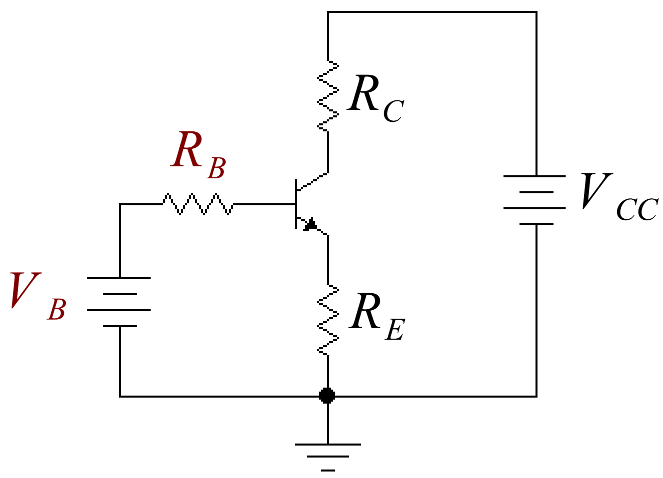
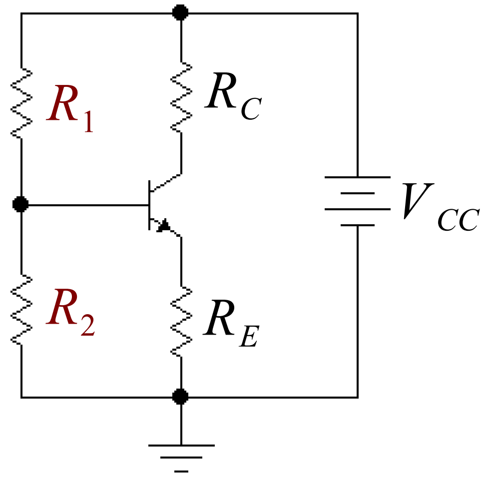
BJT Biasing Cont. & Small Signal Model

- Conservative Bias Design (1/3, 1/3, 1/3 Rule)
- Bias Design Example
- Small-Signal BJT Models
- Small-Signal Analysis

Emitter Feedback Bias Design



Voltage bias circuit



Single power supply version

Thevinen Equivalent

Use Thevinen's theorem:

$$V_{Th}=V_B=\frac{R_2}{R_1+R_2}V_{CC}$$

Let: $\rho=\frac{R_1}{R_2}$

$$V_B=\frac{1}{\rho+1}V_{CC}$$

$$R_B=\frac{R_1 R_2}{R_1+R_2}=\frac{R_1}{\rho+1}$$

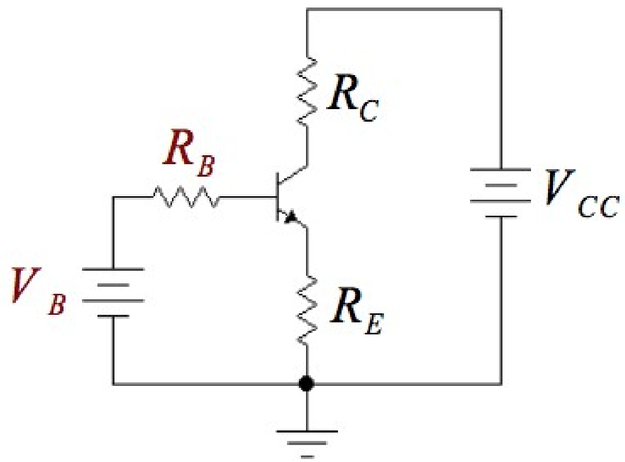
Since we specify V_B and R_B, the inverse is needed:

