamma_F = \frac{v_f}{v_o}$$

The quantity $A\Gamma_F$ = *loop-gain* and A_{cl} = *closed-loop gain*.

The quantity A = *open-loop gain* and Γ_F = *feedback gain*.

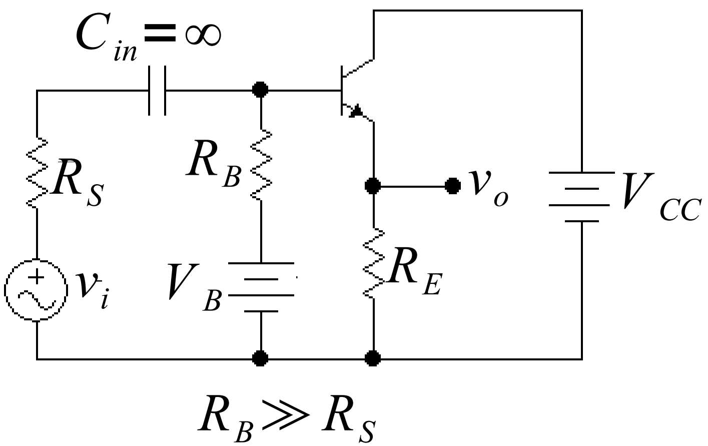
If the loop gain is much greater than one, i.e.

$$A\Gamma_F \gg 1$$

(and the system is stable – a topic to be discussed later!) the *closed-loop gain* approximates $1/\Gamma_F$, *and* is independent of "A"!

$$\boxed{A_{v-cl} = \frac{v_o}{v_i} \approx \frac{1}{\Gamma_F}} \qquad \Gamma_F = \frac{R_2}{R_1 + R_2} \Rightarrow \boxed{\frac{1}{\Gamma_F} = 1 + \frac{R_1}{R_2}}$$

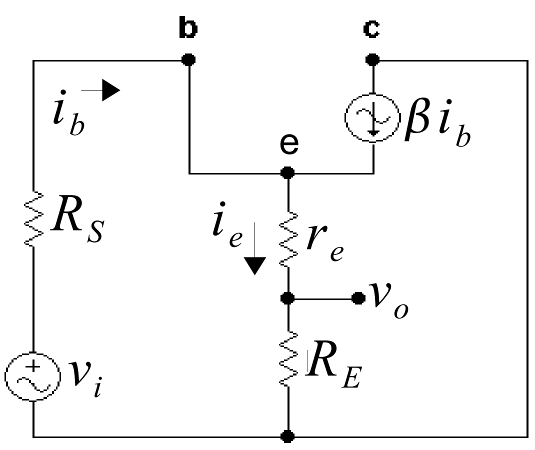
Feedback in the Emitter Follower



Small-signal ac emitter current equation:

$$i_e=\frac{1}{\frac{R_S}{\beta+1}+r_e+R_E}v_i$$

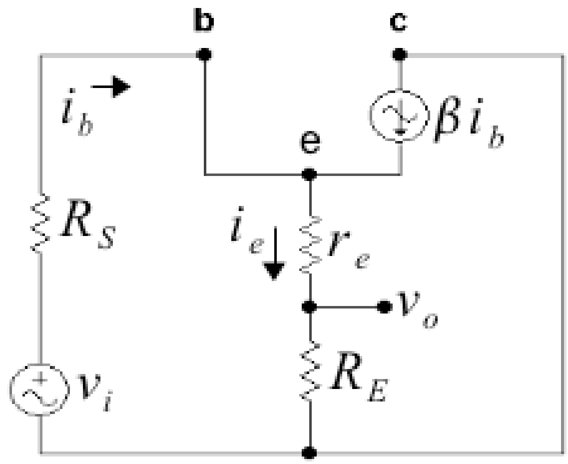
$$v_o=R_E i_e=\frac{R_E}{\frac{R_S}{\beta+1}+r_e+R_E}v_i$$



Create an "artificial" feedback equation, multiply "num" & "denom" by $(\beta+1)/R_S$ and multiply "denom" by R_E/R_E:

$$v_o=\frac{\frac{R_E(\beta+1)}{R_S}}{1+\frac{R_E(\beta+1)}{R_S}\frac{(r_e+R_E)}{R_E}}v_i=\frac{A}{1+A\Gamma_F}v_i$$

Emitter Follower Emitter Current



The forward gain open-loop term, A:

$$A=\frac{R_E(\beta+1)}{R_S}$$

The feedback term, Γ_F:

$$\Gamma_F=\frac{r_e+R_E}{R_E}$$

$$A\Gamma_F=\frac{R_E(\beta+1)}{R_S}\frac{r_e+R_E}{R_E}=(\beta+1)\frac{r_e+R_E}{R_s}\gg 1$$

$$v_o=\frac{\dfrac{R_E(\beta+1)}{R_S}}{1+\dfrac{R_E(\beta+1)}{R_S}\dfrac{(r_e+R_E)}{R_E}}v_i=\frac{A}{1+A\Gamma_F}v_i$$

If the "loop gain" is large, $A\Gamma_F\gg 1$

$$v_o\approx\frac{1}{\Gamma_F}v_i=\frac{R_E}{r_e+R_E}v_i$$

Dependence on β is eliminated.

Loop Gain Sensitivities For:

$$A_{cl}=\frac{A}{1+A\Gamma_F}$$

calculus $\frac{d\,f(x)g(x)}{d\,x}=g(x)\frac{d\,f(x)}{d\,x}+f(x)\frac{d\,g(x)}{d\,x}$

Open-loop gain:

$$\frac{dA_{cl}}{dA}=\frac{1}{(1+A\Gamma_F)^2}$$

$$\frac{dA_{cl}}{A_{cl}}=\frac{1}{(1+A\Gamma_F)^2}\frac{1+A\Gamma_F}{A}dA$$

$$\frac{dA_{cl}}{A_{cl}}=\frac{1}{(1+A\Gamma_F)}\frac{dA}{A}\underset{A\Gamma_F\to\infty}{\to}0$$

Feedback gain:

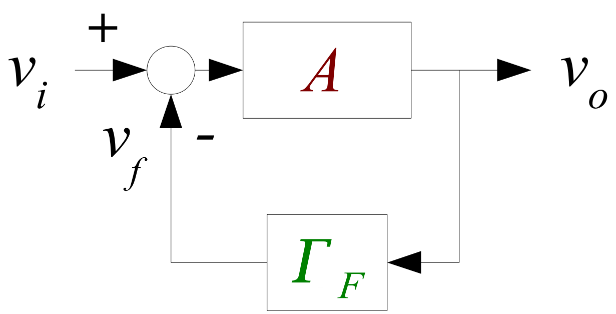
$$\frac{dA_{cl}}{d\,\Gamma_F}=\frac{-A^2}{(1+A\Gamma_F)^2}$$

$$\frac{dA_{cl}}{A_{cl}}=\frac{-A^2}{(1+A\Gamma_F)^2}\frac{1+A\Gamma_F}{A}d\,\Gamma_F$$

$$\frac{dA_{cl}}{A_{cl}}=\frac{-A\Gamma_F}{(1+A\Gamma_F)}\frac{d\,\Gamma_F}{\Gamma_F}\underset{A\Gamma_F\to\infty}{\to}-\frac{d\,\Gamma_F}{\Gamma_F}$$

High loop gain makes system insensitive to A, but sensitive to Γ_F !

Feedback - One-pole $A(s)$



$$A_{cl}(s,\Gamma_F)=\frac{v_o}{v_i}=\frac{A(s)}{1+A(s)\Gamma_F}$$

pole(s) of $A_{cl}(s)$ are the roots of $1+A\Gamma_F=0$, or

$$A(s)\Gamma_F=-1=1*e^{j\pi\pm 2k\pi}$$ for $k = 0, 1, \ldots$

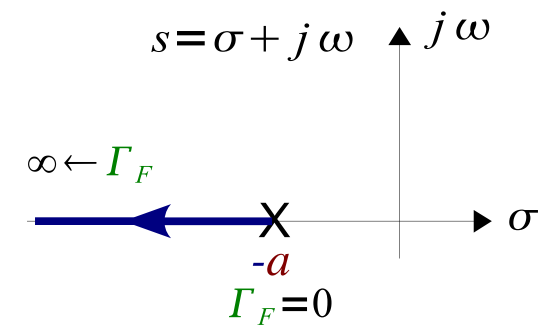
Consider the case where:

$$A(s)=\frac{aK_0}{s+a}$$ open-loop pole

$$A_{cl}(s,\Gamma_F)=\frac{\frac{aK_0}{s+a}}{1+\Gamma_F aK_0\frac{1}{s+a}}=\frac{aK_0}{s+a(1+\Gamma_F K_0)}$$

Where a, Γ_F and K_0 are positive real quantities.

Root Locus Plot for $A_{cl}(s,\Gamma_F)$



closed-loop pole

stable for all Γ_F!

Feedback - One-pole $A(s)$ cont.

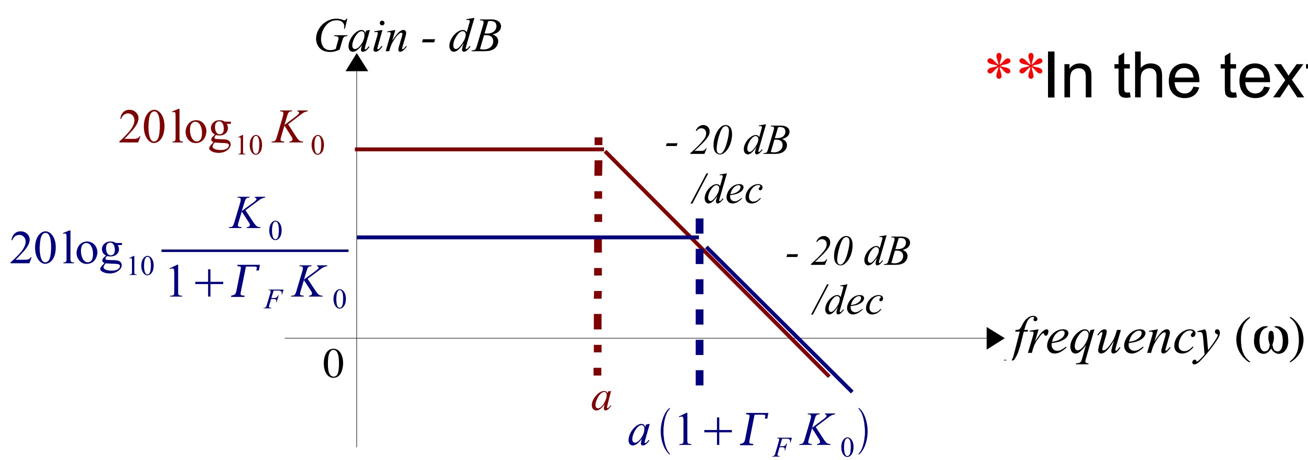
$$A(s)=\frac{aK_0}{s+a}$$

open-loop (OL)

$$A_{cl}(s)=\frac{aK_0}{s+a(1+\Gamma_F K_0)}=\frac{N(s)}{D(s)}$$

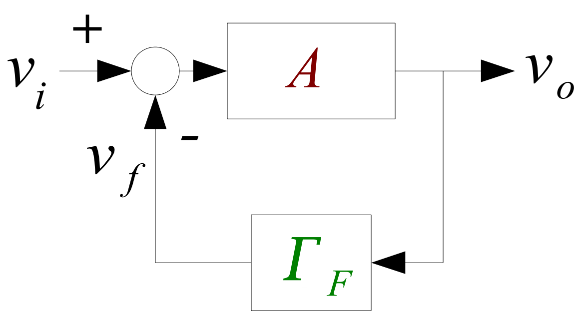
closed-loop (CL)

In the text $\beta=\Gamma_F$



$$GBW_{OL}=GBW_{CL}=aK_0$$

Quick Review



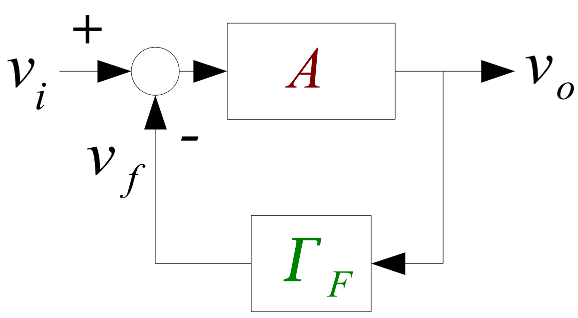
The quantity A = ? and Γ_F = ?

The *loop-gain* = ?

The *closed-loop gain* = ?

If the *loop gain >> 1* what is the closed-loop gain ?

Quick Review



The quantity A = *open-loop gain* and Γ_F = feedback gain.

$$A = \frac{v_o}{v_i}\Big|_{v_f=0} \qquad \Gamma_F = \frac{v_f}{v_o}$$

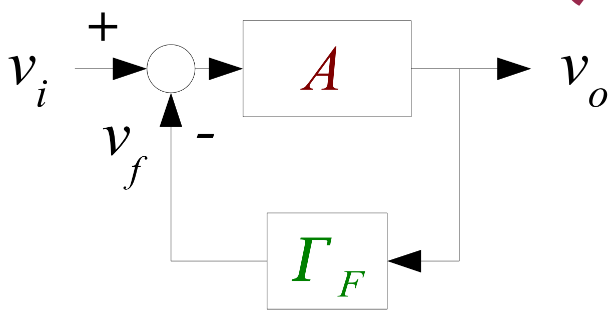
The *loop-gain* = $A\Gamma_F$

The *closed-loop gain* = $A_{cl} = \frac{v_o}{v_i} = \frac{A}{1 + A\Gamma_F}$

If the *loop gain >>* 1 what is the closed-loop gain ?

$$A_{v-cl} = \frac{v_o}{v_i} \approx \frac{1}{\Gamma_F} \qquad A\Gamma_F \gg 1$$

Quick Review



$$A_{cl}(s, \Gamma_F) = \frac{v_o}{v_i} = \frac{A(s)}{1 + A(s)\Gamma_F}$$

$$A(s) = \frac{a K_0}{s + a}$$

a, K_0 and Γ_F are positive real.

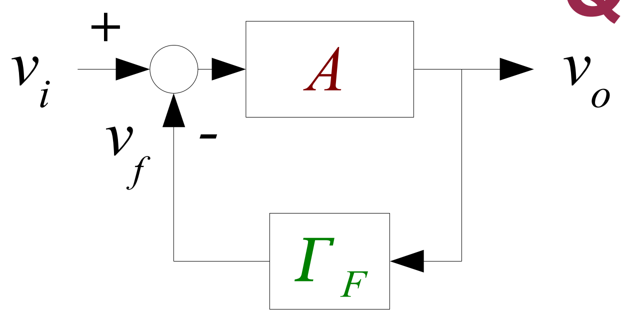
QUESTIONS:

1. $A_{cl}(s, \Gamma_F)$?

2. What is a test of the stability of $A_{cl}(s, \Gamma_F)$?

3. Is $A_{cl}(s, \Gamma_F)$ stable? absolutely stable?

Quick Review



$$A_{cl}(s,\Gamma_F)=\frac{v_o}{v_i}=\frac{A(s)}{1+A(s)\Gamma_F}$$

$$A(s)=\frac{aK_0}{s+a}$$

a, K_0 and Γ_F are positive real.

QUESTIONS:

1. $A_{cl}(s, \Gamma_F)$?

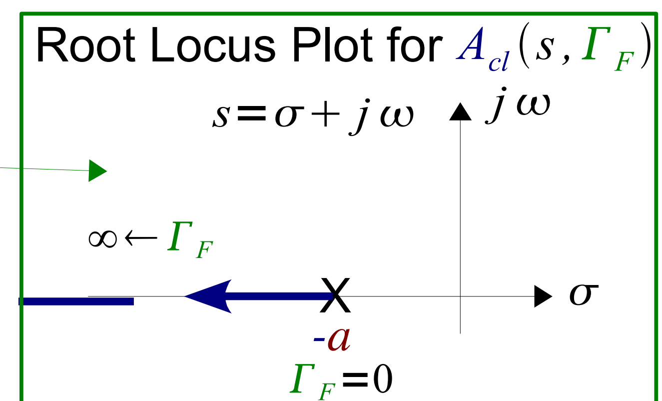
$$A_{cl}(s,\Gamma_F)=\frac{\frac{aK_0}{s+a}}{1+\Gamma_F a K_0\frac{1}{s+a}}=\frac{aK_0}{s+a(1+\Gamma_F K_0)}$$

2. What is a test of the stability of $A_{cl}(s, \Gamma_F)$?

Poles of $A_{cl}(s, \Gamma_F)$ in LHP for each or all Γ_F.

