



Penn
UNIVERSITY of PENNSYLVANIA

ESE319 Introduction to Microelectronics

BJT Intro and Large Signal Model

Why BJT?

What's the competition to BJT and bipolar technologies?

What advantages does the competition have over BJT?

What advantages does BJT and bipolar have over their competition?

What circuit applications benefit from BJT and bipolar technologies?

Why BJT

What's the competition to BJT and bipolar technologies?

MOSFET, in particular CMOS is the leading competitor

What advantages does the competition have over BJT?

Small size (die area), low cost and low power dissipation

What advantages does BJT and bipolar have over their competition?

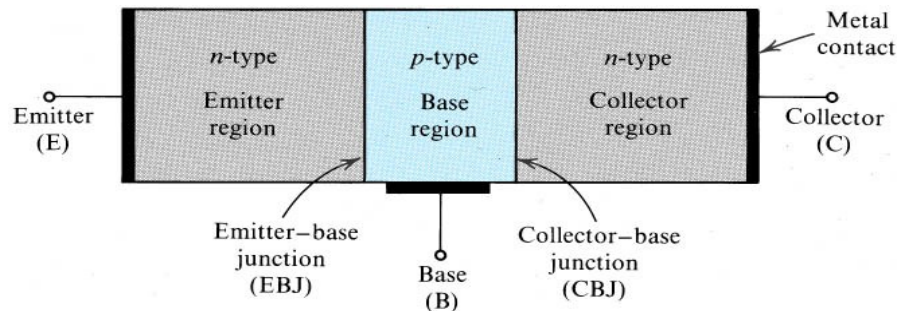
High frequency operation, high current drive, high reliability in severe environmental conditions.

What circuit applications benefit from BJT and bipolar technologies?

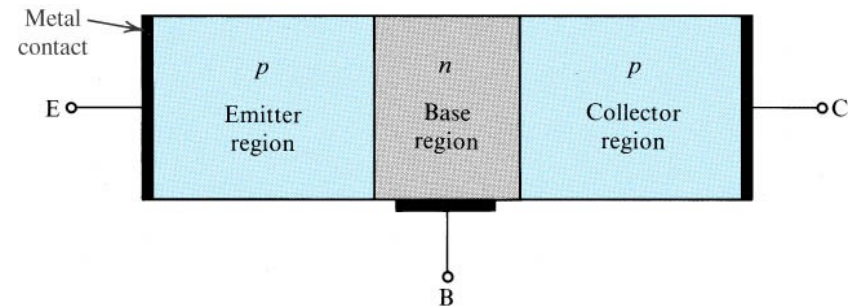
RF analog and digital circuits, power electronics and power amplifiers, automobile electronics, radiation hardened electronics

BJT Physical Configuration

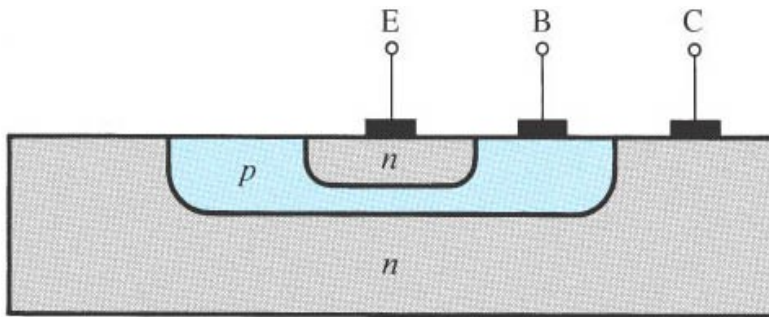
NPN



PNP

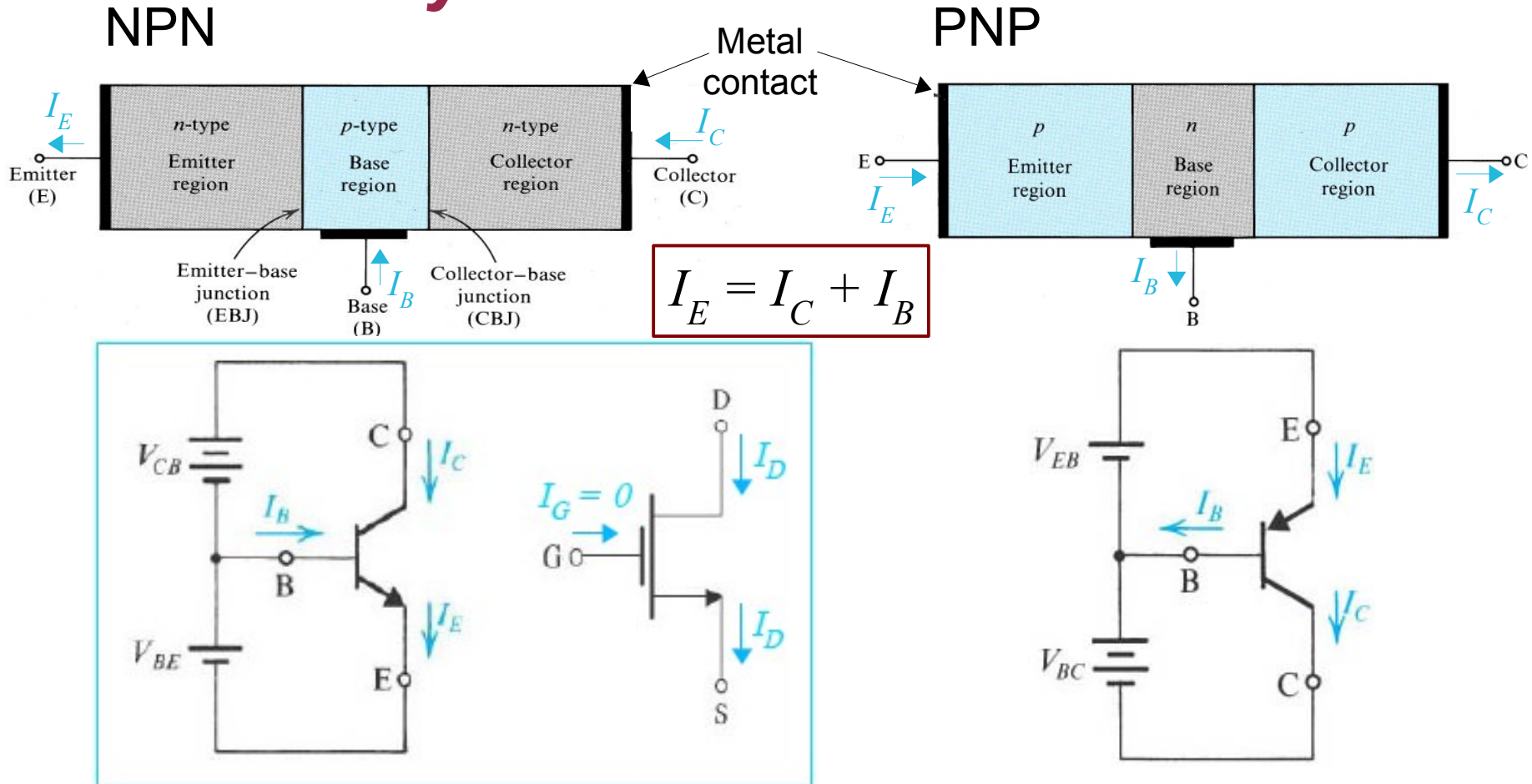


Closer to actual layout



Each transistor looks like two back-to-back diodes, but each behaves much differently!

