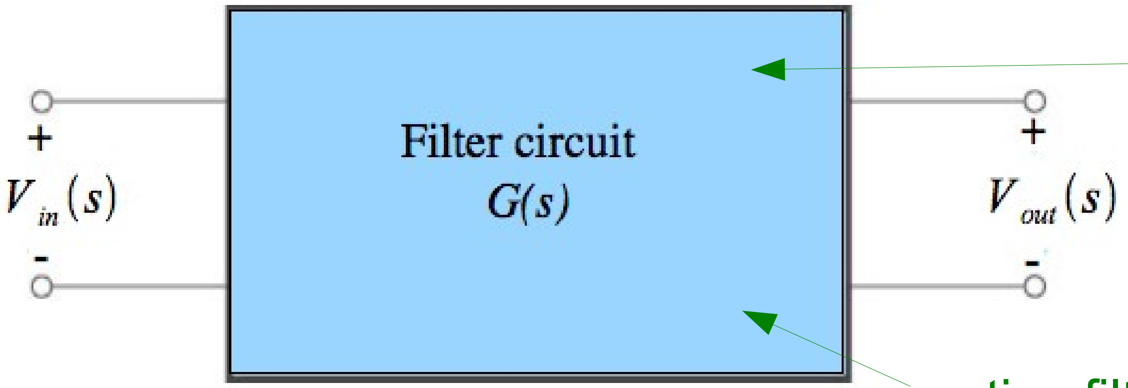
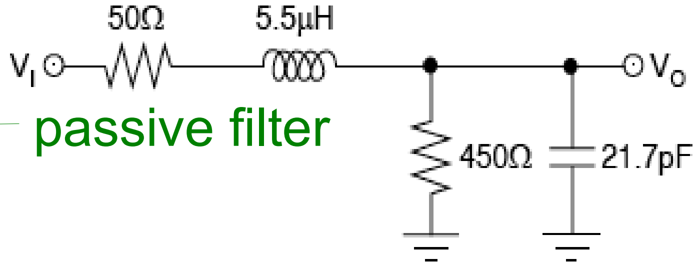
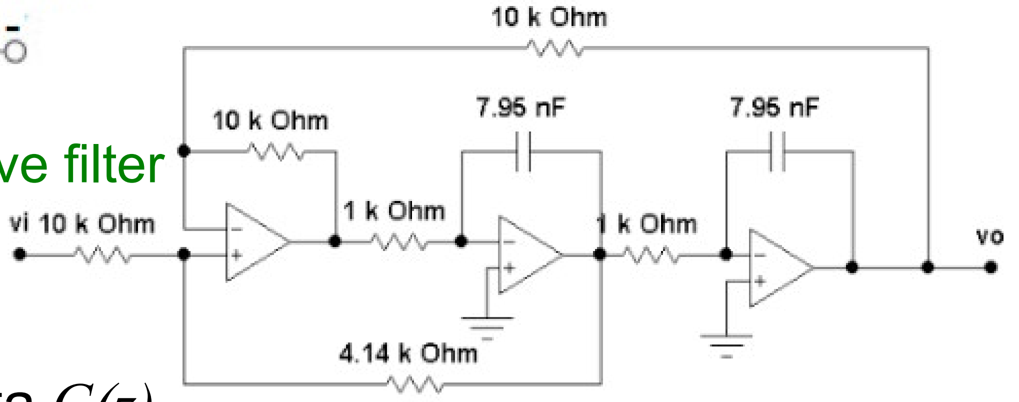
Active Filters – an Introduction





passive filter

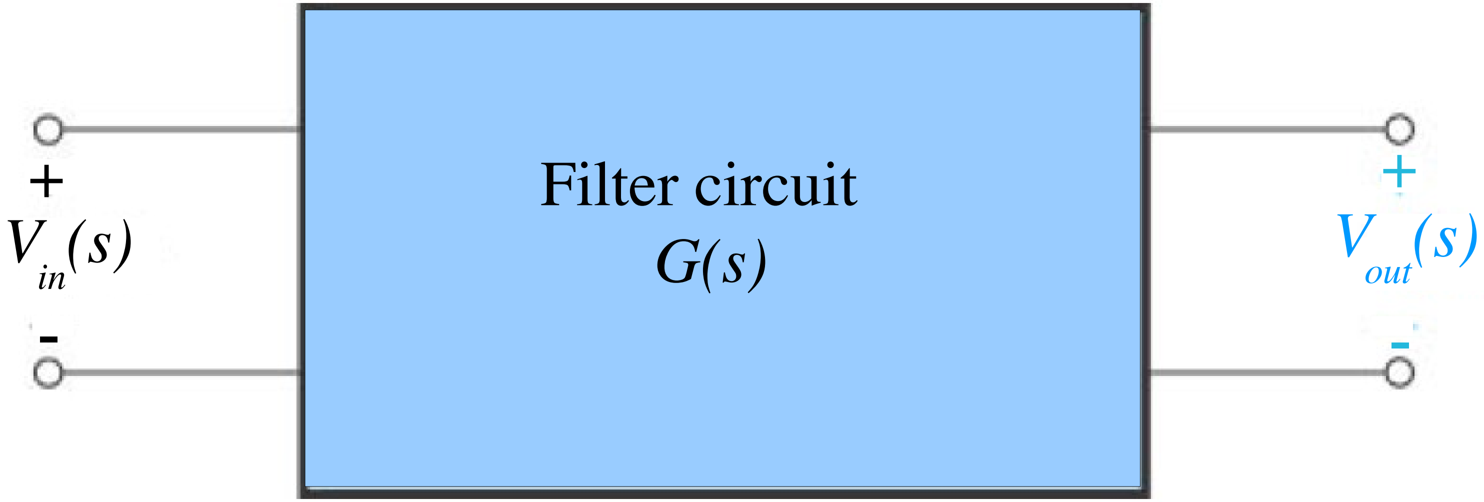


active filter

Active Filter Characteristics

1. Continuous-time $G(s)$ or Sampled-data $G(z)$
2. Employ active elements (e.g. transistors, amplifiers, op-amps)
   a. inductor-less (continuous-time)
   b. inductor-less & resistor-less (sample-data)
   c. $|G(jf)| \geq 1$ in passband

Active Filters – an Introduction



$$G(s)=\frac{V_{out}(s)}{V_{in}(s)}=\frac{a_M s^M+a_{M-1}s^{M-1}+....+a_1 s+a_0}{s^N+b_{N-1}s^{N-1}+....+b_1 s+b_0}$$

a_k = real

b_k = positive real

$$G(s)=\frac{a_M(s+z_1)(s+z_2)........(s+z_M)}{(s+p_1)(s+p_2)........(s+p_N)}$$

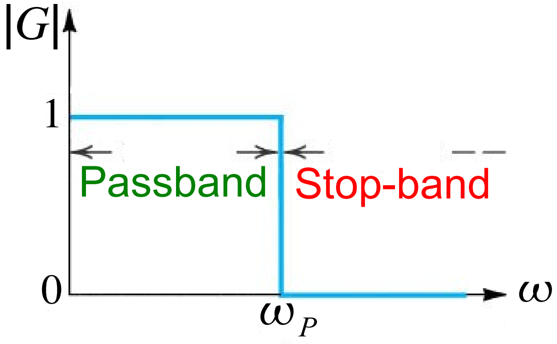
z_k = real or complex

p_k = r.h.p. real of complex

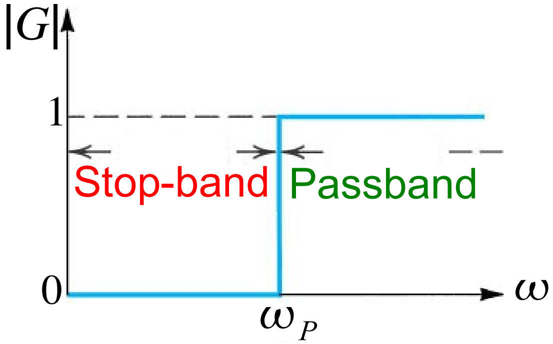
$M \le N$

$\boxed{\text{Filter Order} = N}$

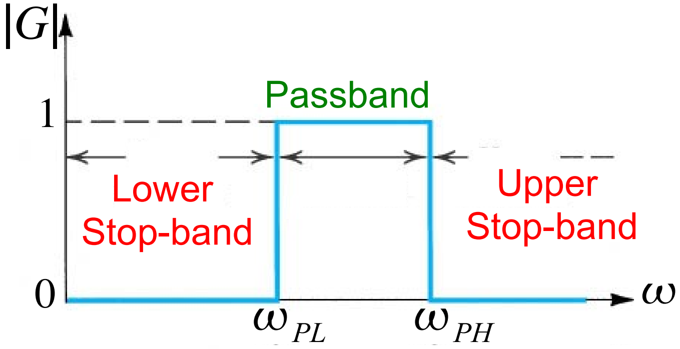
Ideal Filter Response Characteristics



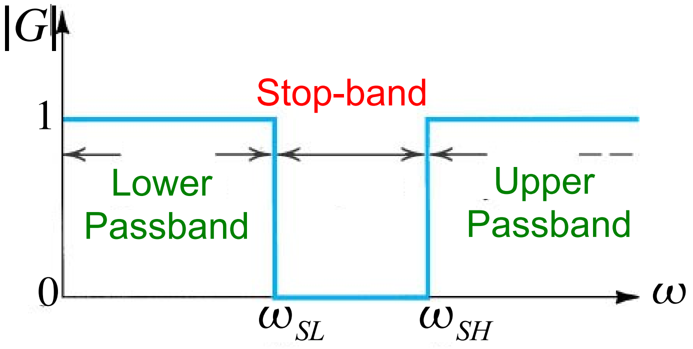
Low-pass (LP)



High-pass (HP)



