BJT Biasing Cont. & Small Signal Model

- Conservative Bias Design (1/3, 1/3, 1/3 Rule)
- Bias Design Example
- Small-Signal BJT Models
- Small-Signal Analysis
Emitter Feedback Bias Design

Voltage bias circuit

Single power supply version
**Thevenin Equivalent**

Use Thevenin's theorem:

\[ V_{Th} = V_B = \frac{R_2}{R_1 + R_2} V_{CC} \]

Let: \( \rho = \frac{R_1}{R_2} \)

\[ V_B = \frac{1}{\rho + 1} V_{CC} \]

\[ R_B = \frac{R_1 R_2}{R_1 + R_2} = \frac{R_1}{\rho + 1} \]

Since we specify \( V_B \) and \( R_B \), the inverse is needed:

\[ \frac{1}{1 + \rho} = \frac{V_B}{V_{CC}} \]

\[ \rho = \frac{V_{CC}}{V_B} - 1 \]

\[ R_1 = (1 + \rho) R_B \]

\[ R_2 = \frac{R_1}{\rho} \]
Conservative Bias Procedure:

1. Bias so that $V_{CC}$ is split equally across $R_C$, $V_{CE}$ (or $V_{CB}$), and $R_E$.
2. Select desired collector current.
3. Assume $I_E = I_C$ to determine $R_C$ and $R_E$.
4. Add 0.7 $V$ to $V_{CC}/3$ to find $V_B$. Assume base current through $R_B$ is negligible.
5. Choose $R_B$ approximately equal to $\beta R_E/10$. (Use lowest value of $\beta$.)
6. Finally, compute $R_1$ and $R_2$. 

Or choose

$$R_T = R_1 + R_2 = \frac{V_{CC}}{I_1} = \frac{V_{CC}}{(I_C/10)}$$
Example

\[ V_{CC} = 12\text{V} \]
\[ V_{RE} = V_{RC} = 4\text{V} \]
\[ I_C = 1\text{mA} \]

Then:

\[ R_C = R_E = \frac{4}{10^{-3}} = 4 \cdot 10^3 = 4\text{ k}\Omega \]
\[ V_B = 4 + 0.7 = 4.7\text{V} \]
\[ R_B = \frac{(\beta_{\text{min}} + 1)R_E}{10} = \frac{50 \cdot 4000}{10} = 20\text{ k}\Omega \]

For a single power supply:

\[ \rho = \frac{V_{CC}}{V_B} - 1 = \frac{12}{4.7} - 1 = 1.55 \]
\[ R_1 = (1 + \rho)R_B = 2.55 \cdot 20 = 51\text{ k}\Omega \]
\[ R_2 = \frac{R_1}{\rho} = \frac{51}{1.55} = 32.9\text{ k}\Omega \]

50 \leq \beta \leq 100
Completed Bias Design

Our design differs from the simulation because we neglected the base current.

There is no point in including the base current, since we will build the circuit using resistors that come only in standard sizes and with 5% tolerances attached to their values. The closest available resistors in the RCA Lab are 47 kΩ, 33 kΩ, and 3.3 kΩ.
Conceptually, the signal $v_s$ we wish to amplify is connected in series with the bias source and is of quite small amplitude.

We will linearize the signal analysis to simplify our mathematics – to avoid having to deal with the nonlinear exponential collector current $i_C$ vs. $v_{BE}$ characteristic.
Linearization Process

Split the total base-emitter voltage and total collector current into bias and signal components:

\[ i_C = I_c + i_c = I_s e^{\frac{v_{BE}}{V_T}} = I_s e^{\frac{V_{BE} + v_{be}}{V_T}} = I_s e^{\frac{V_{BE}}{V_T}} e^{\frac{v_{be}}{V_T}} = I_c \]

Identify and substitute the bias current into the expression:

\[ i_C = I_c e^{\frac{v_{be}}{V_T}} \]

