BJT Biasing Cont. & Small Signal Model

- Bias Design Example using “1/3, 1/3, 1/3 Rule”
- Small Signal BJT Models
- Small Signal Analysis
**Emitter Feedback Bias Design**

Two power supply version

\[ V_B = \frac{R_2}{R_1 + R_2} \cdot V_{CC} \]

\[ R_B = \frac{R_1 R_2}{R_1 + R_2} \]

Single power supply version

Specify \( V_{CC}, V_B \), and \( R_B \), and solve for \( R_1 \) and \( R_2 \):

\[ R_1 = R_B \left( \frac{V_{CC}}{V_B} \right) \]

\[ R_2 = R_1 \left( \frac{V_B}{V_{CC} - V_B} \right) \]
“1/3, 1/3, 1/3 Rule” Bias Procedure:

1. Bias so that $V_{CC}$ is split equally across $R_C$, $V_{CE}$, and $R_E$.

2. Specify the desired collector current.

3. Assume $I_E = I_C$ to determine $R_C$ & $R_E$.

4. Add 0.7 $V$ to $V_{RE} = V_{CC}/3$ to find $V_B$.

Assume base current through $R_B$ is negligible; hence $V_B \approx V_{BE} + V_{RE}$.

5. Choose $R_B$ approximately equal to $(\beta_{min} + 1) R_E / 10$ ($\beta_{min}$ is lowest value of $\beta$)

6. Finally compute $R_I$ and $R_2$, and check that $I_B \ll I_I \ll I_C$.

Or choose $R_T = R_1 + R_2 = V_{CC} I_1 = V_{CC} (I_C/10)$

Check $R_B \ll (\beta_{min} + 1) R_E$
Example

\[ V_{CC} = 12V \]

Then:
\[ R_C = R_E = \frac{4}{10^{-3}} = 4 \cdot 10^3 = 4 \, k\Omega \]
\[ V_B = \frac{V_{CC}}{3} + 0.7V = 4V + 0.7V = 4.7V \]
\[ R_B = \frac{(\beta_{min} + 1) R_E}{10} \approx \frac{50 \cdot 4000}{10} = 20 \, k\Omega \]

For a single power supply:
\[ R_1 = \frac{R_B V_{CC}}{V_B} = (20 \, k\Omega) \frac{12V}{4.7V} = 51 \, k\Omega \]
\[ R_2 = R_1 \frac{V_B}{V_{CC} - V_B} = 51 \, k\Omega \frac{4.7V}{7.3V} = 32.9 \, k\Omega \]
\[ R_T = R_1 + R_2 = 83.9 \, k\Omega \Rightarrow I_1 = 0.14 \, mA \]
Completed Bias Design

Why does our design differ from the simulation?

Electronics Workbench simulation results

\[ R_T = R_1 + R_2 = 83.9 \, k \Omega \]
Completed Bias Design

Electronics Workbench simulation results

Our design differs from the simulation because we neglected the base current.

There is no point in including the base current, since we will build the circuit using resistors that come only in standard sizes and with 5% tolerances attached to their values. The closest available resistors in the Detkin Lab are $47 \, k\Omega$, $33 \, k\Omega$, and $3.3 \, k\Omega$ (or $4.7 \, k\Omega$).

Voltage & currents are specified as ranges ($V_{min} \leq V \leq V_{max}$), not absolute values.
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Voltage & currents are specified as ranges ($V_{\text{min}} \leq V \leq V_{\text{max}}$), not absolute values.
Conceptually, the signal we wish to amplify is connected in series with the bias source and is of small amplitude.