3. Is $A_{cl}(s, \Gamma_F)$ stable? Yes;

absolutely stable? Yes



One-pole Feedback - Root Locus

$$A(s)=\frac{aK_0}{s+a} \qquad A_{cl}(s)=\frac{N(s)}{D(s)}=\frac{A(s)}{1+A(s)\Gamma_F}=\frac{aK_0}{s+a(1+\Gamma_F K_0)}$$

Pole of $A_{cl}(s)$: $D(s)=1+A(s)\Gamma_F=s+a(1+\Gamma_F K_0)=0$ => $s=-a(1+\Gamma_F K_0)$

ALTERNATIVELY

$$s+a(1+\Gamma_F K_0)=0 \Rightarrow \frac{a\Gamma_F K_0}{s+a}=-1=1*e^{j\pi\pm 2k\pi}$$ for $k = 0, 1, \ldots$

$s = r_1$ is a root of $D(s) = 0$ or $A(s)\Gamma_F = -1$ iff

$$\phi_{(r_1+a)}=\pi\pm 2k\pi$$

and

$$\frac{a\Gamma_{F1}K_0}{|r_1+a|}=1$$

for p.r. a, K_0 and Γ_{F1}

Root locus algorithm for 1-pole Amplifier

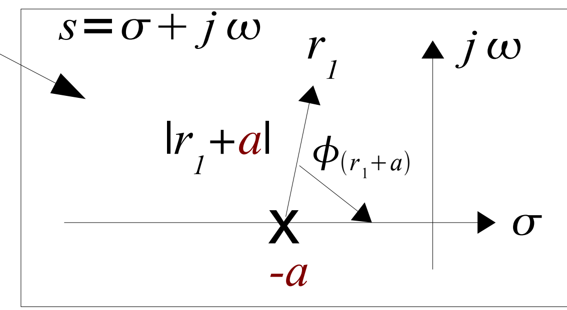
a, K_0 and Γ_F being positive-real => r_1 is positive-real

One-pole Feedback - Root Locus

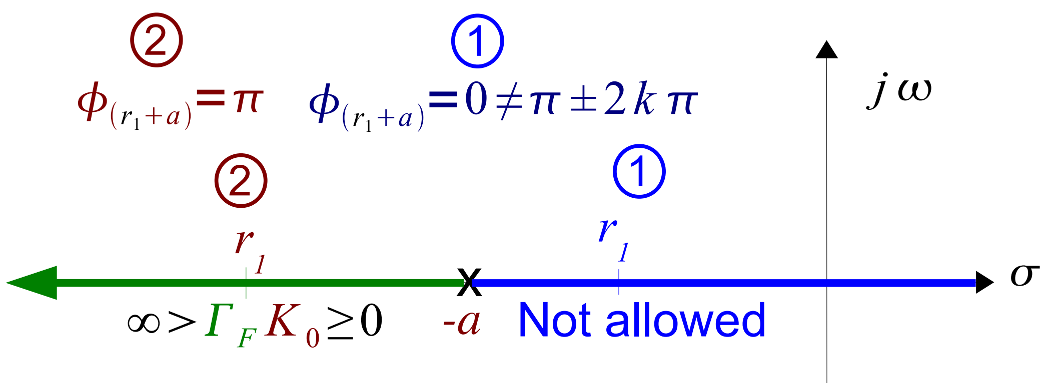
$$A_{cl}(s)=\frac{N(s)}{D(s)}=\frac{A(s)}{1+A(s)\Gamma_F}=\frac{aK_0}{s+a(1+\Gamma_F K_0)}$$

$s=r_1$ is a root of $D(s)=0$ or $A(s)\Gamma_F=-1$ iff

$$\phi_{(r_1+a)}=\pi\pm 2k\pi \quad \text{and} \quad \frac{a\Gamma_{F1}K_0}{|r_1+a|}=1$$



a, K_0 and Γ_F being positive-real => r_1 is positive-real



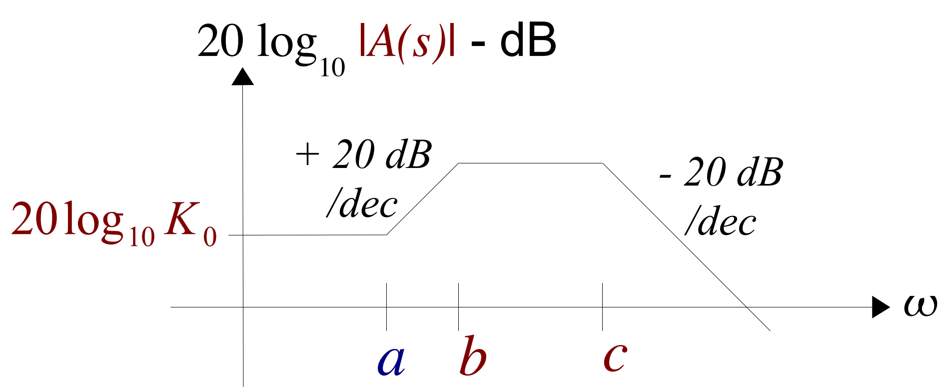
Frequency-Dependent Feedback

Consider the case where the open-loop gain is:

$$A(s)=K\frac{(s+a)}{(s+b)(s+c)} \qquad a<b<c$$

dc Gain: $A(s)_{s=0}=K\frac{(0+a)}{(0+b)(0+c)}=K\frac{a}{bc}=K_0$

A sketch of the Bode plot would look something like:



open-loop (OL)

$$A(s)=K\frac{(s+a)}{(s+b)(s+c)}$$

$$a<b<c$$

Frequency-Dependent Feedback

$$A_{cl}(s) = \frac{A(s)}{1+\Gamma_F A(s)} = \frac{K_0 \frac{(s+a)}{(s+b)(s+c)}}{1+\Gamma_F K_0 \frac{(s+a)}{(s+b)(s+c)}}$$

where a, b, c, K_0 & Γ_F are p.r.

Rationalizing this expression leads to:

$$A_{cl}(s) = \frac{K_0(s+a)}{(s+b)(s+c)+\Gamma_F K_0(s+a)} = \frac{N(s)}{D(s)}$$

The numerator is factored, but the denominator is not. We have a new quadratic polynomial $D(s)$ for $A_{cl}(s)$. $D(s)$ can be factored using the quadratic formula.

$$D(s) = (s+b)(s+c)+\Gamma_F K_0(s+a) = (s+r_1)(s+r_2) = 0$$

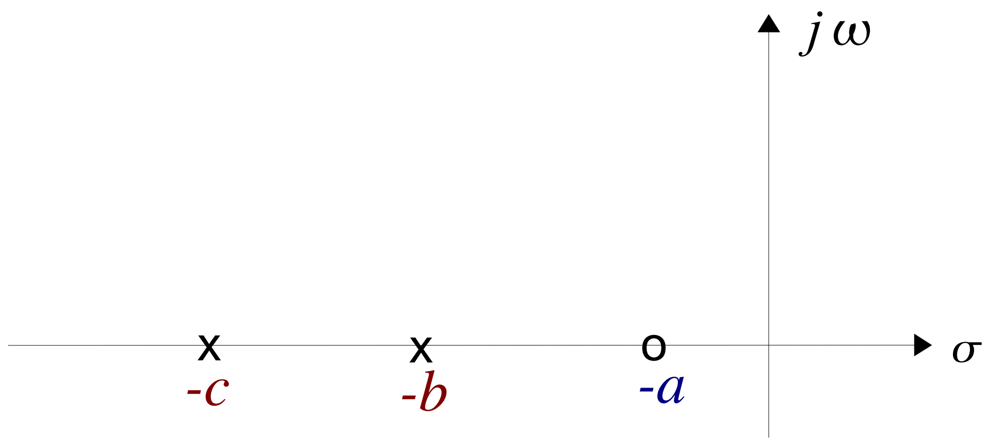
where r_1 and r_2 are either real or complex-conjugate roots

What's the Root Locus for $A_{cl}(s)$?

Is $A_{cl}(s)$ Stable for all Γ_F?

$$A_{cl}(s)=\frac{K_0\dfrac{(s+a)}{(s+b)(s+c)}}{1+\Gamma_F K_0\dfrac{(s+a)}{(s+b)(s+c)}}=K_0\frac{(s+a)}{(s+b)(s+c)+\Gamma_F K_0(s+a)} \qquad a<b<c$$

for p.r. a, b, c, K_0 and Γ_F



What's the Root Locus for $A_{cl}(s)$?

Is $A_{cl}(s)$ Stable for all Γ_F?

$$(s+b)(s+c)+\Gamma_F K_0 (s+a)=0$$

Since the poles of $A_{cl}(s)$ will not equal any of the poles of $A(s)$, we can divide by $(s+b)(s+c)$ and obtain:

$$1+\boxed{\Gamma_F K_0 \frac{(s+a)}{(s+b)(s+c)}}=0$$

loop-gain

$$\Gamma_F K_0 \frac{(s+a)}{(s+b)(s+c)}=\Gamma_F K_0 f(s)=-1=1*e^{j\pi \pm 2k\pi} \quad \text{for } k=0, 1, \ldots$$

The "loop-gain" terms, Γ_F and K_0, are positive-real numbers, so for a root $s = r_i$ to exist, the value of $f(s)$ must be negative-real when evaluated at $s = r_i$, where r_i is in general complex.

What's the Root Locus for $A_{cl}(s)$?

Is $A_{cl}(s)$ Stable for all Γ_F?

$$\Gamma_F K_0 \frac{(s+a)}{(s+b)(s+c)} = -1 = 1 * e^{j\pi \pm 2k\pi}$$ where $k = 0, 1, \ldots$

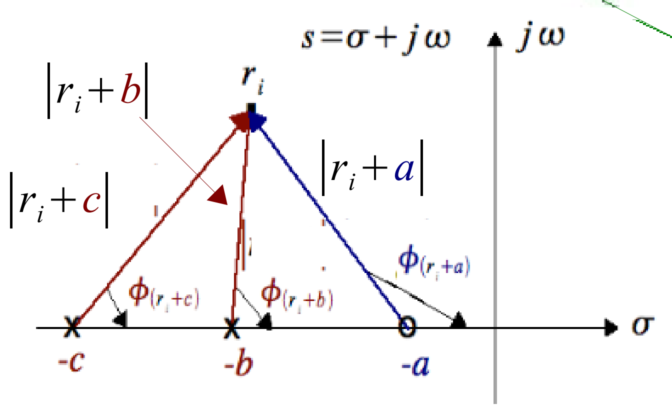
Working with the complex numbers in polar form:

$$\phi_{(r_i+a)} - \phi_{(r_i+b)} + \phi_{(r_i+c)} = \pi \pm 2k\pi$$

and

$$\frac{\Gamma_{FI}\, a K_0 |r_i+a|}{|r_i+b||r_i+c|} = 1$$

for p.r. a, b, c, K_0 and Γ_F



To test a potential root at say $s = r_i$, we first add the angles for zeros to the point "r_i" in the complex s-plane and subtract the angles for poles (open-loop).

What's the Root Locus for $A_{cl}(s)$?

Is $A_{cl}(s)$ Stable for all Γ_F?

Only $D(s)$ root locations where the angles with respect to open-loop pole/zero locations of $A(s)$ are odd multiples of $\pm 180^o$ are candidates.

$$D(s=-r_i) = (r_i + b)(r_i + c) + \Gamma_F (r_i + a) = 0 \qquad \text{for } i = 1, 2$$

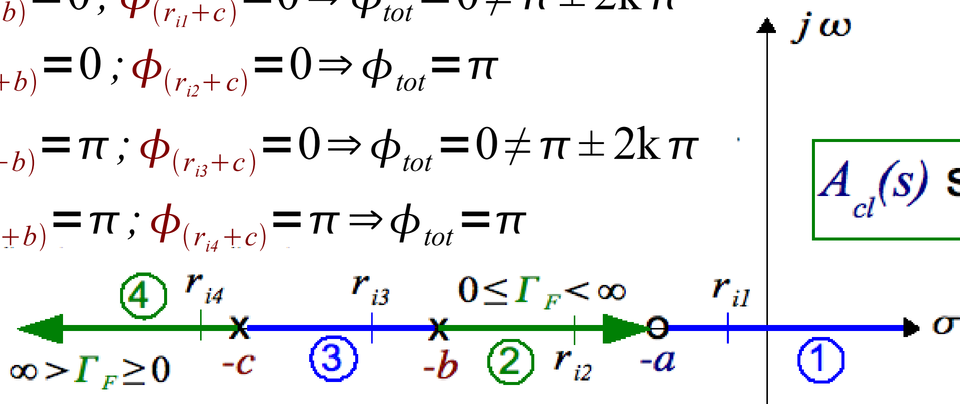
$$\phi_{tot} = \phi_{(r_i+a)} - \phi_{(r_i+b)} - \phi_{(r_i+c)} = \pi \pm 2k\pi$$

① $\phi_{(r_{i1}+a)} = 0 ; \phi_{(r_{i1}+b)} = 0 ; \phi_{(r_{i1}+c)} = 0 \Rightarrow \phi_{tot} = 0 \neq \pi \pm 2k\pi$

② $\phi_{(r_{i2}+a)} = \pi ; \phi_{(r_{i2}+b)} = 0 ; \phi_{(r_{i2}+c)} = 0 \Rightarrow \phi_{tot} = \pi$

③ $\phi_{(r_{i3}+a)} = \pi ; \phi_{(r_{i3}+b)} = \pi ; \phi_{(r_{i3}+c)} = 0 \Rightarrow \phi_{tot} = 0 \neq \pi \pm 2k\pi$

④ $\phi_{(r_{i4}+a)} = \pi ; \phi_{(r_{i4}+b)} = \pi ; \phi_{(r_{i4}+c)} = \pi \Rightarrow \phi_{tot} = \pi$



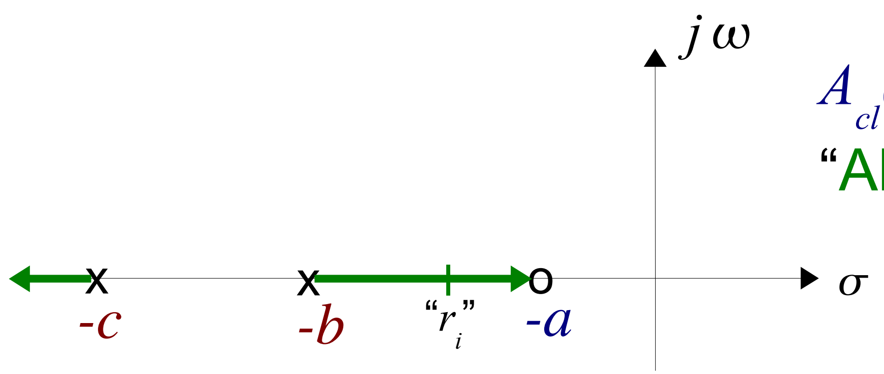
$A_{cl}(s)$ stable for all Γ_F!

1. All roots of $D(s)$ must lie along the "green" loci.
2. Bandwidth of $A_{cl}(s)$ > bandwidth of $A(s)$
3. If an $r_i = -a$ there is a zero/pole cancellation

What's the Root Locus for $A_{cl}(s)$?

Is $A_{cl}(s)$ Stable for all Γ_F?

$$A_{cl}(s)=\frac{K_0\dfrac{(s+a)}{(s+b)(s+c)}}{1+\Gamma_F K_0\dfrac{(s+a)}{(s+b)(s+c)}}=K_0\frac{(s+a)}{(s+b)(s+c)+\Gamma_F K_0(s+a)} \qquad a<b<c$$



$A_{cl}(s)$ <u>IS</u> stable for all Γ_F!
"ABSOLUTELY STABLE"

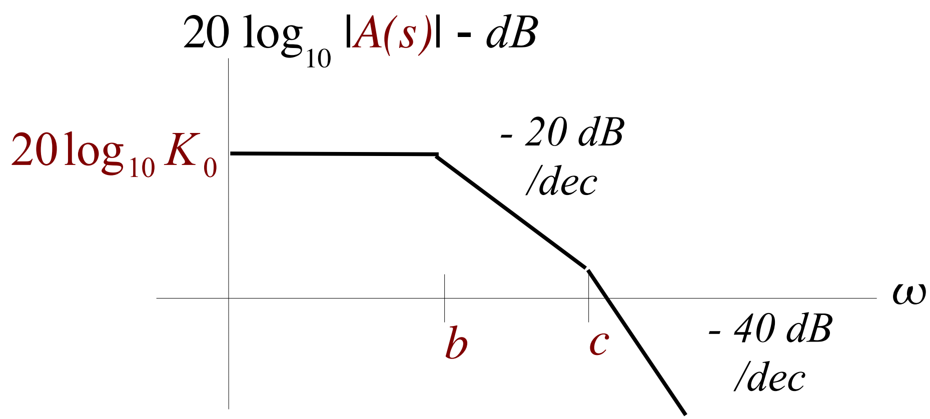
1. zeros of $A_{cl}(s)$ = zeros of $A(s)$ and are independent of feedback
2. poles of $A_{cl}(s)$ ≠ poles of $A(s)$ are a function of the feedback

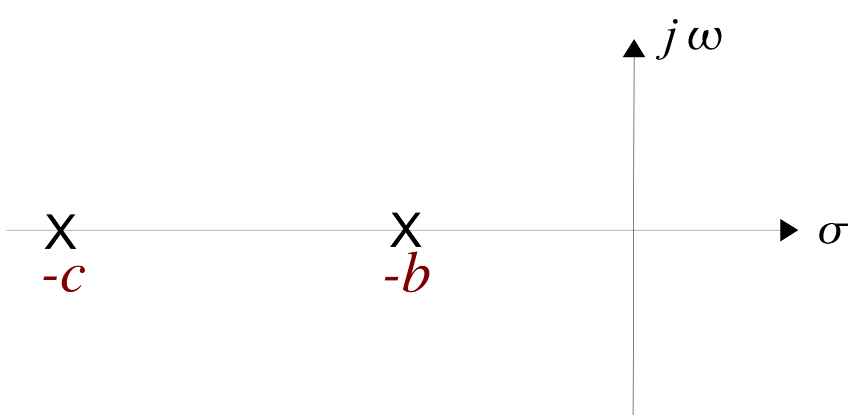
What's the Root Locus for $A_{cl}(s)$?

