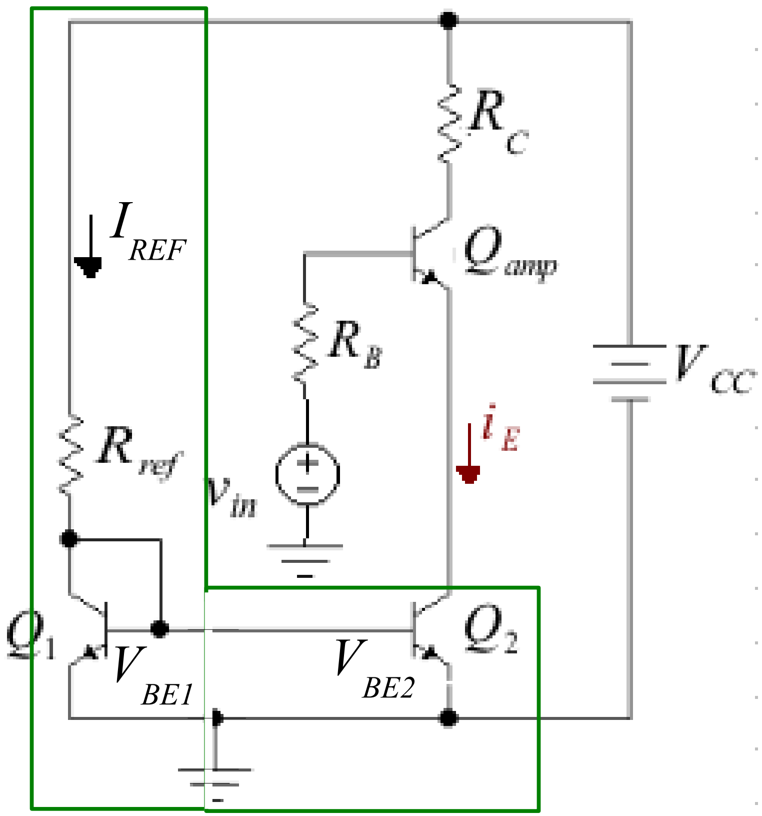
Emitter Current Bias Quick Review



What is the circuit in the green box?

What is the relation between R_{ref} and I_{REF}?

What is the approximate relation between I_{REF} and I_E?

Does $V_{BE1} = V_{BE2}$?

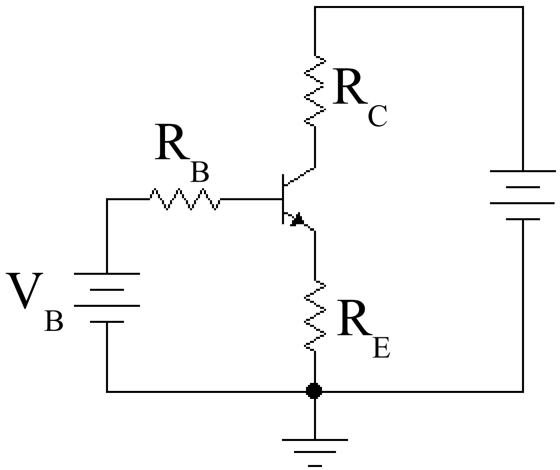
Does the Early Voltage effect the relation between I_{REF} and I_E? How?

What is the output resistance of Q2?

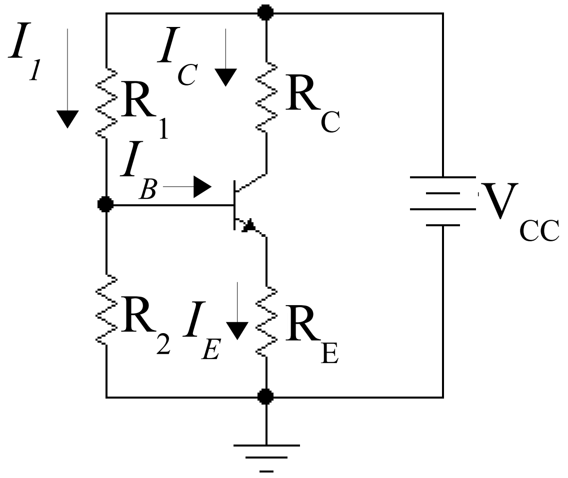
BJT Biasing Cont. & Small Signal Model

- Bias Design Example using “1/3, 1/3, 1/3 Rule”
- Small Signal BJT Models
- Small Signal Analysis

Emitter Feedback Bias Design



Voltage bias circuit



Single power supply version

Thevinen Equivalent

Use Thevinen's theorem:

$$V_{Th} = V_B = \frac{R_2}{R_1 + R_2} V_{CC}$$

Let: $\rho = \frac{R_1}{R_2}$

$$V_B = \frac{1}{\rho + 1} V_{CC}$$

$$R_B = \frac{R_1 R_2}{R_1 + R_2} = \frac{R_1}{\rho + 1}$$

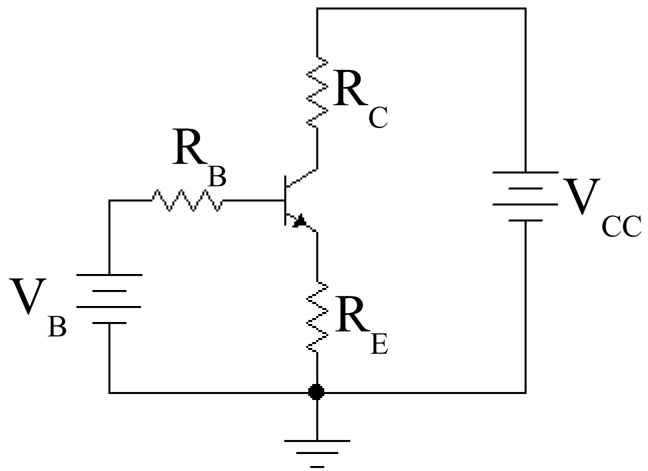
Let's specify V_B and R_B, and solve for ρ then R_1 and R_2:

$$\frac{1}{1 + \rho} = \frac{V_B}{V_{CC}}$$

$$\rho = \frac{V_{CC}}{V_B} - 1$$

$$R_1 = (1 + \rho) R_B$$

$$R_2 = \frac{R_1}{\rho}$$



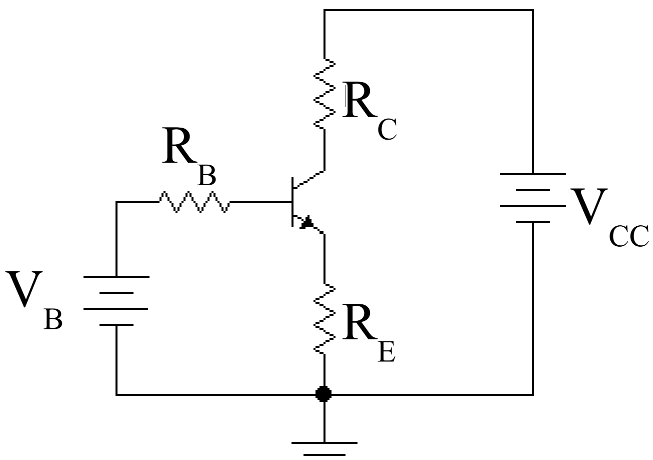
"1/3, 1/3, 1/3 Rule" Bias Procedure:

1. Bias so that V_{CC} is split equally across R_C, V_{CE} *(or V_{CB})*, and R_E *(or V_B)*.
2. Select desired collector current.
3. Assume $I_E = I_C$ to determine R_C & R_E.
4. Add $0.7\ V$ to $V_{Re} = V_{CC}/3$ to find V_B. Assume base current through R_B is negligible; hence $V_B \approx V_{BE} + V_{Re}$
5. Choose R_B approximately equal to $\beta R_E/10$ (Use lowest value of β)
6. Finally, compute ρ then R_1 and R_2.

