ESE 3400: Medical Devices Lab

Lec 11: October 19, 2022

Data Converters Pt 1

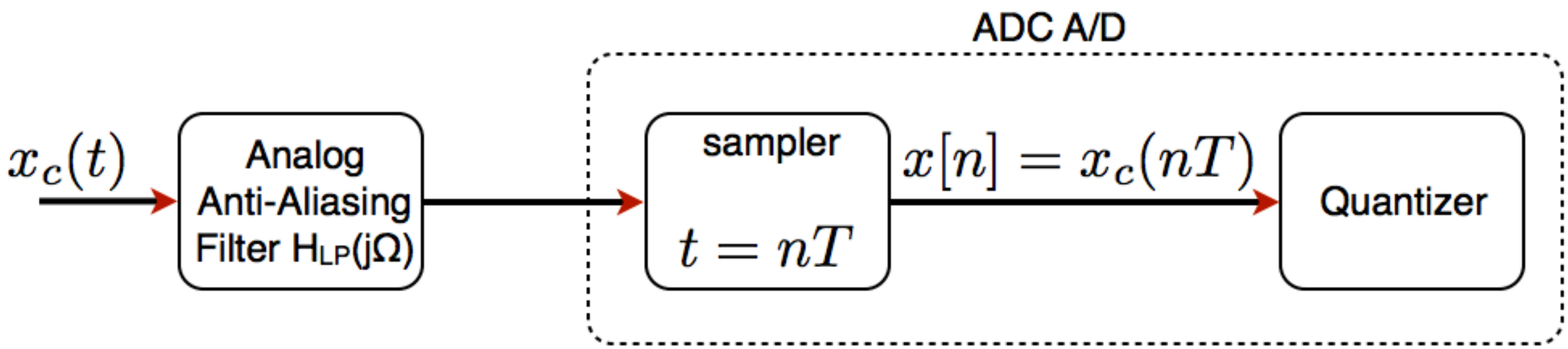
Lecture Outline

- DTFT vs DFT
- Sampling Examples
- Quantization noise
- Oversampling

ADC

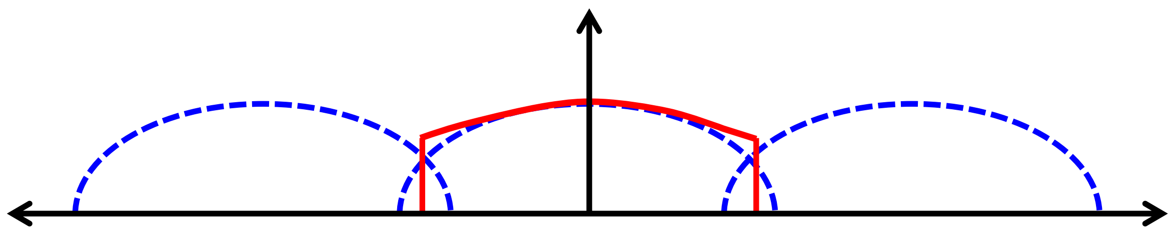
Analog to Digital Converter

Anti-Aliasing Filter with ADC



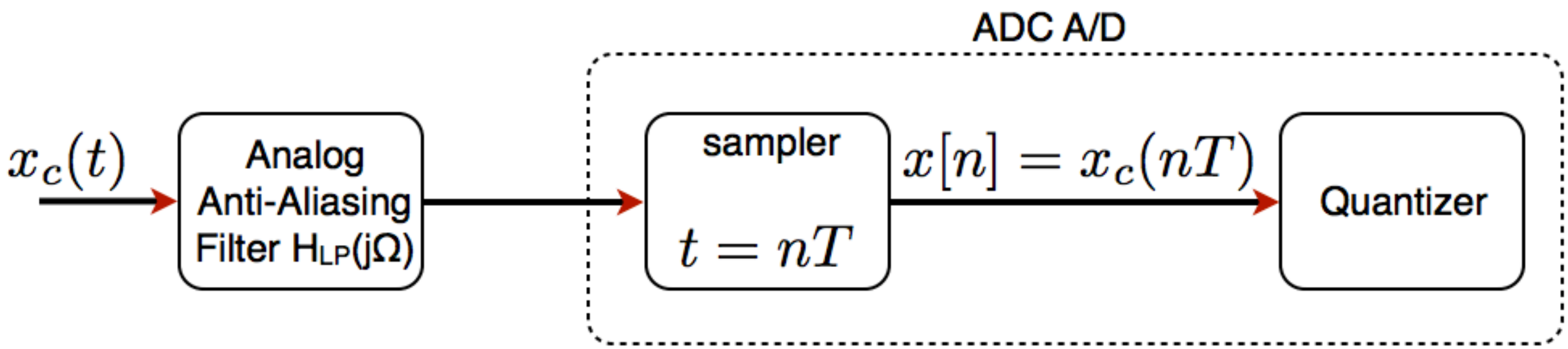
Aliasing

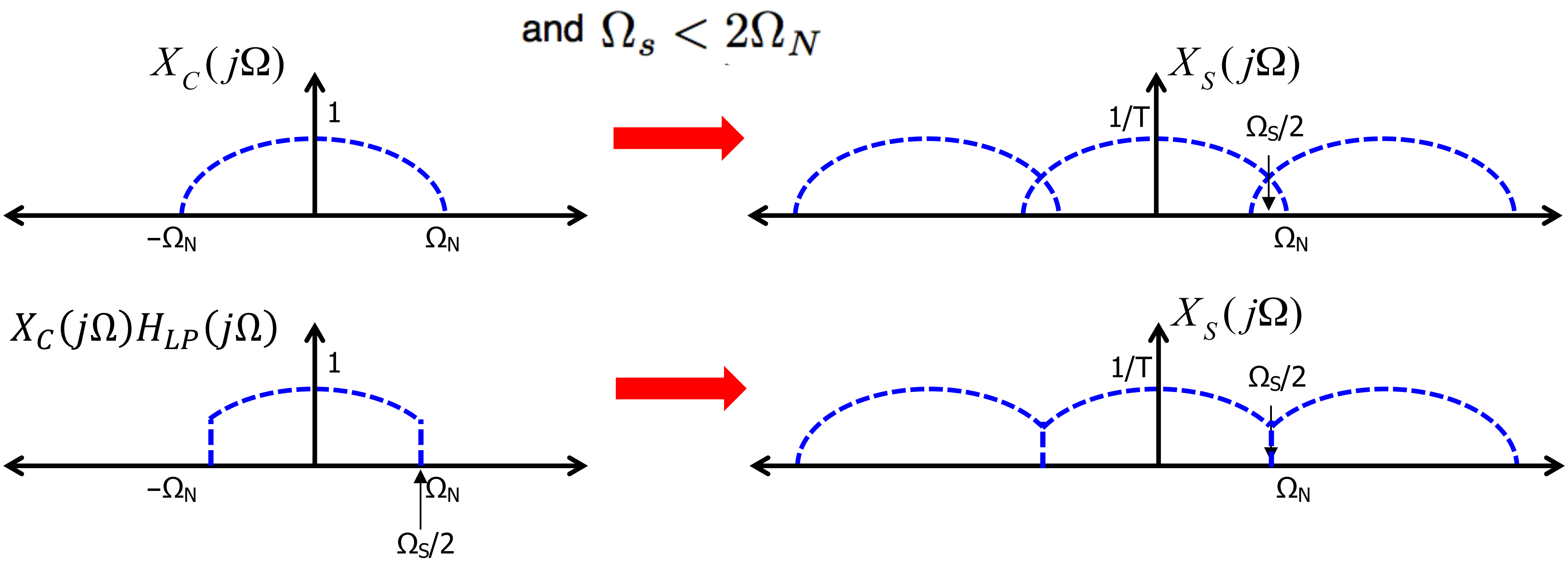
- If $\Omega_N > \Omega_s/2$, $x_r(t)$ an aliased version of $x_c(t)$



$$X_r(j\Omega) = \begin{cases} TX_s(j\Omega) & \text{if} \quad |\Omega| \leq \Omega_s/2 \\ 0 & \text{otherwise} \end{cases}$$

Anti-Aliasing Filter with ADC





DTFT Vs. DFT

DTFT:

$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} x[k]e^{-j\omega k}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