BJT Symbols and Conventions



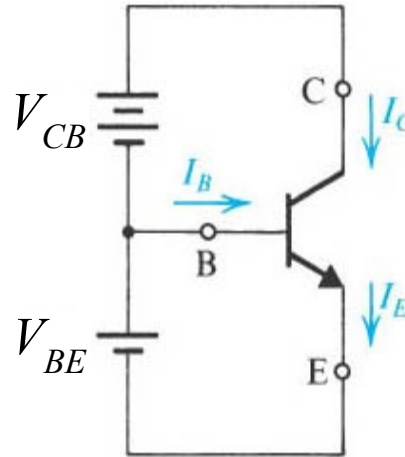
Note reversal in current directions and voltage signs for PNP vs. NPN!

NPN BJT Modes of Operation

$$i_E = i_C + i_B$$

$$v_{CE} = v_{CB} + v_{BE}$$

v_{XY}	$=$	V_{XY}	$+$	v_{xy}
large signal		dc bias		ac signal



Forward-Active Mode

EBJ forward bias ($V_{BE} > 0$)

CBJ reverse bias ($V_{BC} < 0$)

Mode	V_{BE}	V_{BC}
Forward-Active	> 0	< 0
Reverse-Active	< 0	> 0
Cutoff	< 0	< 0
Saturation	> 0	> 0

$$V_{BC} = -V_{CB}$$

← Not Useful!

NPN BJT Forward-Active Mode Basic Model

Collector-base diode
is **reverse biased**

$$V_{CB} > 0 \quad (\text{or } V_{BC} < 0)$$

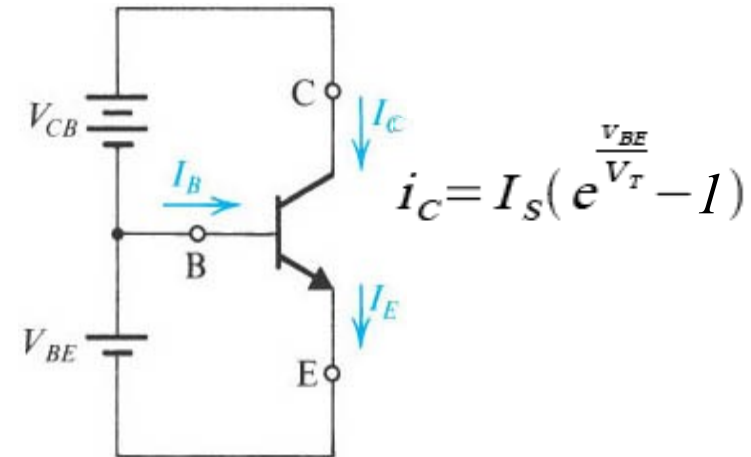
Base-emitter diode is
forward biased

$$V_{BE} \approx 0.7$$

$$I_s = \frac{A_E q D_n n_i^2}{N_A W}$$

saturation current

$$i_C \approx I_s e^{\frac{v_{BE}}{V_T}}$$



$$V_T = \frac{kT}{q} \approx 25 \text{ mV} @ 25^\circ \text{C}$$

$$i_B = \frac{i_C}{\beta}$$

- A_E Area of base-emitter junction
- W Width of base region
- N_A Doping concentration in base
- D_n Electron diffusion constant
- n_i Intrinsic carrier concentration = f(T)

$$i_E = i_B + i_C = (\beta + 1) i_B$$

$\beta = \text{common-emitter current gain}$



NPN BJT Forward-Active Beta (β)

A_E -> Area of base-emitter junction

W -> Width of base region

N_A -> Doping concentration in base

N_D -> Doping concentration in emitter

D_n -> Electron diffusion constant

D_p -> Hole diffusion constant

L_p -> Hole diffusion length in emitter

τ_b -> Minority-carrier lifetime

n_i -> Intrinsic carrier concentration = $f(T)$

$$\beta = \frac{1}{\frac{D_p N_A W}{D_n N_D L_p} + \frac{1}{2} \frac{W^2}{D_n \tau_b}}$$

Large β =>

N_A -> small

N_D -> large

W -> small

Note that the i_C equation looks like that of a forward-biased diode collector-base. Is it?

$$i_C = I_S \left(e^{\frac{v_{BE}}{V_T}} - 1 \right)$$

Using Eqs. $i_E = i_B + i_C$ and $i_C = \beta i_B$ we can answer this question, i.e.

$$i_E = i_B + i_C = \left(\frac{1}{\beta} + 1 \right) i_C = \frac{\beta + 1}{\beta} i_C$$

$\beta = \text{common-emitter current gain}$

$\alpha = \text{common-base current gain}$

and write:

$$i_E = \frac{\beta + 1}{\beta} I_S \left(e^{\frac{v_{BE}}{V_T}} - 1 \right) = \frac{1}{\alpha} I_S \left(e^{\frac{v_{BE}}{V_T}} - 1 \right) = \frac{i_C}{\alpha} \quad \text{where} \quad \alpha = \frac{\beta}{1 + \beta}$$

AhHa! This i_E equation describes a forward-biased emitter-base “diode”.