Expand the exponential in a Taylor series:

\[ f(x) = \sum_{n=0}^{\infty} f^{(n)}(0) \frac{x^n}{n!} \]
Analysis Using Collector Current Model

\[ i_C = I_C e^{\frac{v_{be}}{V_T}} \]

\[ \frac{i_C}{I_C} = e^{\frac{v_{be}}{V_T}} \]

\[ \log_{10}\left(\frac{i_C}{I_C}\right) = \left(\frac{v_{be}}{V_T}\right) \log_{10}e \]

Let \( i_C \) be doubled, i.e. \( \frac{i_C}{I_C} = 2 \):

\[ 0.301 = \left(\frac{v_{be}}{V_T}\right) 0.4343 \]

\[ v_{be} = \frac{0.301}{0.4343} 0.025 \approx 0.017 V \]

Useful constants:

\[ \log_{10}(2) = 0.301 \]

\[ \log_{10}(e) = 0.4343 \]
Linearization Using Taylor Series

Expand in a Taylor series: 
\[ f(x) = \sum_{n=0}^{\infty} f^{(n)}(0) \frac{x^n}{n!} \]

For the exponential function: 
\[ e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \]
where \( x = \frac{v_{be}}{V_T} \)

\[ e^{\frac{v_{be}}{V_T}} = \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{v_{be}}{V_T} \right)^n \]

or:
\[ e^{\frac{v_{be}}{V_T}} \approx 1 + \frac{v_{be}}{V_T} + \frac{1}{2} \left( \frac{v_{be}}{V_T} \right)^2 + \frac{1}{6} \left( \frac{v_{be}}{V_T} \right)^3 + \ldots \]

non-linear terms
Linearization Continued

Recall: $v_{be}$ of about 17 mV causes a 2x change in collector current. Let's expand in Taylor series for this value of $v_{be}$.

$$\frac{v_{be}}{V_T} = \frac{0.017}{0.025} = 0.68$$

$$e^\frac{v_{be}}{V_T} = 1 + 0.68 + \frac{1}{2} (0.68)^2 + \frac{1}{6} (0.68)^3 + \ldots$$

$$e^\frac{v_{be}}{V_T} \approx 1 + 0.68 + 0.2312 + 0.0524 = 1.9444$$

Compare with: $e^{0.68} = 1.973$ exact!

Four term expansion is accurate to about 1.5%, two term (linear) expansion is accurate to about 15%, i.e.

$$v_{be} \approx 0.17 \text{ V may not be such a small signal}$$
Small-Signal Model

Using the grouping into bias and signal voltages/currents:

\[ i_C = I_C e^{\frac{v_{be}}{V_T}} \]

And using the first two terms of the Taylor series expansion:

\[ i_C \approx I_C \left( 1 + \frac{1}{V_T} v_{be} \right) = I_C + \frac{I_C}{V_T} v_{be} = I_C + i_c \]

We define transconductance and small-signal (or ac) current as:

\[ g_m = \frac{I_C}{V_T} \]

\[ i_c = g_m v_{be} \]
Small-Signal BJT Model

\[ i_B = I_B + i_b = \frac{1}{\beta} i_c = \frac{1}{\beta} (I_C + i_c) \]

\[ i_B = \frac{1}{\beta} i_c = \frac{1}{\beta} I_C + \frac{1}{\beta} \left( \frac{I_C}{V_T} \right) v_{be} \]

Define the small-signal base current and base resistance:

\[ i_b = \left( \frac{1}{\beta} \right) i_c = \frac{1}{\beta} \left( \frac{I_C}{V_T} \right) v_{be} = \frac{1}{r_{\pi}} v_{be} \]

bias current

\[ r_{\pi} = \beta \frac{V_T}{I_C} = \frac{\beta}{g_m} \]
Small-Signal BJT Equivalent Models

Equations (1):

\[ i_c = g_m v_{be} \]
\[ i_b = \frac{v_{be}}{r_\pi} \]
\[ i_e = i_b + i_c \]

Equations (2):

\[ i_c = \beta i_b \]
\[ i_b = \frac{i_e}{\beta + 1} \]
\[ i_e = (\beta + 1) i_b = (\beta + 1) \frac{v_{be}}{r_\pi} = \frac{v_{be}}{r_e} \]