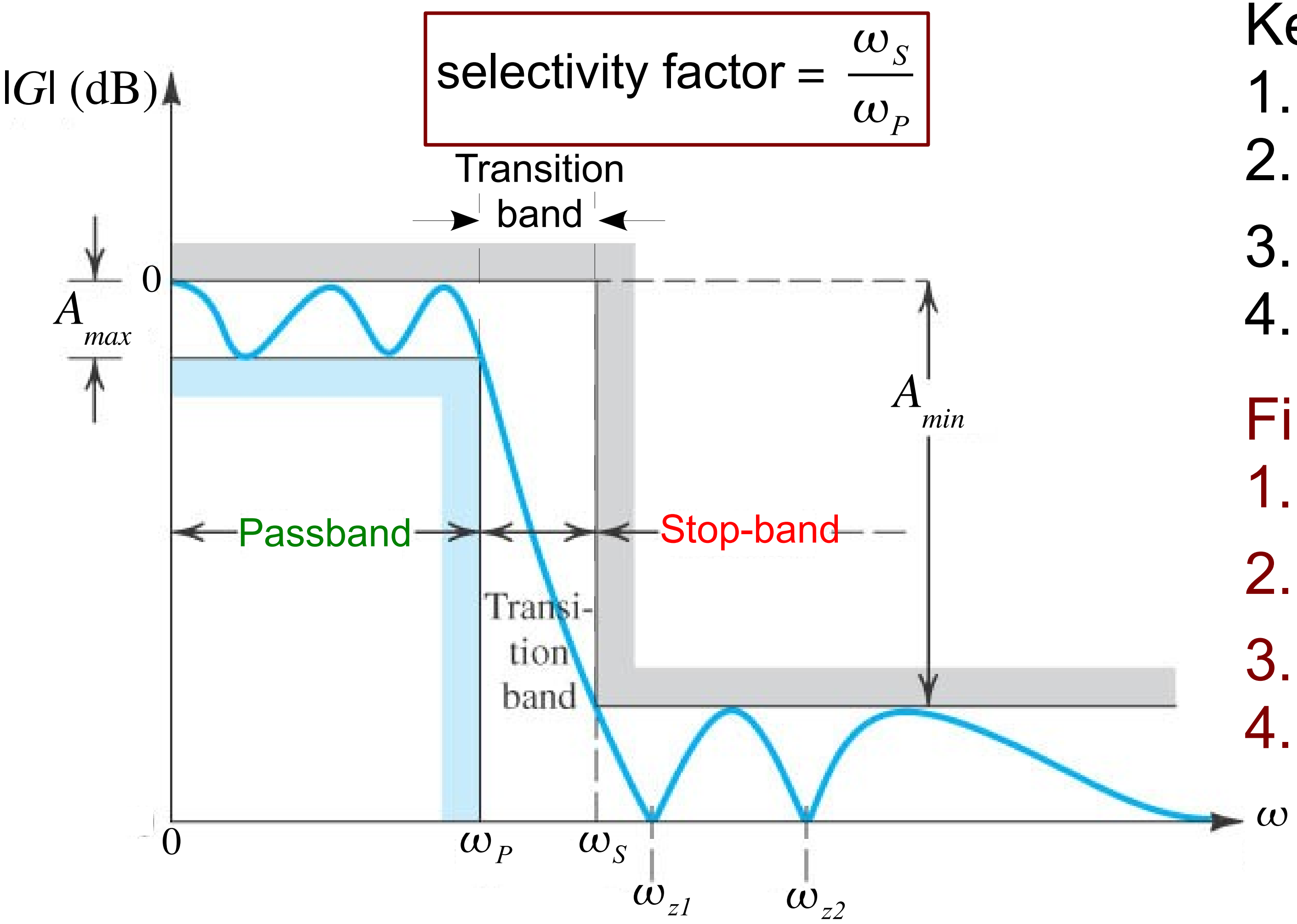
Bandpass (BP)



Bandstop (BS)

$$|G| = |G(j\omega)| = \left| \frac{V_{out}(j\omega)}{V_{in}(j\omega)} \right|$$

Practical Lowpass Filter Specification



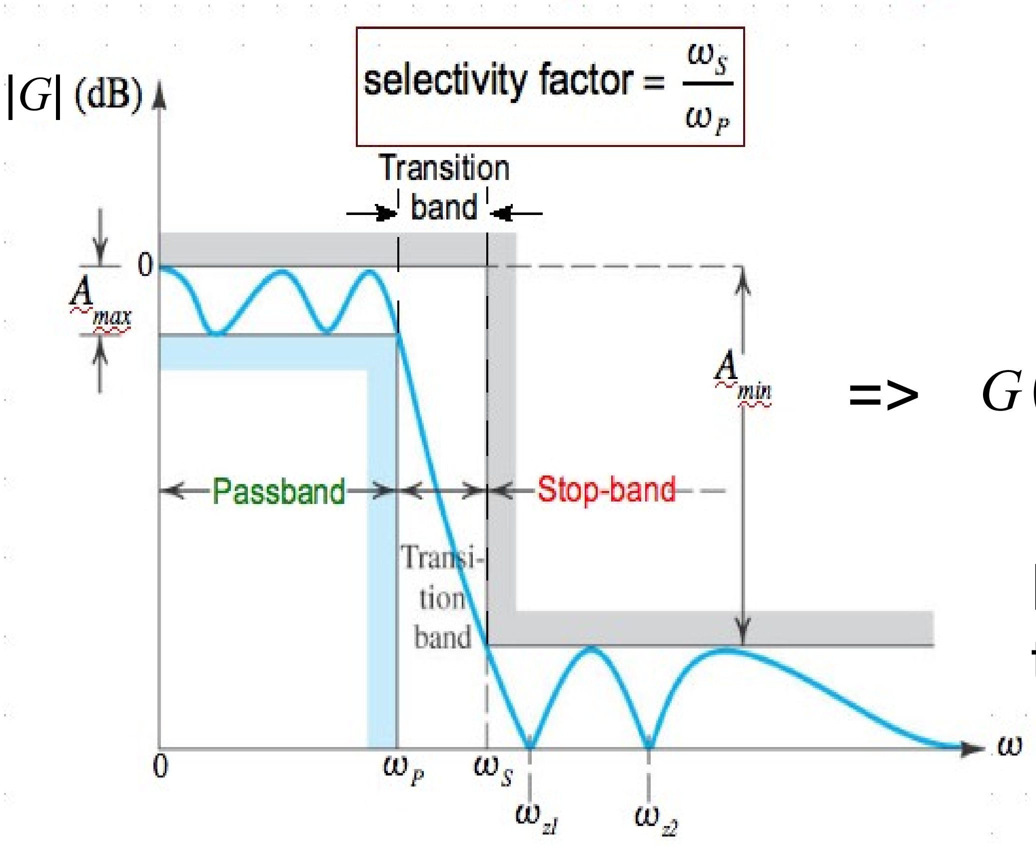
Key specs:

1. $f_B = \omega_P / 2\pi$
2. A_{max}
3. $f_S = \omega_S / 2\pi$
4. A_{min}

Filter cost increases!

1. A_{max} -> lower
2. A_{min} -> larger
3. ω_P -> larger
4. ω_S / ω_P -> 1

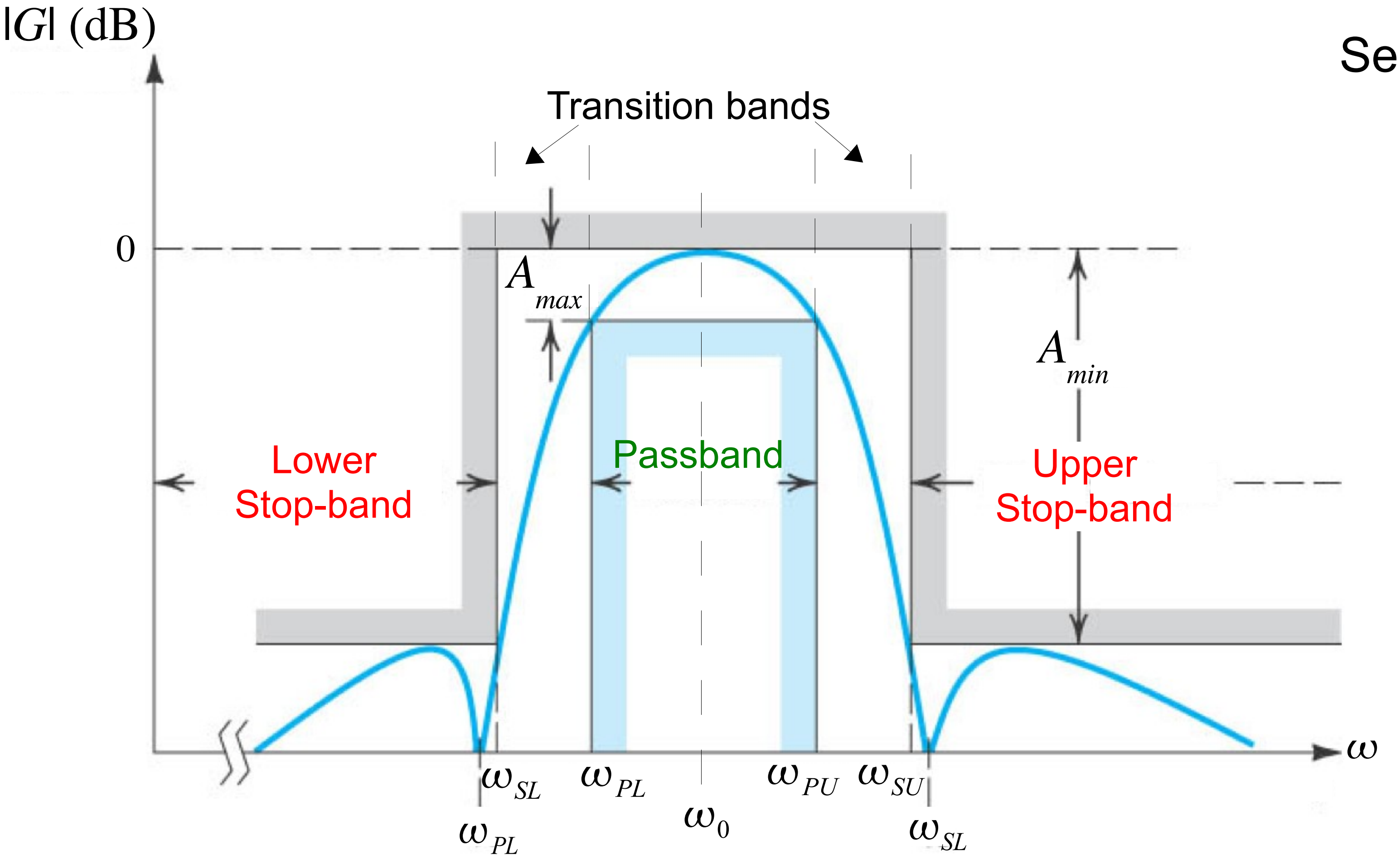
Filter Approximation – Design G(s)



$$\Rightarrow \quad G(s) = \frac{a_M(s+z_1)(s+z_2)........(s+z_M)}{(s+p_1)(s+p_2)........(s+p_N)}$$

MatLab is a good tool for this task.

Practical Bandpass Filter Specification



Selectivity factors

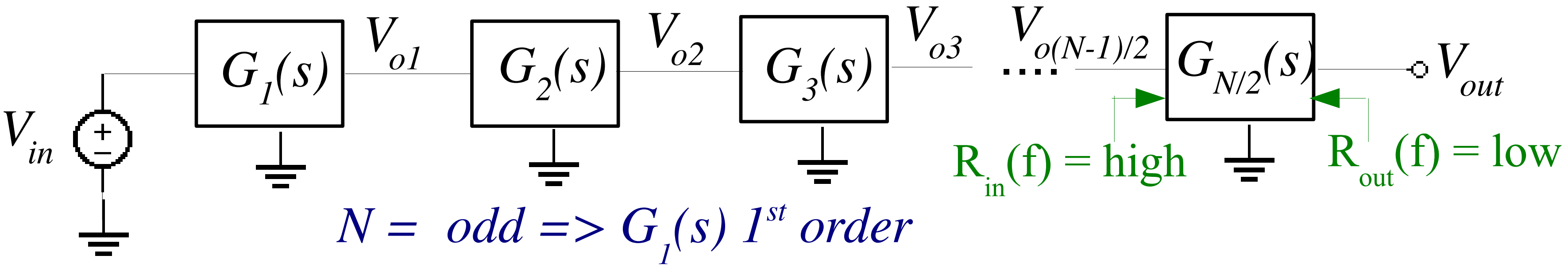
$$\frac{\omega_{SL}}{\omega_{PL}} \neq \frac{\omega_{SU}}{\omega_{PU}}$$

Symmetric bandpass filter

$$\frac{\omega_{SL}}{\omega_{PL}} = \frac{\omega_{SU}}{\omega_{PU}}$$

$$Q = \frac{\omega_{PU} - \omega_{PL}}{\omega_0}$$

Cascade Filter Design



$N =$ odd => $G_1(s)$ 1st order

$N =$ even => $G_1(s)$ 2nd order

$$G(s)=\frac{V_{out}}{V_{in}}=\frac{a_M s^M+a_{M-1}s^{M-1}+....+a_1 s+a_0}{s^N+b_{N-1}s^{N-1}+....+b_1 s+b_0}=\prod_{i=1}^{N/2} G_i(s)$$

If $N =$ even $\quad G_i(s)=\frac{a_{2i}s^2+a_{1i}s+a_{0i}}{s^2+b_{1i}s+b_{0i}}$ for $1\le i\le N/2$

If $N =$ odd $\quad G_1(s)=\frac{a_{10}s+a_{00}}{s+b_{10}} \quad G_i(s)=\frac{a_{2i}s^2+a_{1i}s+a_{0i}}{s^2+b_{1i}s+b_{0i}}$ for $2\le i\le N/2$

The factorization of $G(s)$ into $G_i(s)$ can be done with Matlab.

