BJT Intro and Large Signal Model
Why BJT?

What's the competition to BJT and bipolar technologies?

What advantages does the competition have over BJT?

What advantages does BJT and bipolar have over their competition?

What circuit applications benefit from BJT and bipolar technologies?
Why BJT

What's the competition to BJT and bipolar technologies?
MOSFET, in particular CMOS is the leading competitor

What advantages does the competition have over BJT?
Small size (die area), low cost and low power dissipation

What advantages does BJT and bipolar have over their competition?
High frequency operation, high current drive, high reliability in severe environmental conditions.

What circuit applications benefit from BJT and bipolar technologies?
RF analog and digital circuits, power electronics and power amplifiers, automobile electronics, radiation hardened electronics

2009 by Kenneth R. Laker, update 17Sep09 KRL
Each transistor looks like two back-to-back diodes, but each behaves much differently!
BJT Symbols and Conventions

Note reversal in current directions and voltage signs for PNP vs. NPN!
### NPN BJT Modes of Operation

Forward-Active Mode
- EBJ forward bias ($V_{BE} > 0$)
- CBJ reverse bias ($V_{BC} < 0$)

\[ i_E = i_C + i_B \]
\[ v_{CE} = v_{CB} + v_{BE} \]

\[ v_{XY} = V_{XY} + v_{xy} \]

<table>
<thead>
<tr>
<th>Mode</th>
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<th>$V_{BC}$</th>
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\[ V_{BC} = -V_{CB} \]

Not Useful!
**NPN BJT Forward-Active Mode Basic Model**

Collector-base diode is reverse biased

\[ V_{CB} > 0 \quad \text{ (or} \quad V_{BC} < 0) \]

Base-emitter diode is forward biased

\[ V_{BE} \approx 0.7 \]

\[ I_s = \frac{A_E q D_n n_i^2}{N_A W} \]

- \( A_E \) Area of base-emitter junction
- \( W \) Width of base region
- \( N_A \) Doping concentration in base
- \( D_n \) Electron diffusion constant
- \( n_i \) Intrinsic carrier concentration = f(T)

Saturation current

\[ i_C \approx I_s e^{\frac{v_{BE}}{V_T}} \]

\[ V_T = \frac{kT}{q} \approx 25 \text{ mV} @ 25^\circ C \]

\[ i_B = \frac{i_C}{\beta} \]

\[ i_E = i_B + i_C = (\beta + 1)i_B \]

\( \beta = \text{common-emitter current gain} \)
NPN BJT Forward-Active Beta ($\beta$)

\[
\beta = \frac{1}{D_p N_A W + \frac{1}{2} \frac{W^2}{D_n N_D L_p + \frac{1}{2} D_n \tau_b}}
\]

- $A_E$ -> Area of base-emitter junction
- $W$ -> Width of base region
- $N_A$ -> Doping concentration in base
- $N_D$ -> Doping concentration in emitter
- $D_n$ -> Electron diffusion constant
- $D_p$ -> Hole diffusion constant
- $L_p$ -> Hole diffusion length in emitter
- $\tau_b$ -> Minority-carrier lifetime
- $n_i$ -> Intrinsic carrier concentration = $f(T)$

Large $\beta$ =>

- $N_A$ -> small
- $N_D$ -> large
- $W$ -> small
Note that the $i_C$ equation looks like that of a forward-biased diode collector-base. Is it?

$$i_C = I_S \left( e^{\frac{v_{BE}}{V_T}} - 1 \right)$$

Using Eqs. $i_E = i_B + i_C$ and $i_C = \beta i_B$ we can answer this question, i.e.

$$i_E = i_B + i_C = \left( \frac{1}{\beta} + 1 \right) i_C = \frac{\beta + 1}{\beta} i_C$$

and write:

$$i_E = \frac{\beta + 1}{\beta} I_S \left( e^{\frac{v_{BE}}{V_T}} - 1 \right) = \frac{1}{\alpha} I_S \left( e^{\frac{v_{BE}}{V_T}} - 1 \right) = \frac{i_C}{\alpha}$$

where $\alpha = \frac{\beta}{1 + \beta}$

AhHa! This $i_E$ equation describes a forward-biased emitter-base “diode”.