5.Or choose

$$R_T = R_1 + R_2 = \frac{V_{CC}}{I_1} = \frac{V_{CC}}{(I_C/10)}$$

Example



$$50 \le \beta \le 100$$

$$V_{CC} = 12\text{V}$$

$$V_{RE} = V_{RC} = \frac{V_{CC}}{3} = 4\,V$$

$$I_C = 1\,mA$$

Then:

$$R_C = R_E = \frac{4}{10^{-3}} = 4 \cdot 10^3 = 4\,k\,\Omega$$

$$V_B = \frac{V_{CC}}{3} + 0.7\,V = 4\,V + 0.7\,V = 4.7\,V$$

$$R_B = \frac{(\beta_{min} + 1)\,R_E}{10} \approx \frac{50 \cdot 4000}{10} = 20\,k\,\Omega$$

For a single power supply:

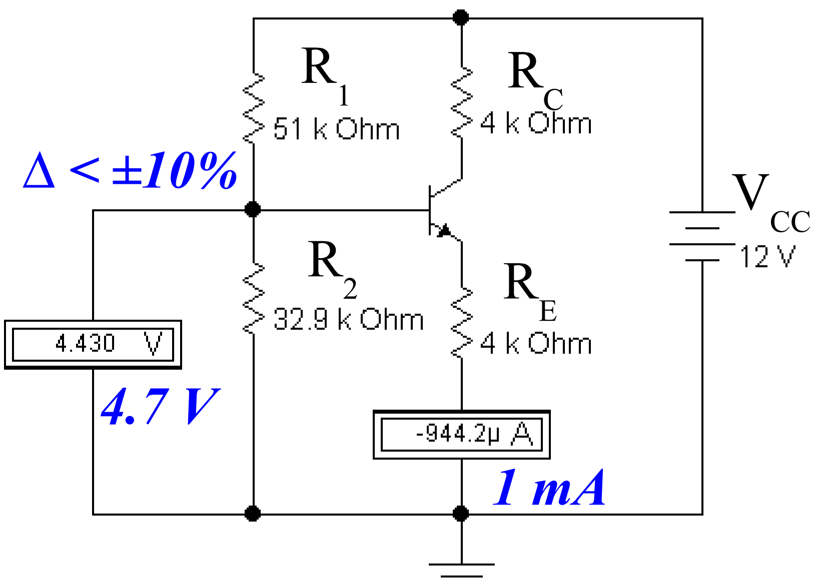
$$\rho = \frac{V_{CC}}{V_B} - 1 = \frac{12}{4.7} - 1 = 1.55$$

$$R_1 = (1 + \rho)\,R_B = 2.55 \cdot 20 = 51\,k\,\Omega$$

$$R_2 = \frac{R_1}{\rho} = \frac{51}{1.55} = 32.9\,k\,\Omega$$

Completed Bias Design

Electronics Workbench simulation results



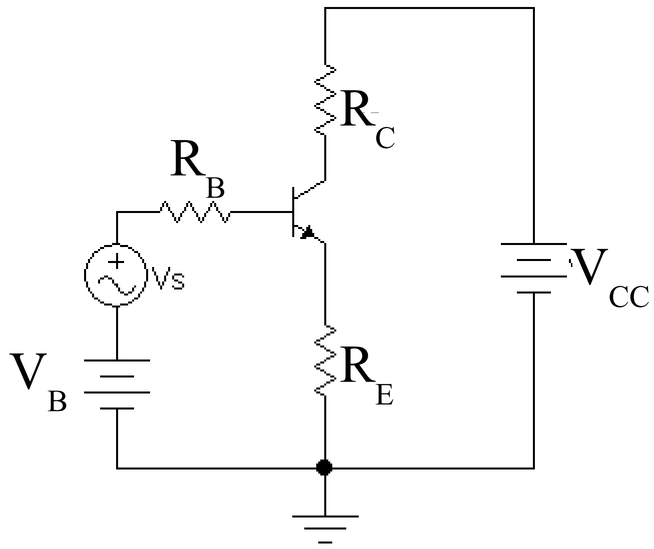
$$R_T = R_1 + R_2 = 83.9\,k\,\Omega$$

$$\Rightarrow I_1 = \frac{V_B}{R_T} = 0.056\,mA < \frac{I_B}{10}$$

Our design differs from the simulation because we neglected the base current.

There is no point in including the base current, since we will build the circuit using resistors that come only in standard sizes and with *5%* tolerances attached to their values. The closest available resistors in the RCA Lab are $47\,k\Omega$, $33\,k\Omega$, and $3.3\,k\Omega$.