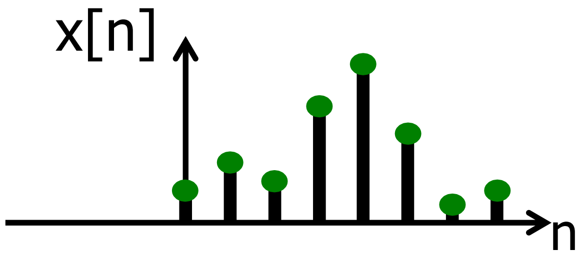
DFT:

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]W_N^{-kn}$$

$$X[k] = \sum_{n=0}^{N-1} x[n]W_N^{kn}$$

DFT Intuition

Time

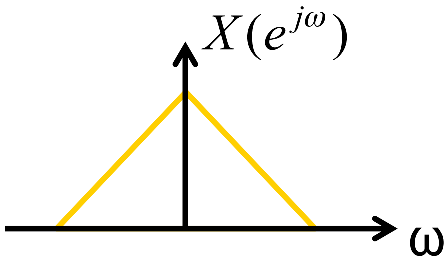


Transform

DTFT

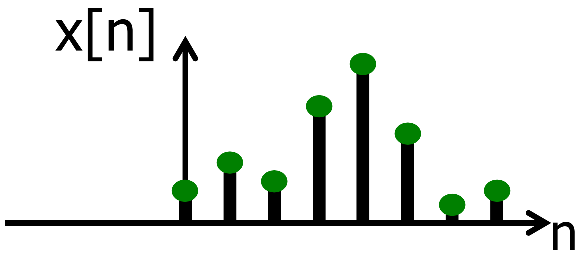
$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

Frequency



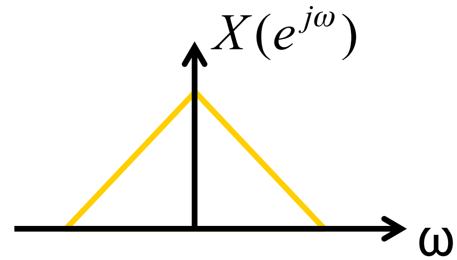
DFT Intuition

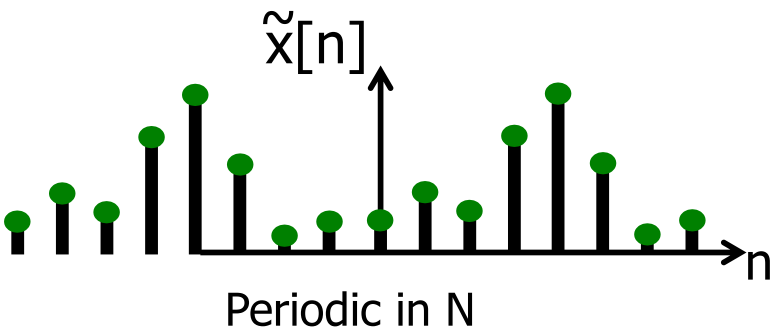
Time | Transform | Frequency



DTFT

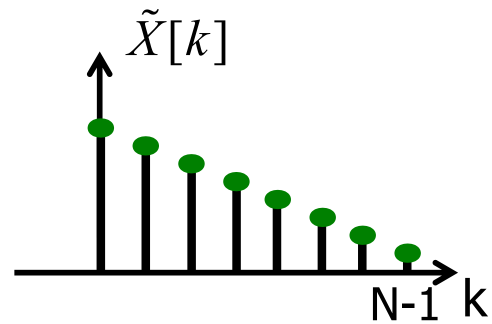
$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$