$$\frac{1}{1+\rho}=\frac{V_B}{V_{CC}}$$

$$\rho=\frac{V_{CC}}{V_B}-1$$

$$R_1=(1+\rho)R_B$$

$$R_2=\frac{R_1}{\rho}$$



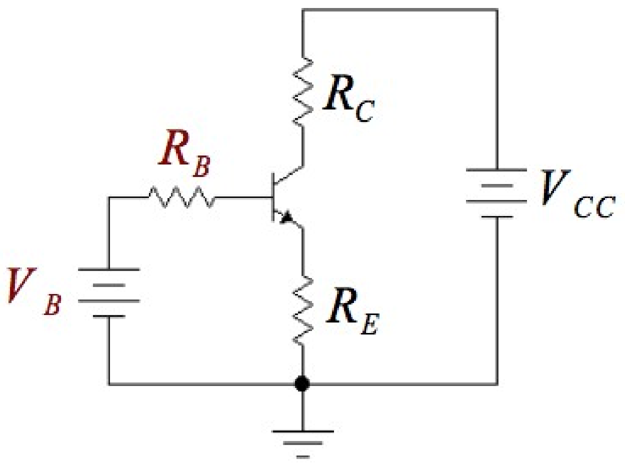
Conservative Bias Procedure:

1. Bias so that V_{CC} is split equally across R_C, V_{CE} $(or\ V_{CB})$, and R_E.
2. Select desired collector current.
3. Assume $I_E = I_C$ to determine R_C and R_E.
4. Add $0.7\ V$ to $V_{CC}/3$ to find V_B. Assume base current through R_B is negligible.
5. Choose R_B approximately equal to $\beta R_E/10$. (Use lowest value of β.)
6. Finally, compute R_1 and R_2.

5.Or choose

$$R_T = R_1 + R_2 = \frac{V_{CC}}{I_1} = \frac{V_{CC}}{(I_C/10)}$$

Example



$$50 \le \beta \le 100$$

$$V_{CC} = 12\text{V}$$

$$V_{RE} = V_{RC} = 4\,V$$

$$I_C = 1\,mA$$

Then:

$$R_C = R_E = \frac{4}{10^{-3}} = 4 \cdot 10^3 = 4\,k\,\Omega$$

$$V_B = 4 + 0.7 = 4.7\,V$$

$$R_B = \frac{(\beta_{min} + 1)R_E}{10} = \frac{50 \cdot 4000}{10} = 20\,k\,\Omega$$

For a single power supply:

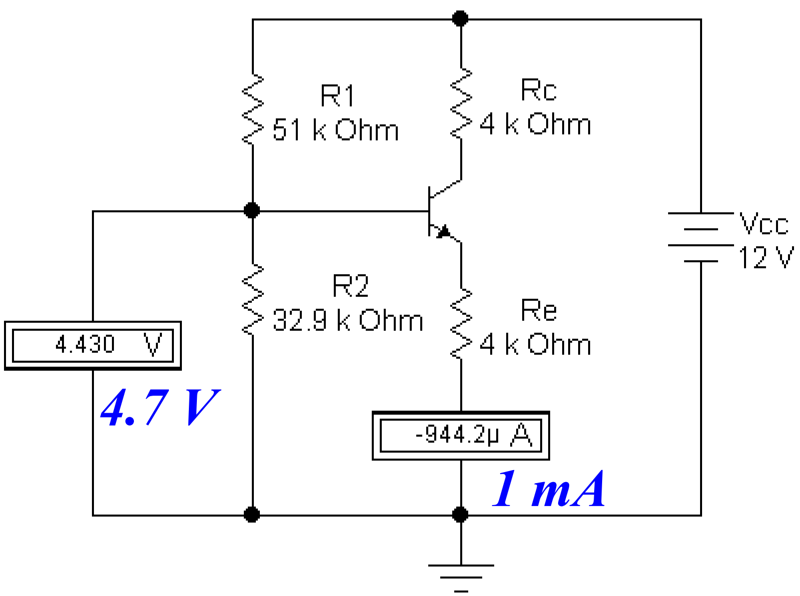
$$\rho = \frac{V_{CC}}{V_B} - 1 = \frac{12}{4.7} - 1 = 1.55$$