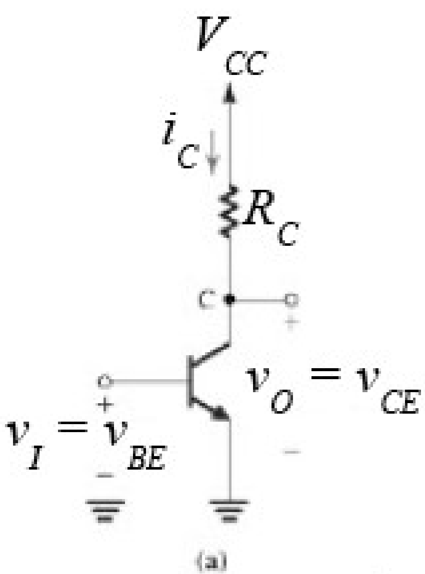
BJT Small Signal Models

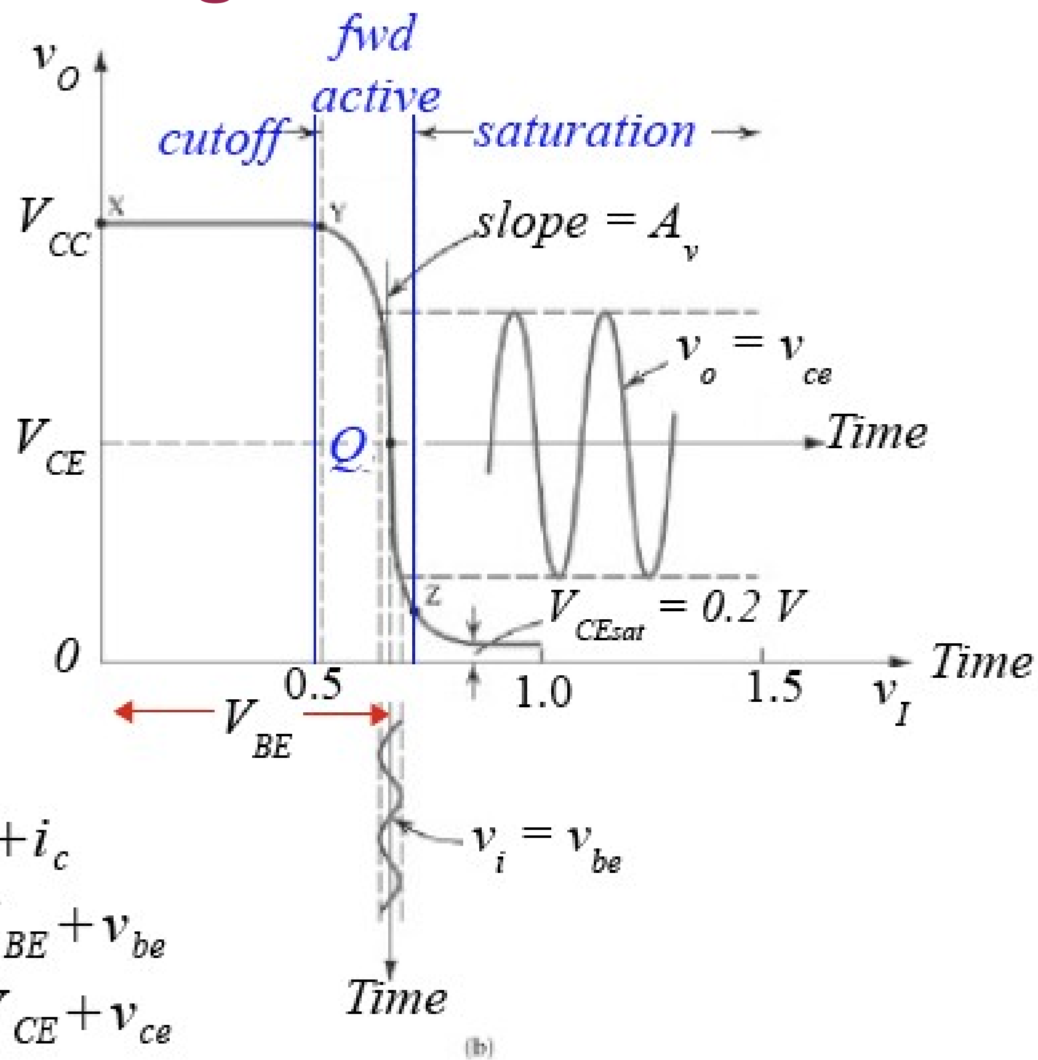


Conceptually, the signal we wish to amplify is connected in series with the bias source and is of small amplitude.

We will linearize the signal analysis to simplify our mathematics – to avoid having to deal with the nonlinear exponential collector Characteristic ($e^{\frac{v_{BE}}{V_T}}$).

BJT Small Signal Models





$$i_C = I_C + i_c$$

$$v_{BE} = V_{BE} + v_{be}$$

$$v_{CE} = V_{CE} + v_{ce}$$

Linearization Process

Split the total base-emitter voltage and total collector current into bias and signal components:

$$i_C = I_C + i_c = I_S e^{\frac{v_{BE}}{V_T}} = I_S e^{\frac{V_{BE}+v_{be}}{V_T}} = \underbrace{I_S e^{\frac{V_{BE}}{V_T}}}_{I_C} e^{\frac{v_{be}}{V_T}}$$

Identify and substitute the bias current into the model:

$$i_C = I_C e^{\frac{v_{be}}{V_T}}$$

Expand the exponential in a Taylor series:

$$f(x) = \sum_{n=0}^{\infty} f^{(n)}(0) \frac{x^n}{n!}$$

Analysis Using Collector Current Model

$$i_C = I_C e^{\frac{v_{be}}{V_T}}$$

$$\frac{i_C}{I_C} = e^{\frac{v_{be}}{V_T}}$$

Useful constants:

$$\log_{10}(2) = 0.301$$

$$\log_{10}(e) = 0.4343$$

$$\log_{10}\left(\frac{i_C}{I_C}\right) = \left(\frac{v_{be}}{V_T}\right)\log_{10} e$$

Let i_C be $2x\ I_C$, i.e. $i_C / I_C = 2$:

$$0.301 = \left(\frac{v_{be}}{V_T}\right) 0.4343$$

$$v_{be} = \frac{0.301}{0.4343} 0.025 \approx 0.017\ V$$

Linearization Using Taylor Series

Expand in a Taylor series: $$f(x)=\sum_{n=0}^{\infty} f^{(n)}(0)\frac{x^n}{n!}$$

For the exponential function: $$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}$$

$$e^{\frac{v_{be}}{V_T}}=\sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{v_{be}}{V_T}\right)^n$$

or:

$$e^{\frac{v_{be}}{V_T}}\approx 1+\frac{v_{be}}{V_T}+\frac{1}{2}\left(\frac{v_{be}}{V_T}\right)^2+\frac{1}{6}\left(\frac{v_{be}}{V_T}\right)^3+\ldots$$

Linearization Continued

Recall: v_{be} of about $17\ mV$ causes a $2x$ change in collector current.
Let's expand in Taylor series for this value of v_{be}.

$$\frac{v_{be}}{V_T} = \frac{0.017}{0.025} = 0.68$$

$$e^{\frac{v_{be}}{V_T}} = 1 + 0.68 + \frac{1}{2}(0.68)^2 + \frac{1}{6}(0.68)^3 + \ldots$$

$$e^{\frac{v_{be}}{V_T}} \approx 1 + 0.68 + 0.2312 + 0.0524 = 1.9444$$

Compare with: $e^{0.68} = 1.973$

Four term expansion is accurate to about 1.5%, two term expansion is only accurate to about 15%, i.e.

$$v_{be} \approx 0.017\, V \textit{ is not such a small signal}$$

Quick Small Signal Model Review

$$i_C = I_S e^{\frac{v_{BE}}{V_T}} = I_C e^{\frac{v_{be}}{V_T}}$$

What is i_C, I_C, v_{BE}, v_{be}?

$$e^{\frac{v_{be}}{V_T}} \approx 1 + \frac{v_{be}}{V_T} + \frac{1}{2}\left(\frac{v_{be}}{V_T}\right)^2 + \frac{1}{6}\left(\frac{v_{be}}{V_T}\right)^3 + \ldots$$

Under what condition(s) can one justifiably approximate the above infinite series as

$$e^{\frac{v_{be}}{V_T}} \approx 1 + \frac{v_{be}}{V_T}$$

Why is this important?