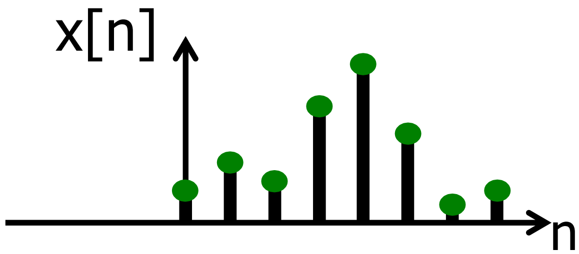
DFS

$$\tilde{x}[n] = \frac{1}{N}\sum_{k=0}^{N-1}\tilde{X}[k]W_N^{-kn}$$



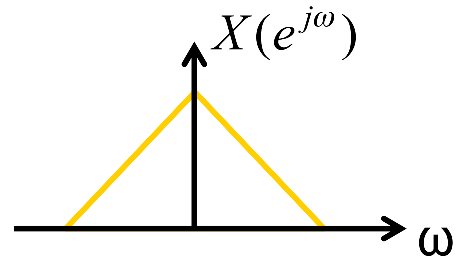
DFT Intuition

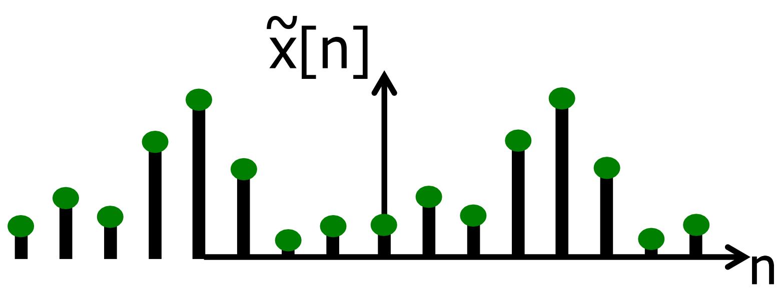
Time | **Transform** | **Frequency**



DTFT

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}\,d\omega$$



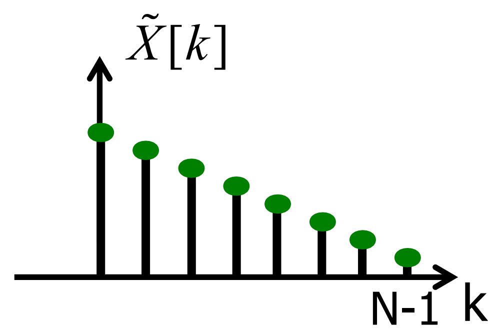


Periodic in N

DFS

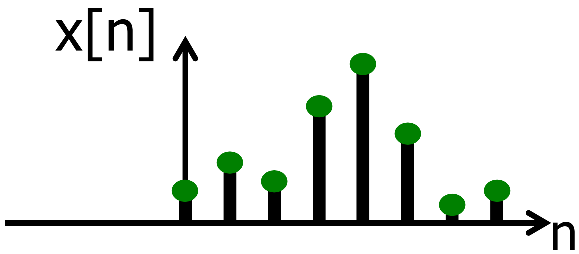
$$\tilde{x}[n] = \frac{1}{N}\sum_{k=0}^{N-1} \tilde{X}[k] W_N^{-kn}$$