2nd Order (Biqudratic) Filter Section

$$G_i(s) = \frac{a_{2i} s^2 + a_{1i} s + a_{0i}}{s^2 + b_{1i} s + b_{0i}}$$

1. $G_i(s)$ is the most important active-filter building block.
2. Numerous realizations of $G_i(s)$, most using op-amp based circuits.
    - a. Multiple Op-amp Circuits with R's & C's (i.e. no L's)
    - b. Single Op-amp Circuits with R's & C's (i.e. no L's)
    - c. Cost advantage depends on how the filter is fabricated.

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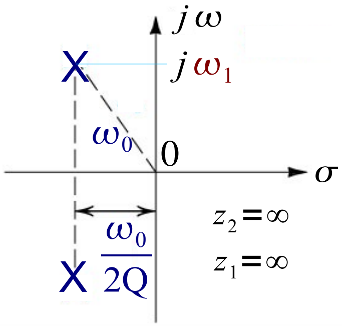
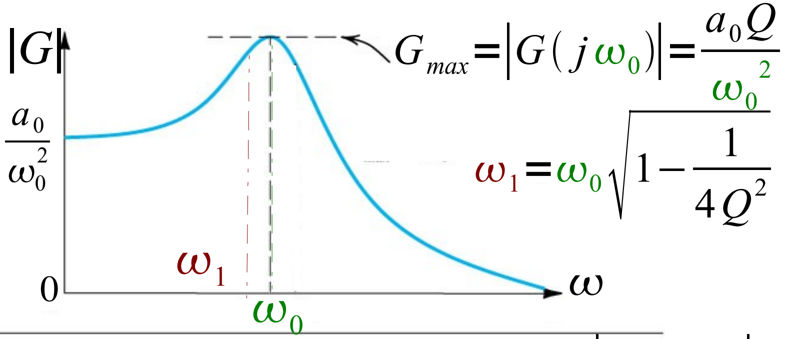
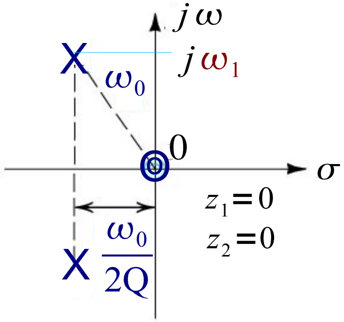
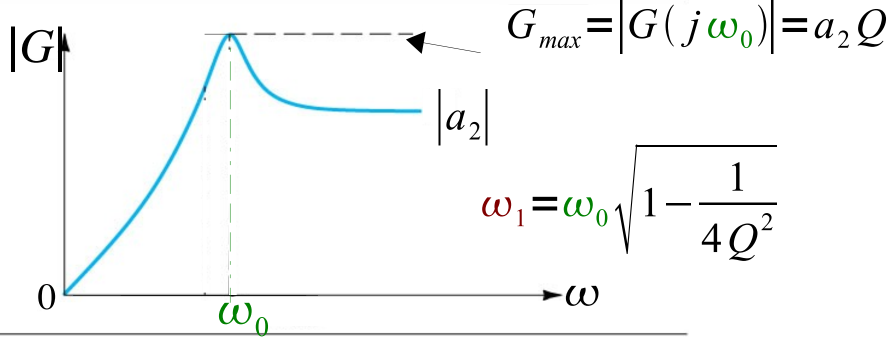
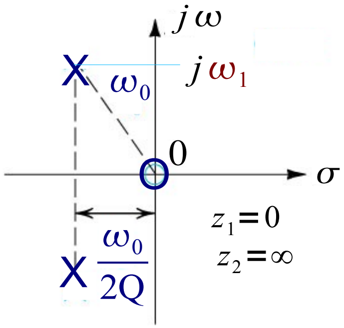
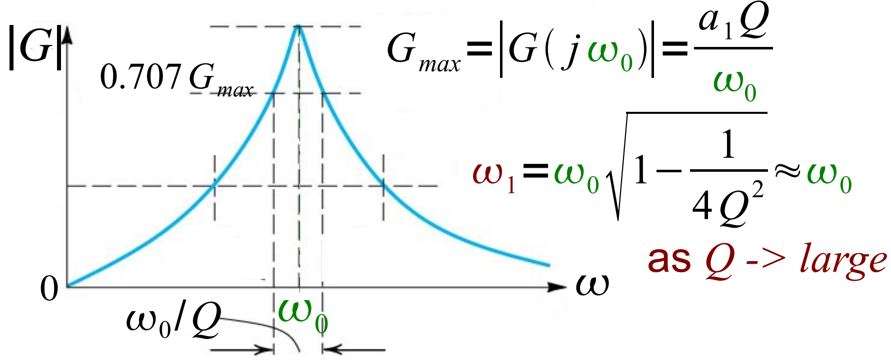
Filter Type	s-plane zeros/poles	$\lvert G\rvert$
2nd order low-pass (LP) $G(s)=\dfrac{a_0}{s^2+s\dfrac{\omega_0}{Q}+\omega_0^2}$ $\lvert G(j0)\rvert=\dfrac{a_0}{\omega_0^2}$	 $z_2=\infty$ $z_1=\infty$	 $G_{max}=\lvert G(j\omega_0)\rvert=\dfrac{a_0Q}{\omega_0{}^2}$ $\omega_1=\omega_0\sqrt{1-\dfrac{1}{4Q^2}}$
2nd order high-pass (HP) $G(s)=\dfrac{a_2s^2}{s^2+s\dfrac{\omega_0}{Q}+\omega_0^2}$ $\lvert G(j\infty)\rvert=a_2$	 $z_1=0$ $z_2=0$	 $G_{max}=\lvert G(j\omega_0)\rvert=a_2Q$ $\omega_1=\omega_0\sqrt{1-\dfrac{1}{4Q^2}}$
2nd order bandpass (BP) $G(s)=\dfrac{a_1s}{s^2+s\dfrac{\omega_0}{Q}+\omega_0^2}$ $\lvert G(j\omega_0)\rvert=\dfrac{a_1Q}{\omega_0}$	 $z_1=0$ $z_2=\infty$	 $G_{max}=\lvert G(j\omega_0)\rvert=\dfrac{a_1Q}{\omega_0}$ $\omega_1=\omega_0\sqrt{1-\dfrac{1}{4Q^2}}\approx\omega_0$ as Q -> *large*

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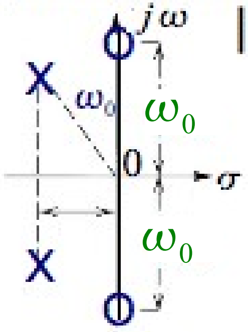
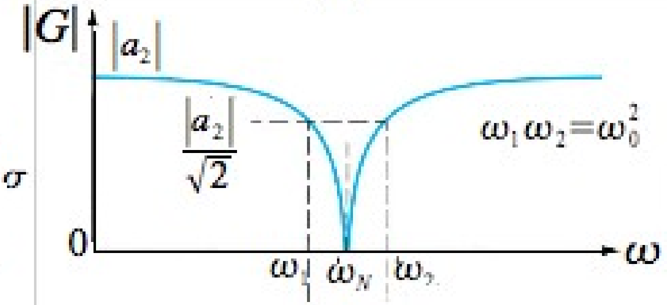
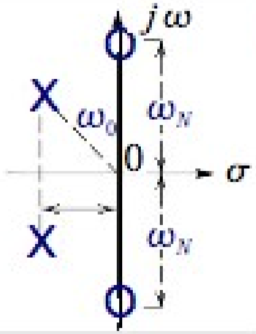
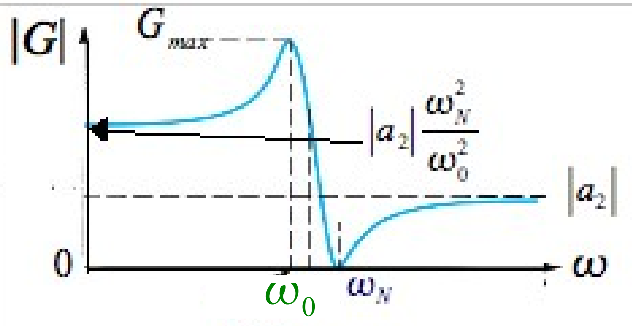
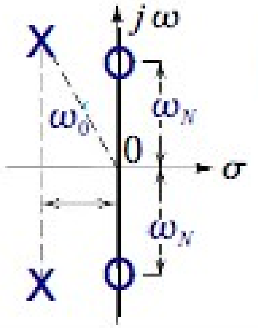
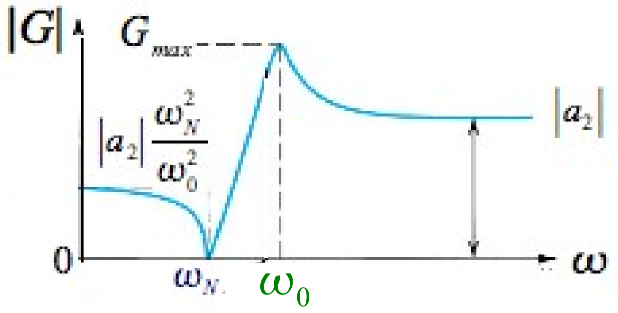
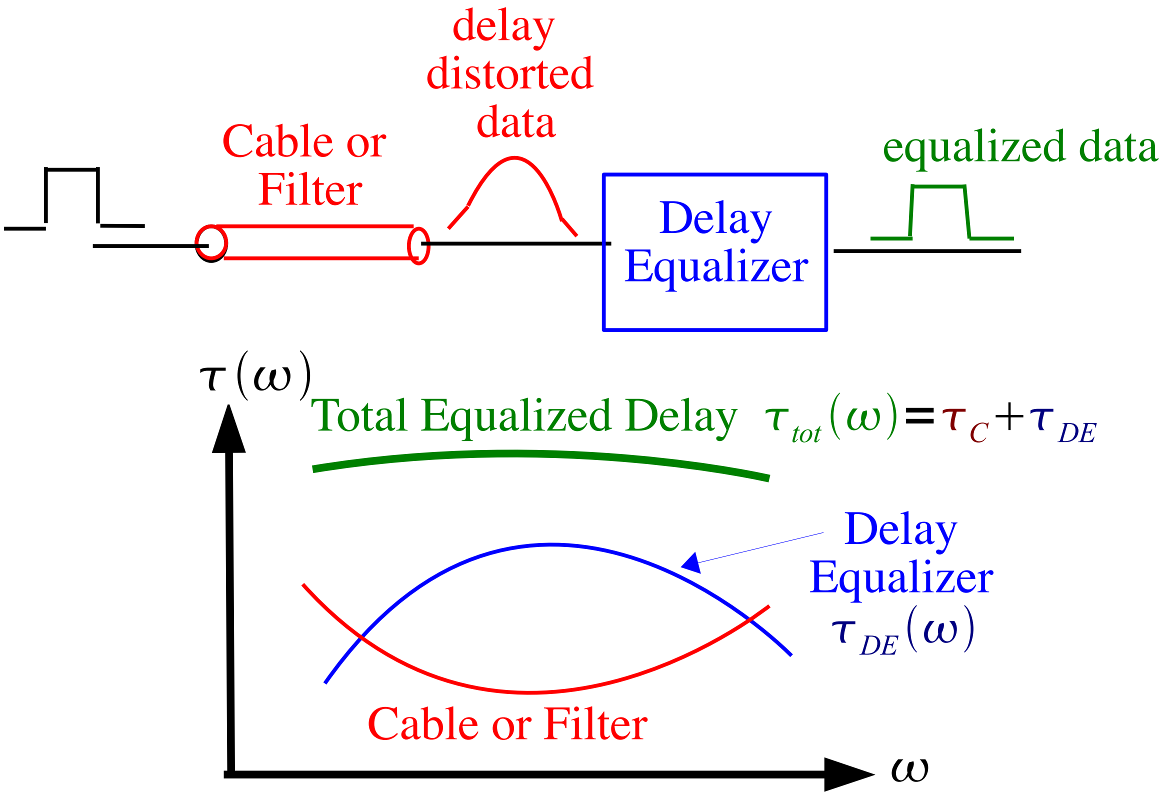
Filter Type	s-plane zeros/poles	$\lvert G\rvert$
2nd order Notch (N) $G(s)=a_2\frac{s^2+\omega_N^2}{s^2+s\frac{\omega_0}{Q}+\omega_0^2}$ $\omega_N=\omega_0$		 $\lvert G(j0)\rvert=\lvert G(j\infty)\rvert=\lvert a_2\rvert$
2nd order LP Notch (LPN) $G(s)=a_2\frac{s^2+\omega_N^2}{s^2+s\frac{\omega_0}{Q}+\omega_0^2}$ $\omega_N>\omega_0$		 $\lvert G(j0)\rvert=\lvert a_2\rvert\frac{\omega_N^2}{\omega_0^2}$ $\lvert G(j\infty)\rvert=\lvert a_2\rvert$
2nd order HP Notch (HPN) $G(s)=a_2\frac{s^2+\omega_N^2}{s^2+s\frac{\omega_0}{Q}+\omega_0^2}$ $\omega_N<\omega_0$		 $\lvert G(j0)\rvert=\lvert a_2\rvert\frac{\omega_N^2}{\omega_0^2}$ $\lvert G(j\infty)\rvert=\lvert a_2\rvert$