$$R_1 = (1 + \rho) R_B = 2.55 \cdot 20 = 51\,k\,\Omega$$

$$R_2 = \frac{R_1}{\rho} = \frac{51}{1.55} = 32.9\,k\,\Omega$$

Completed Bias Design

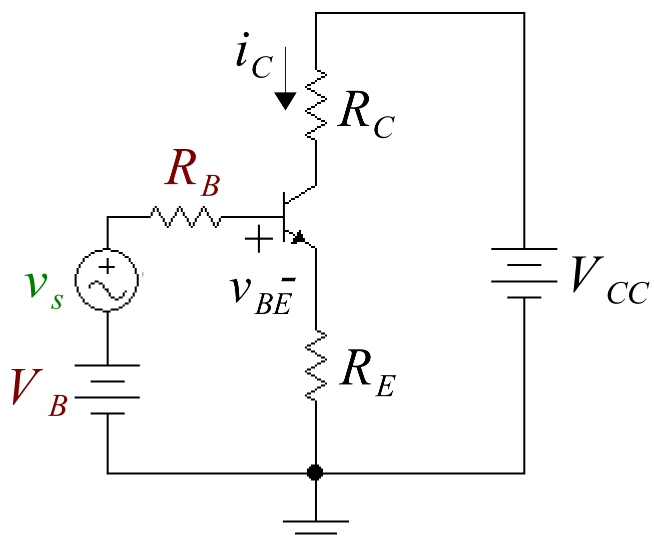
Electronics Workbench simulation results



Our design differs from the simulation because we neglected the base current.

There is no point in including the base current, since we will build the circuit using resistors that come only in standard sizes and with 5% tolerances attached to their values. The closest available resistors in the RCA Lab are $47\ k\Omega$, $33\ k\Omega$, and $3.3\ k\Omega$.

BJT Small-Signal Models



$$i_C = I_S e^{\frac{v_{BE}}{V_T}}$$

Conceptually, the signal v_s we wish to amplify is connected in series with the bias source and is of quite small amplitude.

We will linearize the signal analysis to simplify our mathematics – to avoid having to deal with the nonlinear exponential collector current i_C vs. v_{BE} characteristic.

Linearization Process

Split the total base-emitter voltage and total collector current into bias and signal components:

$$i_C = I_C + i_c = I_S e^{\frac{v_{BE}}{V_T}} = I_S e^{\frac{V_{BE}+v_{be}}{V_T}} = \underbrace{I_S e^{\frac{V_{BE}}{V_T}}}_{I_C} e^{\frac{v_{be}}{V_T}}$$

Identify and substitute the bias current into the expression:

$$i_C = I_C e^{\frac{v_{be}}{V_T}}$$

Expand the exponential in a Taylor series:

$$f(x) = \sum_{n=0}^{\infty} f^{(n)}(0) \frac{x^n}{n!}$$

Analysis Using Collector Current Model

$$i_C = I_C e^{\frac{v_{be}}{V_T}}$$

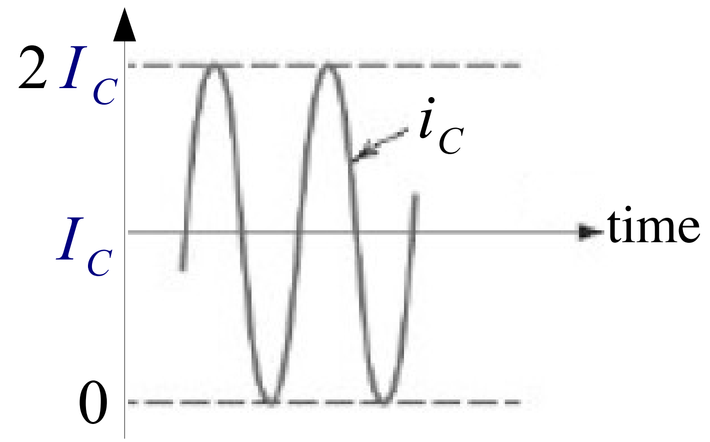
$$\frac{i_C}{I_C} = e^{\frac{v_{be}}{V_T}}$$

$$\log_{10}\left(\frac{i_C}{I_C}\right) = \left(\frac{v_{be}}{V_T}\right)\log_{10} e$$