Is $A_{cl}(s)$ Stable for all Γ_F?

$$A(s)=\frac{K_0 bc}{(s+b)(s+c)}$$ two-pole $b<c$

$$A_{cl}(s)=\frac{K_0 bc}{(s+b)(s+c)+\Gamma_F K_0 bc}=\frac{N(s)}{D(s)}$$



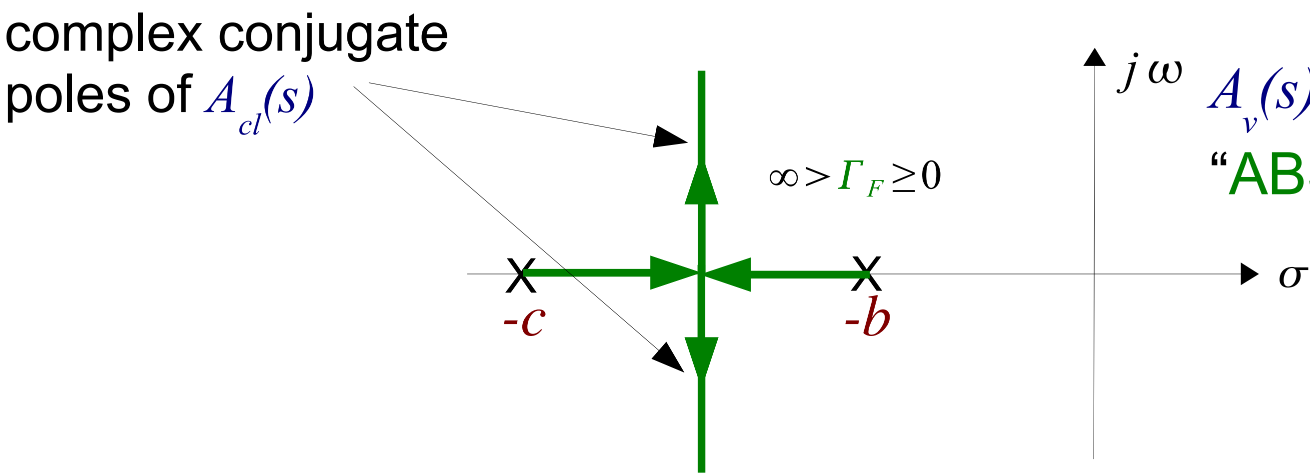


What's the Root Locus for $A_{cl}(s)$?

Is $A_{cl}(s)$ Stable for all Γ_F?

$$A(s)=\frac{K_0 bc}{(s+b)(s+c)} \qquad A_{cl}(s)=\frac{K_0 bc}{(s+b)(s+c)+\Gamma_F K_0 bc}=\frac{N(s)}{D(s)} \qquad b<c$$

where b, c, K_0 & Γ_F are p.r.



$A_v(s)$ IS stable for all Γ_F!
"ABSOLUTELY STABLE"

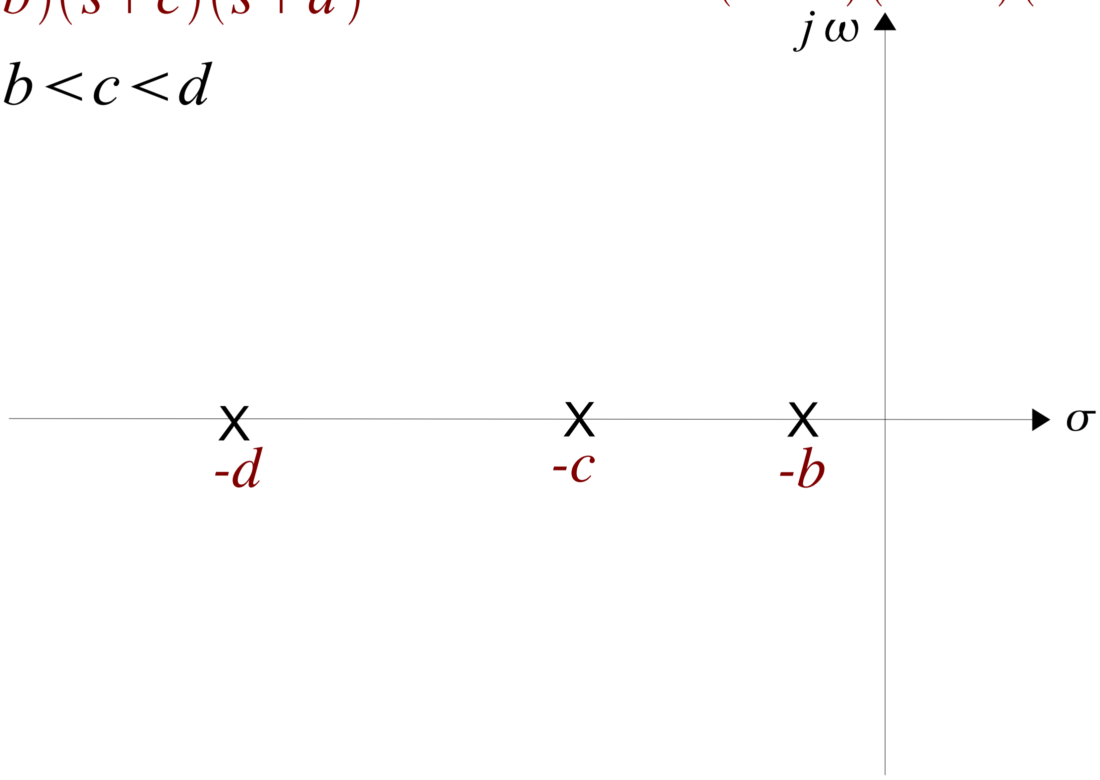
What's the Root Locus for $A_{cl}(s)$?

Is $A_{cl}(s)$ Stable for all Γ_F?

$$A(s)=\frac{K_0 bcd}{(s+b)(s+c)(s+d)}$$

$$b<c<d$$

$$A_{cl}(s)=\frac{K_0 bcd}{(s+b)(s+c)(s+d)+\Gamma_F K_0 bcd}=\frac{N(s)}{D(s)}$$



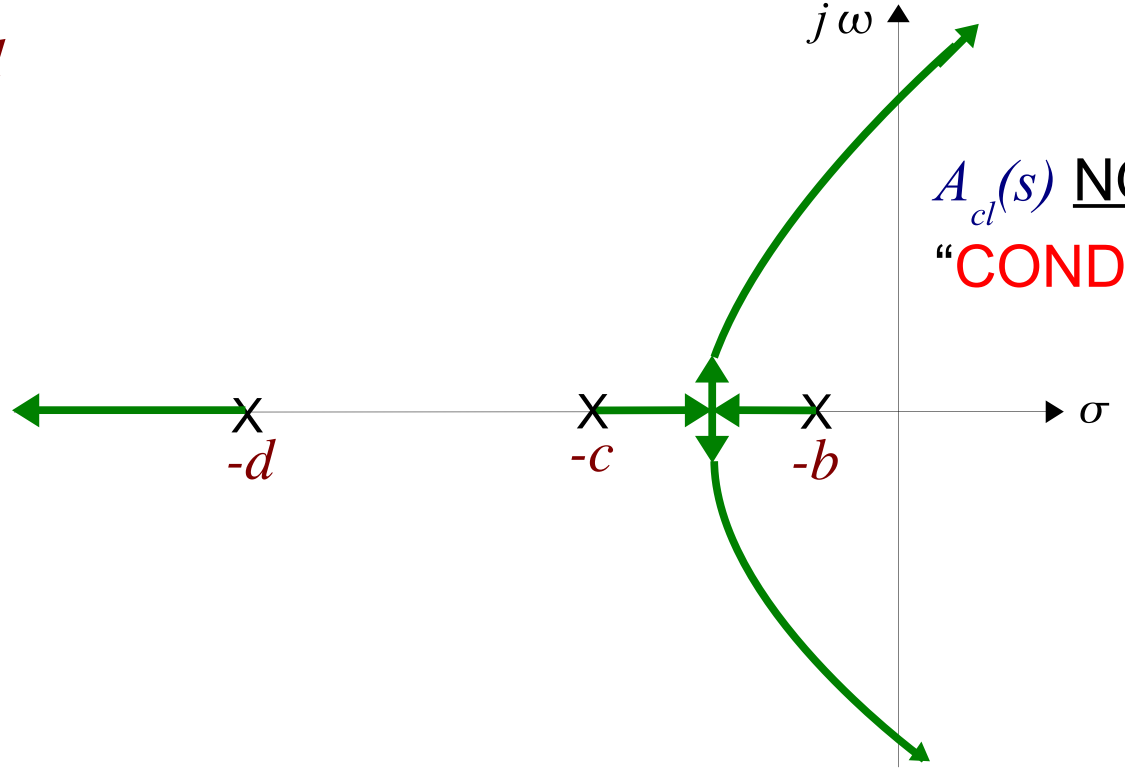
What's the Root Locus for $A_{cl}(s)$?

Is $A_{cl}(s)$ Stable for all Γ_F?

$$A(s)=\frac{K_0 bcd}{(s+b)(s+c)(s+d)}$$

$$A_{cl}(s)=\frac{K_0 bcd}{(s+b)(s+c)(s+d)+\Gamma_F K_0 bcd}=\frac{N(s)}{D(s)}$$

$$b<c<d$$



$A_{cl}(s)$ NOT stable for all Γ_F!

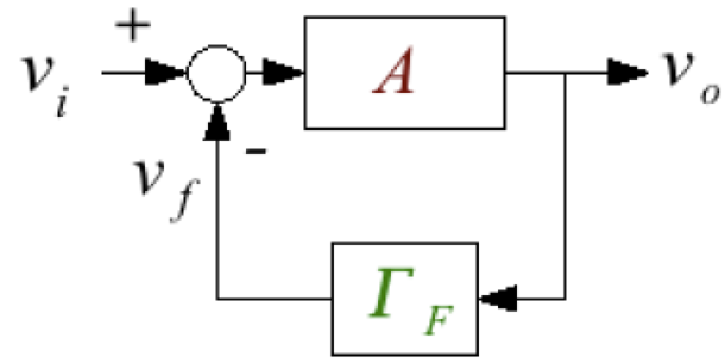
"CONDITIONALLY STABLE"

The Root Locus Method

This graphical method for finding the roots of a polynomial is known as the *root locus method*. It was developed before computers were available. It is still used because it gives valuable insight into the behavior of feedback systems as the loop gain is varied. Matlab (control systems toolbox) will plot root loci.

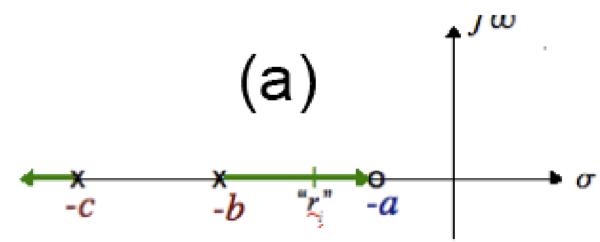
In the frequency-dependent feedback (two-pole & one-zero) example, we noted that increasing feedback increases the CL bandwidth – i.e. the low & high frequency break points moved in opposite directions as Γ_F increases. Higher Γ_F, as a trade-off, reduces the closed-loop mid-band gain.

Quick Review

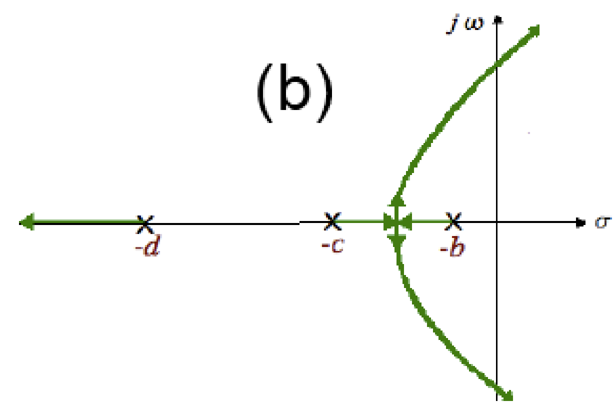


1. $s = r_i$ is a pole of $A_{cl}(s)$ iff ?

2. Let $s = r_i$ is a pole of $A_{cl}(s)$. What insight does the root-locus provide?



3. What can be concluded from the root-locus (a); from root-locus (b)?



Quick Review

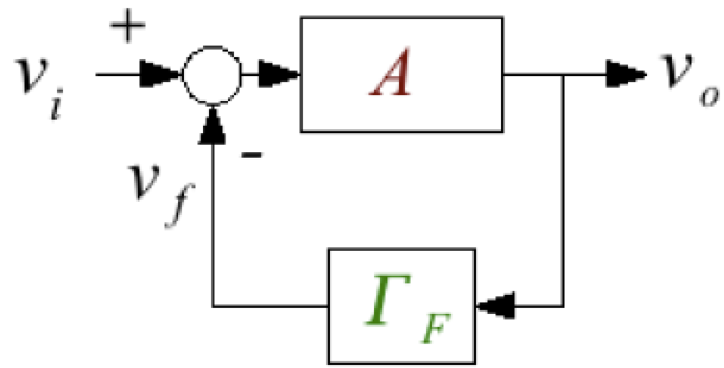
1. $s = r_i$ is a pole of $A_{cl}(s)$ iff ?

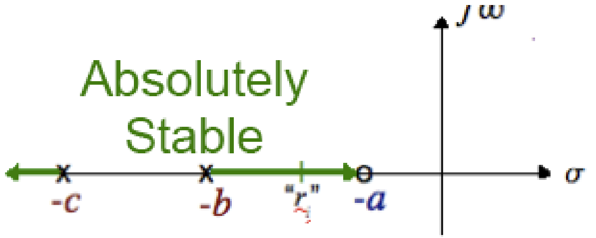
$$Loop\text{-}gain = A(s)\Gamma_F\Big|_{s=r_i} = -1 = 1 * e^{j\pi \pm 2k\pi}$$

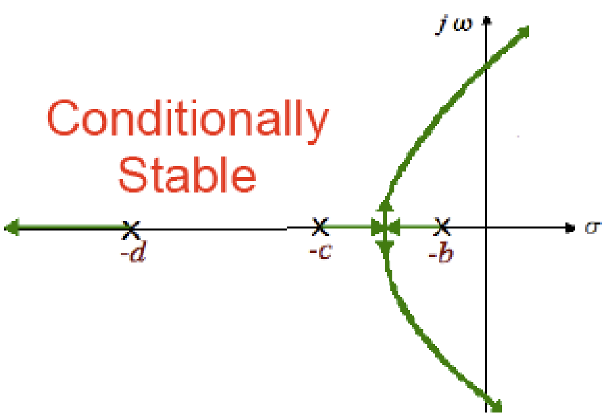
2. Let $s = r_i$ is a pole of $A_{cl}(s)$. What insight does the root-locus provide?

The trajectories of this and all other poles of $A_{cl}(s)$ for values of $0 \le \Gamma_F < \infty$. Stability for any or all values of Γ_F.

3. What can be concluded from the root-locus (a); from root locus (b)?



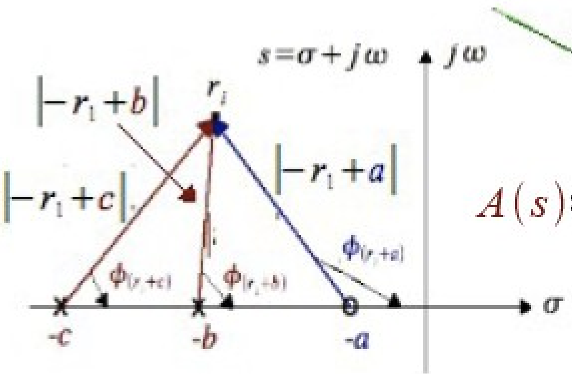




Stability – Gain and Phase Margins

$s = r_i$ is a root of $A_{cl}(s)$ iff

$$A(s)\Gamma_F\Big|_{s=r_i} = -1 = 1 * e^{j\pi \pm 2k\pi}$$



$$A(s) = K_0 \frac{(s+a)}{(s+b)(s+c)}$$

If the root $s = r_i$ is a stable root, what is the margin for the stability?

- Root-Locus provides qualitative insight, but little if any quantitative information.
- Designers need both!
- Root-locus is tedious to determine in general, i.e. one must know the location of all open-loop poles and zeros.

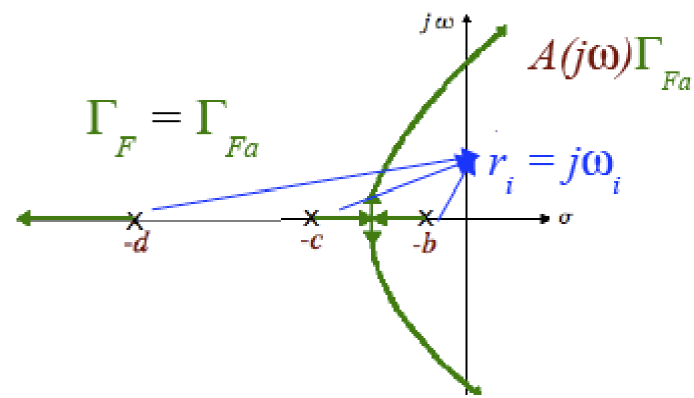
Stability – Gain and Phase Margins

Stability can also be determined quantitatively by plotting the gain and phase of the loop-gain = $A(s=j\omega_i)\Gamma_F$, i.e.

$$A_{cl}(j\omega)=\frac{V_o(j\omega)}{V_i(j\omega)}=\frac{A(j\omega)}{1+A(j\omega)\Gamma_F}$$

Let's test for roots on the $j\omega$ axis, i.e. $r_i=j\omega_i$

$A(j\omega_k)\Gamma_F=-1=1e^{j\pi\pm 2k\pi}$ => oscillation or instability



Does $A(j\omega_i)\Gamma_F=1e^{j\pi\pm 2k\pi}$ for any values of ω_i?