Equations (3):

\[ i_c = g_m v_{be} \]
\[ i_b = \frac{i_c}{\beta} \]
\[ i_e = i_b + i_c \]

\[ i_c = g_m r_\pi i_b = \frac{I_C}{V_T} \beta \frac{V_T}{I_C} i_b = \beta i_b \]

\[ r_e = \frac{r_\pi}{\beta + 1} = \frac{\beta}{\beta + 1} \frac{V_T}{I_C} \]

Choose the model that simplifies the circuit analysis.
Small-Signal BJT Equivalent Circuits

Equations (1):

\[ i_b = \frac{v_{be}}{r_\pi} \]
\[ i_c = g_m v_{be} + \frac{v_{ce}}{r_0} \]
\[ r_\pi = \frac{V_T}{I_B} = \beta \]
\[ i_e = i_b + i_c \]

\[ g_m = \frac{I_C}{V_T} \]
\[ r_0 = \frac{V_A}{I_C} \]

Equations (2):

\[ i_b = \frac{i_e}{\beta + 1} \]
\[ i_c = \beta i_b + \frac{v_{ce}}{r_0} \]
\[ i_e = \frac{v_{be}}{r_e} \]
\[ r_e = \frac{V_T}{I_E} = \frac{\alpha}{g_m} = \frac{r_\pi}{\beta + 1} \]

Equations (3):

\[ i_c = g_m v_{be} + \frac{v_{ce}}{r_0} \]
\[ i_b = \frac{i_c}{\beta} \]
\[ i_e = i_b + i_c \]
Small-Signal Circuit Analysis

The small signal model will replace the large signal model and be used for (approximate) signal analysis once the transistor is biased.

Transistor

Biased circuit amplifier circuit
Biased Circuit Small Signal Analysis

Let: \( \beta = 100 \) and \( I_C = 1 \text{ mA} \)

\[
r_{\pi} = \beta \frac{V_T}{I_C} = 100 \frac{0.025}{0.001} = 2.5 \text{ k} \Omega
\]

Note: \( g_m = \frac{I_C}{V_T} = \frac{0.001}{0.025} = 0.04 \text{ S} = 40 \text{ mS} \)

\[
v_s = i_b (R_B + r_{\pi}) + i_e R_E
\]

\[
v_s = i_b (R_B + r_{\pi}) + i_b (\beta + 1) R_E
\]

\[
i_b = \frac{v_s}{R_B + r_{\pi} + (\beta + 1) R_E}
\]

\[
v_{be} = i_b r_{\pi} = \frac{r_{\pi}}{R_B + r_{\pi} + (\beta + 1) R_E} v_s
\]

\( S = \text{ siemen} \)
Small Signal Analysis - Continued

\[ v_{be} = i_b r_\pi = \frac{r_\pi}{R_B + r_\pi + (\beta + 1) R_E} v_s \]

\[ i_c = g_m v_{be} = \frac{g_m r_\pi}{R_B + r_\pi + (\beta + 1) R_E} v_s \]

\[ v_c = -R_C i_c = \frac{-\beta R_C}{R_B + r_\pi + (\beta + 1) R_E} v_s \]

\[ \beta \approx 100 \]

\[ A_v = \frac{v_c}{v_s} \approx -\frac{R_C}{R_E} \]
Multisim Model of “Bias Design”

1. $R_E$ needs to be large to achieve good op. pt. Stability!
2. Consequence: $|A_v|$ is low.

$A_v = \frac{v_c}{v_s} \approx -\frac{R_C}{R_E} = -1$

$R_C = 4k \, \Omega$
$R_B = 20k \, \Omega$
$V_{cc} = 12V$
$V_B = 4.7V$
$V_{cc} = 12V$
$A_v = \frac{v_c}{v_s} \approx -\frac{R_C}{R_E} = -1$
Multisim Input-output Plot

\[ A_v = \frac{v_c}{v_s} \approx -\frac{R_C}{R_E} = -1 \]
Conclusions

Conservative voltage bias for best operating point stability and signal swing works, but returns unity or low voltage gain.

How does one obtain operating point stability, and simultaneously achieve a more respectable voltage gain?