Useful constants:

$$\log_{10}(2) = 0.301$$

$$\log_{10}(e) = 0.4343$$



Let i_C be doubled, i.e. $i_C / I_C = 2$:

$$0.301 = \left(\frac{v_{be}}{V_T}\right)0.4343$$

$$v_{be} = \frac{0.301}{0.4343} 0.025 \approx 0.017\, V$$

Linearization Using Taylor Series

Expand in a Taylor series: $$f(x)=\sum_{n=0}^{\infty} f^{(n)}(0)\frac{x^n}{n!}$$

For the exponential function: $$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}$$ where $x=\frac{v_{be}}{V_T}$

$$e^{\frac{v_{be}}{V_T}}=\sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{v_{be}}{V_T}\right)^n$$

or:

$$e^{\frac{v_{be}}{V_T}}\approx 1+\frac{v_{be}}{V_T}+\boxed{\frac{1}{2}\left(\frac{v_{be}}{V_T}\right)^2+\frac{1}{6}\left(\frac{v_{be}}{V_T}\right)^3+\ldots}$$

non-linear terms

Linearization Continued

Recall: v_{be} of about *17 mV* causes a *2x* change in collector current.
Let's expand in Taylor series for this value of v_{be}.

$$\frac{v_{be}}{V_T} = \frac{0.017}{0.025} = 0.68$$

$$e^{\frac{v_{be}}{V_T}} = 1 + 0.68 + \frac{1}{2}(0.68)^2 + \frac{1}{6}(0.68)^3 + \ldots$$

linear approximation →

$$e^{\frac{v_{be}}{V_T}} \approx 1 + 0.68 + 0.2312 + 0.0524 = 1.9444$$

Compare with: $e^{0.68} = 1.973$ exact!

Four term expansion is accurate to about *1.5%*, two term (linear) expansion is accurate to about *15%*, i.e.

$$v_{be} \approx 0.17\,V \textit{ may not be such a small signal}$$

Small-Signal Model

Using the grouping into bias and signal voltages/currents:

$$i_C = I_C e^{\frac{v_{be}}{V_T}}$$

And using the first two terms of the Taylor series expansion:

$$i_C \approx I_C\left(1+\frac{1}{V_T}v_{be}\right) = I_C + \frac{I_C}{V_T}v_{be} = I_C + i_c$$

We define *transconductance* and *small-signal (or ac) current as*:

bias current → I_C

$$g_m = \frac{I_C}{V_T}$$

$$i_c = g_m v_{be}$$

Small-Signal BJT Model

$$i_B = I_B + i_b = \frac{1}{\beta} i_C = \frac{1}{\beta}(I_C + i_c)$$

$$i_B = \frac{1}{\beta} i_C = \frac{1}{\beta} I_C + \frac{1}{\beta}\left(\frac{I_C}{V_T}\right) v_{be}$$

Define the *small-signal base current* and *base resistance*:

$$i_b = \left(\frac{1}{\beta}\right) i_c = \frac{1}{\beta}\left(\frac{I_C}{V_T}\right) v_{be} = \frac{1}{r_\pi} v_{be}$$

$$r_\pi = \beta \frac{V_T}{I_C} = \frac{\beta}{g_m}$$

bias current → I_C

Small-Signal BJT Equivalent Models

Equations (1):

$$i_c = g_m v_{be}$$
$$i_b = \frac{v_{be}}{r_\pi}$$
$$i_e = i_b + i_c$$

Equations (2):

$$i_c = \beta i_b$$
$$i_b = \frac{i_e}{\beta + 1}$$
$$i_e = (\beta + 1) i_b = (\beta + 1) \frac{v_{be}}{r_\pi} = \frac{v_{be}}{r_e}$$