$A(j\omega_i)\Gamma_F$ for particular $\Gamma_F=\Gamma_{Fa}$ and all ω_i is obtained from theory, sim or experiment.

Stability - Gain and Phase Margins

Since $A(j\omega_k)\Gamma_F = -1 = 1\,e^{j\pi \pm 2k\pi}$ => instability

stable close-loop amplifier => $|A(j\omega_{180})|\Gamma_{Fa} < 1$

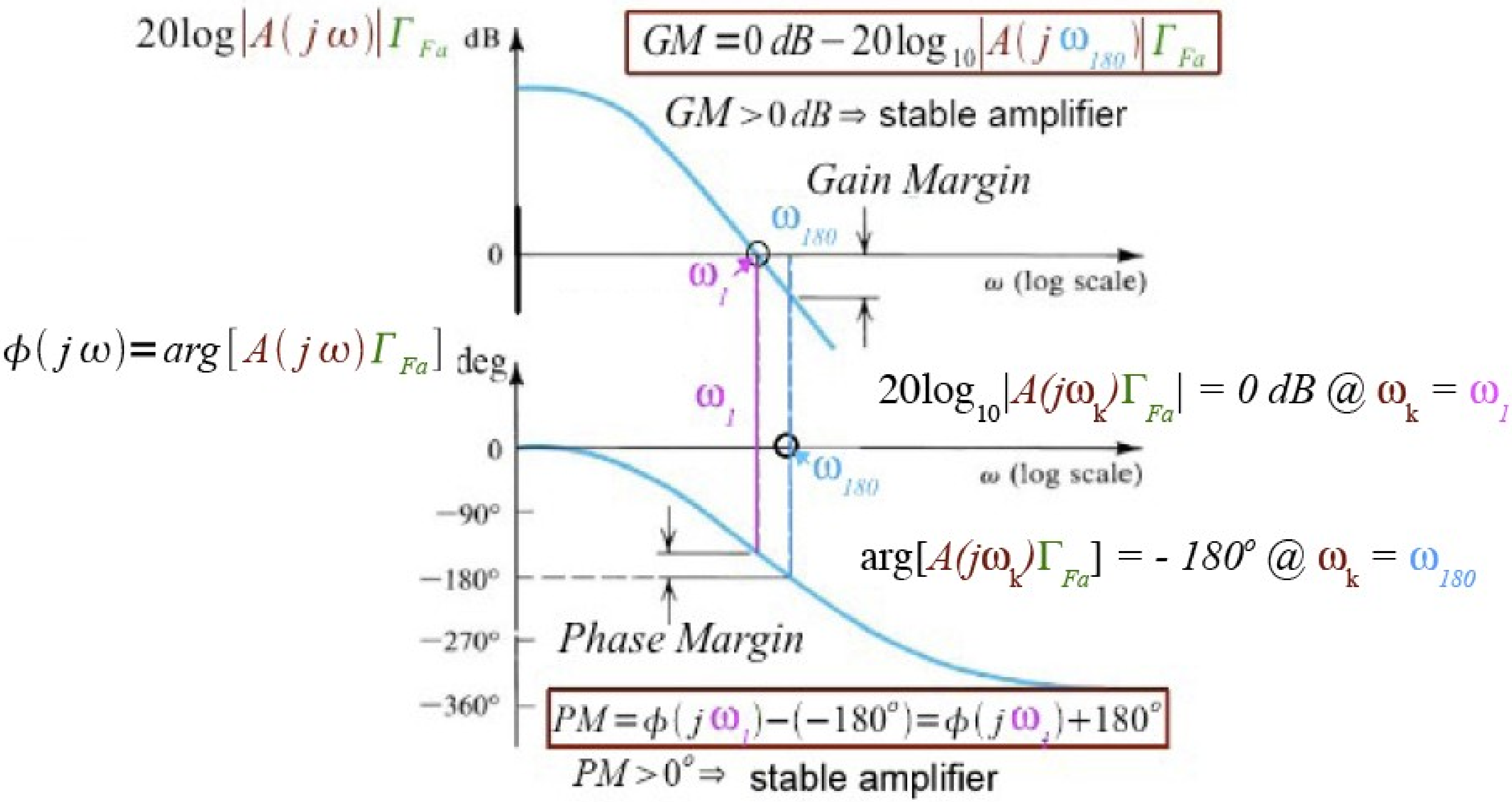
$$20\log|A(j\omega_{180})|\Gamma_{Fa} < 0\,dB \Rightarrow GM > 0\,dB$$
$$\phi(\omega_1) = arg[A(j\omega_1)\Gamma_{Fa}] > -180^o \Rightarrow PM > 0^o$$

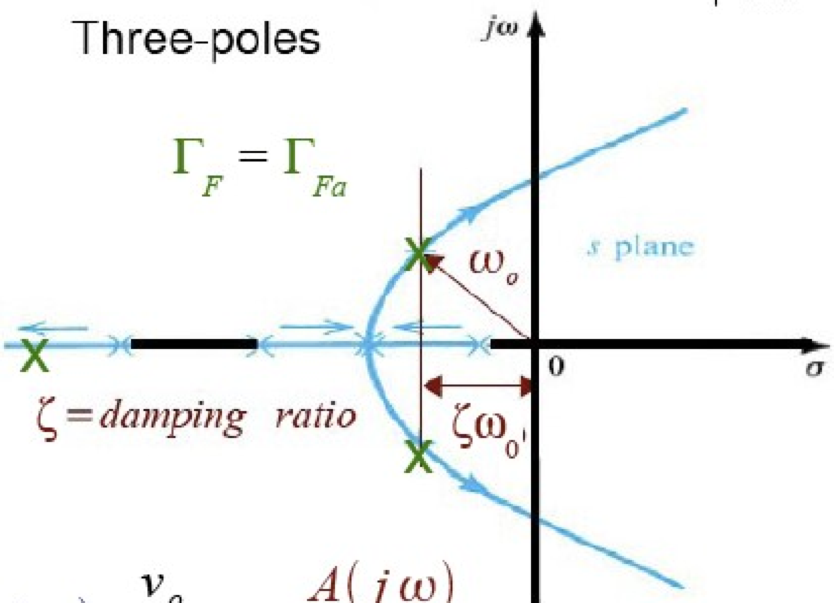
mutually consistent stability conditions

$\arg[A(j\omega_k)\Gamma_{Fa}] = -180^o$ @ $\omega_k = \omega_{180}$ 180^o phase-crossover freq.

$20\log_{10}|A(j\omega_k)\Gamma_{Fa}| = 0\,dB$ @ $\omega_k = \omega_1$ Unity gain-crossover freq.

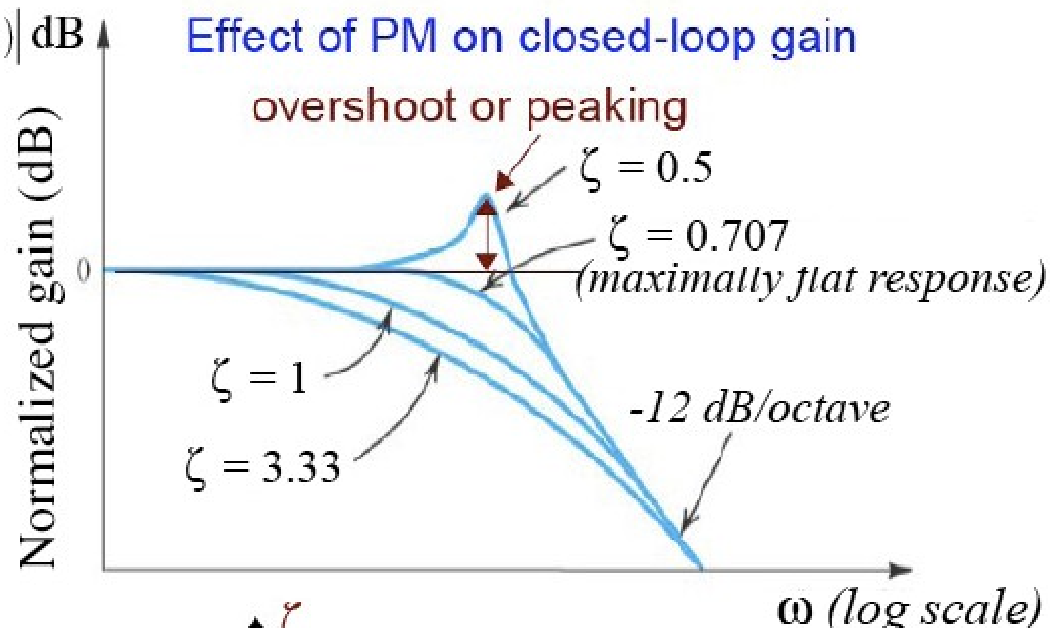
Stability - Gain and Phase Margins



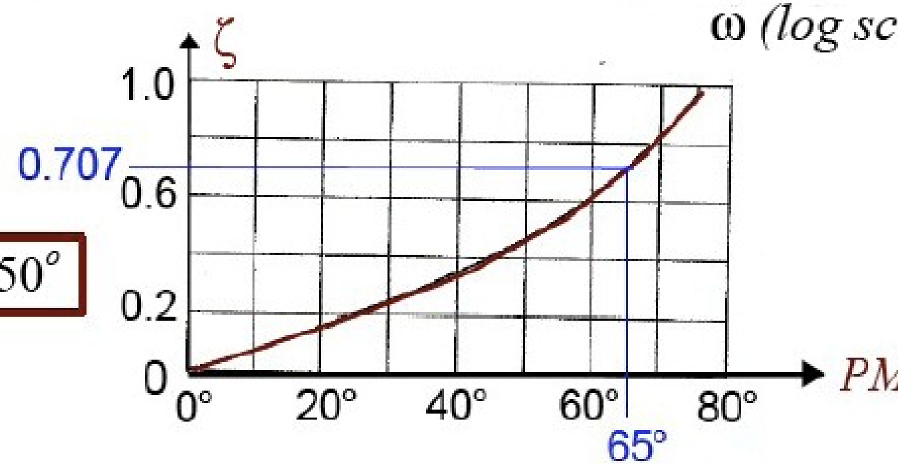


$$A_{cl}(j\omega) = \frac{v_o}{v_i} = \frac{A(j\omega)}{1 + A(j\omega)\Gamma_F}$$

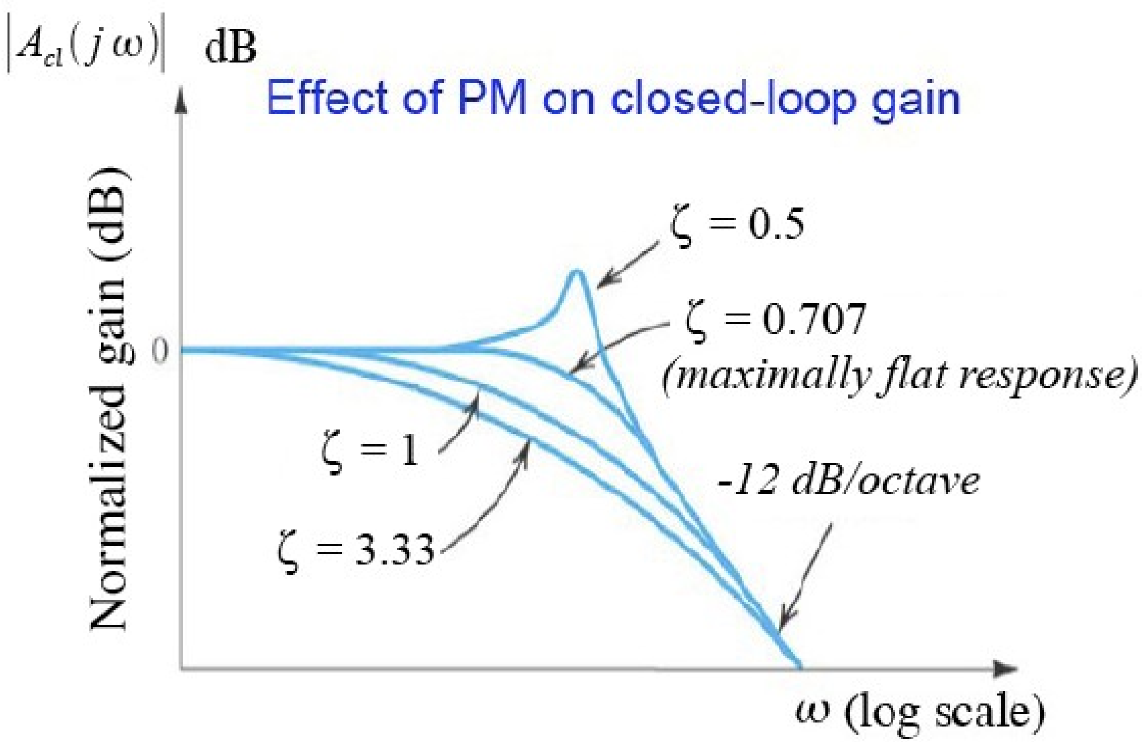
$$\zeta = \frac{1}{2Q}$$



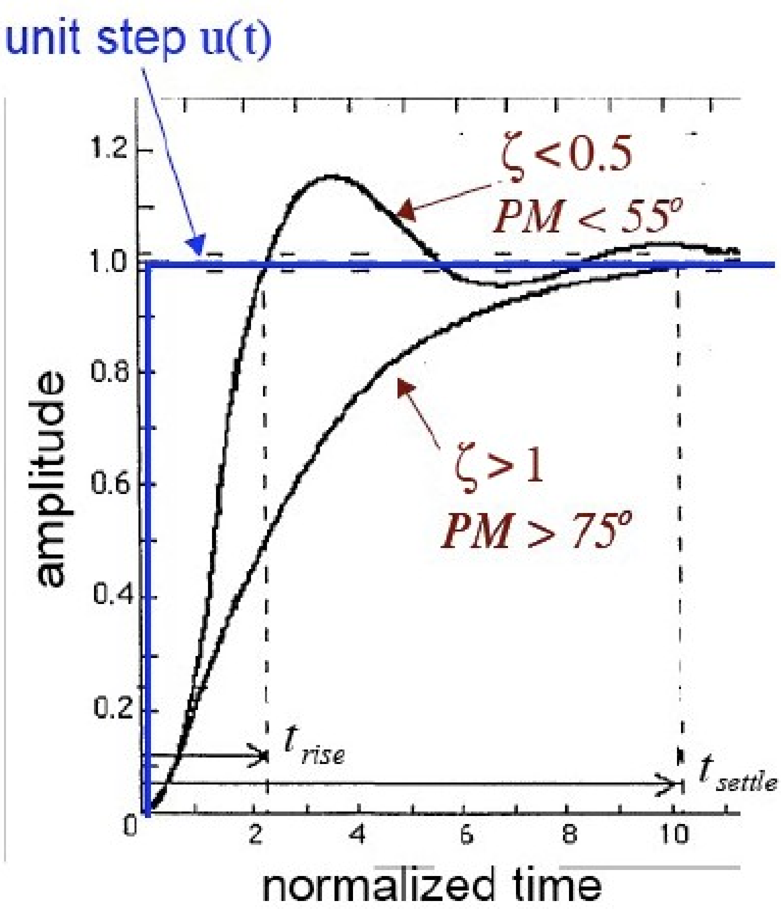
For acceptable overshoot: $PM > 50^o$



Frequency-Domain



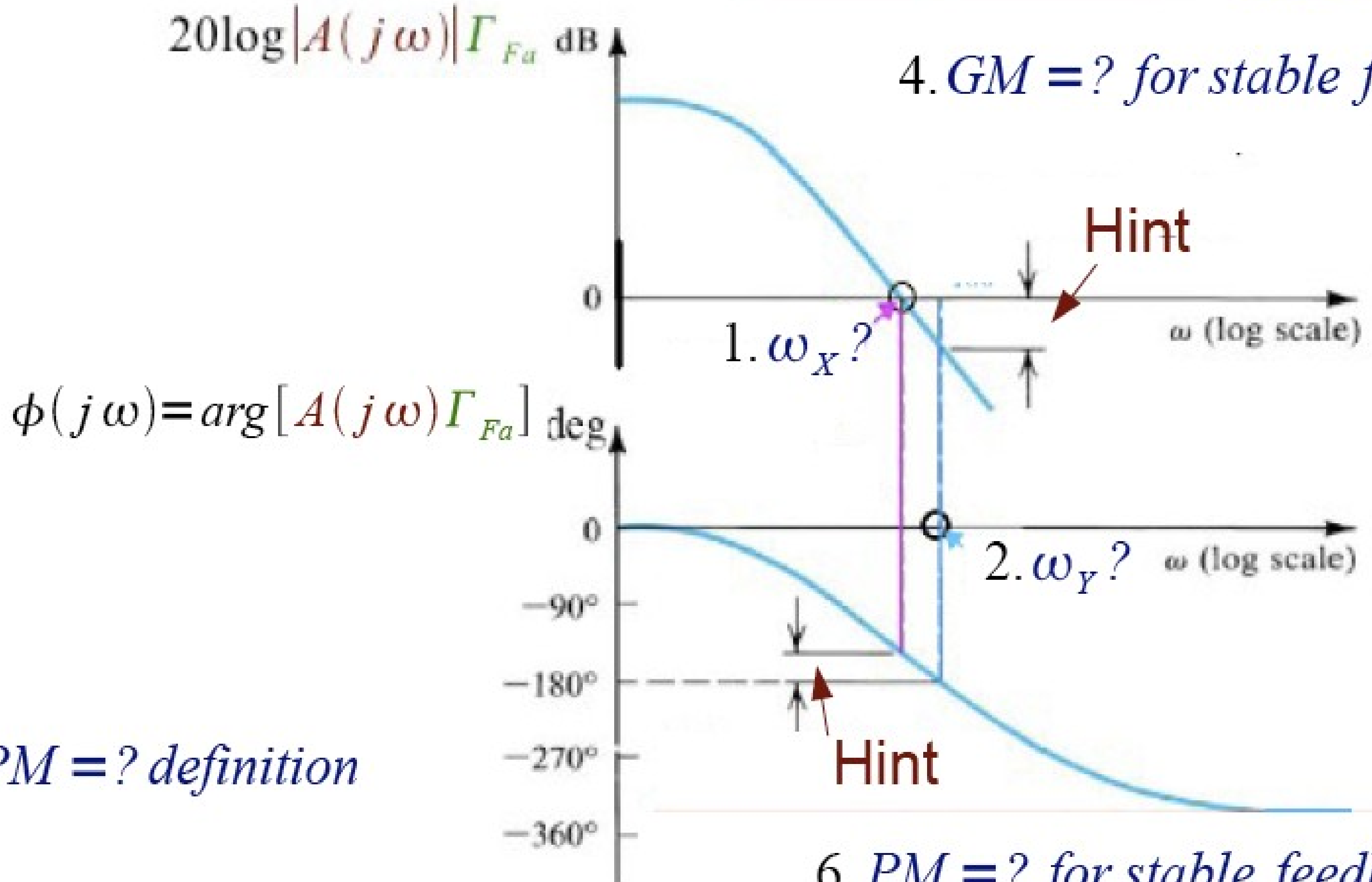
Time-Domain



$$70^o > PM > 55^o$$

For best compromise of overshoot vs. t_{rise} & t_{settle}.