Small Signal Model

Using the grouping into bias and signal voltages/currents:

$$i_C = I_C + i_c = I_C e^{\frac{v_{be}}{V_T}}$$

And using the first two terms of the Taylor series expansion:

$$i_C \approx I_C \left(1 + \frac{1}{V_T} v_{be} \right) = I_C + \frac{I_C}{V_T} v_{be} = I_C + i_c$$

We define *transconductance* and *incremental (or ac) current as*:

$$g_m = \left(\frac{d\, i_C}{d\, v_{BE}} \right)_{i_C = I_C} = \frac{I_C}{V_T}$$

$$i_c = g_m v_{be}$$

bias current

Incremental (small-signal) BJT Model

$$i_B = I_B + i_b = \frac{1}{\beta} i_C = \frac{1}{\beta}(I_C + i_c)$$

$$i_B = \frac{1}{\beta} i_C = \underbrace{\frac{1}{\beta} I_C}_{I_B} + \underbrace{\frac{1}{\beta}\left(\frac{I_C}{V_T}\right) v_{be}}_{i_b}$$

Define the *incremental base current* and *base resistance*:

$$i_b = \left(\frac{1}{\beta}\right) i_c = \frac{1}{\beta}\left(\frac{I_C}{V_T}\right) v_{be} = \frac{1}{r_\pi} v_{be}$$

$$r_\pi = \beta \frac{V_T}{I_C} = \frac{\beta}{g_m}$$ where $I_C = V_T g_m$

bias current → I_C

Equivalent Models

Equations (1):

$$i_c = g_m v_{be}$$
$$i_b = \frac{v_{be}}{r_\pi}$$
$$i_e = i_b + i_c$$

Equations (2):

$$i_c = \beta i_b$$
$$i_b = \frac{i_e}{\beta + 1}$$
$$i_e = (\beta+1) i_b = (\beta+1) \frac{v_{be}}{r_\pi} = \frac{v_{be}}{r_e}$$

Equations (3):

$$i_c = g_m v_{be}$$
$$i_b = \frac{i_c}{\beta}$$
$$i_e = i_b + i_c$$

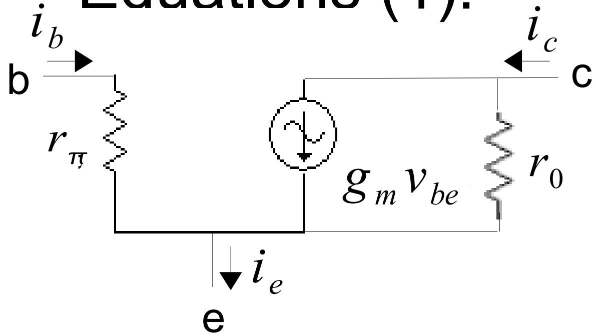
$$i_c = g_m r_\pi i_b = \frac{I_C}{V_T} \beta \frac{V_T}{I_C} i_b = \beta i_b$$

$$r_e = \frac{r_\pi}{\beta+1} = \frac{\beta}{\beta+1} \frac{V_T}{I_C}$$

Choose the model that simplifies the circuit analysis.

Equivalent Circuits

Equations (1):



$$i_c = g_m v_{be} + \frac{v_{ce}}{r_0}$$

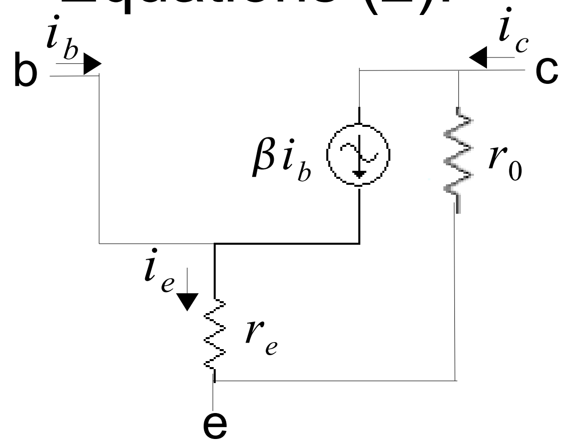
$$g_m = \frac{I_C}{V_T} \qquad r_0 = \frac{V_A}{I_C}$$

$$i_b = \frac{v_{be}}{r_\pi}$$

$$r_\pi = \frac{V_T}{I_B} = \beta \frac{V_T}{I_C} = \frac{\beta}{g_m}$$

$$i_e = i_b + i_c$$

Equations (2):



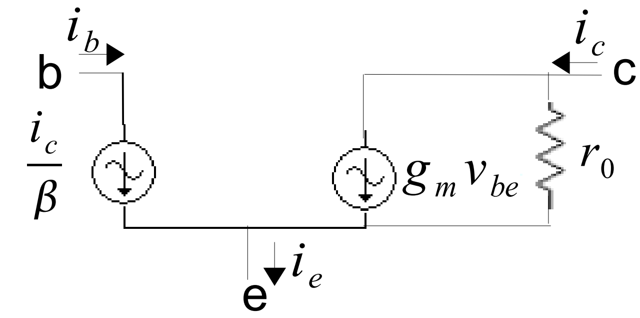
$$i_c = \beta i_b + \frac{v_{ce}}{r_0}$$

$$i_b = \frac{i_e}{\beta + 1}$$

$$i_e = \frac{v_{be}}{r_e}$$

$$r_e = \frac{V_T}{I_E} = \frac{\alpha}{g_m} = \frac{r_\pi}{\beta + 1}$$

Equations (3):

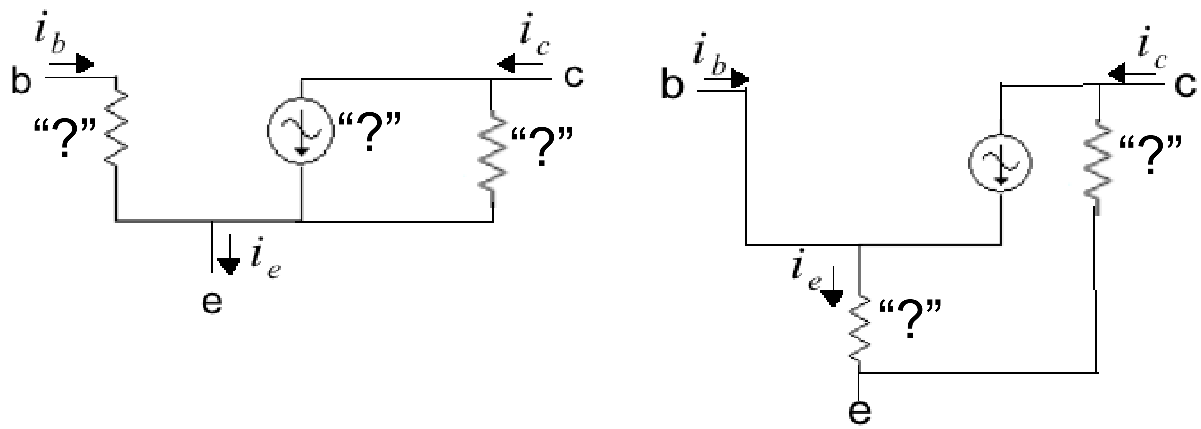


$$i_c = g_m v_{be} + \frac{v_{ce}}{r_0}$$

$$i_b = \frac{i_c}{\beta}$$

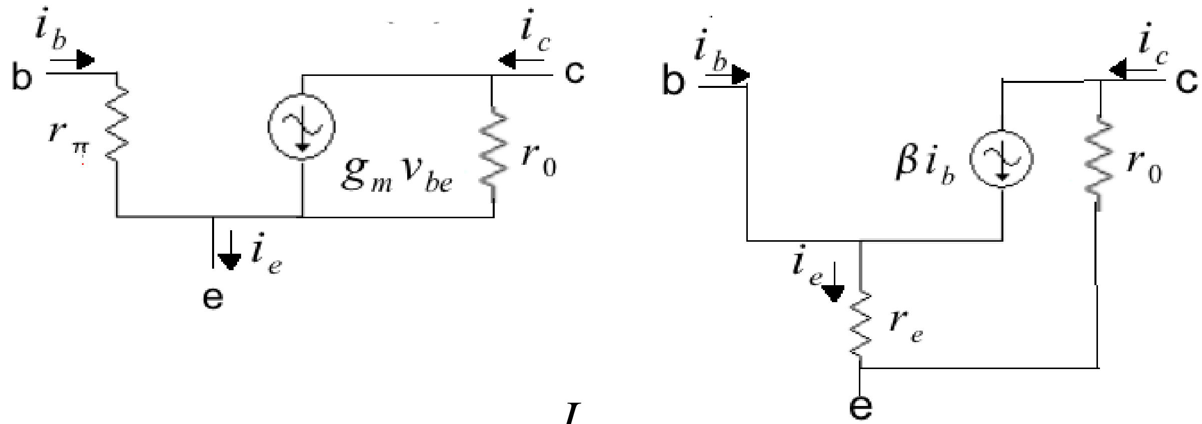
$$i_e = i_b + i_c$$

Quick Review



What are each of the "?" parameters?