Equations (3):

$$i_c = g_m v_{be}$$
$$i_b = \frac{i_c}{\beta}$$
$$i_e = i_b + i_c$$

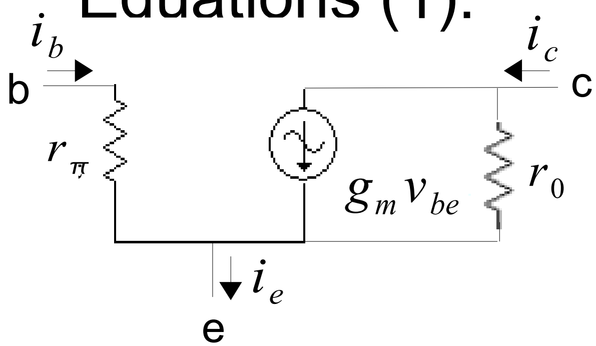
$$i_c = g_m r_\pi i_b = \frac{I_C}{V_T} \beta \frac{V_T}{I_C} i_b = \beta i_b$$

$$r_e = \frac{r_\pi}{\beta + 1} = \frac{\beta}{\beta + 1} \frac{V_T}{I_C}$$

Choose the model that simplifies the circuit analysis.

Small-Signal BJT Equivalent Circuits

Equations (1):



$$i_c = g_m v_{be} + \frac{v_{ce}}{r_0}$$

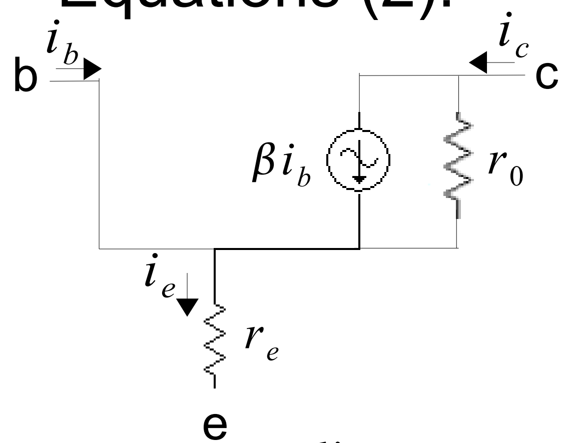
$$g_m = \frac{I_C}{V_T} \qquad r_0 = \frac{V_A}{I_C}$$

$$i_b = \frac{v_{be}}{r_\pi}$$

$$r_\pi = \frac{V_T}{I_B} = \frac{\beta}{g_m}$$

$$i_e = i_b + i_c$$

Equations (2):



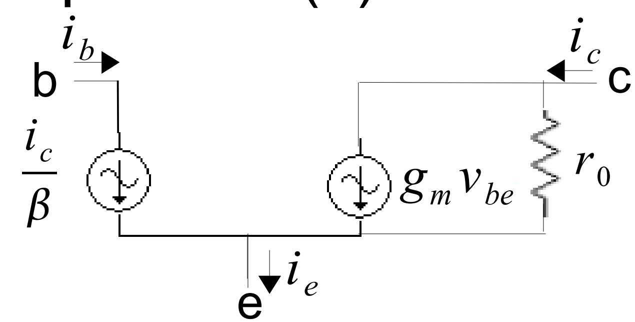
$$i_c = \beta i_b + \frac{v_{ce}}{r_0}$$

$$i_b = \frac{i_e}{\beta + 1}$$

$$i_e = \frac{v_{be}}{r_e}$$

$$r_e = \frac{V_T}{I_E} = \frac{\alpha}{g_m} = \frac{r_\pi}{\beta + 1}$$

Equations (3):



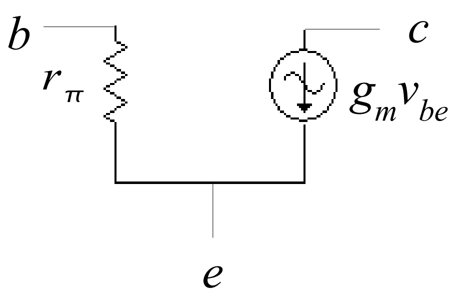
$$i_c = g_m v_{be} + \frac{v_{ce}}{r_0}$$