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So the new set of equations is:

\[ i_E = \frac{I_S}{\alpha} (e^{\frac{v_{BE}}{V_T}} - 1) \]

\[ i_C = I_S (e^{\frac{v_{BE}}{V_T}} - 1) = \alpha i_E \]

\[ i_B = \frac{i_C}{\beta} \]

\[ I_s = \frac{A_E q D_n n_i^2}{N_A W} \]

Where:

\[ \alpha = \frac{\beta}{\beta + 1} \]

Typically:

\[ 50 < \beta < 200 \Rightarrow 0.980 < \alpha < 0.995 \]

\[ 10^{-18} < I_s < 10^{-12} \text{ } \text{A.} \]

\( I_s \) is strongly temperature-dependent, doubling for a 5 degree Celsius increase in ambient temperature!
Two equivalent large signal circuit models for the forward-active mode NPN BJT:

\[ i_C = I_S \left( e^{\frac{v_{BE}}{V_T}} - 1 \right) \approx I_S e^{\frac{v_{BE}}{V_T}} \]

Nonlinear VCCS

\[ \alpha = \alpha_F \]

Key Eqs.

\[ i_C \approx I_S e^{\frac{v_{BE}}{V_T}} = \alpha_F i_E \]

\[ i_E \approx \frac{I_S}{\alpha_F} e^{\frac{v_{BE}}{V_T}} \]
Yet another NPN BJT large signal model

\[ i_C = \beta i_B \approx I_S e^{\frac{v_{BE}}{V_T}} \Rightarrow i_B \approx \frac{I_S}{\beta} e^{\frac{v_{BE}}{V_T}} \]

This “looks like” a diode between base and emitter and the equivalent circuit becomes:

Note that in this model, the diode current is represented in terms of the base current. In the previous ones, it was represented in terms of the emitter current.
NPN BJT Operating in the Reverse-Active Mode

Recall for NPN Reverse-Active Mode $V_{BE} < 0 \& V_{BC} > 0$

- We also have weak transistor action if we:
  - Forward bias the base-collector junction and
  - Reverse bias the base emitter junction
  - Collector and emitter reverse roles
- The physical construction of the transistor results
  - Weak reverse-active performance
    - Small values of $\beta$ on the order of 0.01 to 1
    - Correspondingly smaller values of $\alpha$, e.g.

$$\alpha_R = \frac{\beta_R}{\beta_R + 1} \approx \frac{0.1}{1.1} \approx 0.091 \quad \text{for} \quad \beta_R = 0.1$$
The equivalent large signal circuit model for the reverse-active mode NPN BJT:

- Collector and emitter reverse roles
- BJT is non-symmetrical

Key Eqs.

\[ i_C \approx -I_S e^\frac{v_{BE}}{V_T} = -I_{SC} e^\frac{v_{BC}}{V_T} \]

\[ i_E \approx -I_S e^\frac{v_{BE}}{V_T} = \alpha_R i_C \]

Note that the directions of the reverse-active currents are the reverse of the forward-active currents; hence the “minus” signs.
The Ebers-Moll Large Signal Model

The E-M model combines the FWD & RVRS Active equivalent circuits:

Note that the lower left diode and the upper right controlled current source form the forward-active mode model, while the upper left diode and the lower right source represent the reverse-active mode model.

\[ i_C = \alpha_F i_{DE} - i_{DC} \]
\[ i_E = i_{DE} - \alpha_R i_{DC} \]
\[ i_B = (1 - \alpha_F) I_{DE} + (1 - \alpha_R) I_{DC} \]
\[ i_{DE} = \frac{I_S}{\alpha_F} (e^{v_{BE}/V_T} - 1) \]
\[ i_{DC} = \frac{I_S}{\alpha_R} (e^{v_{BC}/V_T} - 1) \]
Operation in the Saturation Mode

Recall for Saturation Mode \( V_{BE} > 0 \) & \( V_{BC} > 0 \) (or \( V_{CB} < 0 \))

Consider the E-M model for collector current.

\[
i_C = \alpha_F i_{DE} - i_{DC}
\]

\[
i_C = I_S \left( e^{\frac{V_{BE}}{V_T}} - 1 \right) - \frac{I_S}{\alpha_R} \left( e^{\frac{V_{BC}}{V_T}} - 1 \right)
\]

The first term is the forward mode collector current:

\[
\alpha_F i_{DE} = I_S \left( e^{\frac{V_{BE}}{V_T}} - 1 \right)
\]

The second is the reverse mode collector current:

\[
i_{DC} = \frac{I_S}{\alpha_R} \left( e^{\frac{V_{BC}}{V_T}} - 1 \right)
\]
Combining terms:

\[ i_C = I_s \left( e^{\frac{v_{BE}}{V_T}} - 1 \right) - \frac{I_s}{\alpha_R} \left( e^{\frac{v_{BC}}{V_T}} - 1 \right) \]

\[ i_C = \left[ \frac{v_{BE}}{V_T} - 1 \right] - \frac{\beta_R + 1}{\beta_R} \left( e^{-\frac{v_{CB}}{V_T}} - 1 \right) \]