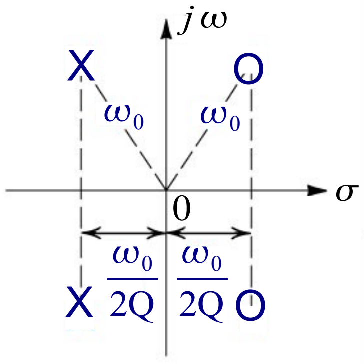
Delay Equalization Concept

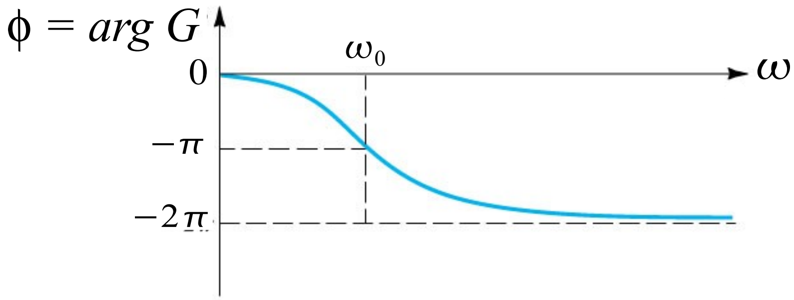


2nd order All-Pass (AP)

$$G(s)=a_2\frac{s^2-s\frac{\omega_0}{Q}+\omega_0^2}{s^2+s\frac{\omega_0}{Q}+\omega_0^2}$$

$$|G(j0)|=|G(j\infty)|=|a_2|$$





Ideal transmission:

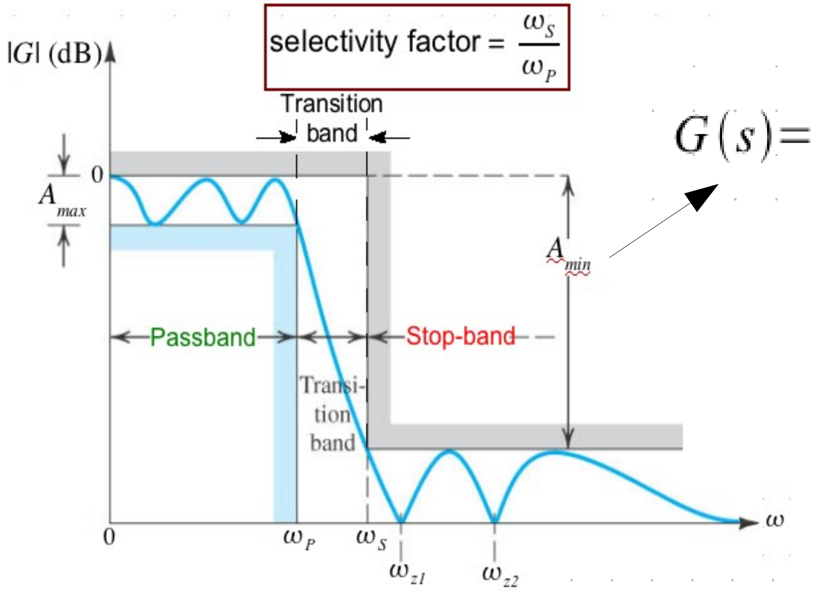
$$v_O(t)=K\,v_I(t-t_d)\Rightarrow G(j\omega)=|G(j\omega)|e^{j\omega t_d}$$

$$\rightarrow |G(j\omega)|=K$$

$$\rightarrow \phi(j\omega)=-\omega t_d$$

Group Delay $\tau(\omega)=\frac{-d\,\phi(j\omega)}{d\,\omega}=t_d$

Quick Review

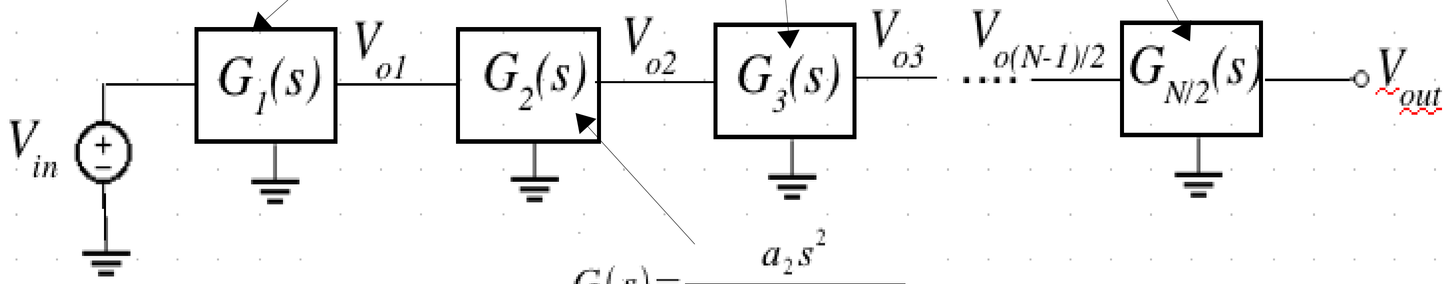


$$G(s)=\frac{V_{out}}{V_{in}}=\frac{a_M s^M + a_{M-1} s^{M-1} + ... + a_1 s + a_0}{s^N + b_{N-1} s^{N-1} + ... + b_1 s + b_0}=\prod_{i=1}^{N/2} G_i(s)$$

$$G(s)=a_2\frac{s^2+\omega_N^2}{s^2+s\frac{\omega_0}{Q}+\omega_0^2}$$

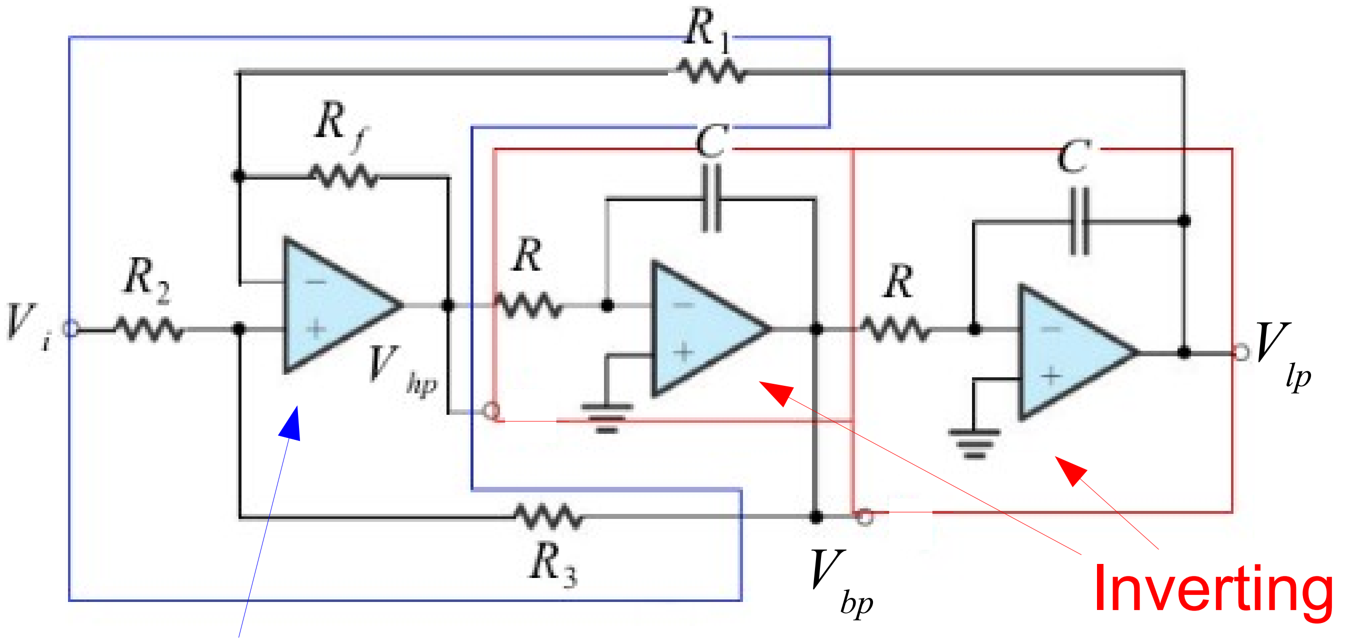
$$G(s)=\frac{a_1 s}{s^2+s\frac{\omega_0}{Q}+\omega_0^2}$$

$$G(s)=\frac{a_0}{s^2+s\frac{\omega_0}{Q}+\omega_0^2}$$



$$G(s)=\frac{a_2 s^2}{s^2+s\frac{\omega_0}{Q}+\omega_0^2}$$

Multiple Op-Amp Biquadratic Circuits



Summer

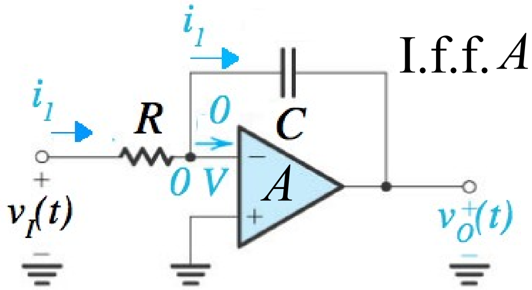
Inverting Integrators

Two-Integrator-Feedback-Loop

OP Amp Circuit Building Blocks

Inverting Integrator

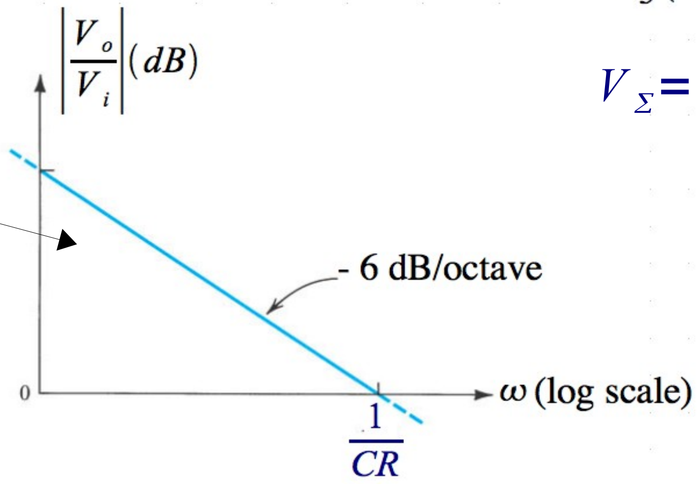
$$v_O(t) = \frac{-1}{CR}\int_0^t v_I(t)\,dt$$