$$i_b = \frac{i_c}{\beta}$$

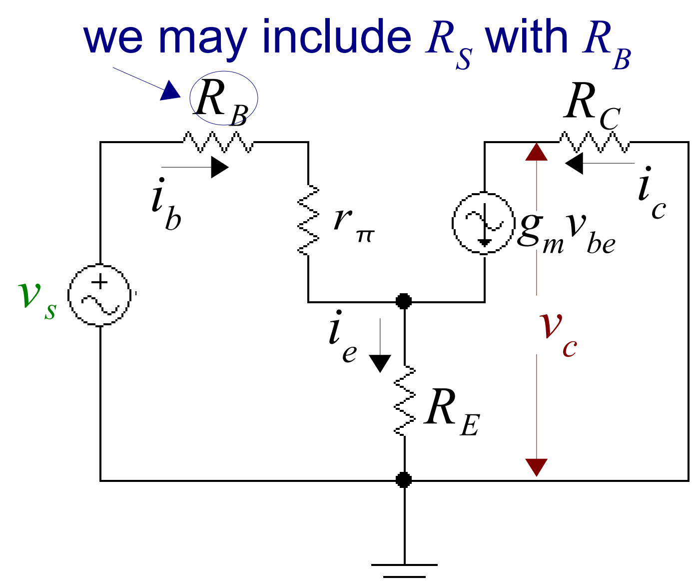
$$i_e = i_b + i_c$$

Small-Signal Circuit Analysis

The small signal model will replace the large signal model and be used for (approximate) signal analysis once the transistor is biased.