Quick Review



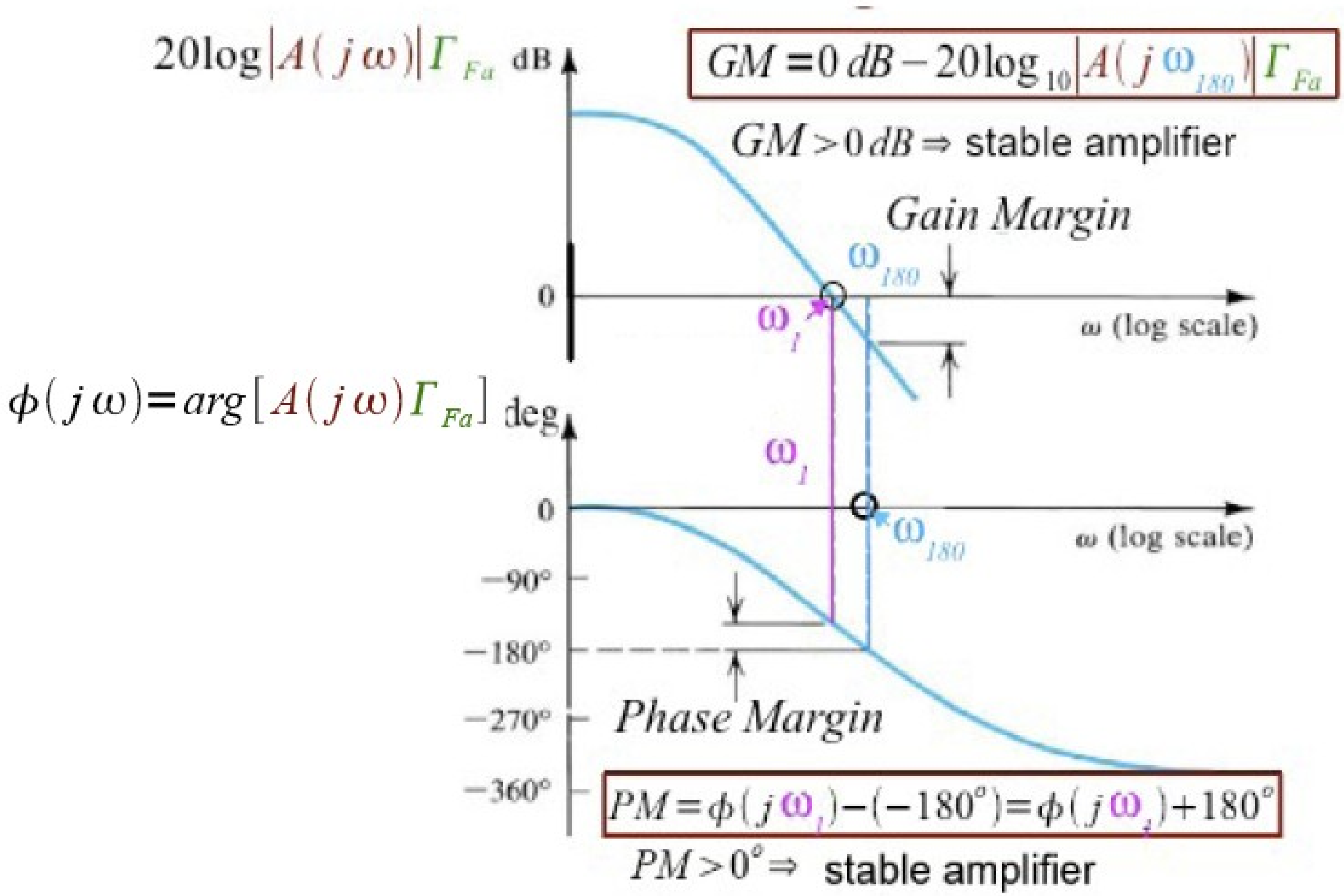
3. $GM = ?$ *definition*

4. $GM = ?$ *for stable feedback amplifier*

5. $PM = ?$ *definition*

6. $PM = ?$ *for stable feedback amplifier*

Quick Review



Alternative Stability Analysis

1. Investigating stability for a variety of feedback gains Γ_F by constructing Bode plots for the loop-gain $A(j\omega)\Gamma_F$ can be tedious and time consuming.

2. A simpler approach involves constructing Bode plots for $A(j\omega)$ and Γ_F (or $1/\Gamma_F$) separately.

$$20\log\left|A(j\omega)\Gamma_F\right| = 20\log\left|A(j\omega)\right| - 20\log\frac{1}{\Gamma_F}$$

$$20\log\Gamma_F = -20\log\frac{1}{\Gamma_F}$$

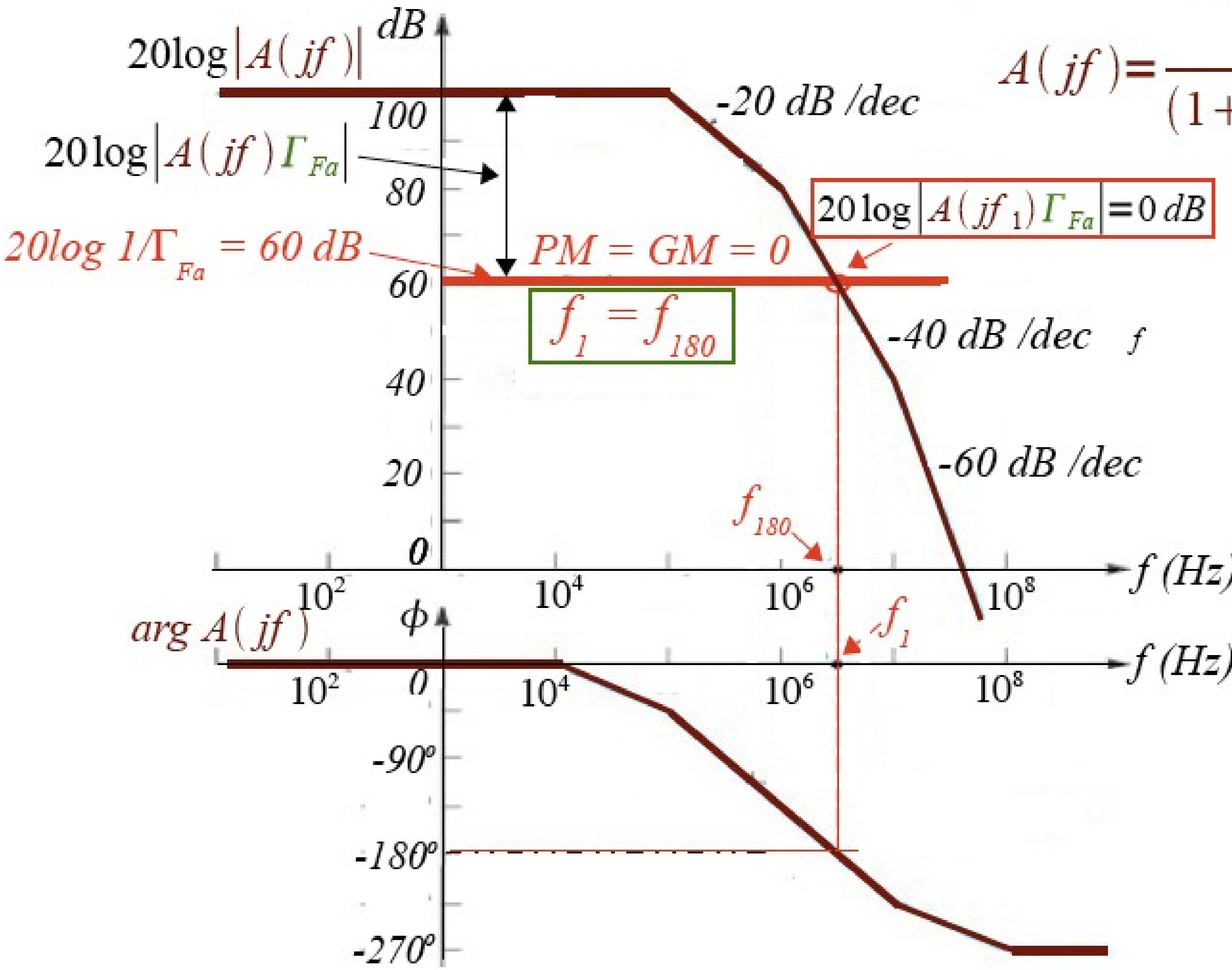
when

$$20\log\left|A(j\omega)\right| = 20\log\frac{1}{\Gamma_F} \Rightarrow 20\log\left|A(j\omega)\Gamma_F\right| = 0\,dB \Rightarrow \omega = \omega_1$$

gain cross-over frequency

Alternative Stability Analysis - cont.

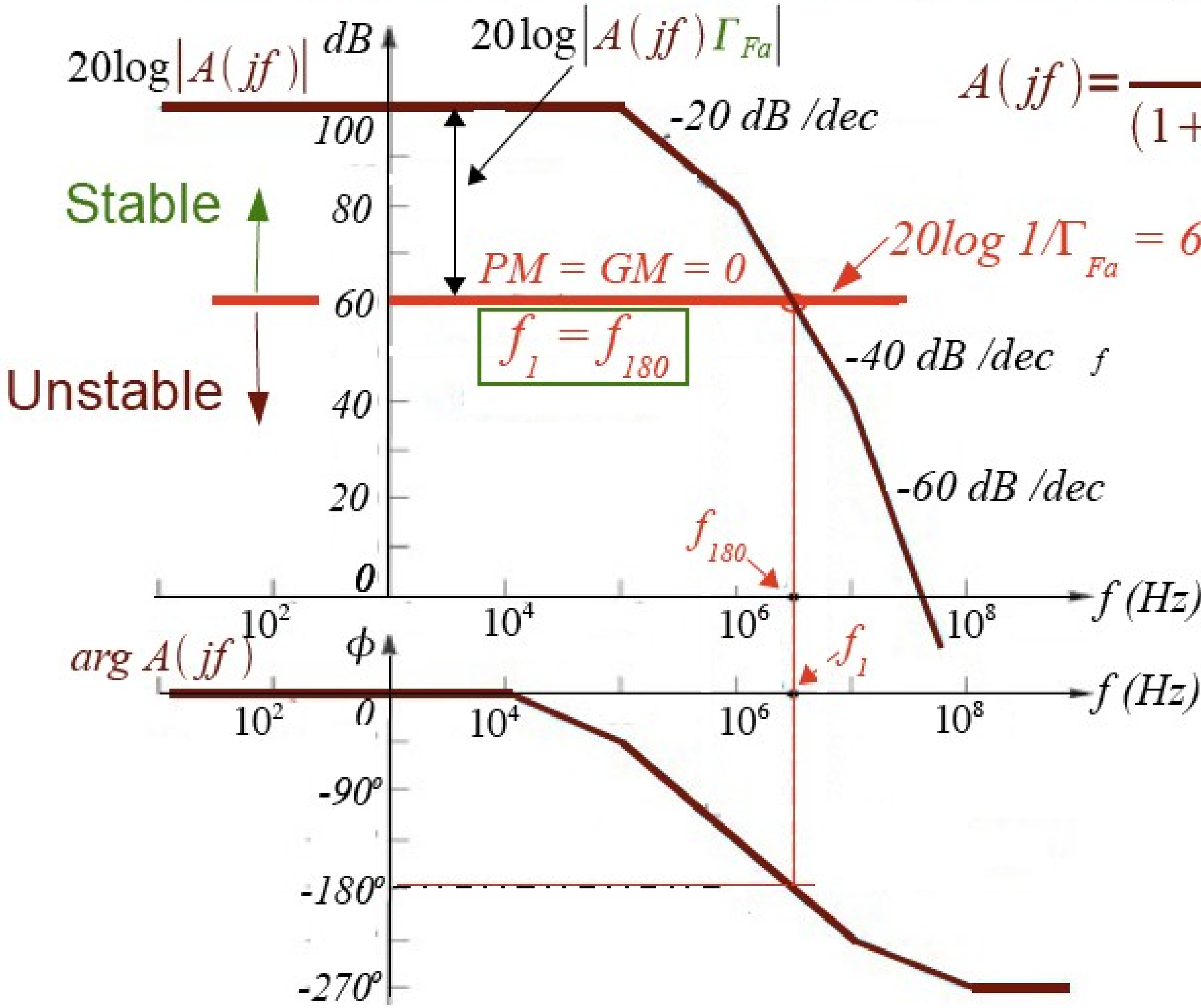
$$A(jf) = \frac{10^5}{(1+jf/10^5)(1+jf/10^6)(1+jf/10^7)}$$



Basic Relation

$$20\log|A(jf)\Gamma_F| = 20\log|A(jf)| - 20\log\frac{1}{\Gamma_F}$$

Alternative Stability Analysis - cont.



$$A(jf) = \frac{10^5}{(1 + jf/10^5)(1 + jf/10^6)(1 + jf/10^7)}$$

Stable iff

$20\log 1/\Gamma_F > 60$ dB

or

$20\log \Gamma_F < -60$ dB

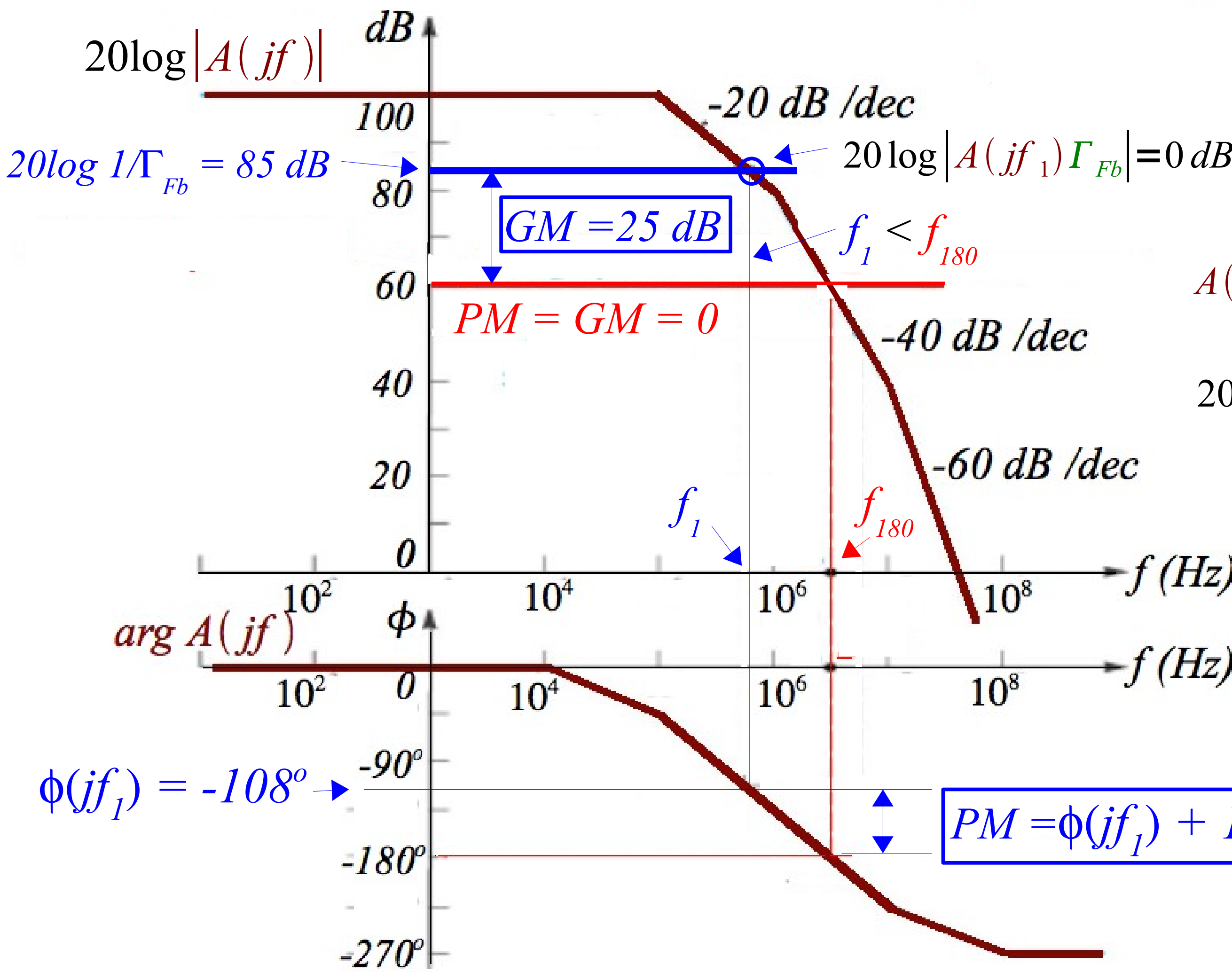
or

$\Gamma_F < 0.001$

$\Rightarrow f_1 < f_{180}$

$\Gamma_F < 0.001 \Rightarrow$ *weakly conditionally stable*

Alternative Stability Analysis - cont.