I.f.f. $A = \infty \Rightarrow v^- = 0$

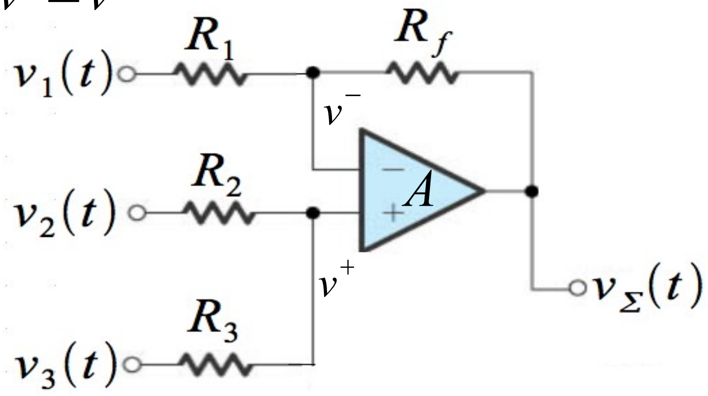
$$\frac{V_o(s)}{V_i(s)} = -\frac{1}{sCR} = -\frac{\omega_0}{s}$$

$$\omega_0 = \frac{1}{CR}$$



Summer

I.f.f. $A = \infty \Rightarrow v^- = v^+$



$$V_\Sigma = \frac{-R_f}{R_1}V_1 + \frac{R_3}{R_2+R_3}\left(1+\frac{R_f}{R_1}\right)V_2 + \frac{R_2}{R_2+R_3}\left(1+\frac{R_f}{R_1}\right)V_3$$

Two-Integrator-Feedback-Loop Active Filter

Let's start with $V_{hp}(s)=\dfrac{K\,s^2}{s^2+s\dfrac{\omega_0}{Q}+\omega_0^2}V_i(s)$

OBJECTIVE: realize $V_{hp}(s)$ using a two-integrator cascade (loop).

1. Divide numerator and denominator of $V_{hp}(s)$ by s^2, i.e.

$$V_{hp}(s)=\frac{K}{1+\dfrac{1}{Q}\dfrac{\omega_0}{s}+\dfrac{\omega_0^2}{s^2}}V_i(s)$$

2. Multiply both sides of $V_{hp}(s)$ by $1+\dfrac{1}{Q}\dfrac{\omega_0}{s}+\dfrac{\omega_0^2}{s^2}$, i.e.

$$V_{hp}(s)[1+\frac{1}{Q}\underbrace{\boxed{\frac{\omega_0}{s}}}_{?}+\underbrace{\boxed{\frac{\omega_0^2}{s^2}}}_{?}]=K\,V_i(s)\Rightarrow \boxed{\underbrace{V_{hp}(s)}_{\text{HP}}+\underbrace{\frac{1}{Q}\frac{\omega_0}{s}V_{hp}(s)}_{\text{BP}}+\underbrace{\frac{\omega_0^2}{s^2}V_{hp}(s)}_{\text{LP}}=K\,V_i(s)}$$

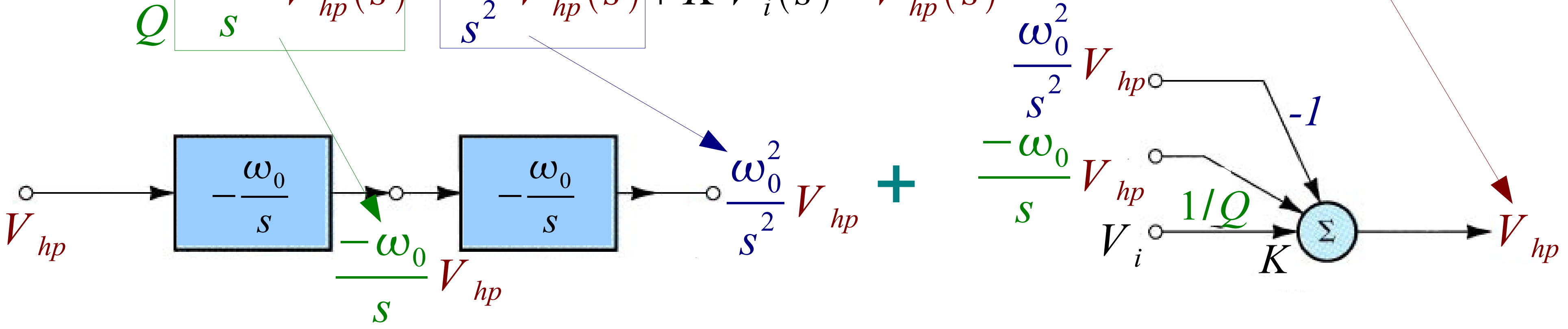
Two-Integrator-Feedback-Loop Active Filter

From previous slide

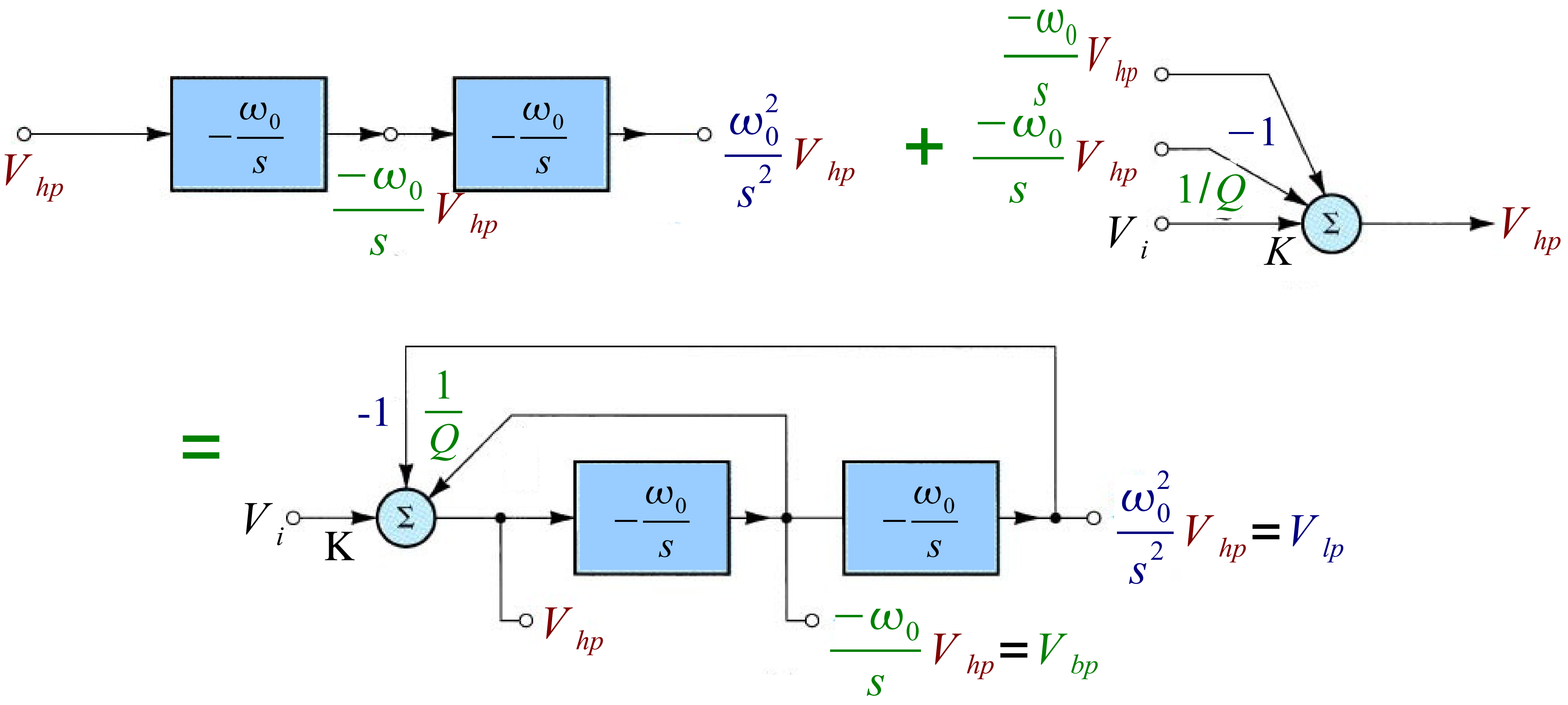
$$V_{hp}(s)+\frac{1}{Q}\frac{\omega_0}{s}V_{hp}(s)+\frac{\omega_0^2}{s^2}V_{hp}(s)=K\,V_i(s)$$