$$W_N = e^{-j\frac{2\pi}{N}}$$



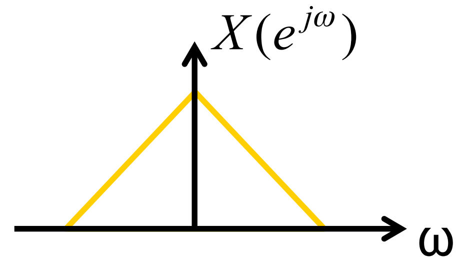
DFT Intuition

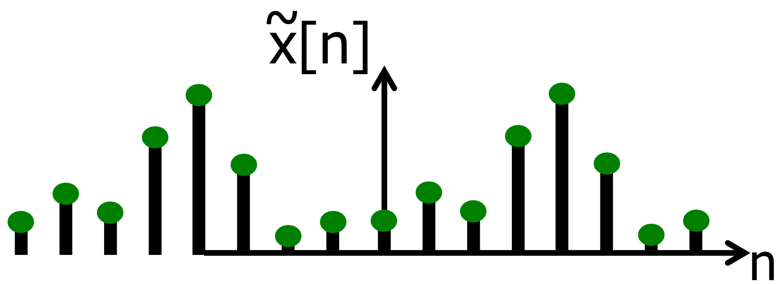
Time	Transform	Frequency



DTFT

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$





Periodic in N

DFS

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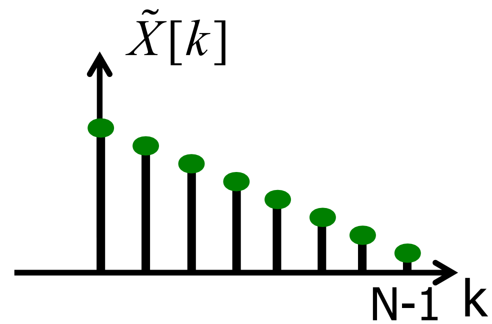
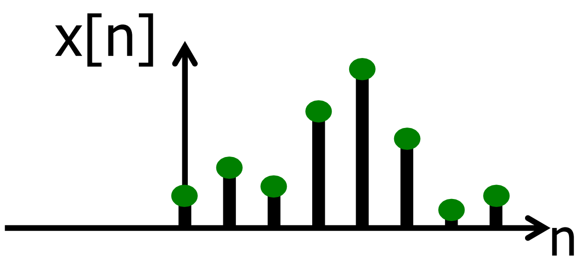
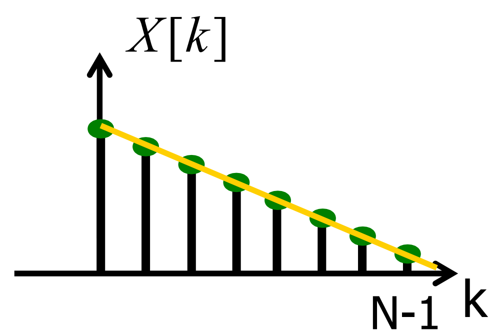




DFT

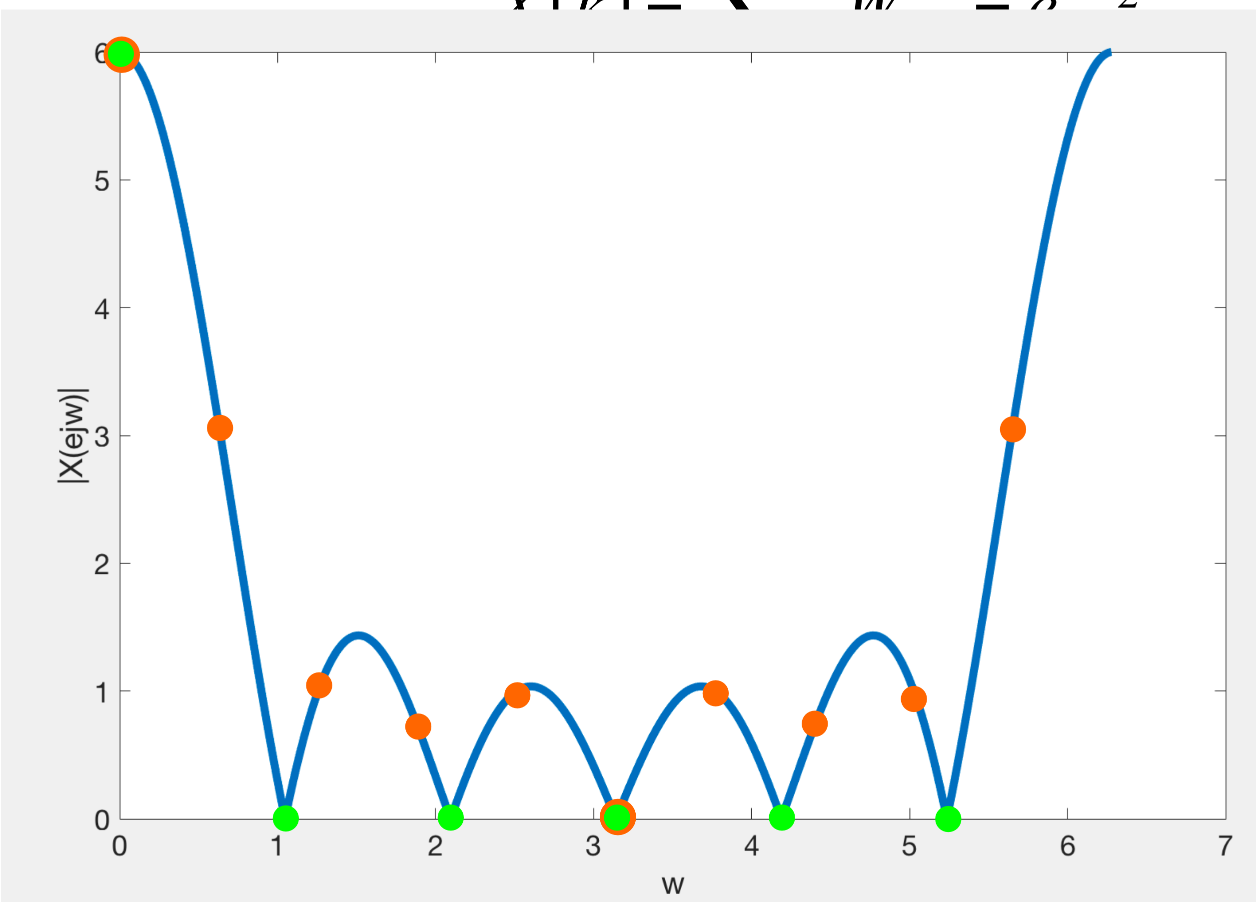
$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1}X[k]W_N^{-kn}$$