$$A(jf) = \frac{10^5}{(1 + jf/10^5)(1 + jf/10^6)(1 + jf/10^7)}$$

$$20\log|A(jf)\Gamma_F| = 20\log|A(jf)| - 20\log\frac{1}{\Gamma_{Fb}}$$

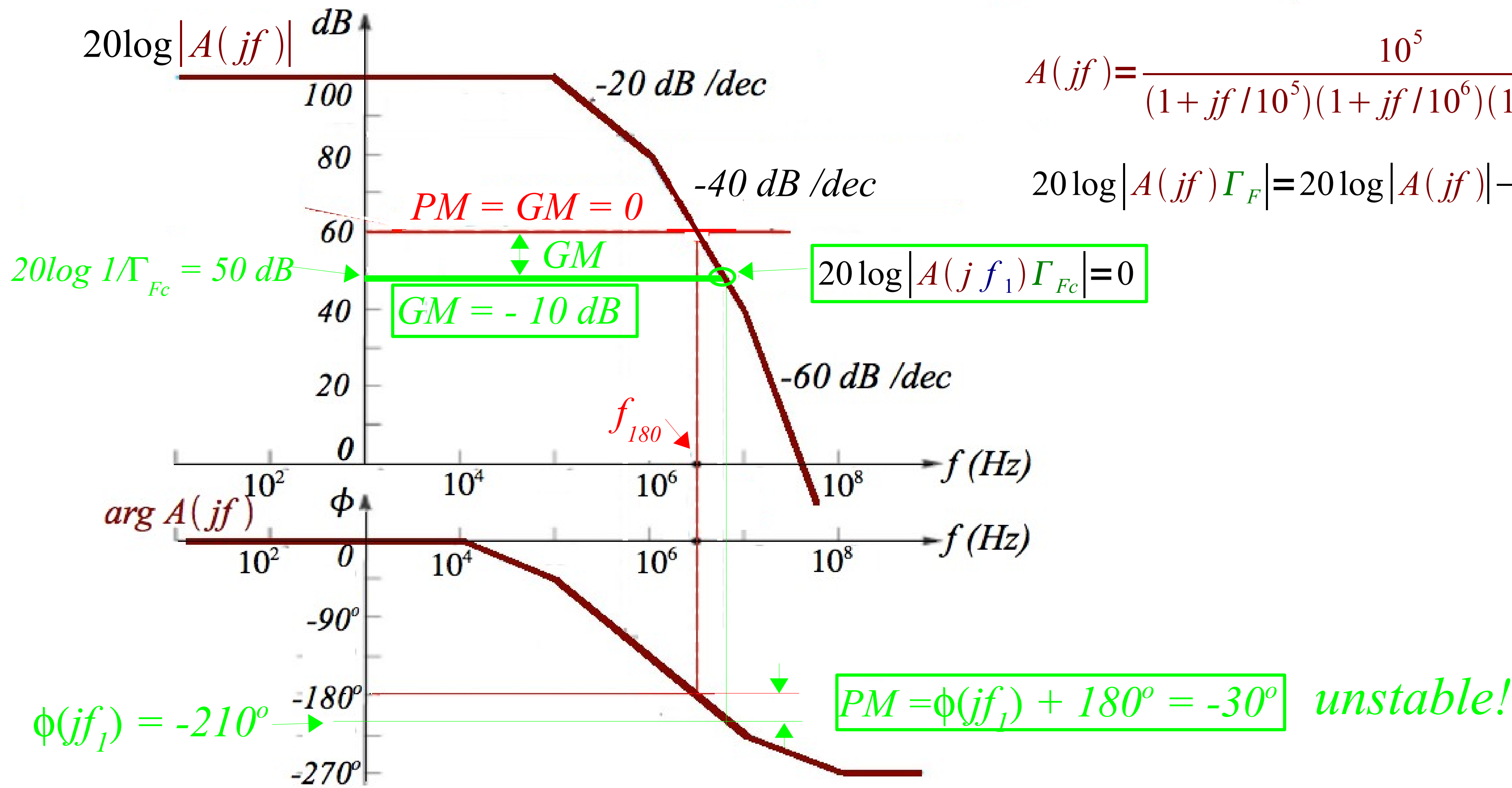
$$\phi(jf_1) = arg[A(jf_1)\Gamma_{Fb}] > -180^o$$

$PM = \phi(jf_1) + 180^o = 72^o$ *stable!*

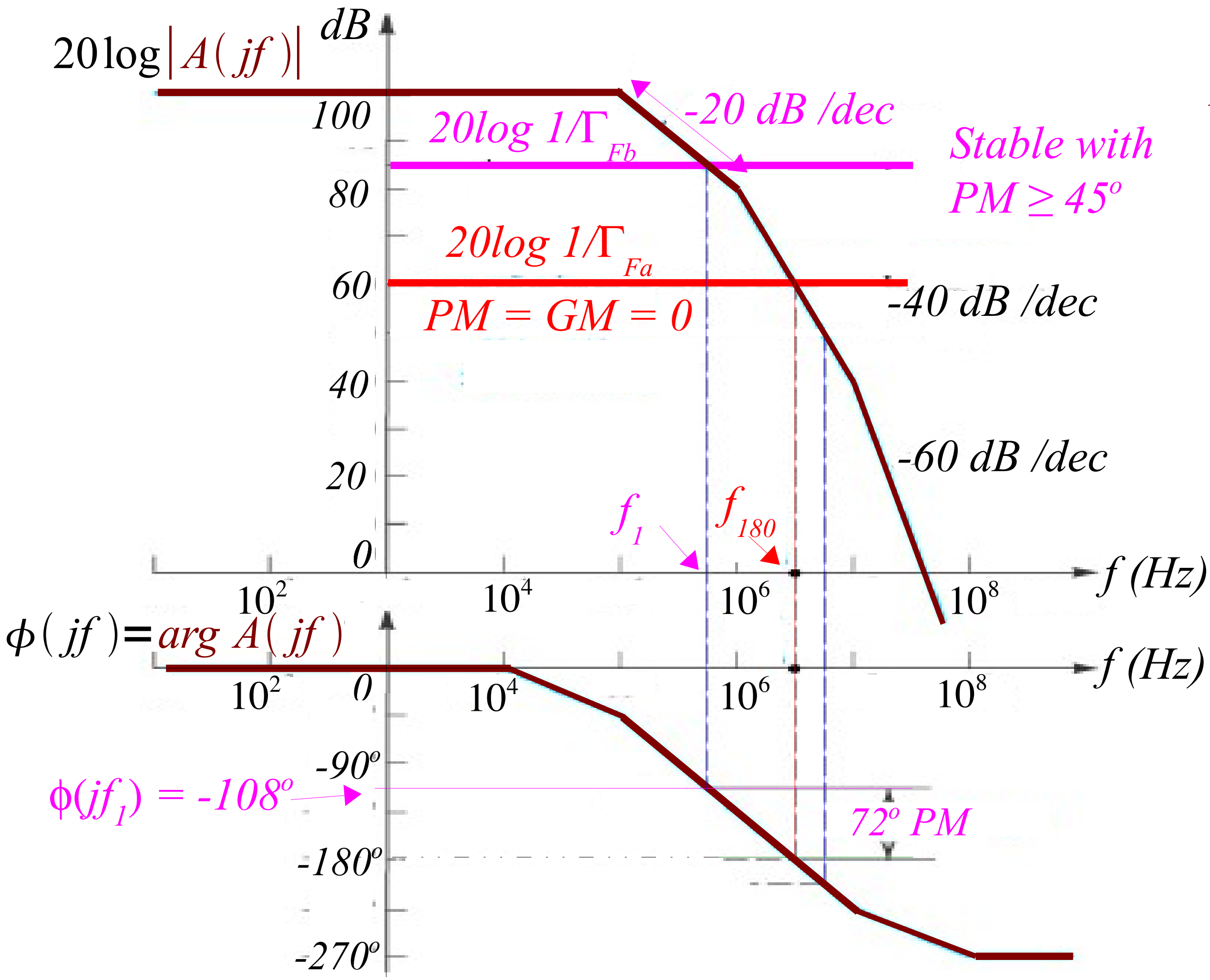
Alternative Stability Analysis - cont.

$$A(jf) = \frac{10^5}{(1+jf/10^5)(1+jf/10^6)(1+jf/10^7)}$$

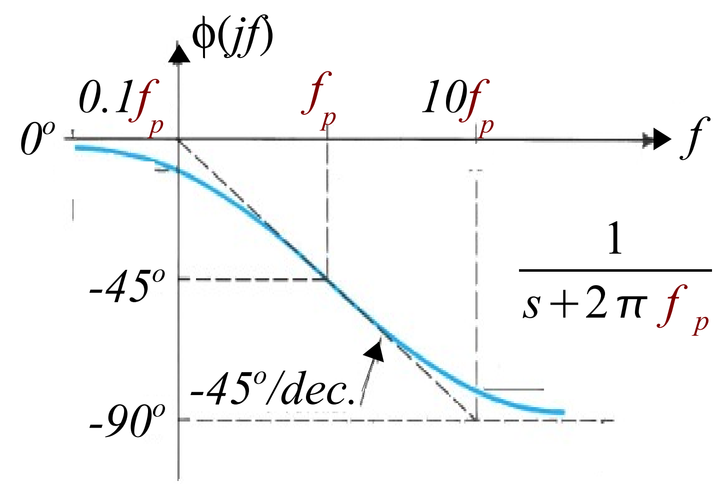
$$20\log|A(jf)\Gamma_F| = 20\log|A(jf)| - 20\log\frac{1}{\Gamma_{Fc}}$$



Alternative Stability Analysis - cont.



$$A(jf)=\frac{10^5}{(1+jf/10^5)(1+jf/10^6)(1+jf/10^7)}$$



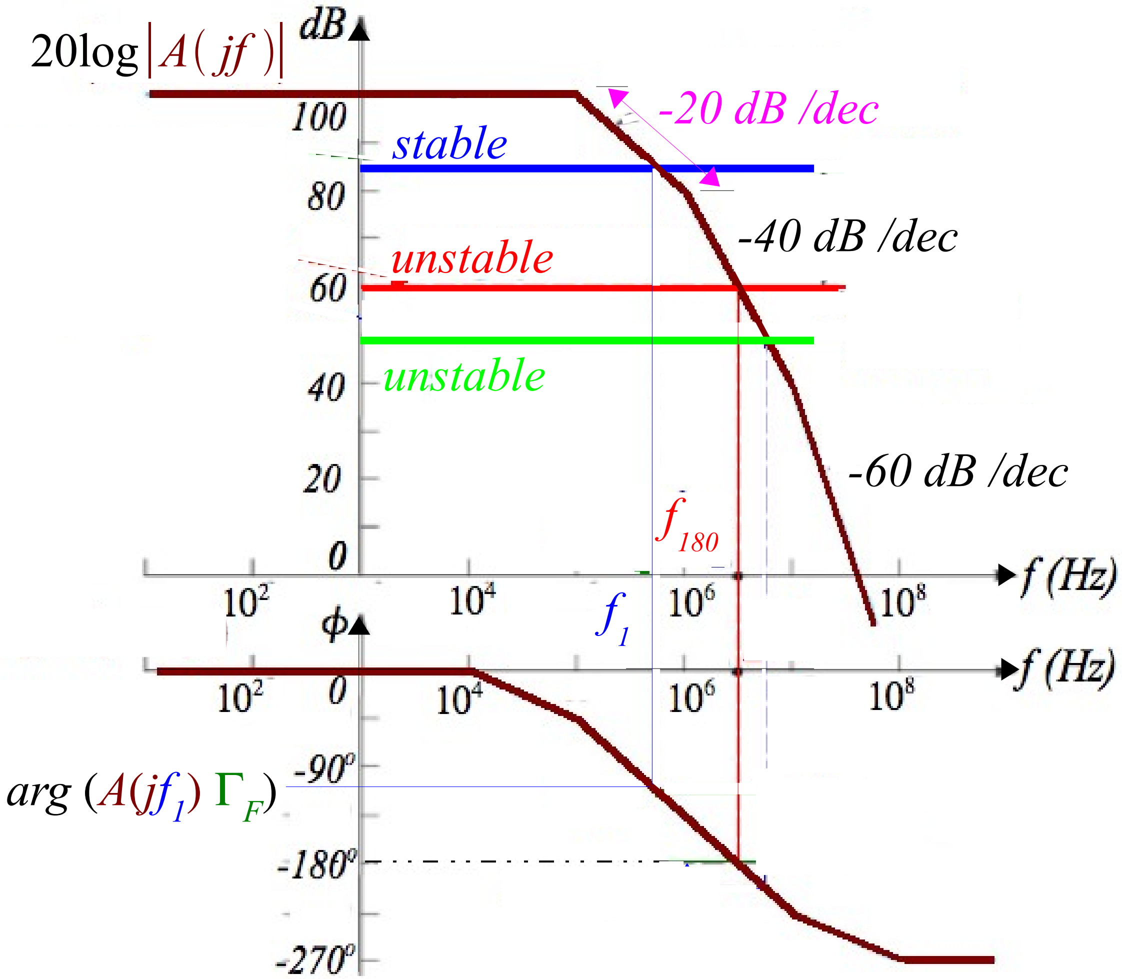
Hence, over $-20dB/dec.$

Segment of $20\log|A(jf)|$

$-45^o \geq \phi(jf) \geq -135^o$

=> $PM = \phi(jf) + 180^o \geq 45^o$

Alternative Stability Analysis - cont.



$$A(jf)=\frac{10^5}{(1+jf/10^5)(1+jf/10^6)(1+jf/10^7)}$$

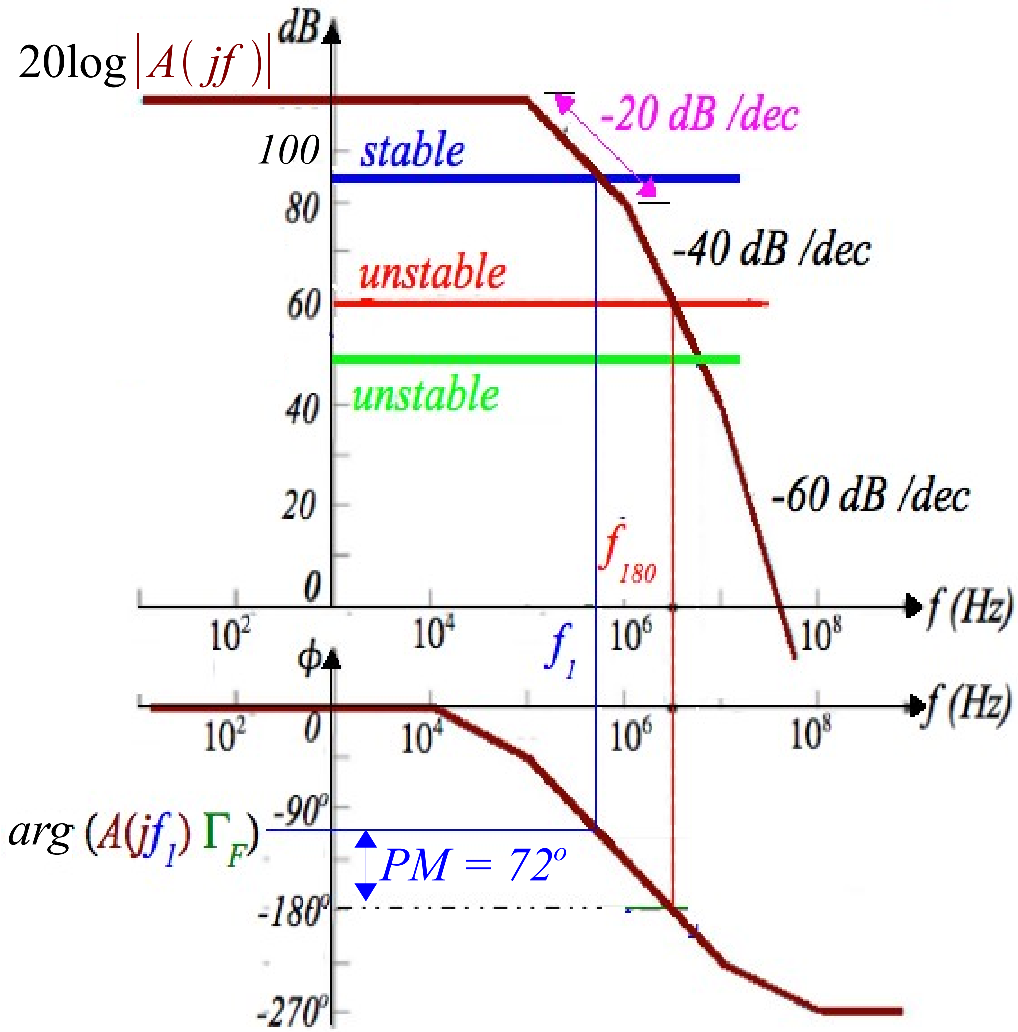
$$20\log|A(jf)\Gamma_F|=20\log|A(jf)|-20\log\frac{1}{\Gamma_F}$$

"Rule of Thumb" – Closed -loop amplifier will be stable if the $20\log 1/\Gamma_F$ line intersects the $20\log|A(jf)|$ curve on the -20 dB/dec segment.