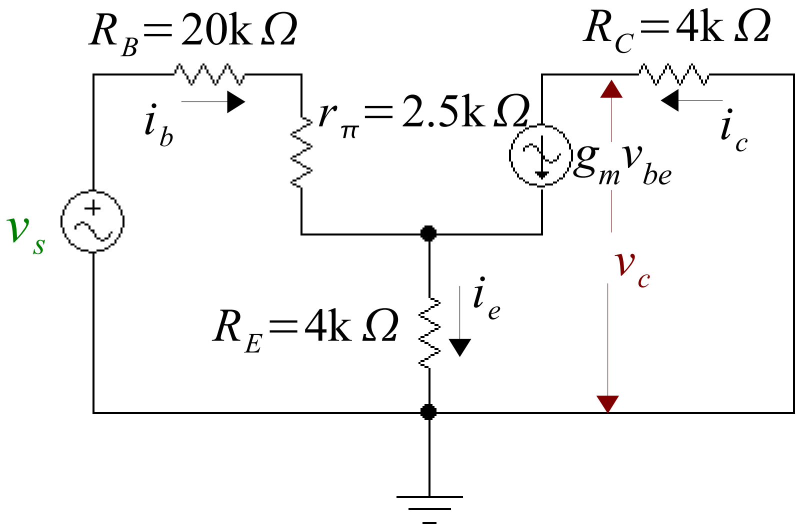


Transistor



Biased circuit amplifier circuit

Biased Circuit Small Signal Analysis



$$v_s = i_b (R_B + r_\pi) + i_e R_E$$

$$v_s = i_b (R_B + r_\pi) + i_b (\beta + 1) R_E$$

$$i_b = \frac{v_s}{R_B + r_\pi + (\beta + 1) R_E}$$

$$v_{be} = i_b r_\pi = \frac{r_\pi}{R_B + r_\pi + (\beta + 1) R_E} v_s$$

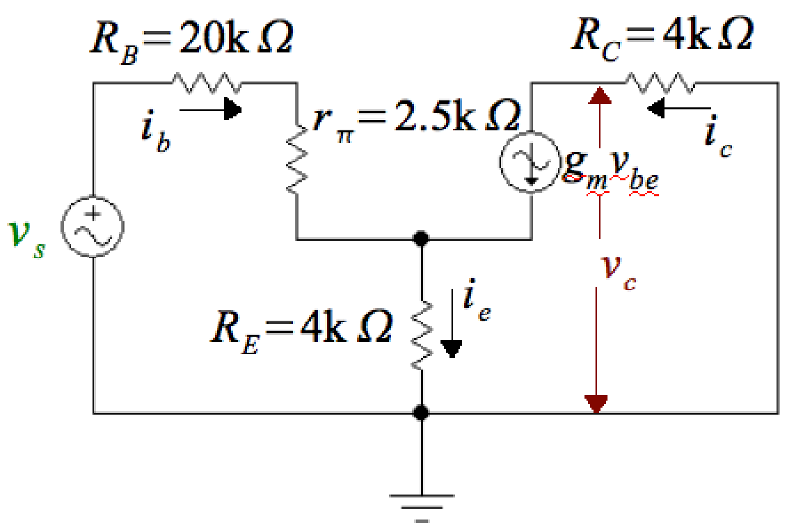
Let: $\beta = 100$ and $I_C = 1\, mA$

$$r_\pi = \beta \frac{V_T}{I_C} = 100 \frac{0.025}{0.001} = 2.5\, k\Omega$$

Note: $g_m = \frac{I_C}{V_T} = \frac{0.001\, A}{0.025\, V} = 0.04\, S = 40\, mS$

S = siemen

Small Signal Analysis - Continued



$$v_{be}=i_b r_\pi=\frac{r_\pi}{R_B+r_\pi+(\beta+1)R_E}v_s$$

$$i_c=g_m v_{be}=\frac{g_m r_\pi}{R_B+r_\pi+(\beta+1)R_E}v_s$$

$$v_c=-R_C i_c=\frac{-\beta R_C}{R_B+r_\pi+(\beta+1)R_E}v_s$$

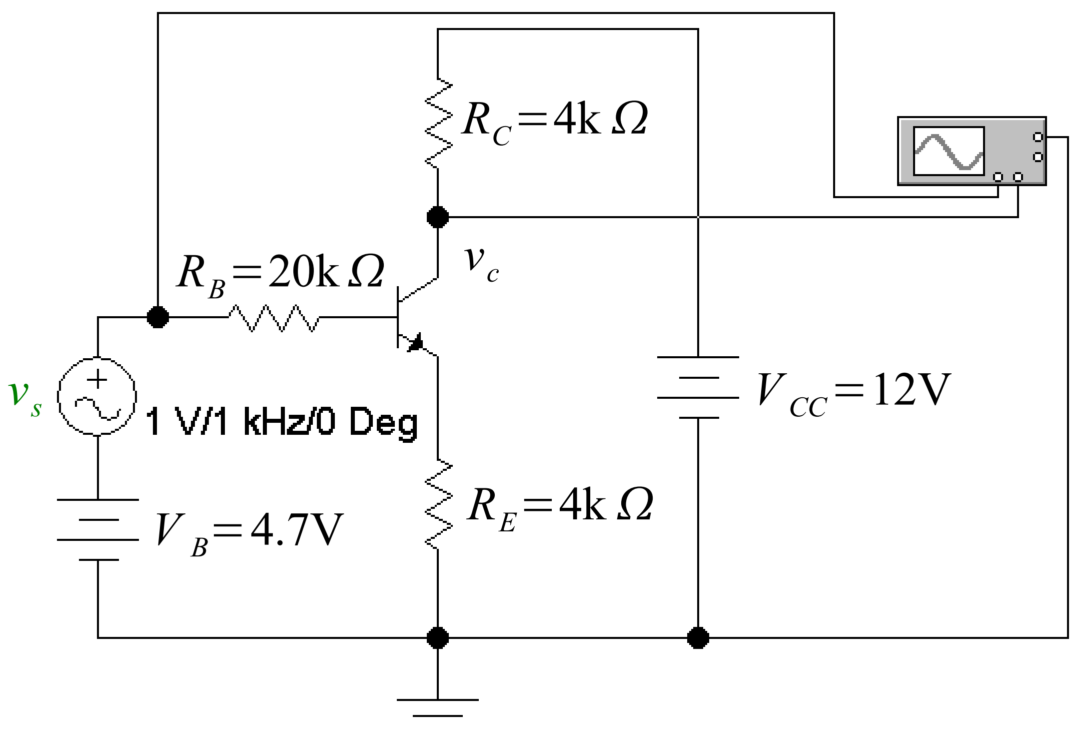
$$g_m r_\pi=\frac{I_C}{V_T}\beta\frac{V_T}{I_C}=\beta$$

$$\beta\approx 100$$

$$\beta R_E \gg R_B+r_\pi \Rightarrow v_c\approx\frac{-\beta R_C}{(\beta+1)R_E}v_s$$