We will linearize the signal analysis to simplify our mathematics – to avoid having to deal with the nonlinear exponential collector current

\[ i_C = I_S e^{\frac{v_{BE}}{V_T}} \]
BJT Small Signal Models

\[ v_I = v_{BE} \]
\[ v_o = v_{CE} \]

\[ i_C = I_C + i_c \]
\[ v_{BE} = V_{BE} + v_{be} \]
\[ v_{CE} = V_{CE} + v_{ce} \]
Linearization Process

Split the total base-emitter voltage and total collector current into bias and signal components:

$$i_C = I_C + i_c = I_S e^{\frac{v_{BE}}{V_T}} = I_S e^{\frac{v_{BE} + v_{be}}{V_T}} = I_S e^{\frac{V_{BE}}{V_T}} e^{\frac{v_{be}}{V_T}}$$

Identify and substitute the bias current into the model:

$$i_C = I_C e^{\frac{v_{be}}{V_T}}$$

Expand the exponential in a Taylor series:

$$f(x) = \sum_{n=0}^{\infty} f^{(n)}(0) \frac{x^n}{n!}$$
Analysis Using Collector Current Model

\[ i_C = I_C e^{\frac{v_{be}}{V_T}} \]

\[ \frac{i_C}{I_C} = e^{\frac{v_{be}}{V_T}} \]

\[ \log_{10}\left(\frac{i_C}{I_C}\right) = \left(\frac{v_{be}}{V_T}\right) \log_{10} e \]

Let \( i_C = I_C + i_c \) be 2x \( I_C \), i.e. \( i_C/I_C = 2 \)

\[ 0.301 = \left(\frac{v_{be}}{V_T}\right) 0.4343 \]

\[ v_{be} = \frac{0.301}{0.4343} 0.025 \approx 0.017 V \]

Useful constants:

\[ \log_{10}(2) = 0.301 \]

\[ \log_{10}(e) = 0.4343 \]
Linearization Using Taylor Series

Expand in a Taylor series:

\[ f(x) = \sum_{n=0}^{\infty} f^{(n)}(0) \frac{x^n}{n!} \]

For the exponential function:

\[ f(x) = e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \]

\[
\frac{v_{be}}{V_T} = e^{\frac{v_{be}}{V_T}} = \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{v_{be}}{V_T} \right)^n
\]

or:

\[
e^{\frac{v_{be}}{V_T}} = 1 + \frac{v_{be}}{V_T} + \frac{1}{2} \left( \frac{v_{be}}{V_T} \right)^2 + \frac{1}{6} \left( \frac{v_{be}}{V_T} \right)^3 + \ldots \approx 1 + \frac{v_{be}}{V_T}
\]

when \[ \frac{1}{2} \left( \frac{v_{be}}{V_T} \right)^2 \ll 1 \]
Linearization Continued

Recall: $v_{be}$ of about 17 mV causes a 2x change in collector current $i_C = I_C + i_c$.

Let's expand in Taylor series for this value of $v_{be}$.

$$\frac{v_{be}}{V_T} = \frac{0.017}{0.025} = 0.68$$

$$e^{\frac{v_{be}}{V_T}} = 1 + 0.68 + \frac{1}{2}(0.68)^2 + \frac{1}{6}(0.68)^3 + \ldots$$

$$e^{\frac{v_{be}}{V_T}} \approx 1 + 0.68 + 0.2312 + 0.0524 = 1.9444$$

Compare with: $e^{0.68} = 1.973$

Four term expansion is accurate to about 1.5%, two term expansion is only accurate to about 15%, i.e.

$$v_{be} \approx 0.017 \text{ } V \text{ is not such a small signal}$$
**Small Signal Model**

Using the grouping into bias and signal voltages/currents:

\[ i_C = I_C + i_c = I_C e^{\frac{v_{be}}{V_T}} \]

And using the first two terms of the Taylor series expansion:

\[ i_C \approx I_C \left( 1 + \frac{1}{V_T} v_{be} \right) = I_C + \frac{I_C}{V_T} v_{be} = I_C + i_c \]

We define *transconductance* and *incremental (or ac) current* as:

\[ g_m = \left( \frac{d i_C}{d v_{BE}} \right)_{i_c = I_C} = \frac{I_C}{V_T} \]

\[ i_c = g_m v_{be} \]