DFT vs DTFT

- Back to example

$$X[k]=\sum^{5} W^{nk}=e^{-j\frac{\pi}{2}k}\frac{\sin\left(\frac{3\pi}{5}k\right)}{\sin\left(\frac{\pi}{10}k\right)}$$

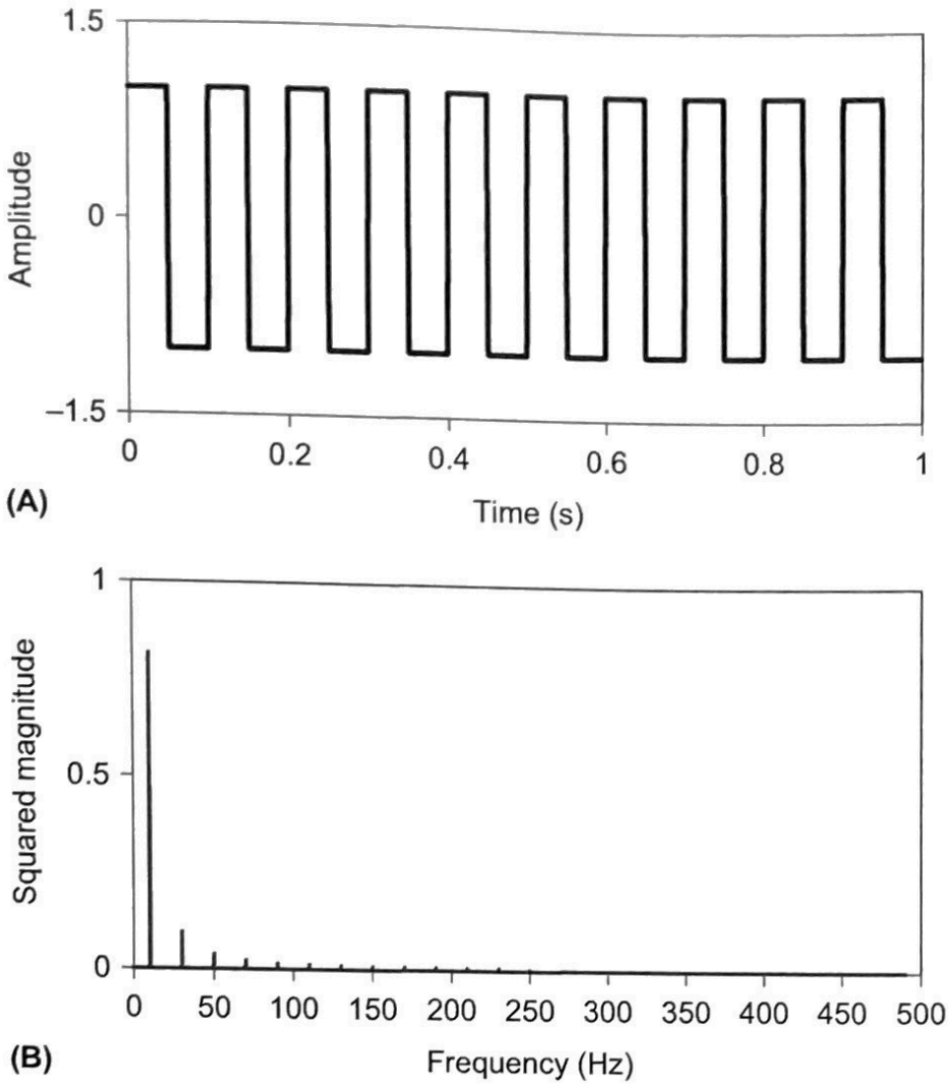


"6-point" DFT

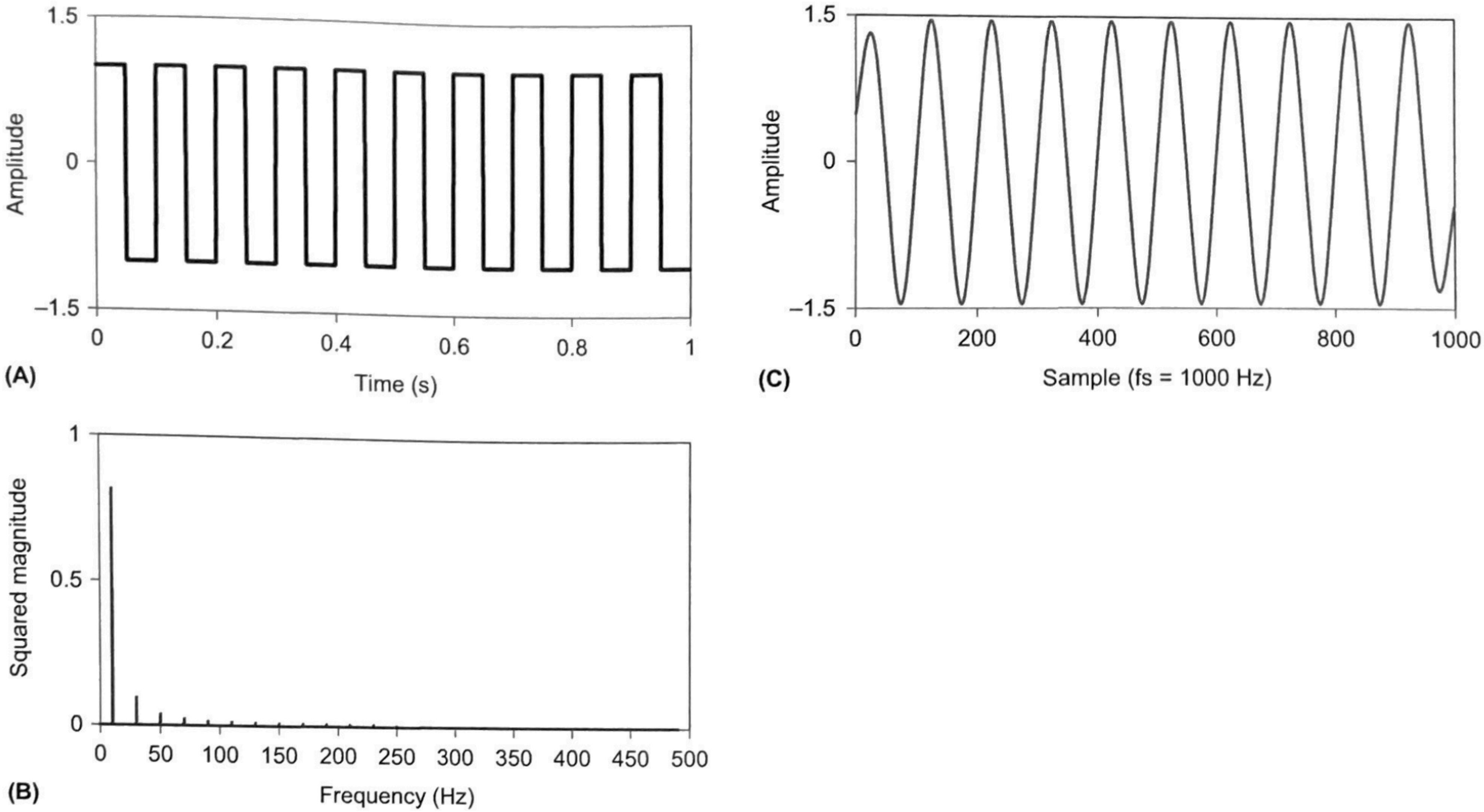