$$A_v=\frac{v_c}{v_s}\approx-\frac{R_C}{R_E}$$

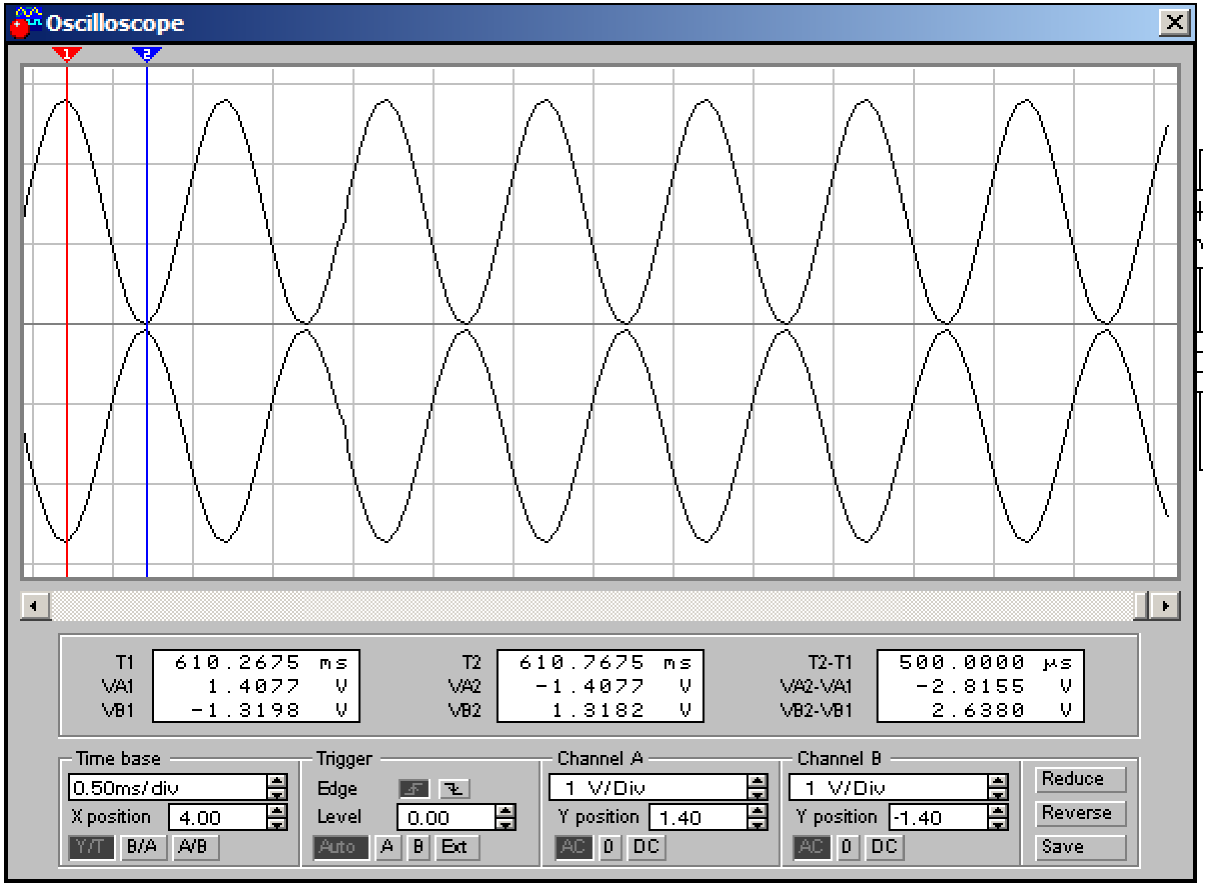
Multisim Model of "Bias Design"



$$A_v = \frac{v_c}{v_s} \approx -\frac{R_C}{R_E} = -1$$

1. R_E needs to be large to achieve good op. pt. Stability!
2. Consequence: $|A_v|$ is low.

Multisim Input-output Plot



$$A_v = \frac{v_c}{v_s} \approx -\frac{R_C}{R_E} = -1$$

Conclusions

Conservative voltage bias for best operating point stability and signal swing works, but returns unity or low voltage gain.

How does one obtain operating point stability, and simultaneously achieve a more respectable voltage gain?