**Bias Current**
Small Signal - Graphical Representation
**Incremental (small-signal) BJT Model**

\[ i_B = I_B + i_b = \frac{1}{\beta} i_C = \frac{1}{\beta} (I_C + i_c) \]

\[ i_B = \frac{1}{\beta} I_C + \frac{1}{\beta} \left( \frac{I_C}{V_T} \right) v_{be} \]

- **Define the incremental base current and base resistance:**

  \[ i_b = \left( \frac{1}{\beta} \right) i_c = \frac{1}{\beta} \left( \frac{I_C}{V_T} \right) v_{be} = \frac{1}{r_\pi} v_{be} \]

  - \[ r_\pi = \beta \frac{V_T}{I_C} = \frac{\beta}{g_m} \]

  - bias current

Kenneth R. Laker, updated 18Sep13 KRL
Equivalent Models

Equations (1):
\[ i_c = g_m v_{be} \]
\[ i_b = \frac{v_{be}}{r_{\pi}} \]
\[ i_e = i_b + i_c \]

Equations (2):
\[ i_c = \beta i_b \]
\[ i_b = \frac{i_e}{\beta + 1} \]
\[ i_e = (\beta + 1)i_b = (\beta + 1) \frac{v_{be}}{r_{\pi}} = \frac{v_{be}}{r_{e}} \]

Equations (3):
\[ i_c = g_m v_{be} \]
\[ i_b = \frac{i_c}{\beta} \]
\[ i_e = i_b + i_c \]

Choose the model that simplifies the circuit analysis.

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Equivalent Circuits

**Equations (1):**

\[ i_b = \frac{v_{be}}{r_\pi} \]
\[ i_c = g_m v_{be} + \frac{v_{ce}}{r_0} \]
\[ g_m = \frac{I_C}{V_T} \quad r_0 = \frac{V_A}{I_C} \]
\[ i_b = \frac{v_{be}}{r_\pi} \]
\[ r_\pi = \frac{V_T}{I_B} = \beta \frac{V_T}{I_C} = \beta \]
\[ i_e = i_b + i_c \]

**Equations (2):**

\[ i_c = \beta i_b + \frac{v_{ce}}{r_0} \]
\[ i_b = \frac{i_e}{\beta + 1} \]
\[ i_e = \frac{v_{be}}{r_e} \]
\[ r_e = \frac{V_T}{I_E} = \frac{\alpha}{g_m} = \frac{r_\pi}{\beta + 1} \]

**Equations (3):**

\[ i_c = g_m v_{be} + \frac{v_{ce}}{r_0} \]
\[ i_b = \frac{i_c}{\beta} \]
\[ i_e = i_b + i_c \]
Quick Small Signal Model Review

\[
i_C = I_S e^{\frac{v_{BE}}{V_T}} = I_C e^{\frac{v_{be}}{V_T}}
\]

What is \( i_C \), \( I_C \), \( v_{BE} \), \( v_{be} \)?

\[
e^{\frac{v_{be}}{V_T}} \approx 1 + \frac{v_{be}}{V_T} + \frac{1}{2} \left( \frac{v_{be}}{V_T} \right)^2 + \frac{1}{6} \left( \frac{v_{be}}{V_T} \right)^3 + \ldots
\]

Under what condition(s) can one justifiably approximate the above infinite series as

\[
e^{\frac{v_{be}}{V_T}} \approx 1 + \frac{v_{be}}{V_T}
\]

Why is this important?
DC & AC Circuit Decomposition

DOES $R_b = R_B = R_1 \parallel R_2$ ?