So the new set of equations is:

$$i_E = \frac{I_S}{\alpha} \left(e^{\frac{v_{BE}}{V_T}} - 1 \right)$$

$$i_C = I_S \left(e^{\frac{v_{BE}}{V_T}} - 1 \right) = \alpha i_E$$

$$i_B = \frac{i_C}{\beta}$$

$$I_s = \frac{A_E q D_n n_i^2}{N_A W}$$

Where:

$$\alpha = \frac{\beta}{\beta + 1}$$

Typically:

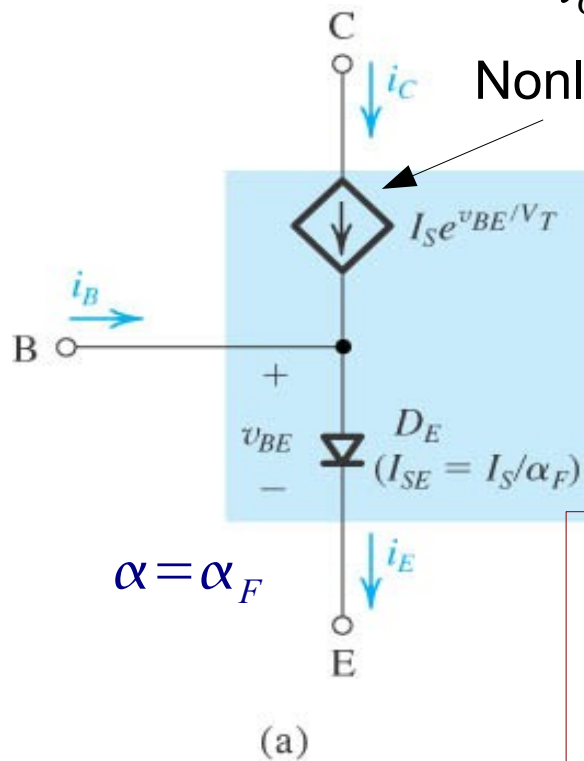
$$50 < \beta < 200 \Rightarrow 0.980 < \alpha < 0.995$$

$$10^{-18} < I_S < 10^{-12} \text{ A.}$$

I_s is strongly temperature-dependent, doubling for a 5 degree Celsius increase in ambient temperature!

Two equivalent large signal circuit models for the **forward-active mode NPN BJT**:

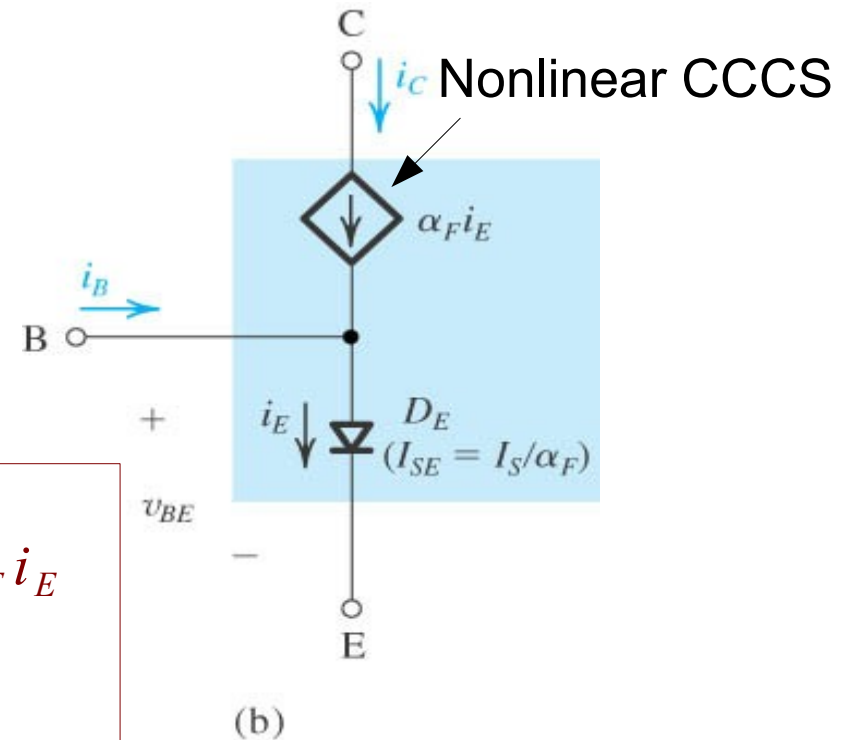
$$i_C = I_S \left(e^{\frac{v_{BE}}{V_T}} - 1 \right) \approx I_S e^{\frac{v_{BE}}{V_T}}$$



Key Eqs.

$$i_C \approx I_S e^{\frac{v_{BE}}{V_T}} = \alpha_F i_E$$

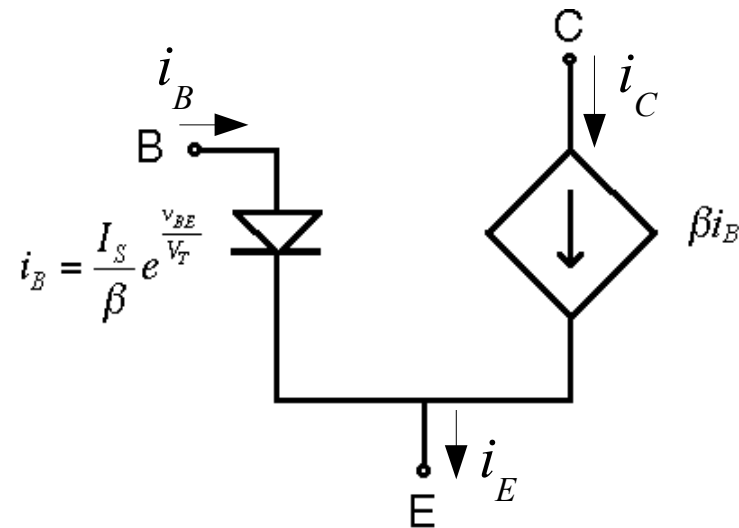
$$i_E \approx \frac{I_S}{\alpha_F} e^{\frac{v_{BE}}{V_T}}$$



Yet another NPN BJT large signal model

$$i_C = \beta i_B \approx I_S e^{\frac{v_{BE}}{V_T}} \Rightarrow i_B \approx \frac{I_S}{\beta} e^{\frac{v_{BE}}{V_T}}$$

This “looks like” a diode between base and emitter and the equivalent circuit becomes:



Note that in this model, the diode current is represented in terms of the *base current*. In the previous ones, it was represented in terms of the *emitter current*.