3. Rearrange terms

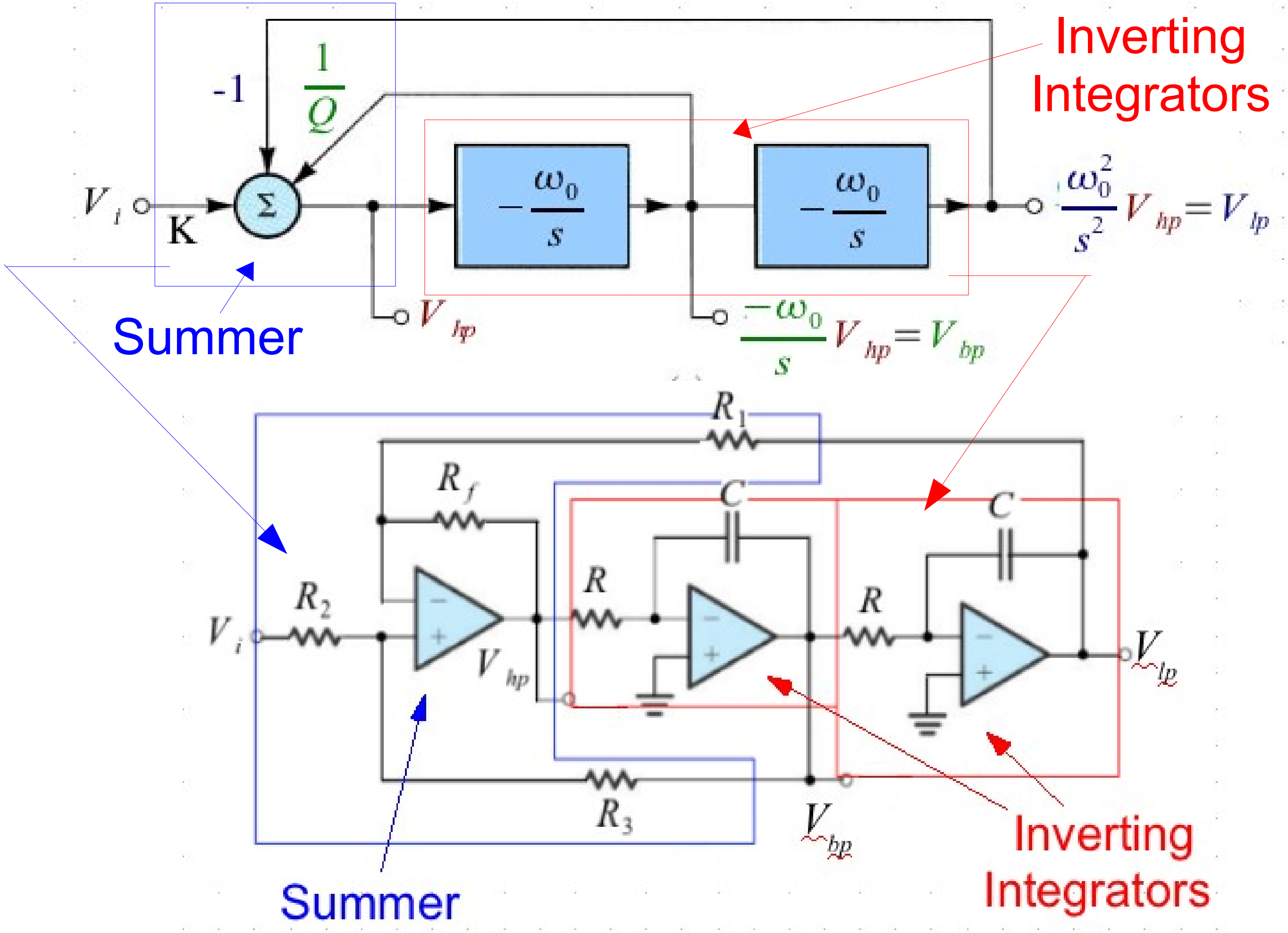
$$\frac{1}{Q}\boxed{\frac{-\omega_0}{s}}V_{hp}(s)-\boxed{\frac{\omega_0^2}{s^2}}V_{hp}(s)+K\,V_i(s)=V_{hp}(s)$$



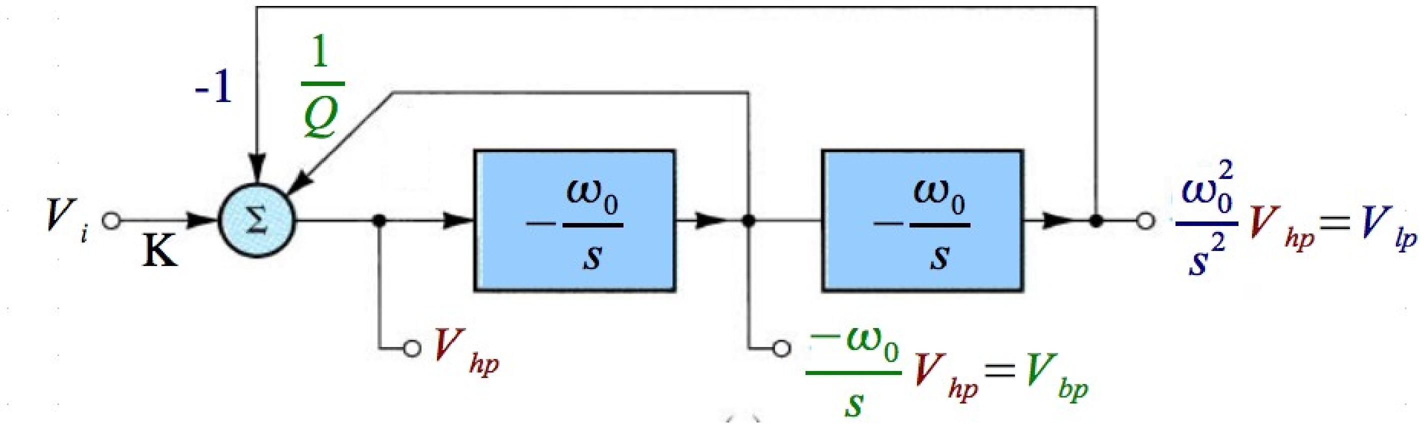
Two-Integrator-Feedback-Loop Active Filter



Two-Integrator-Feedback-Loop Active Filter



Two Integrator Loop cont.



$$V_{hp} = -V_{lp} + \frac{1}{Q} V_{bp} + K V_i \Rightarrow V_{hp} = -\frac{\omega_0^2}{s^2} V_{hp} - \frac{1}{Q}\frac{\omega_0}{s} V_{hp} + K V_i$$

$$\Rightarrow G_{hp}(s) = \frac{V_{hp}}{V_i} = \frac{K}{1 + \frac{1}{Q}\frac{\omega_0}{s} + \frac{\omega_0^2}{s^2}} = \frac{K s^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

Numeric (Spec) Eqs.

Specs: ω_0 or f_0, Q & K

$G_{bp}(s)$ and $G_{lp}(s)$ are also available.

Two Integrator Loop cont.

High Pass Output:

$$G_{hp}(s)=\frac{V_{hp}}{V_i}=\frac{K\,s^2}{s^2+\frac{\omega_0}{Q}s+\omega_0^2}$$

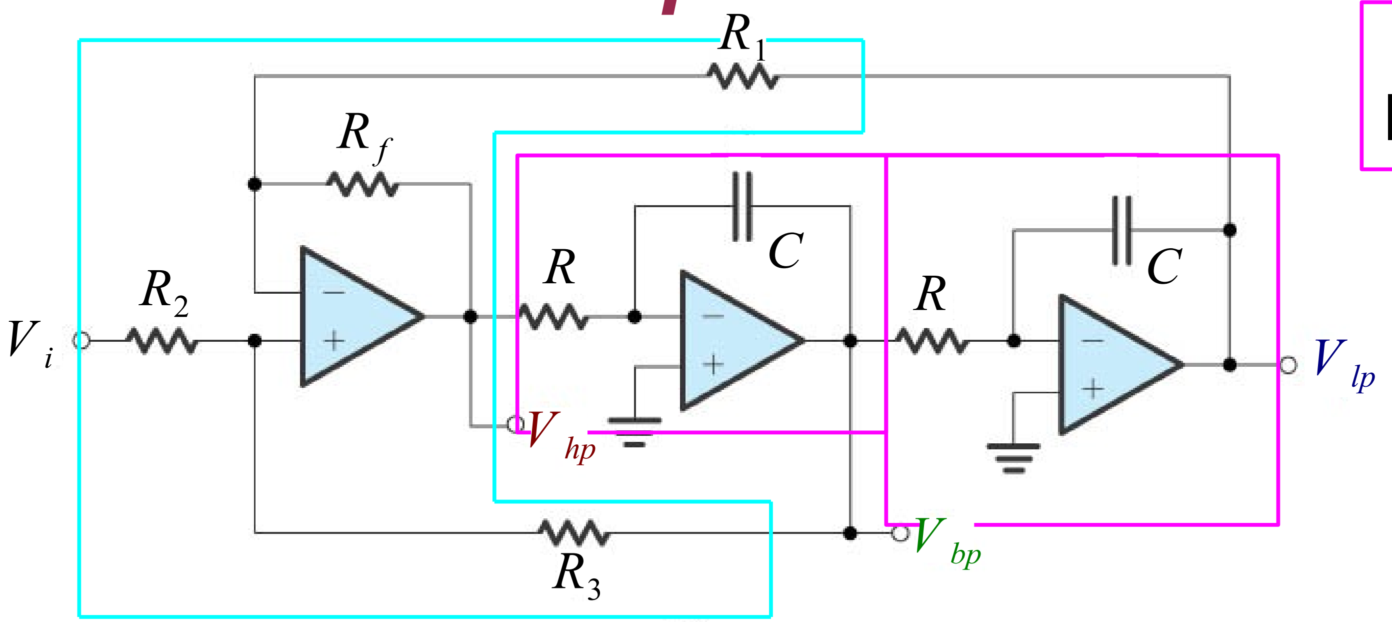
Bandpass Output:

$$G_{bp}(s)=\frac{V_{bp}}{V_i}=\frac{-\omega_0}{s}\frac{V_{hp}}{V_i}=\frac{-K\,\omega_0\,s}{s^2+\frac{\omega_0}{Q}s+\omega_0^2}$$

Lowpass Output:

$$G_{lp}(s)=\frac{V_{lp}}{V_i}=\frac{-\omega_0}{s}\frac{V_{bp}}{V_i}=\frac{K\,\omega_0^2}{s^2+\frac{\omega_0}{Q}s+\omega_0^2}$$

Implementation



Inverting Integrators $\omega_0 = \frac{1}{CR}$

Summing Amp

$$V_{bp} = \frac{-\omega_0}{s} V_{hp}$$

$$V_{lp} = \frac{\omega_0^2}{s^2} V_{hp}$$

$$V_{hp} = \boxed{-\frac{R_f}{R_1}} V_{lp} + \boxed{\frac{R_2}{R_2+R_3}\left(1+\frac{R_f}{R_1}\right)} V_{bp} + \boxed{\frac{R_3}{R_2+R_3}\left(1+\frac{R_f}{R_1}\right)} V_i$$

Symbolic Eq.

Numeric Eq. $V_{hp} = (-1) V_{lp} + \frac{1}{Q} V_{bp} + K V_i$

Numeric Specs: ω_0 or f_0, Q & K

Numeric Eq. = Symbolic Eq.

Implementation cont.

$$V_{hp}=-\frac{R_f}{R_1}V_{lp}+\frac{R_2}{R_2+R_3}\left(1+\frac{R_f}{R_1}\right)V_{bp}+\frac{R_3}{R_2+R_3}\left(1+\frac{R_f}{R_1}\right)V_i$$

Set: $\boxed{R_f=R_1}$ And compare terms:

$$V_{hp}=(-1)V_{lp}+\frac{2R_2}{R_2+R_3}V_{bp}+\frac{2R_3}{R_2+R_3}V_i$$

Symbolic Eq.

$$V_{hp}=(-1)V_{lp}+\frac{1}{Q}V_{bp}+KV_i$$

Numeric Eq.

$$Q=\frac{R_2+R_3}{2R_2}\Rightarrow Q=\frac{1}{2}\left(1+\frac{R_3}{R_2}\right) \quad => \quad \boxed{\begin{array}{l}\frac{R_3}{R_2}=2Q-1=\frac{2}{K}-1\\ \frac{1}{CR}=\omega_0\end{array}} \Rightarrow K=2-\frac{1}{Q}$$

$$K=\frac{2R_3}{R_2+R_3}\Rightarrow\frac{1}{K}=\frac{1}{2}\left(1+\frac{R_2}{R_3}\right)$$

Design Eqs.

Design Equations

$$RC = \frac{1}{\omega_0}$$ Given $\omega_0 = 2\pi f_0$, choose C, calculate R

$$R_f = R_1$$ Choose R_f, Calculate R_1 or vice-versa.