Using typical values:

\[ \beta_R = 0.1 \]
\[ I_s = 10^{-14} A \]
\[ V_T = 0.025 V \]

We obtain:

\[ i_C = \left[ \left( e^{40v_{BE}} - 1 \right) - 11 \left( e^{-40v_{CB}} - 1 \right) \right] 10^{-14} \]

Let's plot \( i_C \) vs. \( v_{BC} \) (or \( v_{CB} \)) with \( v_{BE} = 0.7 V \)
Scilab Saturation Mode Calculation

//Calculate and plot npn BJT collector
//current in saturation mode
vBE=0.7;
VsubT=0.025;
VTinv=1/VsubT;
betaR=0.1;
IsubS=1E-14;
alphaR=betaR/(betaR+1);
alphaInv=1/alphaR;
ForwardExp=exp(VTinv*vBE)-1;
vCB=-0.7:0.001:-0.1;
vBC=-vCB;
ReverseExp=alphaInv*(exp(VTinv*vBC)-1);

\[ i_C = I_S \left( e^{\frac{v_{BE}}{V_T}} - 1 \right) - \frac{I_S}{\alpha_R} \left( e^{\frac{v_{BC}}{V_T}} - 1 \right) \]

iC=(ForwardExp-ReverseExp)*IsubS;
signiC=sign(iC);
iCplus=(iC+signiC.*iC)/2; //Zero negative values
plot(vCB,1000*iCplus); //Current in mA.
Saturation Mode Plot

Recall for Sat. Mode
\( v_{BE} > 0 \)
\&
\( v_{BC} > 0 \)
(or \( v_{CB} < 0 \))

\[ i_C = \left[ (e^{40v_{BE}} - 1) - 11(e^{-40v_{CB}} - 1) \right] 10^{-14} A \]

\( v_{BE} = 0.7V \)

Note: forward-active NPN operation continues for negative \( v_{CB} \) down to -0.5V; i.e. \( v_{CB} > -0.5V \)

Forward-active

Saturation

Saturation Mode Collector Current

\( i_C (ma) \)

\( v_{CB} (V) \)
Scilab Plot of NPN Characteristic

\((i_C \text{ vs. } v_{CE} \text{ and } v_{BE})\)

//Calculate and plot npn BJT collector
//characteristic using Ebers-Moll model

\(V_{subT}=0.025;\)
\(V_{Tinv}=1/V_{subT};\)
\(\beta_{R}=0.1;\)
\(\alpha_{Inv}=(\beta_{R}+1)/\beta_{R};\)
\(I_{subS}=1E^{-14};\)
for \(v_{BE}=0.6:0.02:0.68\)
\(\text{ForwardExp}=\exp(V_{Tinv}*v_{BE})-1;\)
\(v_{CE}=-0:0.001:10;\)
\(v_{BC}=v_{BE}-v_{CE};\)
\(\text{ReverseExp}=\alpha_{Inv}*(\exp(V_{Tinv}*v_{BC})-1);\)
\(i_C=(\text{ForwardExp}-\text{ReverseExp})*I_{subS};\)
\(\text{signiC}=\text{sign}(i_C);\)
\(iCplus=(iC+\text{signiC}.*iC)/2; \quad //Zero \text{ negative } \text{ vals}\)
\(\text{plot}(v_{CE},1000*iCplus); \quad //Current \text{ in mA.}\)
\end

\[ i_C = I_S\left(e^{\frac{v_{BE}}{V_T}} - 1\right) - \frac{I_S}{\alpha_R}\left(e^{\frac{v_{BC}}{V_T}} - 1\right) \]

\[ v_{CE} = v_{CB} + v_{BE} \]
Plot Output

\[ v_{BE} = 0.68 \, V \]

\[ v_{BE} = 0.66 \, V \]

\[ v_{BE} = 0.64 \, V \]

\[ v_{BE} = 0.62 \, V \]

Early effect not included.

\[ v_{CE} = v_{CB} + v_{BE} \]

NPN Transistor Collector Characteristic

<table>
<thead>
<tr>
<th>( i_C ) (ma)</th>
<th>( v_{CE} ) (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>6.6</td>
</tr>
<tr>
<td>6</td>
<td>6.4</td>
</tr>
<tr>
<td>5</td>
<td>6.2</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>5.8</td>
</tr>
<tr>
<td>2</td>
<td>5.6</td>
</tr>
<tr>
<td>1</td>
<td>5.4</td>
</tr>
<tr>
<td>0</td>
<td>5.2</td>
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saturation mode ← forward-active mode
More on NPN Saturation