Quick Review



$$g_m = \frac{I_C}{V_T}$$

$$r_\pi = \frac{V_T}{I_B} = \beta \frac{V_T}{I_C} = \frac{\beta}{g_m}$$

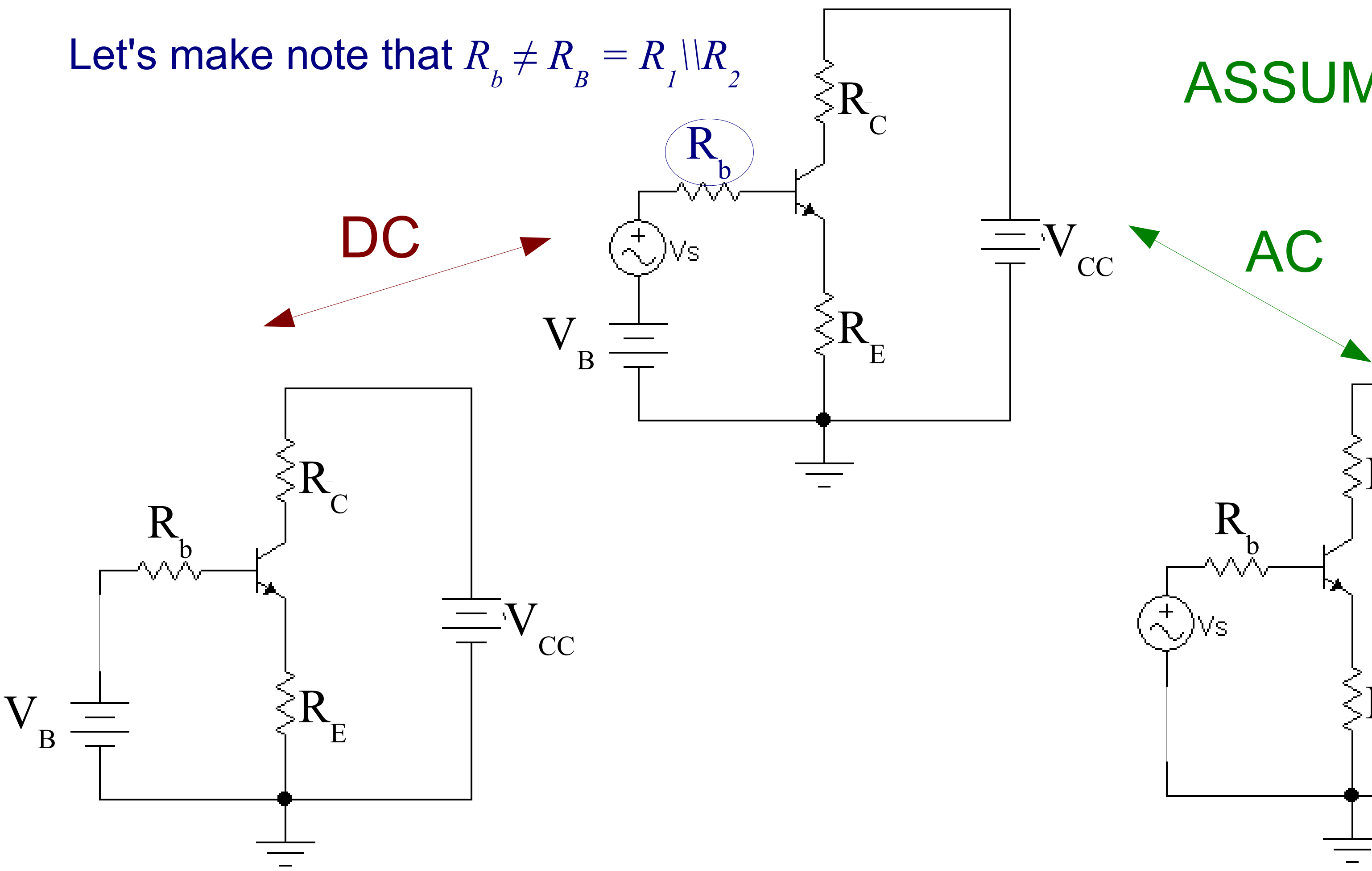
$$r_e = \frac{V_T}{I_E} = \frac{\alpha}{g_m} = \frac{r_\pi}{\beta + 1}$$

$$r_0 = \frac{V_A}{I_C}$$

DC & AC Circuit Decomposition

Let's make note that $R_b \neq R_B = R_1 \| R_2$

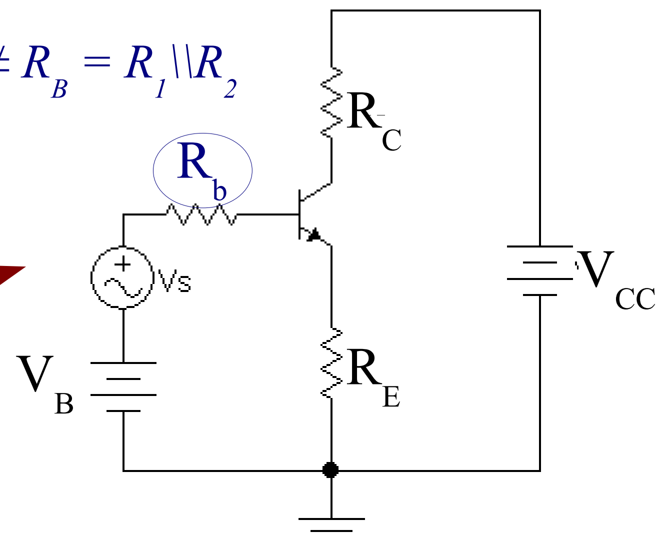
ASSUMPTION: ?