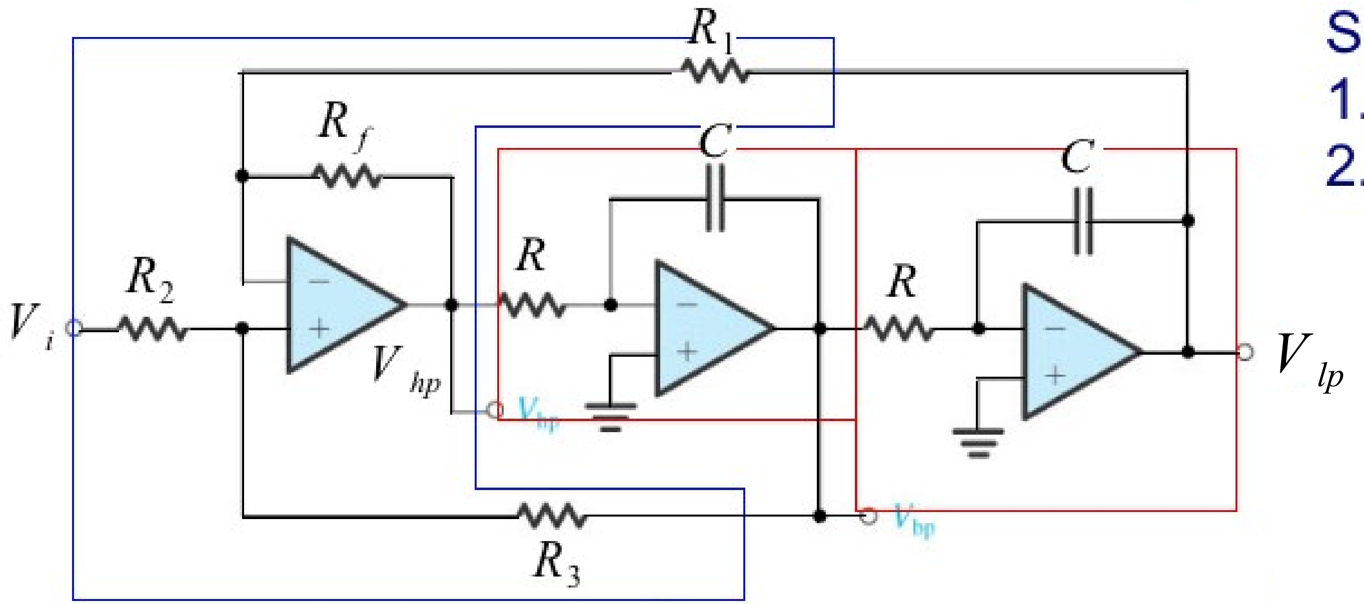
$$\frac{R_3}{R_2} = 2Q - 1$$

Given Q, choose R_2, calculate R_3 or vice-versa.

$$K = 2 - \frac{1}{Q}$$ K is fixed by choice of Q.

- 2 independent requirements (ω_0 and Q)
- 3 independent components (C, R_f, and R_2)
- 3 dependent components (R, R_1, and R_3)

MFM LP Example



Specifications:

1. MFM 2^{nd} $Order\ LP$
2. $f_P = f_0 = 20\,kHz$

symbolic $G_{lp}(s)$

specification $G_{lp}(s)$

$$G_{lp}(s) = \frac{V_{lp}}{V_i} = \frac{\dfrac{R_3}{R_2+R_3}\left(1+\dfrac{R_f}{R_1}\right)\omega_0^2}{s^2 + s\dfrac{R_2}{R_2+R_3}\left(1+\dfrac{R_f}{R_1}\right)\left(\dfrac{1}{RC}\right) + \dfrac{R_f}{R_1}\left(\dfrac{1}{RC}\right)^2} = \frac{K\omega_0^2}{s^2 + s\sqrt{2}\,\omega_0 + \omega_0^2}$$

MFM LP Example - cont.

Choosing $R_f = R_1 = R_2 = 10\,k\,\Omega \;\Rightarrow \dfrac{R_f}{R_1} = 1$

$$G_{lp}(s) = \frac{V_{lp}}{V_i} = \frac{\dfrac{R_3}{R_2+R_3}\left(1+\dfrac{R_f}{R_1}\right)\omega_0^2}{s^2 + s\dfrac{R_2}{R_2+R_3}\left(1+\dfrac{R_f}{R_1}\right)\left(\dfrac{1}{RC}\right) + \dfrac{R_f}{R_1}\left(\dfrac{1}{RC}\right)^2} = \frac{K\,\omega_0^2}{s^2 + s\sqrt{2}\,\omega_0 + \omega_0^2}$$

$$RC = RC = \omega_0 = 2\pi f_0$$

$$\frac{2R_2}{R_2+R_3} = \sqrt{2} \Rightarrow \frac{R_3}{R_2} = \frac{2}{\sqrt{2}} - 1 = \sqrt{2} - 1 = 0.414$$

$$\frac{2R_3}{R_2+R_3} = K \Rightarrow \frac{R_2}{R_3} = \frac{2}{K} - 1 \Rightarrow K = 2\frac{\sqrt{2}-1}{\sqrt{2}} = 0.586$$

MFM LP Example - cont.

Design Eqs.

Choose $R_2 = R_f = R_1 = 10\,k\,\Omega$

$C = 7.95\,nF$

$$\frac{1}{RC} = \omega_0 = 2\pi f_0 = 125.66\,k\,rad/sec$$

$$R = \frac{1}{C(125.66\,k\,rad/sec)} = \frac{1}{7.95\,10^{-9}(125.66\,k\,rad/sec)} = 1\,k\,\Omega$$

$$\frac{2R_2}{R_2+R_3} = \sqrt{2} \Rightarrow \frac{R_3}{R_2} = 0.414 \Rightarrow R_3 = 4.14\,k\,\Omega$$

$$\frac{2R_3}{R_2+R_3} = K \Rightarrow K = 0.586$$

Final Design

$R_2 = R_f = R_1 = 10\,k\,\Omega$

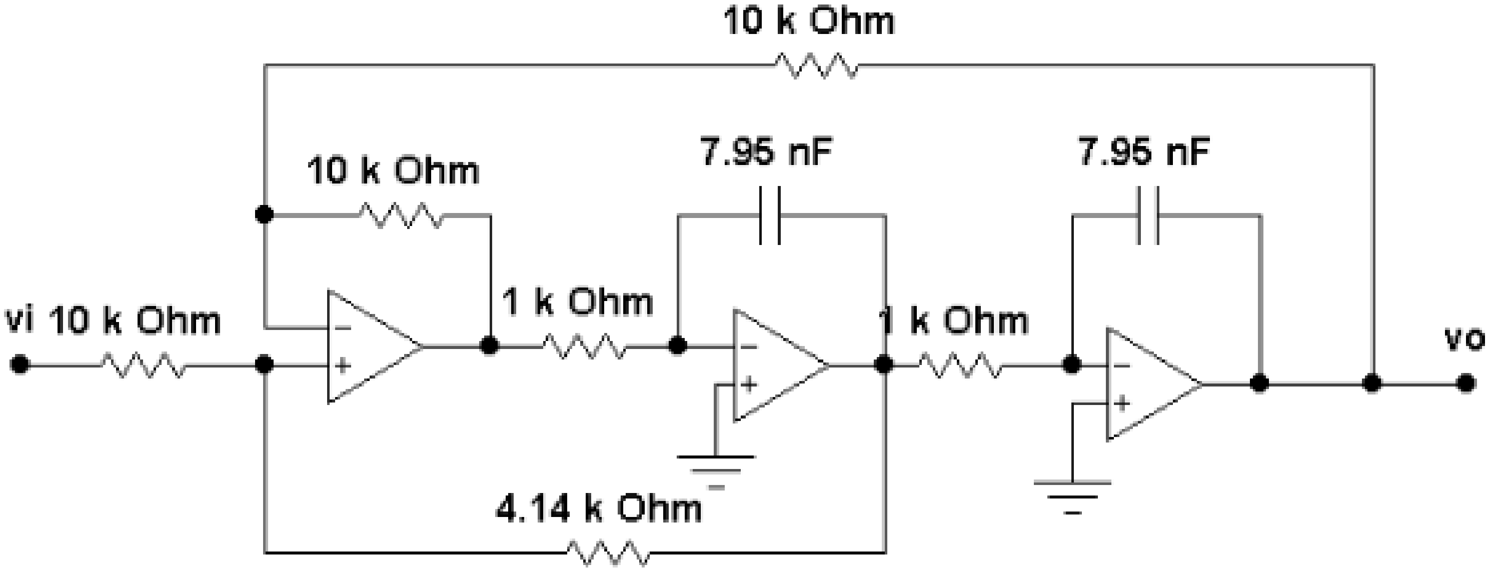
$R_3 = 4.14\,k\,\Omega$

$R = 1\,k\,\Omega$

$C = 7.95\,nF$

$K = 0.586$

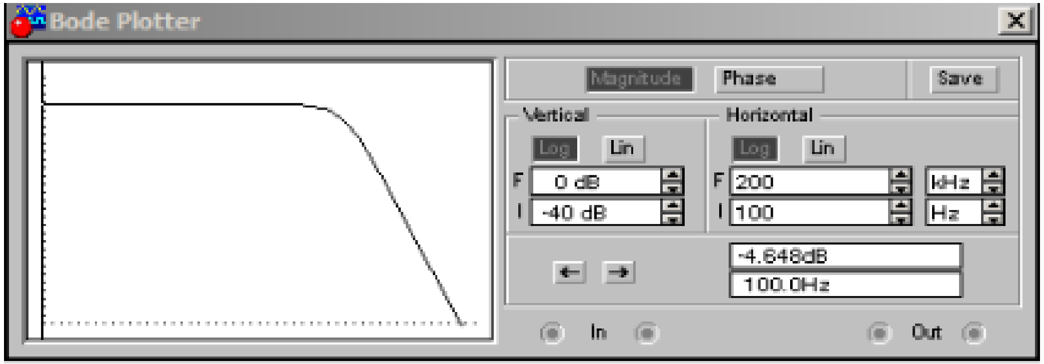
MFM LP Example - cont.



$$K = 0.586$$

$$G_{lp}(j0) = 20\log_{10}(0.586) = -4.64\,dB$$

MFM LP Example - Bode Plots

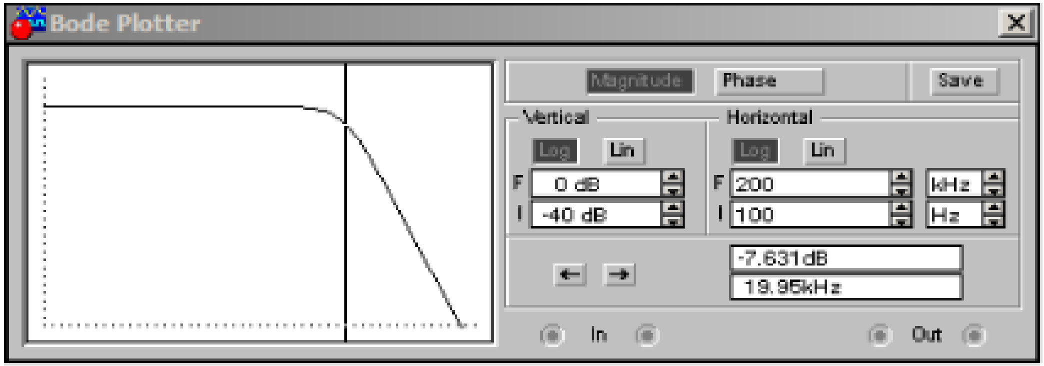


Vertical: Log
F 0 dB
I -40 dB

Horizontal: Log
F 200 kHz
I 100 Hz

-4.648dB
100.0Hz

Passband Gain



Vertical: Log
F 0 dB
I -40 dB

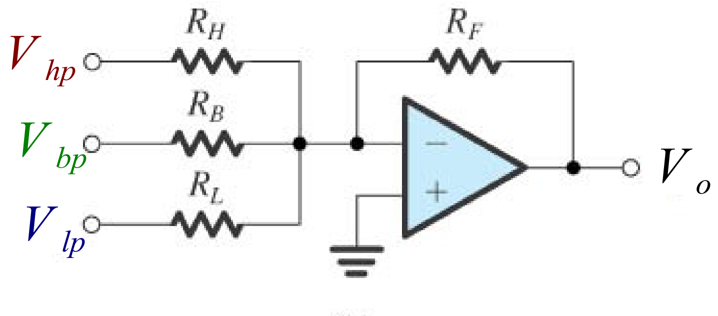
Horizontal: Log
F 200 kHz
I 100 Hz

-7.631dB
19.95kHz

3dB Frequency

Adding Finite Zeros – (Notches)

To be able to create notches in the response, we need a second summing amplifier:



(b)

Where the weighted inputs come from the highpass, bandpass, and lowpass outputs of the feedback circuit.