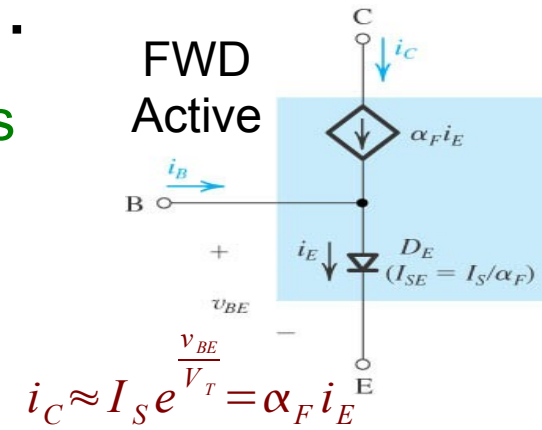
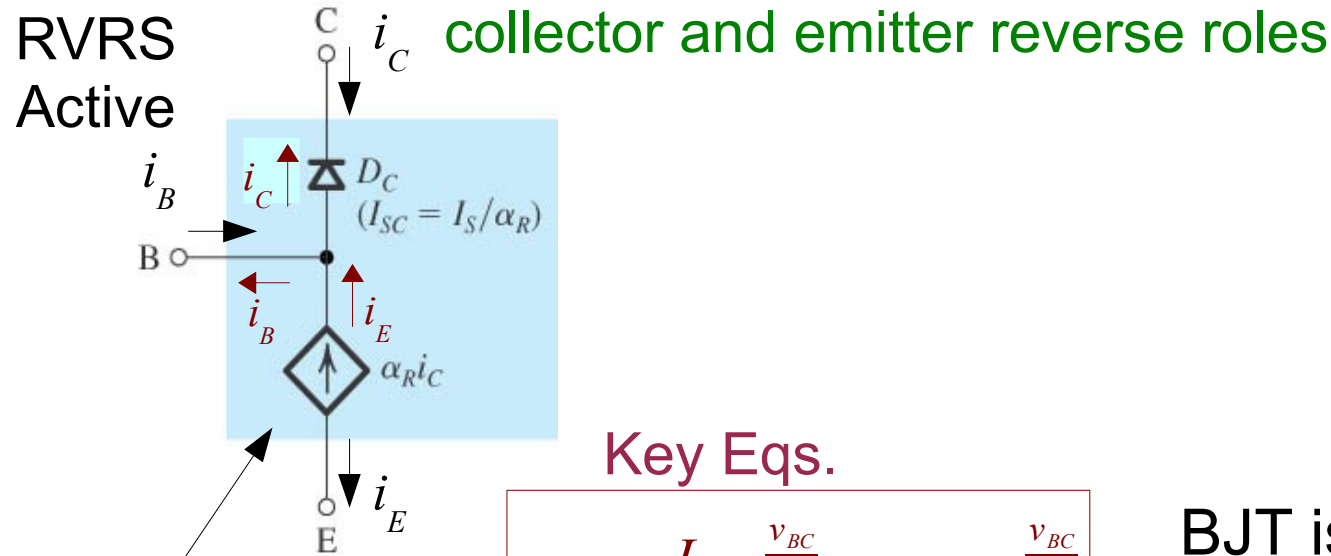
NPN BJT Operating in the Reverse-Active Mode

Recall for NPN Reverse-Active Mode $V_{BE} < 0$ & $V_{BC} > 0$

- We also have weak transistor action if we:
 - Forward bias the base-collector junction and
 - Reverse bias the base emitter junction
 - Collector and emitter reverse roles
- The physical construction of the transistor results
 - Weak reverse-active performance
 - Small values of β on the order of 0.01 to 1
 - Correspondingly smaller values of α , e. g.

$$\alpha_R = \frac{\beta_R}{\beta_R + 1} \approx \frac{0.1}{1.1} \approx 0.091 \quad \text{for} \quad \beta_R = 0.1$$

The equivalent large signal circuit model for the **reverse-active mode** NPN BJT:



Note that the directions of the **reverse-active currents** are the reverse of the forward-active currents; hence the “minus” signs.

Key Eqs.

$$i_C \approx \frac{-I_S}{\alpha_R} e^{\frac{v_{BC}}{V_T}} = -I_{SC} e^{\frac{v_{BC}}{V_T}}$$

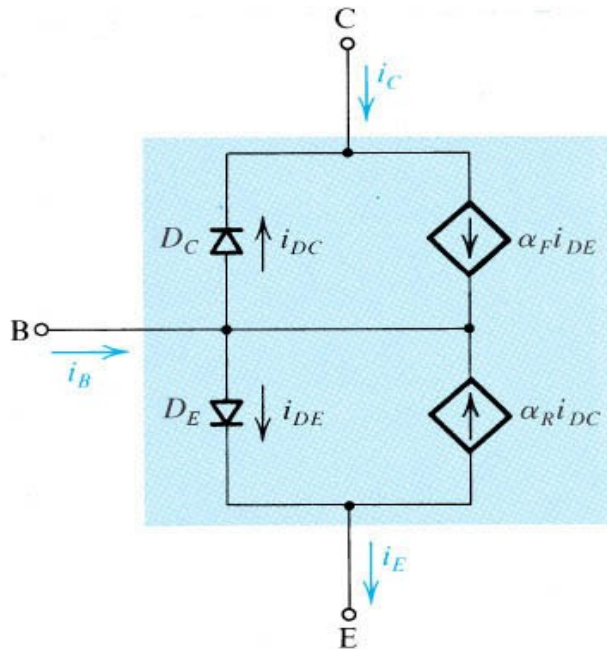
$$i_E \approx -I_S e^{\frac{v_{BC}}{V_T}} = \alpha_R i_C$$

BJT is non-symmetrical

$$\alpha_R \ll \alpha_F \quad \beta_R \ll \beta_F$$

The Ebers-Moll Large Signal Model

The E-M model combines the FWD & RVRS Active equivalent circuits:



Note that the lower left diode and the upper right controlled current source form the forward-active mode model, while the upper left diode and the lower right source represent the reverse-active mode model.

$$i_C = \alpha_F i_{DE} - i_{DC}$$

$$i_E = i_{DE} - \alpha_R i_{DC}$$

$$i_B = (1 - \alpha_F) I_{DE} + (1 - \alpha_R) I_{DC}$$

$$i_{DE} = \frac{I_S}{\alpha_F} \left(e^{\frac{v_{BE}}{V_T}} - 1 \right)$$

$$i_{DC} = \frac{I_S}{\alpha_R} \left(e^{\frac{v_{BC}}{V_T}} - 1 \right)$$

Operation in the Saturation Mode

Recall for Saturation Mode $V_{BE} > 0$ & $V_{BC} > 0$ (or $V_{CB} < 0$)

Consider the E-M model for collector current.

$$i_C = \alpha_F i_{DE} - i_{DC}$$

$$i_C = I_S \left(e^{\frac{v_{BE}}{V_T}} - 1 \right) - \frac{I_S}{\alpha_R} \left(e^{\frac{v_{BC}}{V_T}} - 1 \right)$$