ASSUMPTION: ?
**DC & AC Circuit Decomposition**

NO \( R_b \neq R_B = R_1 \parallel R_2 \)

ac source \( v_s \) has series input resistance

\[ R_b = R_B + R_s \approx R_B \]

ASSUMPTION: signal \( v_s \) is pure AC!
Quick Review

What are each of the “?” parameters?
**Quick Review**

\[ g_m = \frac{I_C}{V_T} \]

\[ r_\pi = \frac{V_T}{I_B} = \beta \frac{V_T}{I_C} = \frac{\beta}{g_m} \]

\[ r_e = \frac{V_T}{I_E} = \frac{\alpha}{g_m} = \frac{r_\pi}{\beta+1} \]

\[ r_0 = \frac{V_A}{I_C} \]
Small Signal Circuit Analysis

The small signal model will replace the large signal model and be used for (approximate) signal analysis once the transistor is biased.

\[ R_b = R_B + R_s \approx R_B \]

Assume for time being that \( R_b = R_B = R_1 \parallel R_2 \)

actual circuit, AC + DC

AC small signal circuit (Bias circuit set to “zero” (using superposition))
**Biased Circuit Small Signal Analysis**

\[ V_s = i_b (R_B + r_\pi) + i_e R_E \]

\[ V_s = i_b (R_B + r_\pi) + i_b (\beta + 1) R_E \]

\[ i_b = \frac{V_s}{R_B + r_\pi + (\beta + 1) R_E} \]

\[ V_{be} = i_b r_\pi = \frac{r_\pi}{R_B + r_\pi + (\beta + 1) R_E} V_s \]

Let: \( \beta = 100 \) and \( I_C = 1 \text{ mA} \)

\[ r_\pi = \beta \frac{V_T}{I_C} = 100 \frac{0.025}{0.001} = 2.5 \text{ k}\Omega \]

Note: \( g_m = \frac{I_C}{V_T} = \frac{0.001 A}{0.025 V} = 0.04 \text{ } S = 40 \text{ mS} \)

S = siemen
Small Signal Analysis - Continued

\[ v_{be} = i_b r_{\pi} = \frac{r_{\pi}}{R_B + r_{\pi} + (\beta + 1) R_E} v_s \]

\[ i_c = g_m v_{be} = \frac{g_m r_{\pi}}{R_B + r_{\pi} + (\beta + 1) R_E} v_s \]

\[ v_c = -R_C i_c = \frac{-\beta R_C}{R_B + r_{\pi} + (\beta + 1) R_E} v_s \]

\[ (\beta + 1) R_E \gg R_B + r_{\pi} \Rightarrow v_c \approx -\frac{\beta R_C}{(\beta + 1) R_E} v_s \]

\[ A_v = \frac{v_c}{v_s} \approx -\frac{R_C}{R_E} \]
Multisim Model of “Bias Design”

1. $R_E$ needs to be large to achieve good op. pt. Stability!
2. Consequence: $|A_v|$ is low.
Multisim Input-output Plot

\[ A_v = \frac{v_c}{v_s} \approx -\frac{R_C}{R_E} = -1 \]
AC Source with low output impedance upsets bias. Thevenin equivalent at base.
What, if anything, is wrong with this amplifier design?

If there is something(s) wrong, how can it (they) be remedied?
What, if anything, is wrong with this amplifier design?

1. $V_B$ is effectively shorted out by $v_s$.
2. DC on $v_s$ corrupts the op. pt. (i.e. $V_C$ & $I_C$).
3. The voltage gain is $-\frac{R_C}{R_E}$. 
If there is something(s) wrong, how can it (they) be remedied?

1. Insert blocking capacitor $C_{in}$ in series with $R_S$.
2. Insert bypass capacitor $C_{byp}$ in parallel with all or part of $R_E$. 
Conclusions

Conservative voltage bias for best operating point stability and signal swing works, but returns unity voltage gain.

How does one obtain operating point stability, and simultaneously achieve a respectable voltage gain?
Conclusion Cont.

How does one obtain operating point stability, and simultaneously achieve a respectable voltage gain?

“Blocking Capacitor”

“Bypass Capacitor”