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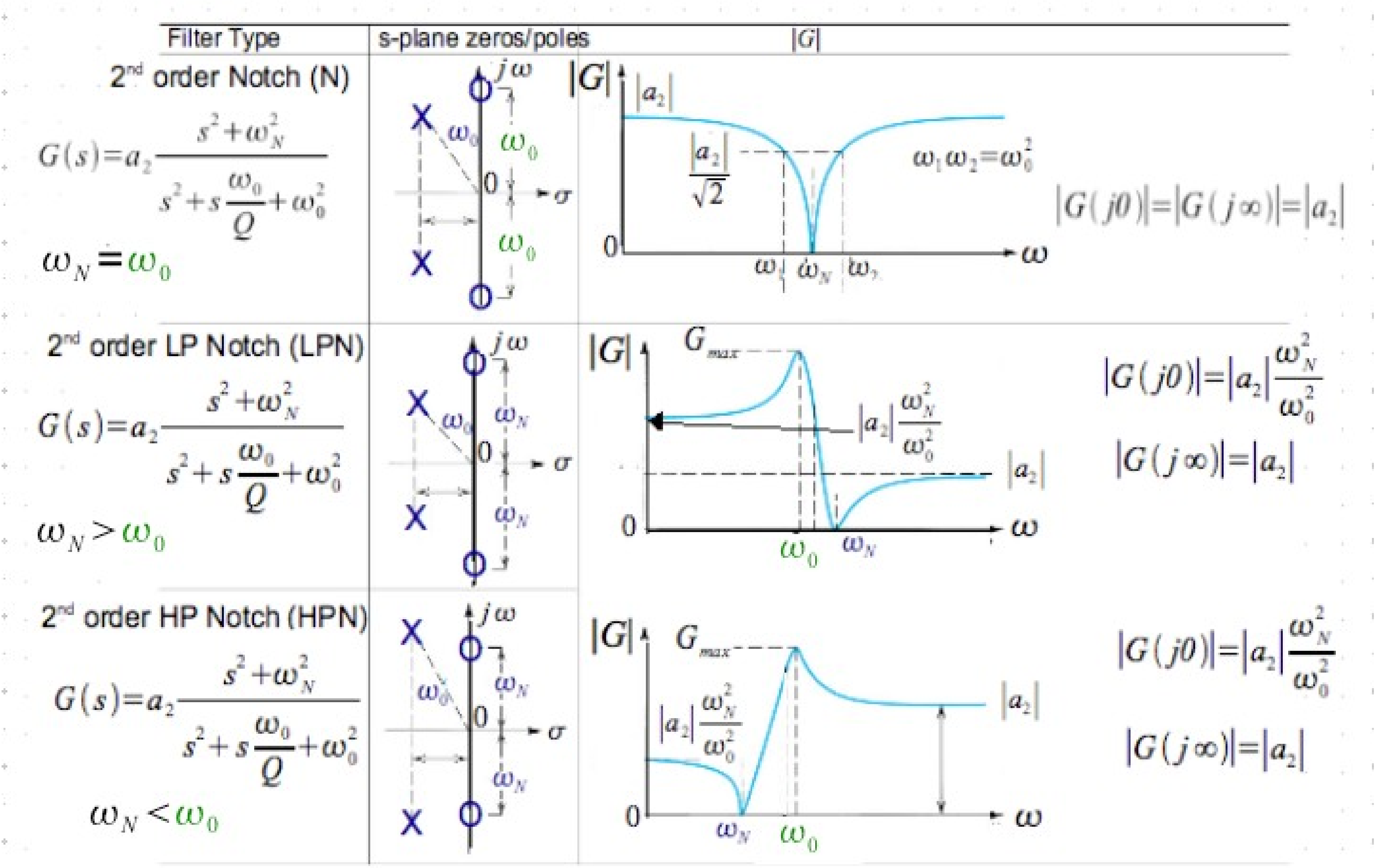
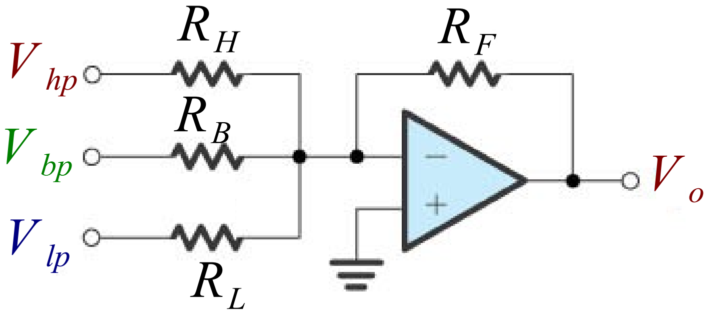


Filter Type	s-plane zeros/poles	|G|
2nd order Notch (N) $G(s)=a_2\frac{s^2+\omega_N^2}{s^2+s\frac{\omega_0}{Q}+\omega_0^2}$ $\omega_N=\omega_0$		$\omega_1\omega_2=\omega_0^2$ $\lvert G(j0)\rvert=\lvert G(j\infty)\rvert=\lvert a_2\rvert$
2nd order LP Notch (LPN) $G(s)=a_2\frac{s^2+\omega_N^2}{s^2+s\frac{\omega_0}{Q}+\omega_0^2}$ $\omega_N>\omega_0$		$\lvert G(j0)\rvert=\lvert a_2\rvert\frac{\omega_N^2}{\omega_0^2}$ $\lvert G(j\infty)\rvert=\lvert a_2\rvert$
2nd order HP Notch (HPN) $G(s)=a_2\frac{s^2+\omega_N^2}{s^2+s\frac{\omega_0}{Q}+\omega_0^2}$ $\omega_N<\omega_0$		$\lvert G(j0)\rvert=\lvert a_2\rvert\frac{\omega_N^2}{\omega_0^2}$ $\lvert G(j\infty)\rvert=\lvert a_2\rvert$

Notch Creation

All the output point transfer functions contain the same denominator, so only the numerator terms will be affected:



$$G(s) = -\left(\frac{R_F}{R_H} V_{hp} + \frac{R_F}{R_B} V_{bp} + \frac{R_F}{R_L} V_{lp}\right)$$

$$G(s) = -K \frac{(R_F/R_H)s^2 - (R_F/R_B)\omega_0 s + (R_F/R_L)\omega_0^2}{s^2 + (\omega_0/Q)s + \omega_0^2}$$

For a notch at $\omega = \omega_N$, no connection is made to V_{bp}, i.e. $R_B = \infty$

"Big Picture" Filter Design Tasks

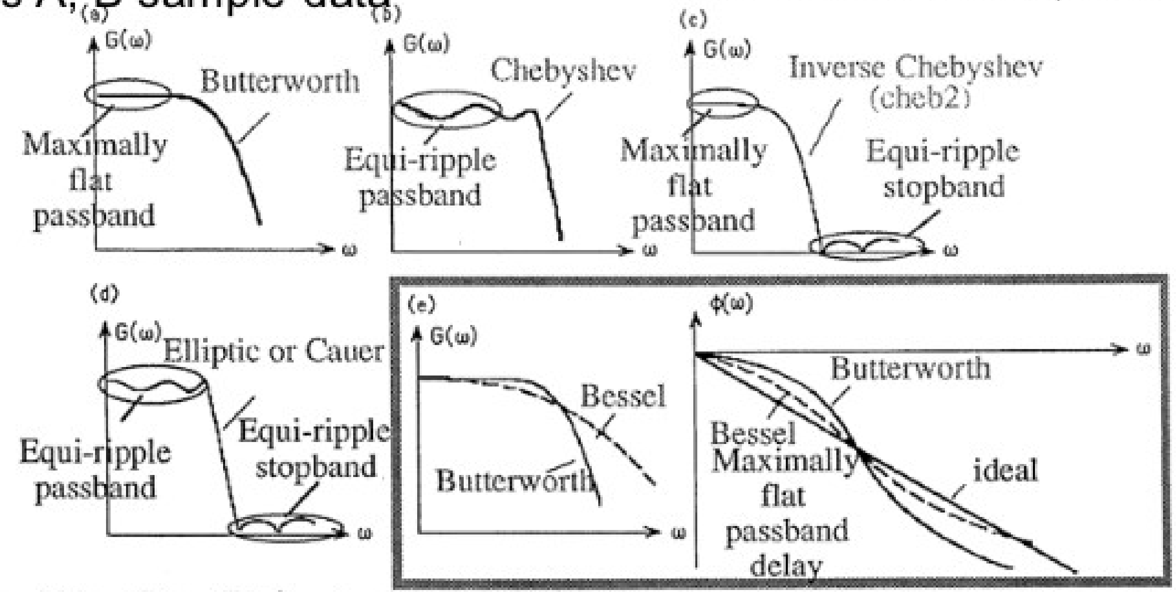
1. Design $G(s)$ from filter specs.
2. Determine filter structure (block diagram) to realize $G(s)$.
3. Determine filter circuit(s) to implement structure.
4. Determine component values.

Filter Design CAD Tools on the Market

1. MatLab - Mathworks
2. FILTER PRO – Texas Instruments
3. Aktiv Filter – New Wave Instruments
4. Filter Lab – Microchip
5. Filter Wiz Pro – Schematica
6. FilterCAD – Linear Technology

Butterworth (MFM)
Matlab:
1. buttord (Wp, Ws, Rp, Rs) => n
2. buttap (n) => poles, zeros, K cont-time
3. butter (n, Wn, ftype) => poles, zeros, K or coefficients A, B sample-data

Chebyshev
Matlab:
1. cheb1ord (Wp, Ws, Rp, Rs) => n
2. cheb1ap (n) => poles, zeros, K cont-time
3. cheb1 (n, Rp, Wn, ftype) => poles, zeros, K or coefficients A, B sample-data



Elliptic
Matlab:
1. ellipord (Wp, Ws, Rp, Rs) => n
2. ellipap (n, Rp, Rs) => poles, zeros, K cont-time
3. ellip (n, Rp, Wn, ftype) => poles, zeros, K or coefficients A, B sample-data