Using "Rule of Thumb":

$$\arg(A(jf_1)\,\Gamma_F) > -135^o$$

=> $PM \geq 45^o$



Rate of Closure (RC)

$$RC = slope\ A(j2\pi f)_{dB/dec} - slope\left(\frac{1}{\Gamma_F}\right)_{dB/dec}$$

RC = - 20 dB/dec – 0 dB/dec = - 20 dB/dec

RC = - 40 dB/dec – 0 dB/dec = - 40 dB/dec

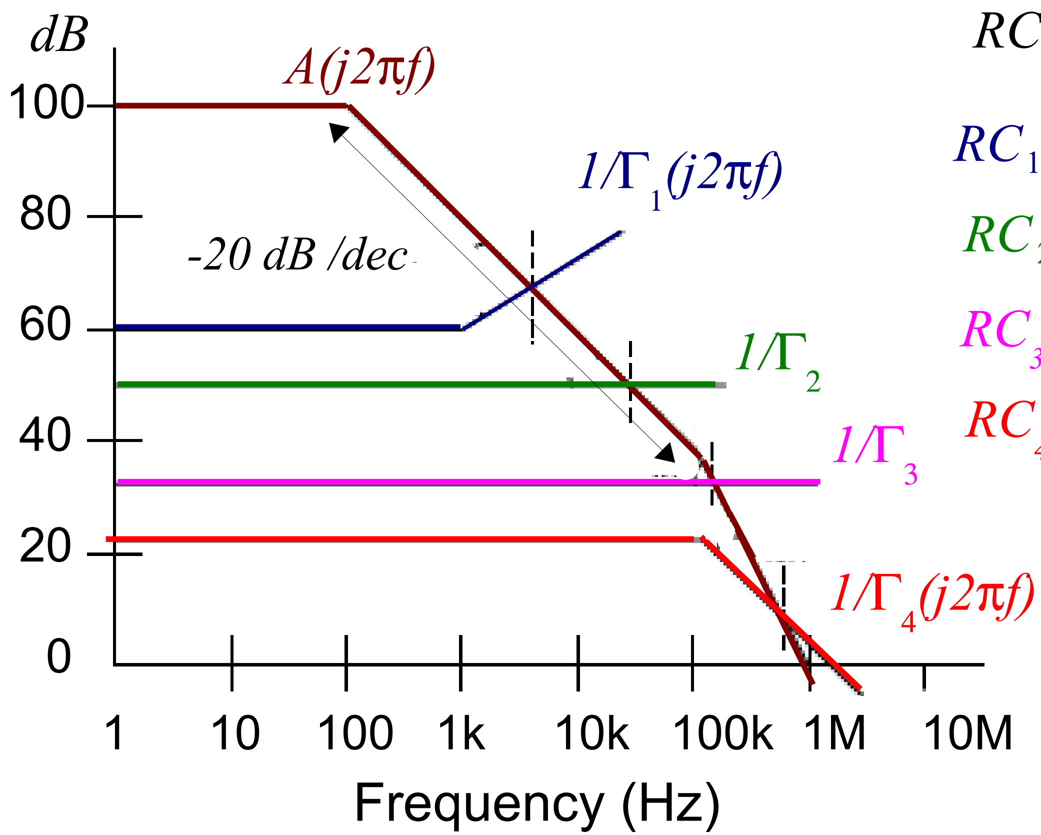
RC = - 40 dB/dec – 0 dB/dec = - 40 dB/dec

From previous slide:
"Rule of Thumb" – Closed-loop amplifier will be stable if $20\log 1/\Gamma_F$ line intersects $20\log|A(jf)|$ curve on the *-20 dB/dec* segment.

"Equivalent Rule of Thumb" – Closed-loop amplifier will be stable if

$$RC = -20\ dB/dec$$

Frequency Dependent Feedback



$$RC = slope\, A(j2\pi f)_{dB/dec} - slope\left(\frac{1}{\Gamma_F}\right)_{dB/dec}$$

$$RC_1 = -20\, dB/dec - (+20\, dB/dec) = -40\, dB/dec$$

$$RC_2 = -20\, dB/dec - (0\, dB/dec) = -20\, dB/dec$$

$$RC_3 = -40\ dB/dec - (0\ dB/dec) = -40\ dB/dec$$

$$RC_4 = -40\ dB/dec - (-20\ dB/dec) = -20\ dB/dec$$

"Equivalent Rule of Thumb" – Closed-loop amplifier will be stable with $PM \geq 45^o$ if

$$RC = -20\, dB/dec$$

Quick Review

$$A_{cl}(s)=\frac{A(s)}{1+\Gamma_F A(s)}$$

Gain & Phase Margins

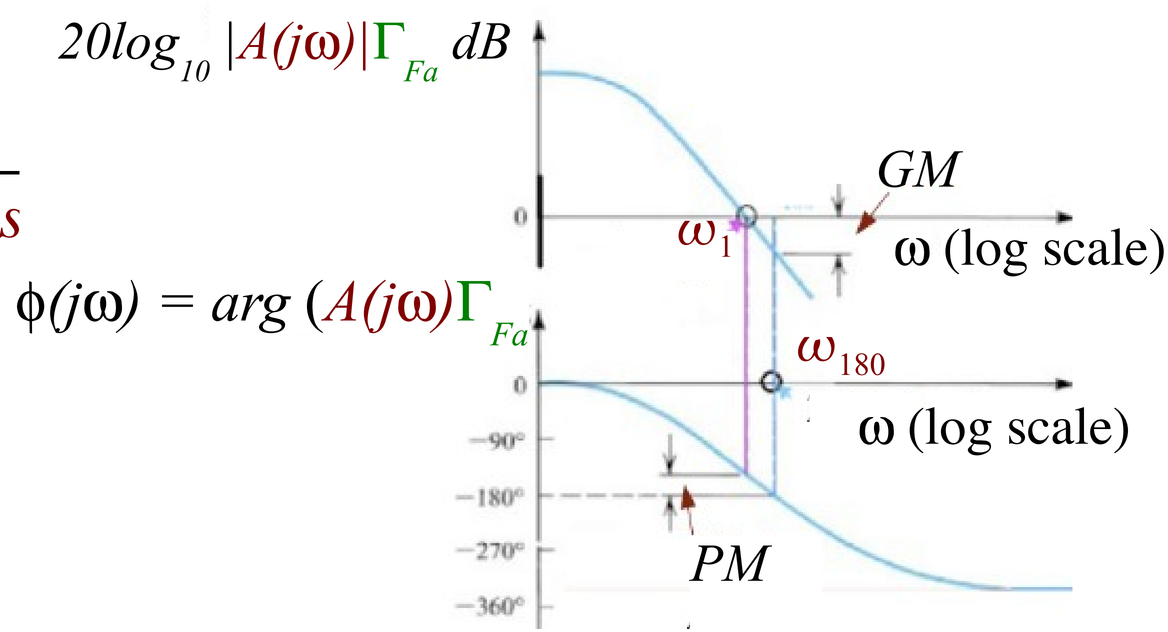
A. What are the two methods for determining stability of closed loop amplifier?

B. What is the "Rule of thumb"?

Quick Review cont.

A. What are the two methods for determining stability of closed loop amplifier?

$$A_{cl}(s)=\frac{A(s)}{1+\Gamma_F A(s)}$$



where $\Gamma_F = \Gamma_{Fa}$

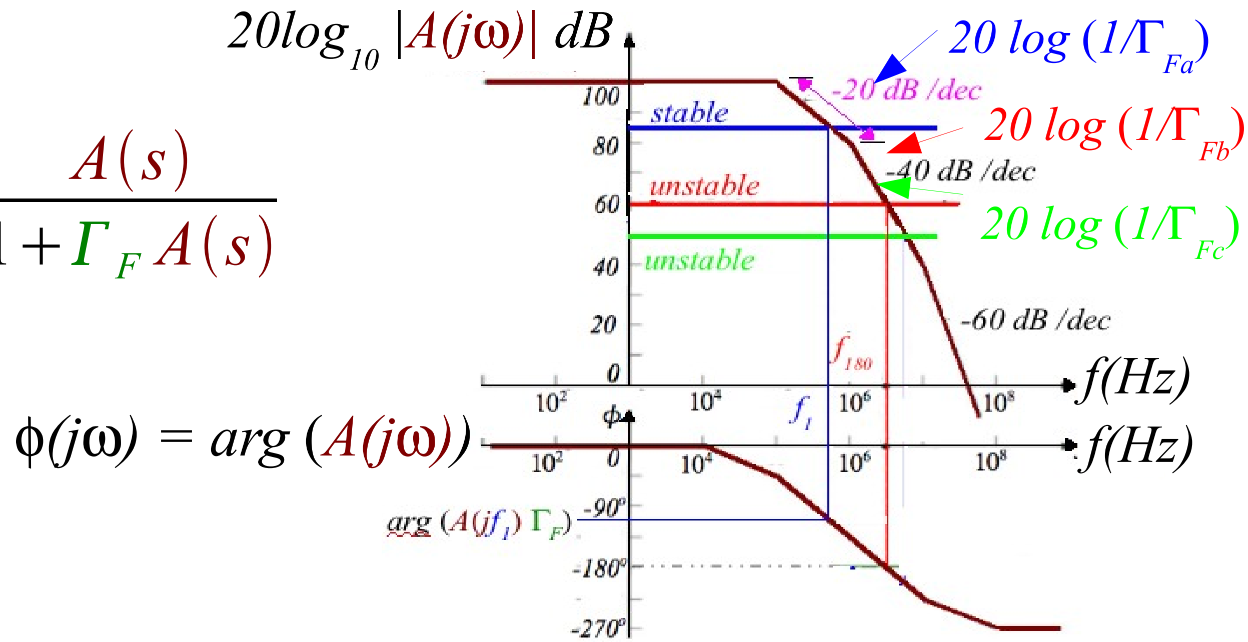
(1) $A(j\omega)\Gamma_F = -1 = 1\, e^{j\pi \pm 2k\pi}$

$$GM = 0\,dB - 20\log_{10}\left|A(j\omega_{180})\right|\Gamma_{Fa} > 0$$

$$PM = arg\left(A(j\omega_1)\Gamma_{Fa}\right) - (-180^o)$$

Quick Review cont.

$$A_{cl}(s)=\frac{A(s)}{1+\Gamma_F A(s)}$$

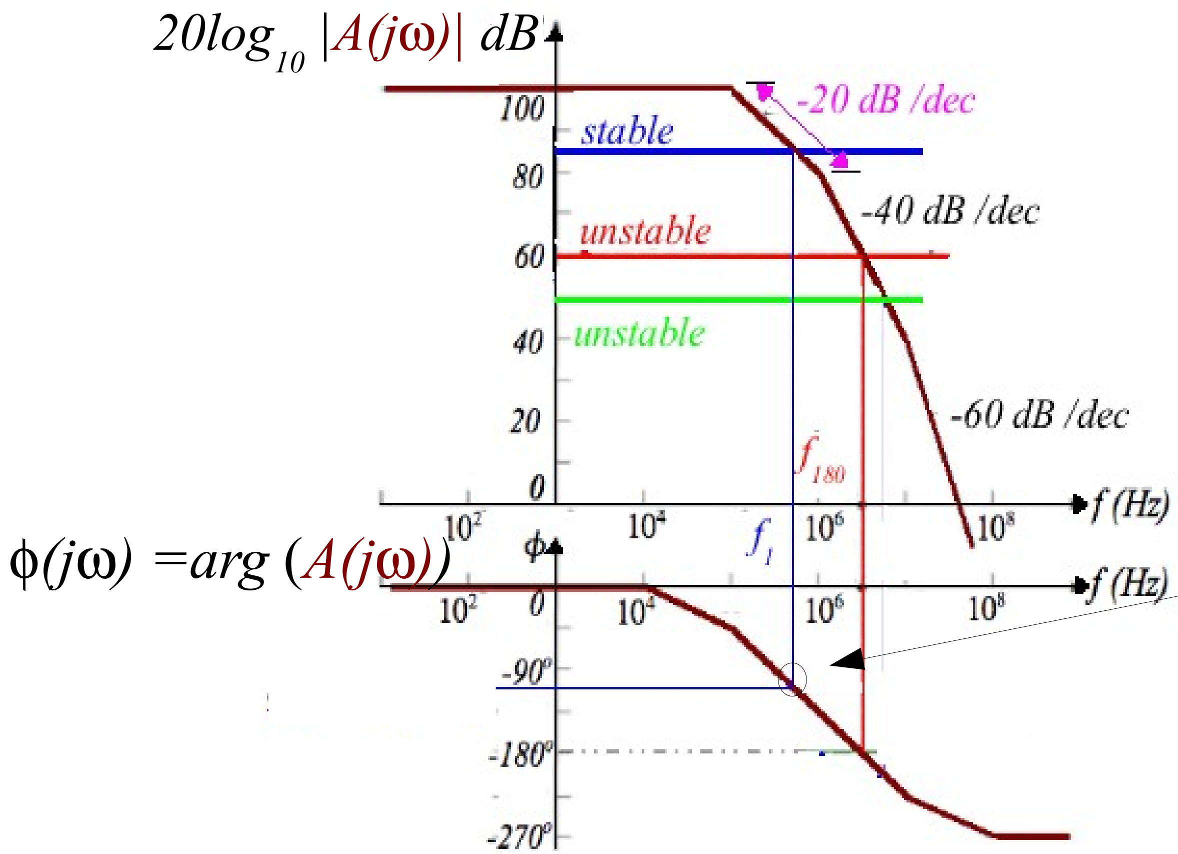


(2) $$20\log\left|A(j\omega)\Gamma_F\right|=20\log\left|A(j\omega)\right|-20\log\frac{1}{\Gamma_F}$$ where $\Gamma_F=\Gamma_{Fa}$

$$20\log\left|A(j\omega_1)\right|=20\log\frac{1}{\Gamma_{Fa}}$$

Quick Review cont.

"Rule of Thumb" – Closed -loop amplifier will be stable if the $20log1/\Gamma_F$ line intersects the $20log|A(jf)|$ curve on the $-20\ dB/dec$ segment and $PM \geq 45^o$.

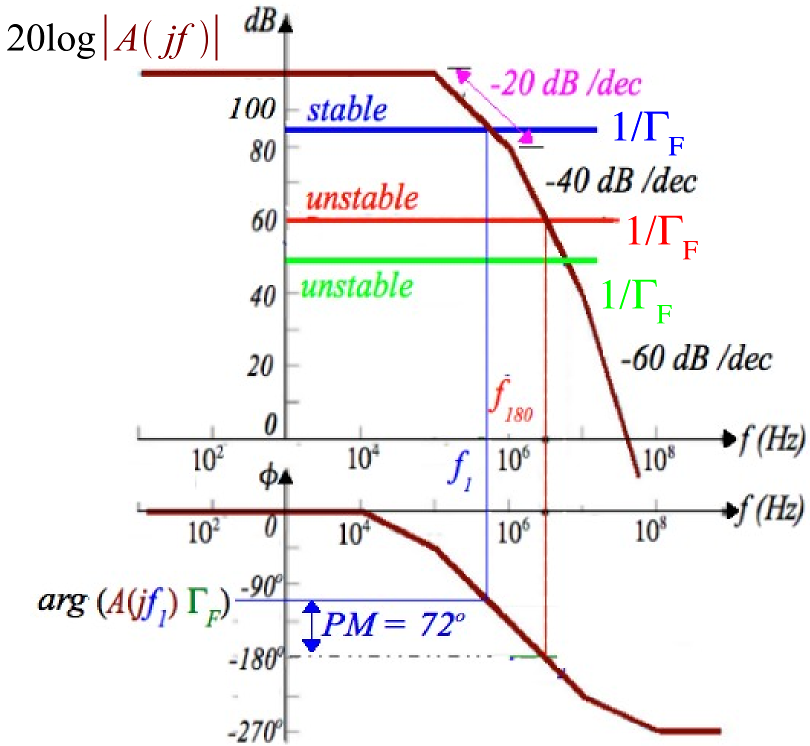


over $-\ 20dB/dec.$ segment

$-45^o \geq arg\ (A(jf_1)\Gamma_F) \geq -135^o$

$\Rightarrow PM = \phi(jf) + 180^o \geq 45^o$

Frequency Compensation



"Rule of Thumb" – Closed -loop amplifier will be stable if the $20\log 1/\Gamma_F$ line intersects the $20\log|A(jf)|$ curve on the -20 dB/dec segment.