- The base-collector diode has much larger area than the base-emitter one.
- Therefore, with the same applied voltage, it will conduct a much larger forward current than will the base-emitter diode.

\[ \begin{align*}
    i_C &= I_S \left( e^{\frac{v_{BE}}{V_T}} - 1 \right) - \frac{I_S}{\alpha_R} \left( e^{\frac{v_{BC}}{V_T}} - 1 \right) \quad \text{where } \alpha_R << 1 \\
    \end{align*} \]

- When the collector-emitter voltage drops below the base-emitter voltage, the base-collector diode is forward biased and conducts heavily.

\[ v_{CB} = v_{CE} - v_{BE} \quad \text{when} \quad v_{CE} \approx V_{CE(sat)} \]
Note that the collector current is zero at about $v_{CE} = 0.06 \, V$, not $0 \, V$!
Also note the large reverse collector-base current for $v_{CE} < 0.06 \, V$. 

$i_C$ Expansion Around Zero $v_{CE}$

$v_{BE} = 0.68 \, V,$
$v_{CE} = 0.25 \, V,$
$v_{BC} = v_{BE} - v_{CE} = 0.43 \, V$

Diode forward voltage
Voltage at Zero Collector Current

\[ i_C = I_S \left( e^{\frac{v_{BE}}{V_T}} - 1 \right) - \frac{I_S}{\alpha_R} \left( e^{\frac{v_{BC}}{V_T}} - 1 \right) \]

\[ \alpha_R \left( e^{\frac{v_{BE}}{V_T}} - 1 \right) = \left( e^{\frac{v_{BE} - v_{CE}}{V_T}} - 1 \right) \]

\[ \alpha_R \left( 1 - e^{\frac{-v_{BE}}{V_T}} \right) = \left( e^{\frac{-v_{CE}}{V_T}} - e^{\frac{-v_{BE}}{V_T}} \right) \]

\[ \frac{v_{BE}}{V_T} \approx 40 \times 0.7 \Rightarrow e^{\frac{-v_{BE}}{V_T}} \ll 1 \]

\[ \alpha_R = e^{\frac{-v_{CE}}{V_T}} \Rightarrow v_{CE} = -V_T \ln(\alpha_R) \]

\[ \alpha_R = e^{\frac{-v_{CE}}{V_T}} \Rightarrow v_{CE} = -V_T \ln(\alpha_R) \]

For \( \beta_R = 0.1 \Rightarrow \alpha_R = 0.09 \Rightarrow v_{CE} = -0.06 V \]

\[ i_C = 0 \Rightarrow I_S \left( e^{\frac{v_{BC}}{V_T}} - 1 \right) = I_S \left( e^{\frac{v_{BE}}{V_T}} - 1 \right) \]

\[ \alpha_R \left( e^{\frac{v_{BC}}{V_T}} - 1 \right) = \left( e^{\frac{v_{BE}}{V_T}} - 1 \right) \]
The PNP Transistor

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$V_{CB} = -V_{BC}$

Not Useful!
PNP BJT Forward-Active Mode Basic Model

Collector-base diode is reverse biased

\[ V_{BC} > 0 \]

Emitter-base diode is forward biased

\[ V_{EB} \approx 0.7 \]

Note reversal in voltage polarity and in current directions!

\[ i_C = I_S \left( e^{\frac{V_{EB}}{V_T}} - 1 \right) \]

\[ i_B = \frac{i_C}{\beta} \]

\[ i_E = i_B + i_C = (\beta + 1) i_B \]
PNP BJT Large Signal Model
FWD. Active

Note reversal in current directions!

\[ i_C = I_S (e^{\frac{v_{EB}}{V_T}} - 1) \]

\[ i_B = \frac{i_C}{\beta} \]

\[ i_E = i_B + i_C = (\beta + 1) i_B \]

Substituting, as in the npn case, we get:

\[ i_E = \frac{I_S}{\alpha} \left( e^{\frac{v_{EB}}{V_T}} - 1 \right) \]
Yet another PNP BJT large signal model

\[ i_C = \beta i_B = I_S \left( e^{\frac{v_{EB}}{V_T}} - 1 \right) \Rightarrow i_B = \frac{I_S}{\beta} \left( e^{\frac{v_{EB}}{V_T}} - 1 \right) \approx \frac{I_S}{\beta} e^{\frac{v_{EB}}{V_T}} \]

This “looks like” a diode between base and emitter and the equivalent circuit becomes:

Again, in this model, the diode carries only base current, not emitter current.