DC & AC Circuit Decomposition

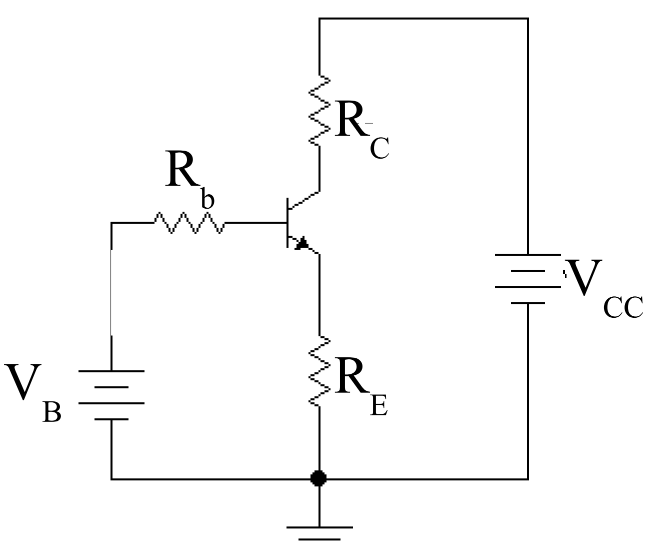
Let's make note that $R_b \neq R_B = R_1 \| R_2$

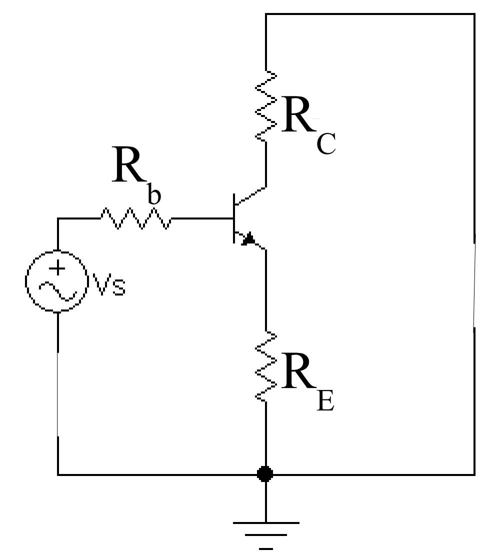
ASSUMPTION: signal v_s is pure AC!



DC

AC

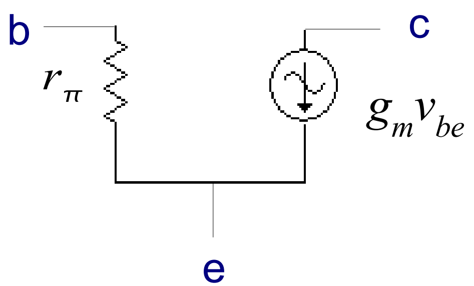




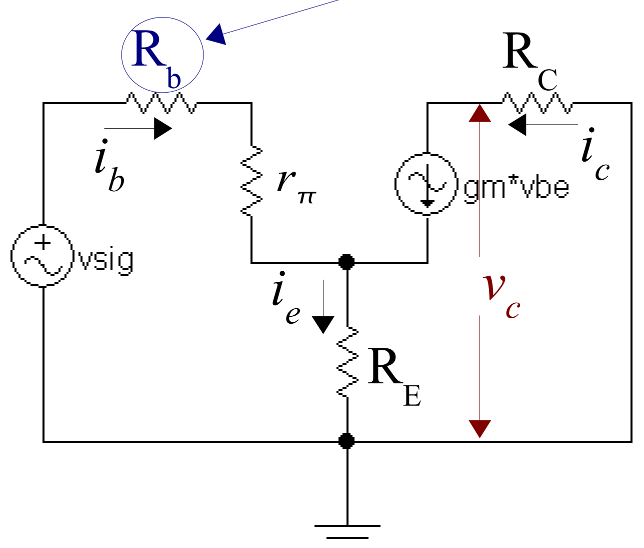
Small Signal Circuit Analysis

The small signal model will replace the large signal model and be used for (approximate) signal analysis once the transistor is biased.

Let's make note that $R_b \neq R_B = R_1 \| R_2$



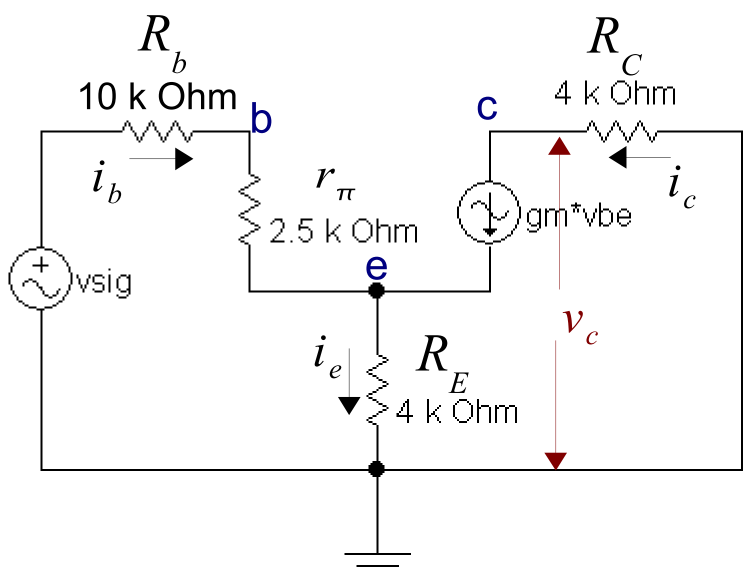
Transistor



Bias circuit

(Set to “zero” by superposition)