The first term is the forward mode collector current:

$$\alpha_F i_{DE} = I_S \left(e^{\frac{v_{BE}}{V_T}} - 1 \right)$$

The second is the reverse mode collector current:

$$i_{DC} = \frac{I_S}{\alpha_R} \left(e^{\frac{v_{BC}}{V_T}} - 1 \right)$$

Combining terms:

$$i_C = I_S \left(e^{\frac{v_{BE}}{V_T}} - 1 \right) - \frac{I_S}{\alpha_R} \left(e^{\frac{v_{BC}}{V_T}} - 1 \right)$$

$$i_C = \left[\left(e^{\frac{v_{BE}}{V_T}} - 1 \right) - \frac{\beta_R + 1}{\beta_R} \left(e^{\frac{-v_{CB}}{V_T}} - 1 \right) \right] I_S$$

$$\frac{1}{\alpha_R} = \frac{\beta_R + 1}{\beta_R}$$

Using typical values:

$$\beta_R = 0.1$$

$$I_S = 10^{-14} \text{ A}$$

$$V_T = 0.025 \text{ V}$$

We obtain:

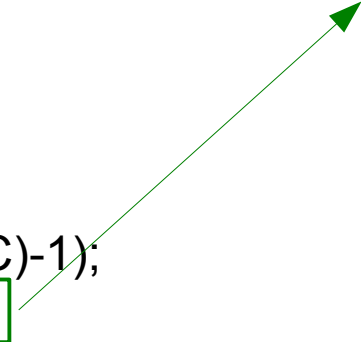
$$i_C = \left[\left(e^{40v_{BE}} - 1 \right) - 11 \left(e^{-40v_{CB}} - 1 \right) \right] 10^{-14}$$

Let's plot i_C vs. v_{BC} (or v_{CB}) with $v_{BE} = 0.7 \text{ V}$

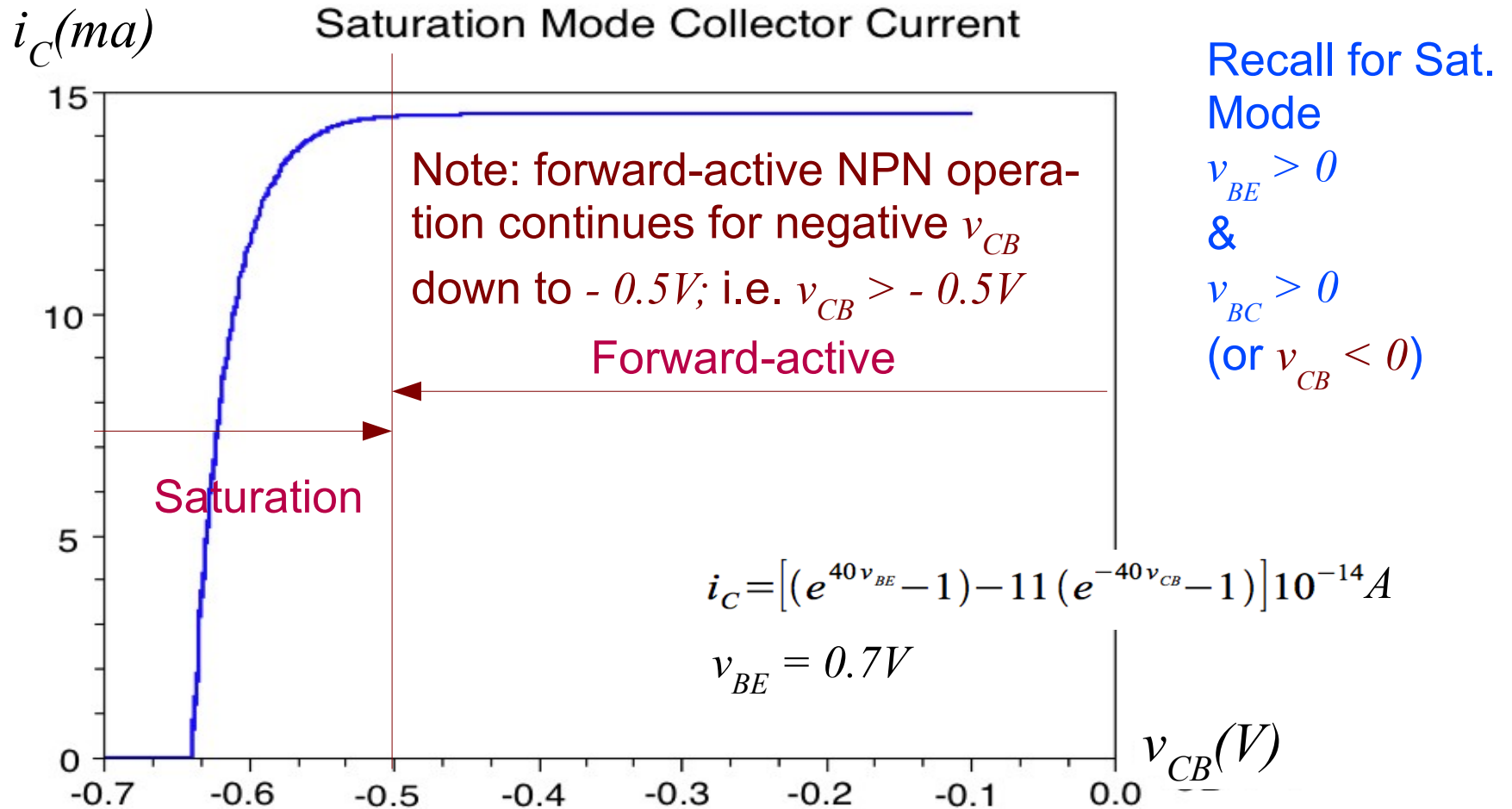
Scilab Saturation Mode Calculation

```
//Calculate and plot npn BJT collector
//current in saturation mode
vBE=0.7;
VsubT=0.025;
VTinv=1/VsubT;
betaR=0.1;
IsubS=1E-14;
alphaR=betaR/(betaR+1);
alphaInv=1/alphaR;
ForwardExp=exp(VTinv*vBE)-1;
vCB=-0.7:0.001:-0.1;
vBC=-vCB;
ReverseExp=alphaInv*(exp(VTinv*vBC)-1);
iC=(ForwardExp-ReverseExp)*IsubS;
signiC=sign(iC);
iCplus=(iC+signiC.*iC)/2; //Zero negative values
plot(vCB,1000*iCplus); //Current in mA.
```

$$i_C = I_S \left(e^{\frac{v_{BE}}{V_T}} - 1 \right) - \frac{I_S}{\alpha_R} \left(e^{\frac{v_{BC}}{V_T}} - 1 \right)$$