Using "Rule of Thumb":

$arg\ (A(jf_1)\ \Gamma_F) > -135^o$

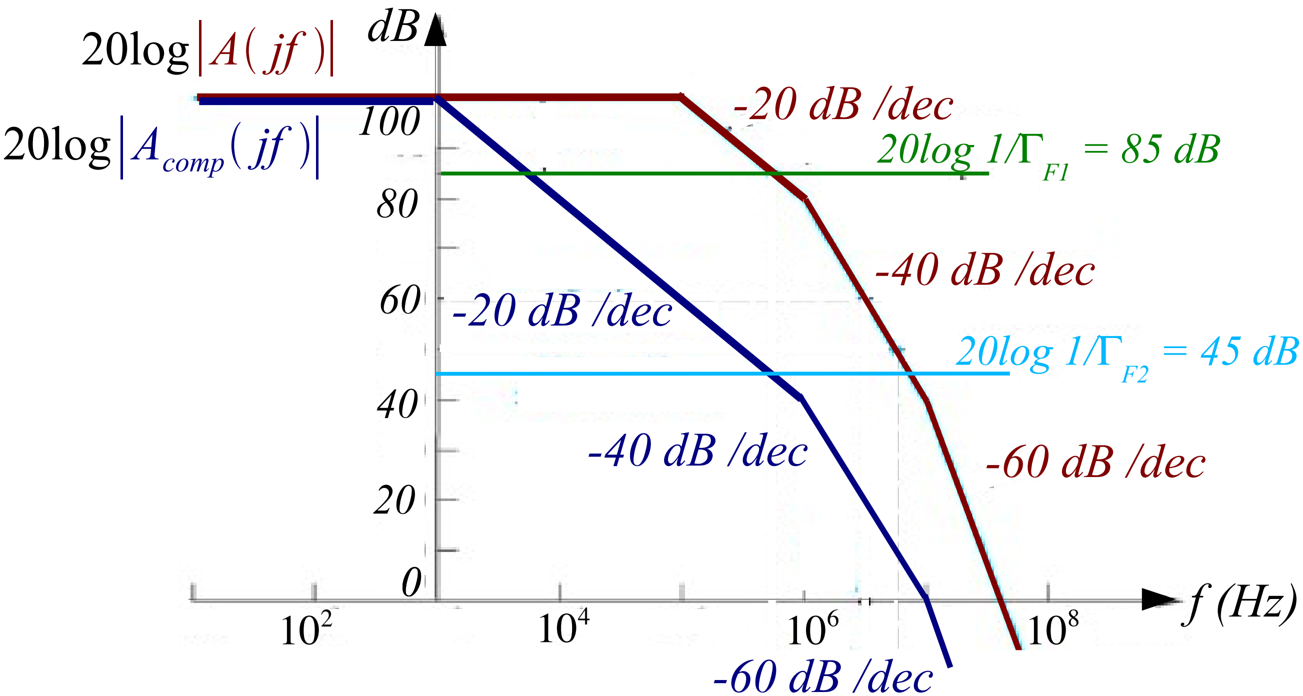
=> $PM \geq 45^o$

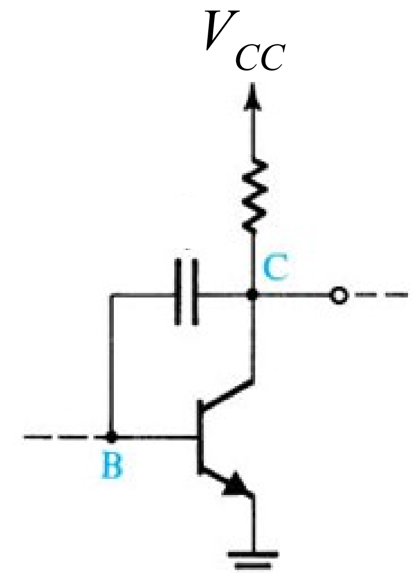
Ex: LM 741 op amp is frequency compensated to be stable with 60^o PM for $20\log|A_{cl}(jf)| = 20\log 1/\Gamma_F = 0$ dB.

frequency compensation – modifying the open-loop $A(s)$ so that the closed-loop $A_{cl}(s)$ is stable for any desired value of $|A_{cl}(jf)|$ by extending the -20 dB/dec segment.

desired implementation - minimum on-chip or external components.

Compensation – What if 1st pole is shifted lower?

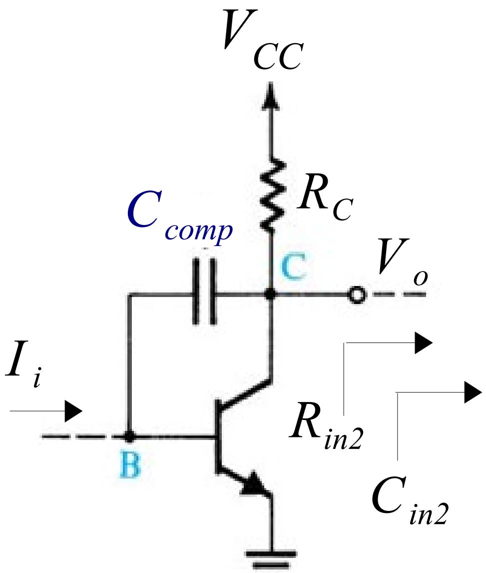


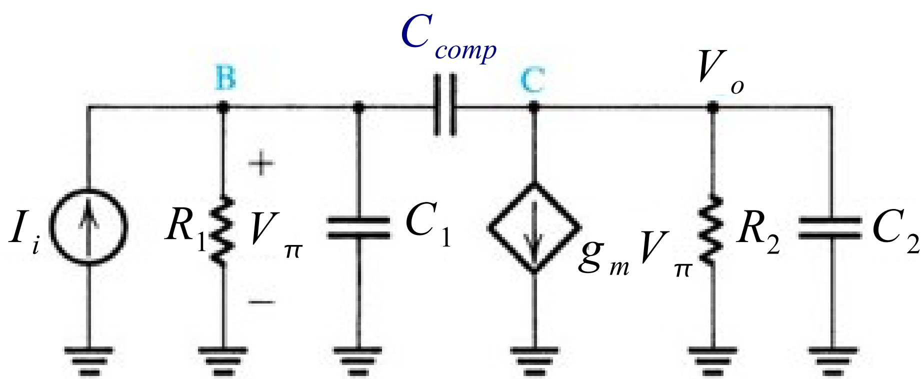


$$A(jf)=\frac{10^5}{(1+jf/10^5)(1+jf/10^6)(1+jf/10^7)}$$

$$A_{comp}(jf)\approx\frac{10^5}{(1+jf/10^3)(1+jf/10^6)(1+jf/10^7)}$$

Frequency Compensation Using Miller Effect





with $C_{omp} = 0$

$$\omega_{p1} = \frac{1}{R_1 C_1}$$

$$\omega_{p2} = \frac{1}{R_2 C_2}$$

$$C_1 = C_\pi + C_\mu (1 + g_m R_2)$$

BJT Miller Capacitance

$$R_1 = R_B \| r_\pi$$

$$R_2 = R_C \| r_o \| R_{in2}$$

$$C_2 = C_\mu + C_{in2}$$

$$\frac{V_o}{I_i} = \frac{(sC_{comp} - g_m) R_1 R_2}{1 + s[C_1 R_1 + C_2 R_2 + C_{comp}(g_m R_1 R_2 + R_1 + R_2)] + s^2 [C_1 C_2 + C_{comp}(C_1 + C_2)] R_1 R_2}$$

$$= \frac{(sC_{comp} - g_m) R_1 R_2}{\left(1 + \frac{s}{\omega_{p1c}}\right)\left(1 + \frac{s}{\omega_{p2c}}\right)} = \frac{(sC_{comp} - g_m) R_1 R_2}{1 + s\left(\frac{1}{\omega_{p1c}} + \frac{1}{\omega_{p2c}}\right) + \frac{s^2}{\omega_{p1c}\omega_{p2c}}}$$

where $\omega_{p2c} \gg \omega_{p1c}$

Compensation Using Miller Effect - cont.

$$\frac{V_o}{I_i}=\frac{(sC_{comp}-g_m)R_1R_2}{1+s\left(\frac{1}{\omega_{p1c}}+\frac{1}{\omega_{p2c}}\right)+\frac{s^2}{\omega_{p1c}\omega_{p2c}}}\approx\frac{(sC_{comp}-g_m)R_1R_2}{1+s\left(\frac{1}{\omega_{p1c}}\right)+s^2\left(\frac{1}{\omega_{p1c}\omega_{p2c}}\right)}$$

assumption

If $\omega_{p2c}\gg\omega_{p1c}$

pole-splitting

and

$$\frac{V_o}{I_i}=\frac{(sC_{comp}-g_m)R_1R_2}{1+s[C_1R_1+C_2R_2+C_{comp}(g_mR_1R_2+R_1+R_2)]+s^2[C_1C_2+C_{comp}(C_1+C_2)]R_1R_2}$$

$$\omega_{p1c}=\frac{1}{C_1R_1+C_2R_2+C_{comp}(g_mR_1R_2+R_1+R_2)}\approx\boxed{\frac{1}{C_{comp}g_mR_2R_1}}$$

Compensation Miller Capacitance

$$\omega_{p2c}=\frac{\omega_{p1c}\omega_{p2c}}{\omega_{p1c}}=\frac{C_1R_1+C_2R_2+C_{comp}(g_mR_1R_2+R_1+R_2)}{[C_1C_2+C_{comp}(C_1+C_2)]R_1R_2}\approx\frac{C_{comp}g_mR_1R_2}{C_{comp}(C_1+C_2)R_1R_2}\approx\boxed{\frac{g_m}{C_1+C_2}}$$

If $C_{comp}(C_1+C_2)\gg C_1C_2$

Compensation Using Miller Effect - cont.

$$A(jf)=\frac{10^5}{(1+\frac{jf}{10^5})(1+\frac{jf}{10^6})(1+\frac{jf}{10^7})}=\frac{10^5}{(1+\frac{jf}{f_{p1}})(1+\frac{jf}{f_{p2}})(1+\frac{jf}{f_{p3}})}$$

$$f_{p1}=\frac{\omega_{p1}}{2\pi}=\frac{1}{R_1 C_1} \qquad f_{p2}=\frac{\omega_{p2}}{2\pi}=\frac{1}{R_2 C_2}$$

Using Miller effect compensation: $f_{p1} \rightarrow f_{p1c} << f_{p1}$ and $f_{p2} \rightarrow f_{p2c} >> f_{p2}$ (pole-splitting)

Also f_{p3}, determined by another stage, is unaffected by the compensation => $f_{p3c}=f_{p3}$

$$A_{comp}(jf)=\frac{10^5}{(1+\frac{jf}{f_{p1c}})(1+\frac{jf}{f_{p2c}})(1+\frac{jf}{f_{p3}})}$$

$$f_{p1c}=\frac{\omega_{p1c}}{2\pi}\approx\frac{1}{2\pi[C_{comp}g_m R_1 R_2]} \qquad f_{p2c}=\frac{\omega_{p2c}}{2\pi}\approx\frac{g_m}{2\pi[C_1+C_2]}$$

Miller Compensation Example

Given:

$$A(jf)=\frac{10^5}{(1+\frac{jf}{R_1C_1})(1+\frac{jf}{R_2C_2})(1+\frac{jf}{10^7})}=\frac{10^5}{(1+\frac{jf}{f_{p1}})(1+\frac{jf}{f_{p2}})(1+\frac{jf}{10^7})}=\frac{10^5}{(1+\frac{jf}{10^5})(1+\frac{jf}{10^6})(1+\frac{jf}{10^7})}$$

where $C_1 = 20\ pF,\ C_2 = 5\ pF,\ g_m = 40\ mS,\ R_1 = 100/2\pi\ k\Omega$ and $R_2 = 200/2\pi\ k\Omega$

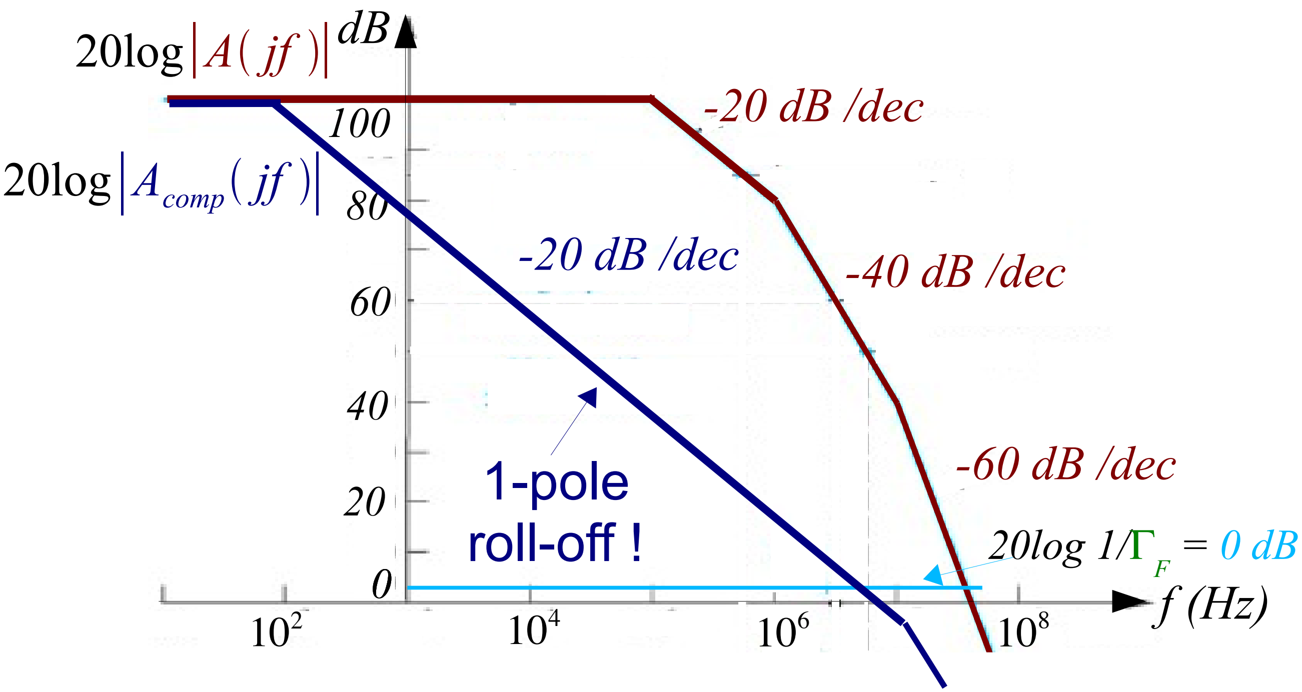
Design Objective: determine C_{comp} s.t. $f_{p1c} = 100\ Hz$ and compute f_{p2c}.

$$A_{comp}(jf)=\frac{10^5}{(1+\frac{jf}{10^2})(1+\frac{jf}{f_{p2c}})(1+\frac{jf}{10^7})}=\frac{10^5}{(1+\frac{jf}{f_{p1c}})(1+\frac{jf}{f_{p2c}})(1+\frac{jf}{10^7})}$$

$$f_{p1c}=100\ Hz\approx\frac{1}{2\pi[C_{comp}g_mR_1R_2]}=\frac{(2\pi)^2}{2\pi[C_{comp}(40\text{x}10^{-3})(10^5)(2\text{x}10^5)]}\Rightarrow \boxed{C_{comp}=78.5\ pF}$$

$$f_{p2c}=\frac{g_m}{2\pi(C_1+C_2)}\approx\boxed{61\ MHz}$$

Compensation Using Miller Effect - cont.



$A_{comp}(jf)$

$$PM > 45^o \; for \left|A_{cl}(jf)\right| = \frac{1}{\Gamma_F} \geq 1$$

$$A(jf) = \frac{10^5}{\left(1+\frac{jf}{10^5}\right)\left(1+\frac{jf}{10^6}\right)\left(1+\frac{jf}{10^7}\right)}$$

$$A_{comp}(jf) \approx \frac{10^5}{\left(1+\frac{jf}{10^2}\right)\left(1+\frac{jf}{6.1*10^7}\right)\left(1+\frac{jf}{10^7}\right)}$$

Summary

Feedback has many desirable features, but it can create unexpected - undesired results if the full frequency-dependent nature (phase-shift with frequency) of the feedback circuit is not taken into account.

Feedback can be used to convert a well-behaved stable circuit into an oscillator. Sometimes, due to parasitics, feedback in amplifiers results in unexpected oscillations.

The root-locus method was used to show how feedback can create the conditions for oscillation and instability.