Biased Circuit Small Signal Analysis



$$v_{sig}=i_b(R_b+r_\pi)+i_e R_E$$

$$v_{sig}=i_b(R_b+r_\pi)+i_b(\beta+1)R_E$$

$$i_b=\frac{v_{sig}}{R_b+r_\pi+(\beta+1)R_E}$$

$$v_{be}=i_b r_\pi=\frac{r_\pi}{R_b+r_\pi+(\beta+1)R_E}v_{sig}$$

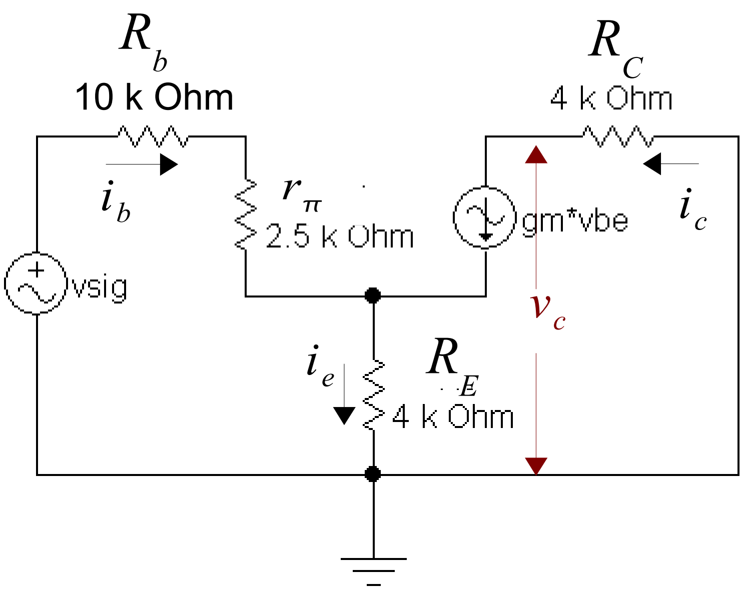
Let: $\beta=100$ and $I_C=1\,mA$

$$r_\pi=\beta\frac{V_T}{I_C}=100\frac{0.025}{0.001}=2.5\,k\,\Omega$$

Note: $g_m=\frac{I_C}{V_T}=\frac{0.001\,A}{0.025\,V}=0.04\,S=40\,mS$

S = siemen

Small Signal Analysis - Continued



$$v_{be}=i_b r_\pi=\frac{r_\pi}{R_b+r_\pi+(\beta+1)R_E}v_{sig}$$

$$i_c=g_m v_{be}=\frac{g_m r_\pi}{R_b+r_\pi+(\beta+1)R_E}v_{sig}$$

$$v_c=-R_C i_c=\frac{-\beta R_C}{R_b+r_\pi+(\beta+1)R_E}v_{sig}$$

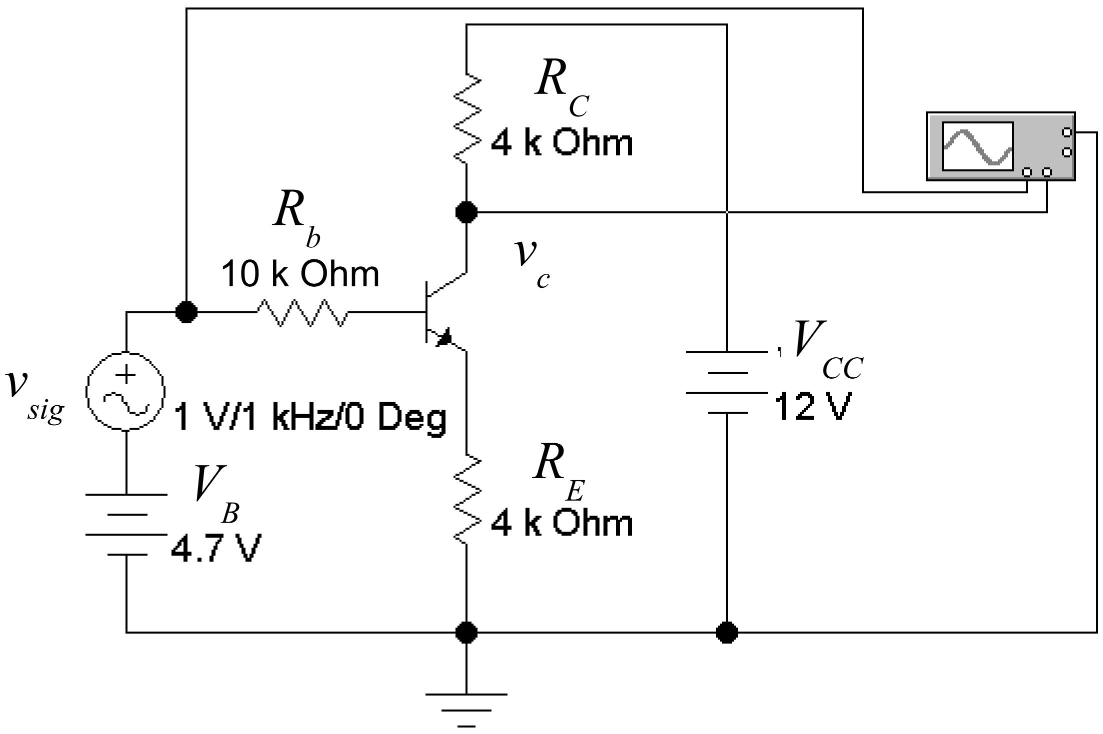
$$g_m r_\pi=\frac{I_C}{V_T}\beta\frac{V_T}{I_C}=\beta$$

$$\beta\approx 100$$

$$(\beta+1)R_E\gg R_b+r_\pi\Rightarrow v_c\approx\frac{-\beta R_C}{(\beta+1)R_E}v_{sig}$$

$$A_v=\frac{v_c}{v_{sig}}\approx-\frac{R_C}{R_E}$$

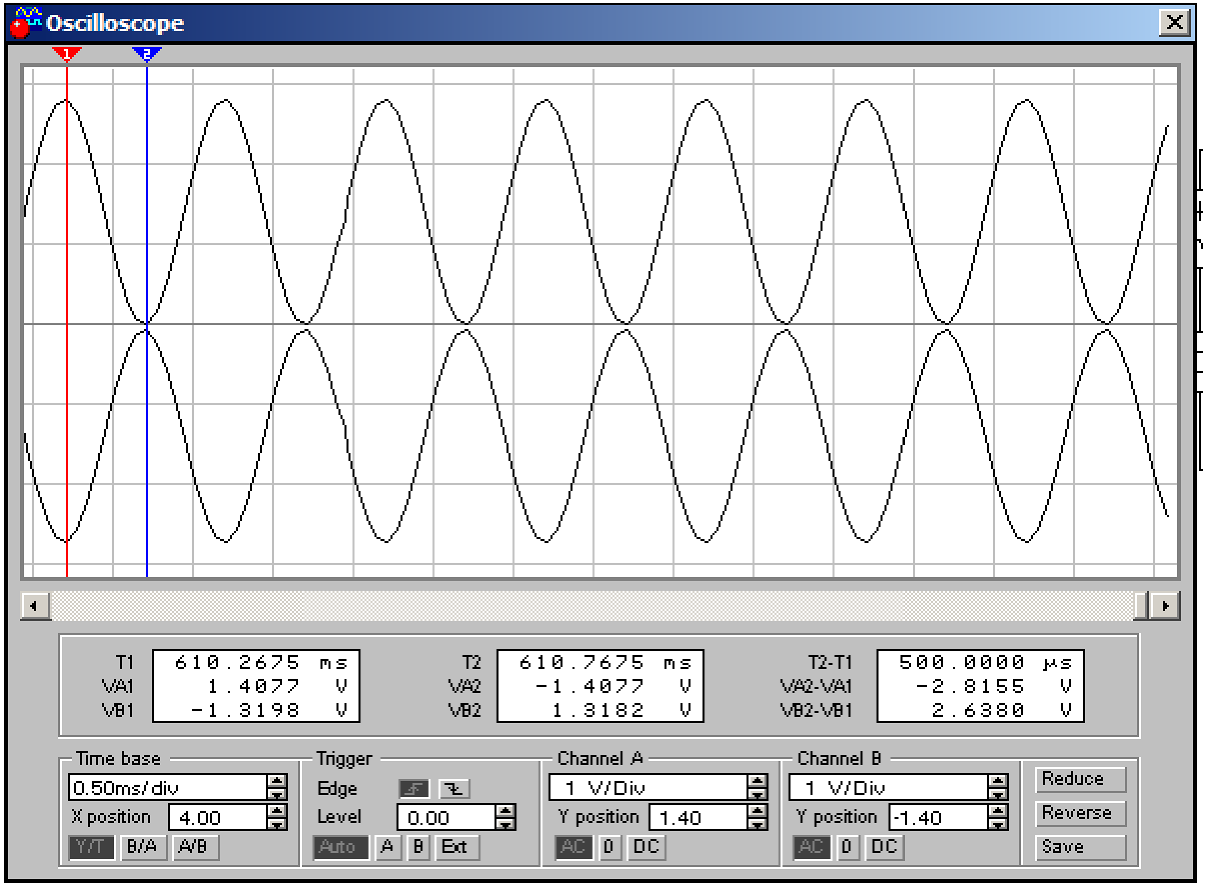
Multisim Model of "Bias Design"