Saturation Mode Plot





Scilab Plot of NPN Characteristic

$(i_C \text{ vs. } v_{CE} \text{ and } v_{BE})$

```
//Calculate and plot npn BJT collector
//characteristic using Ebers-Moll model
```

```
VsubT=0.025;
```

```
VTinv=1/VsubT;
```

```
betaR=0.1;
```

```
alphaInv=(betaR+1)/betaR;
```

```
lsubS=1E-14;
```

```
for vBE=0.6:0.02:0.68
```

```
ForwardExp=exp(VTinv*vBE)-1;
```

```
vCE=-0:0.001:10;
```

```
vBC=vBE-vCE;
```

```
ReverseExp=alphaInv*(exp(VTinv*vBC)-1);
```

```
iC=(ForwardExp-ReverseExp)*lsubS;
```

```
signiC=sign(iC);
```

```
iCplus=(iC+signiC.*iC)/2; //Zero negative vals
```

```
plot(vCE,1000*iCplus); //Current in mA.
```

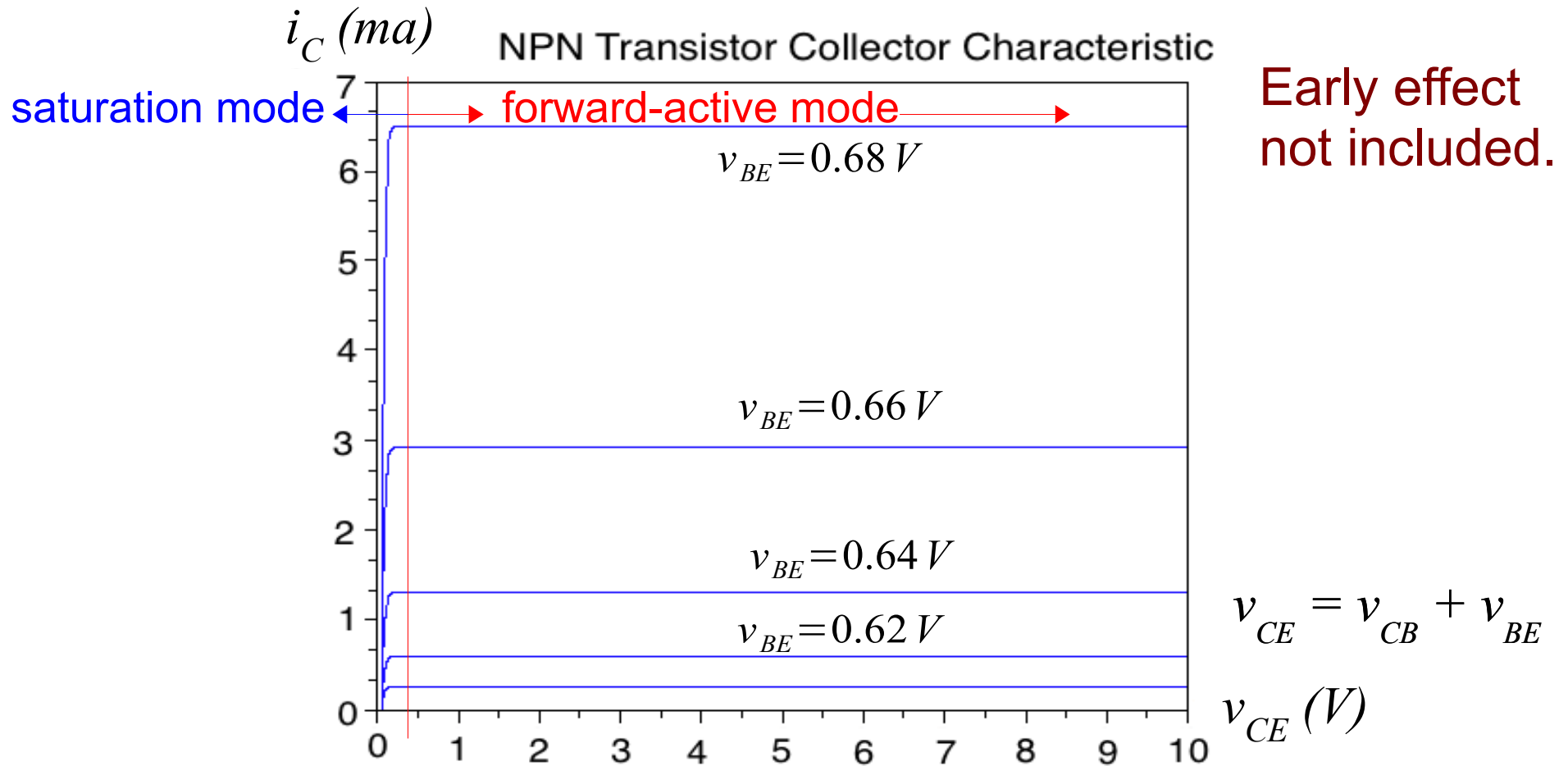
```
end
```

$$i_C = I_S \left(e^{\frac{v_{BE}}{V_T}} - 1 \right) - \frac{I_S}{\alpha_R} \left(e^{\frac{v_{BC}}{V_T}} - 1 \right)$$

$$v_{CE} = v_{CB} + v_{BE}$$

α_R

Plot Output



More on NPN Saturation

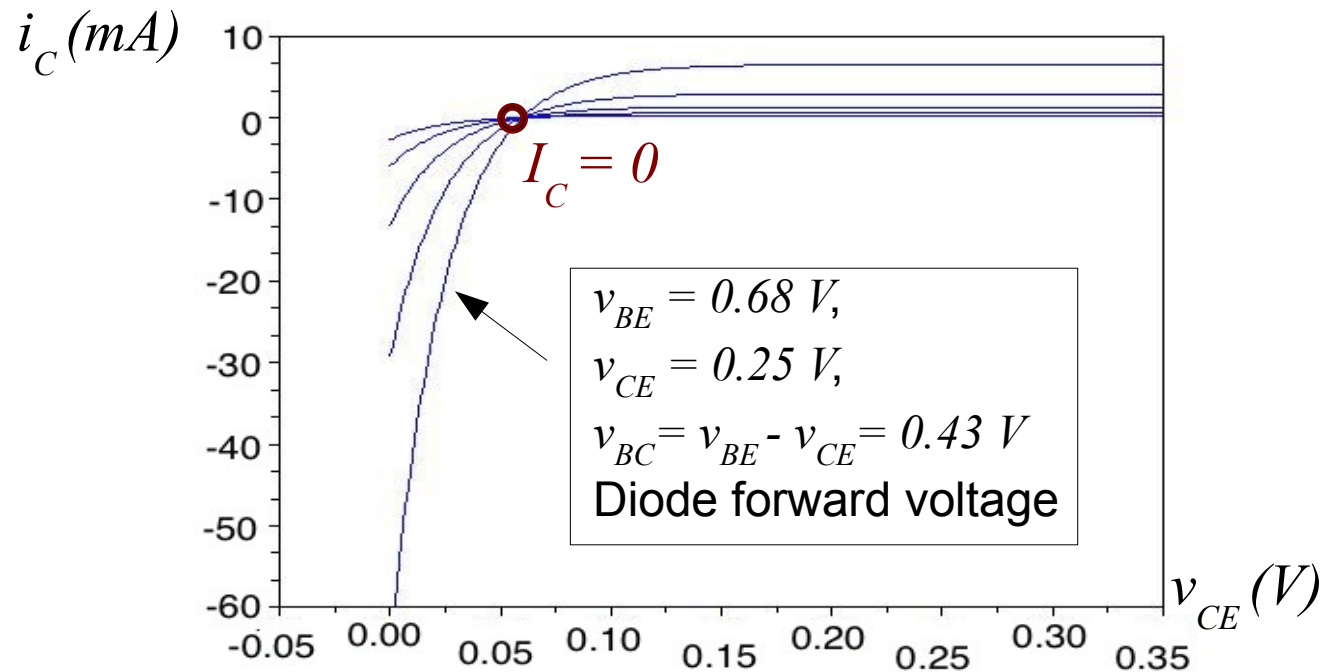
- The base-collector diode has much larger area than the base-emitter one.
- Therefore, with the same applied voltage, it will conduct a much larger forward current than will the base-emitter diode.

$$i_C = I_S \left(e^{\frac{v_{BE}}{V_T}} - 1 \right) - \frac{I_S}{\alpha_R} \left(e^{\frac{v_{BC}}{V_T}} - 1 \right) \quad \text{where } \alpha_R \ll 1$$

- When the collector-emitter voltage drops below the base-emitter voltage, the base-collector diode is forward biased and conducts heavily.

$$v_{CB} = v_{CE} - v_{BE} \quad \text{when} \quad v_{CE} \approx V_{CE}(\text{sat})$$

i_C Expansion Around Zero v_{CE}

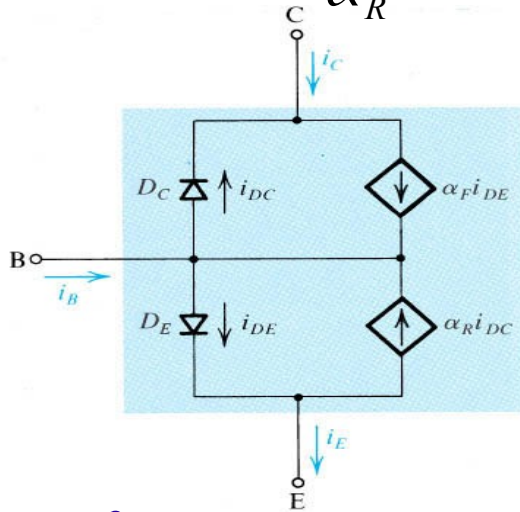


Note that the collector current is zero at about $v_{CE} = 0.06$ V, not 0 V!
 Also note the large reverse collector-base current for $v_{CE} < 0.06$ V.



Voltage at Zero Collector Current

$$i_C = I_S \left(e^{\frac{v_{BE}}{V_T}} - 1 \right) - \frac{I_S}{\alpha_R} \left(e^{\frac{v_{BC}}{V_T}} - 1 \right)$$



$$\alpha_R \left(e^{\frac{v_{BE}}{V_T}} - 1 \right) = \left(e^{\frac{v_{BE} - v_{CE}}{V_T}} - 1 \right)$$

$$\alpha_R \left(1 - e^{-\frac{v_{BE}}{V_T}} \right) = \left(e^{-\frac{v_{CE}}{V_T}} - e^{-\frac{v_{BE}}{V_T}} \right)$$

$$\frac{V_{BE}}{V_T} \approx 40 * 0.7 \Rightarrow e^{-\frac{v_{BE}}{V_T}} \ll 1$$

$$\alpha_R = e^{-\frac{v_{CE}}{V_T}} \Rightarrow v_{CE} = -V_T \ln(\alpha_R)$$

$$i_C = 0 \Rightarrow$$

$$\frac{I_S}{\alpha_R} \left(e^{\frac{v_{BC}}{V_T}} - 1 \right) = I_S \left(e^{\frac{v_{BE}}{V_T}} - 1 \right)$$

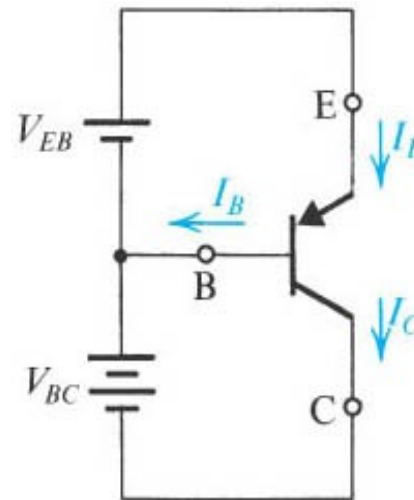
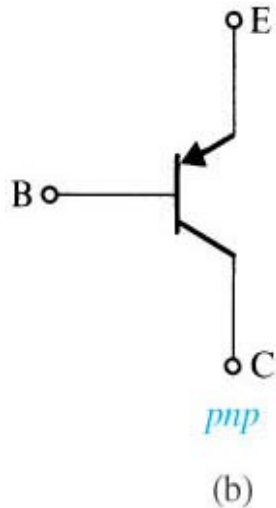
$$\alpha_R \left(e^{\frac{v_{BE}}{V_T}} - 1 \right) = \left(e^{\frac{v_{BC}}{V_T}} - 1 \right)$$

$$\alpha_R = e^{-\frac{v_{CE}}{V_T}} \Rightarrow v_{CE} = -V_T \ln(\alpha_R)$$

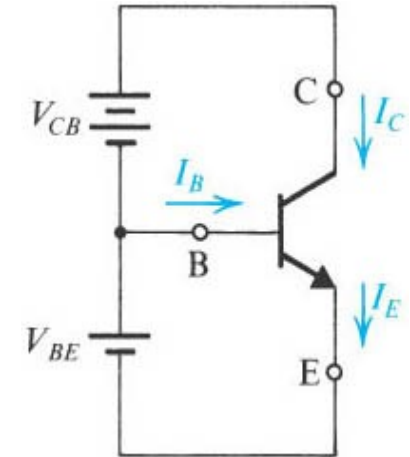
$$\text{For } \beta_R = 0.1 \Rightarrow \alpha_R = 0.09 \Rightarrow v_{CE} = -0.06 \text{ V}$$

The PNP Transistor

pnp



npn



Mode	V_{EB}	V_{CB}
Forward-Active	> 0	< 0
Reverse-Active	< 0	> 0
Cutoff	< 0	< 0
Saturation	> 0	> 0

$$V_{CB} = -V_{BC}$$

← Not Useful!