$$A_v = \frac{v_c}{v_{sig}} \approx -\frac{R_C}{R_E} = -1$$

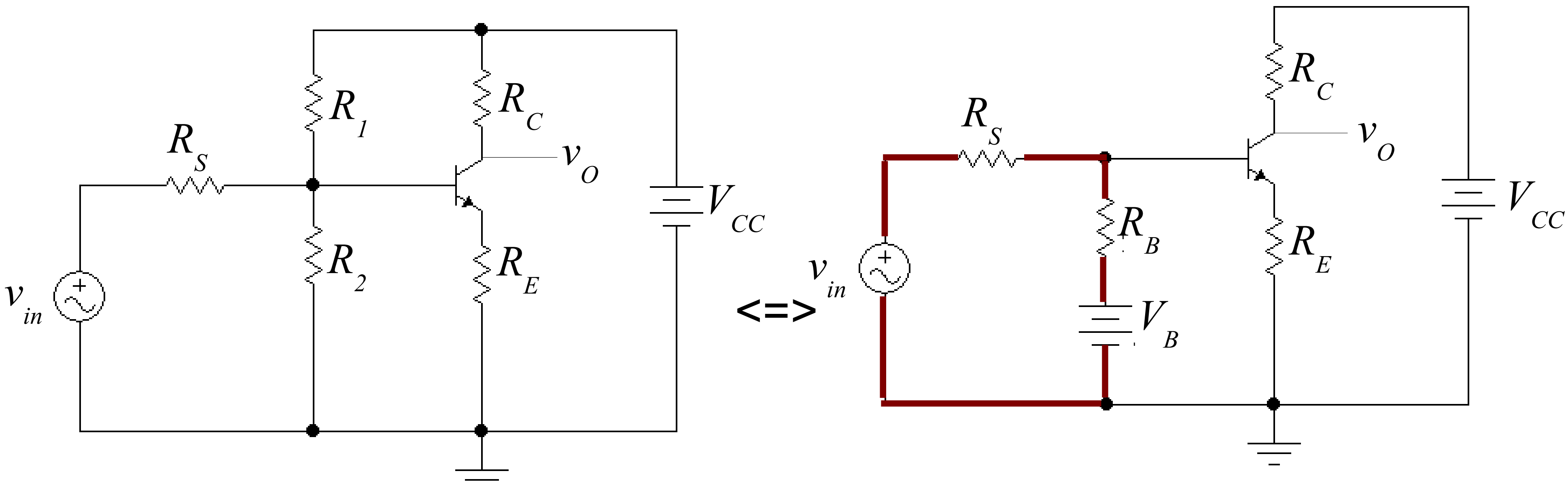
1. R_E needs to be large to achieve good op. pt. Stability!
2. Consequence: $|A_v|$ is low.

Multisim Input-output Plot



$$A_v = \frac{v_c}{v_{sig}} \approx -\frac{R_C}{R_E} = -1$$

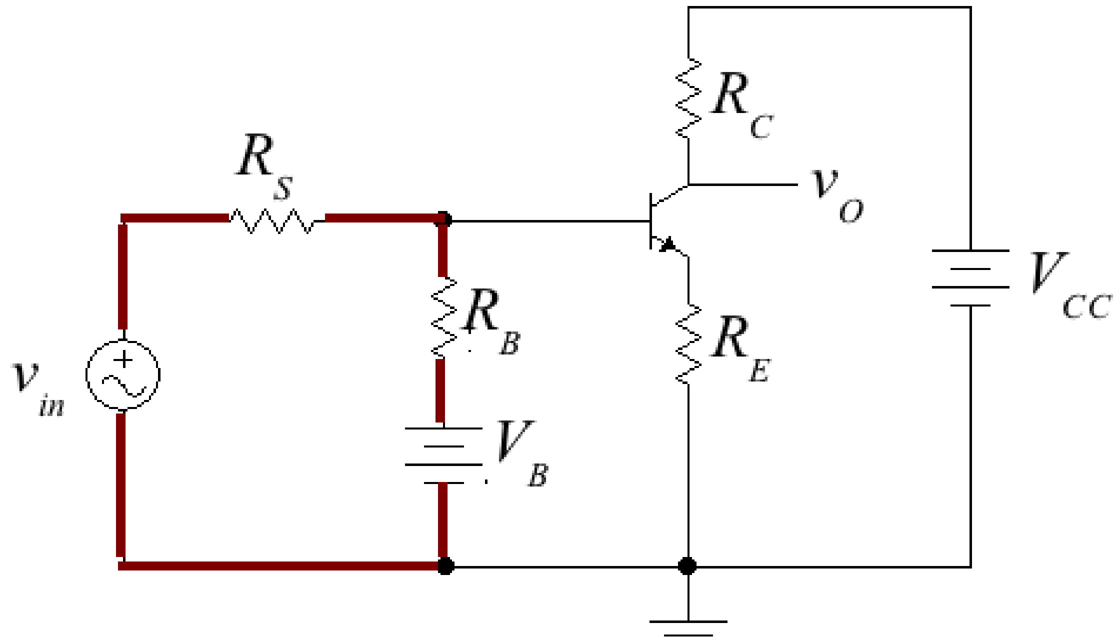
Combining AC and DC Sources



AC Source with low output impedance upsets bias.

Thevenin equivalent at base.

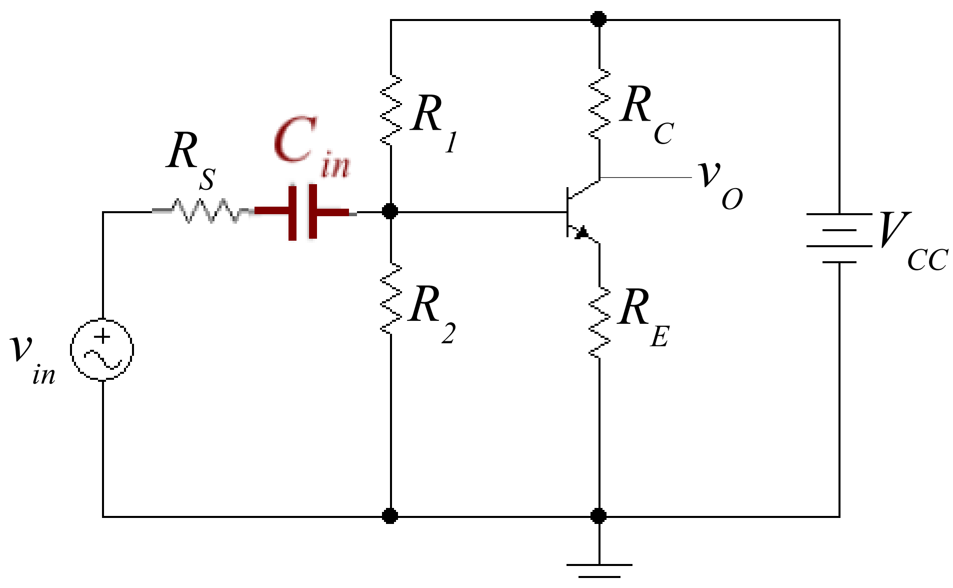
Combining AC and DC Sources Cont.



ISSUES:

1. Signal source v_{in} shorts out V_B.
2. DC bias on v_{in} interferes with op point design.

"Blocking Capacitor" Remedy



What is the purpose of "blocking capacitor" C_{in}?

How does one determine the value of C_{in}?

Conclusions

Conservative voltage bias for best operating point stability and signal swing works, but returns unity voltage gain.

How does one obtain operating point stability, and simultaneously achieve a respectable voltage gain?

Conclusion Cont.

How does one obtain operating point stability, and simultaneously achieve a respectable voltage gain?