PNP BJT Forward-Active Mode Basic Model

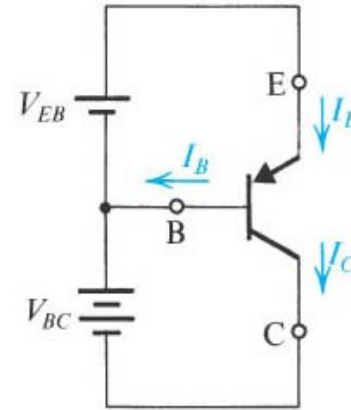
Collector-base diode
is **reverse biased**

$$V_{BC} > 0$$

Emitter-base diode is
forward biased

$$V_{EB} \approx 0.7$$

**Note reversal in voltage
polarity and in current
directions!**

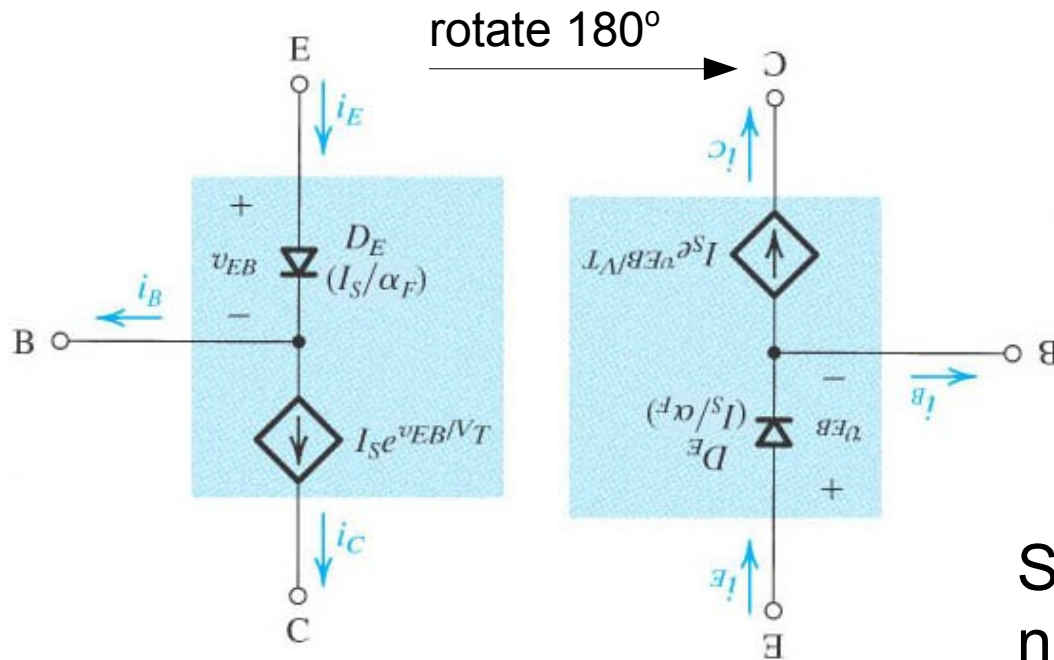


$$i_C = I_S \left(e^{\frac{v_{EB}}{V_T}} - 1 \right)$$

$$i_B = \frac{i_C}{\beta}$$

$$i_E = i_B + i_C = (\beta + 1) i_B$$

PNP BJT Large Signal Model FWD. Active



Note reversal in current directions!

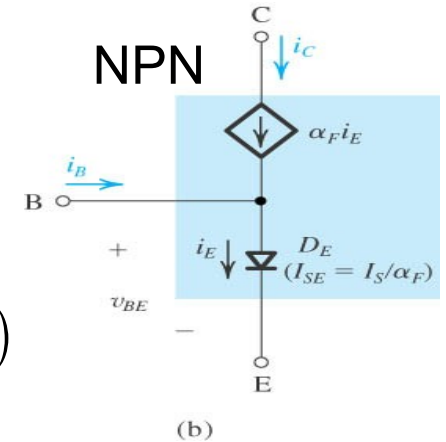
$$i_C = I_S \left(e^{\frac{v_{EB}}{V_T}} - 1 \right)$$

$$i_B = \frac{i_C}{\beta}$$

$$i_E = i_B + i_C = (\beta + 1) i_B$$

Substituting, as in the npn case, we get:

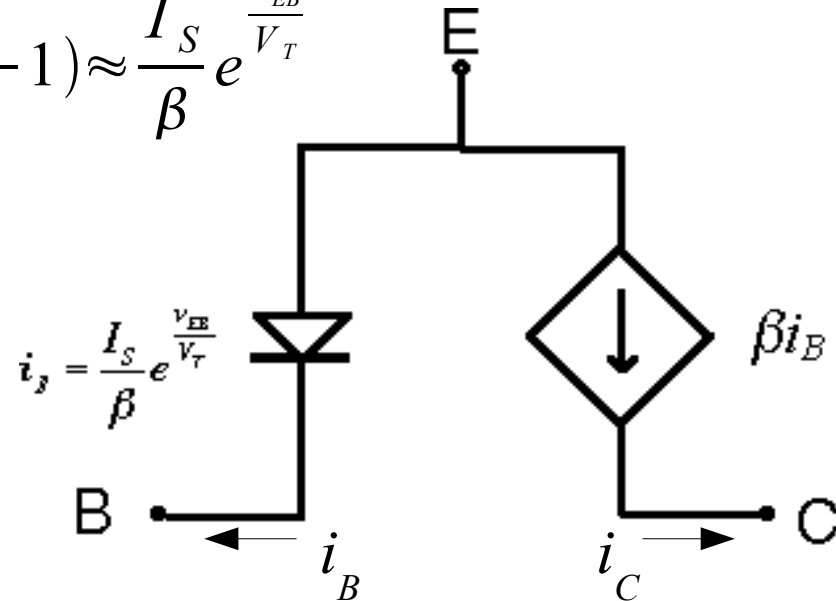
$$i_E = \frac{I_S}{\alpha} \left(e^{\frac{v_{EB}}{V_T}} - 1 \right)$$



Yet another PNP BJT large signal model

$$i_C = \beta i_B = I_S \left(e^{\frac{v_{EB}}{V_T}} - 1 \right) \Rightarrow i_B = \frac{I_S}{\beta} \left(e^{\frac{v_{EB}}{V_T}} - 1 \right) \approx \frac{I_S}{\beta} e^{\frac{v_{EB}}{V_T}}$$

This “looks like” a diode between base and emitter and the equivalent circuit becomes:



Again, in this model, the diode carries only *base current*, not *emitter current*.