"10-point" DFT

Use `fftshift` to center around dc

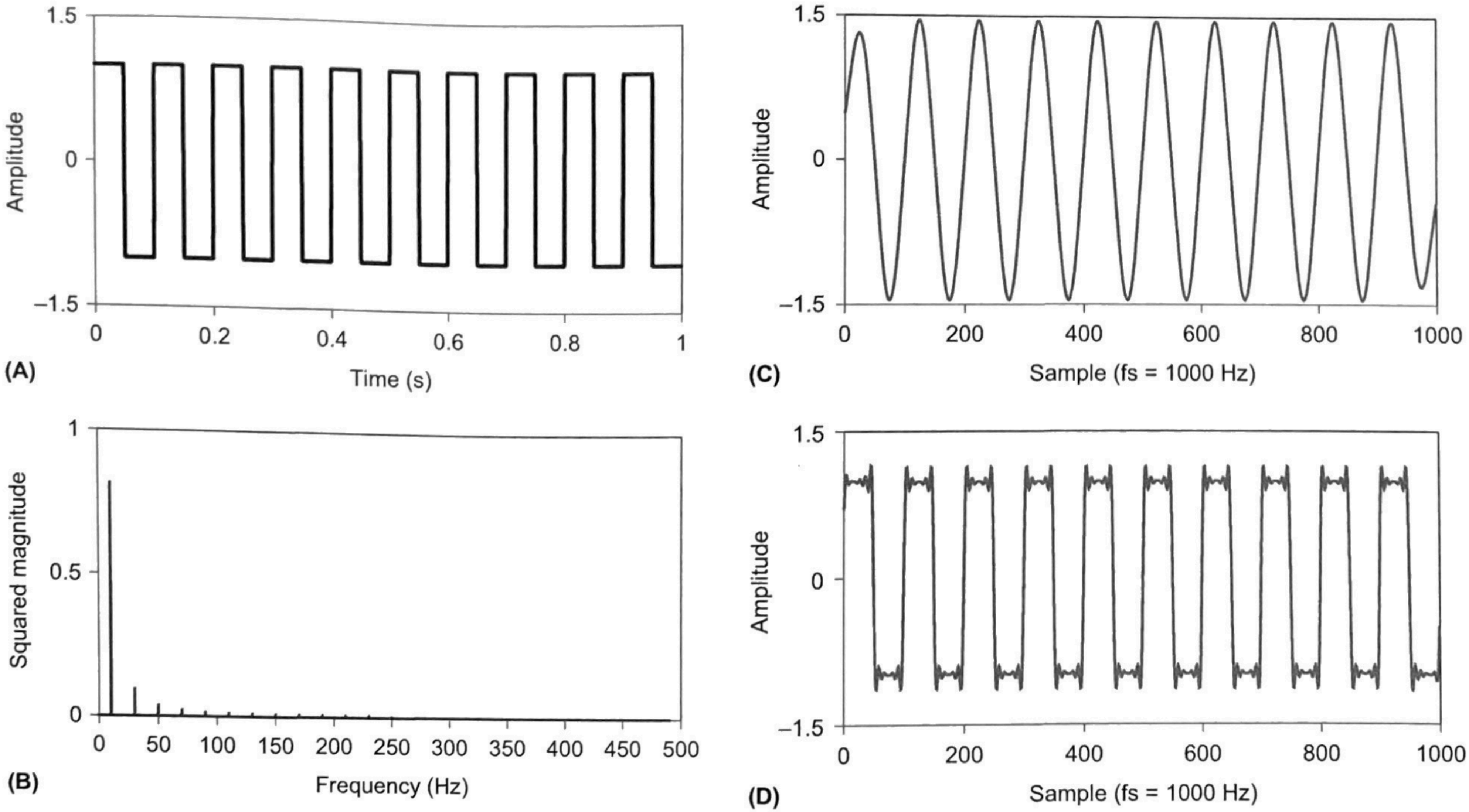
Sampling Example



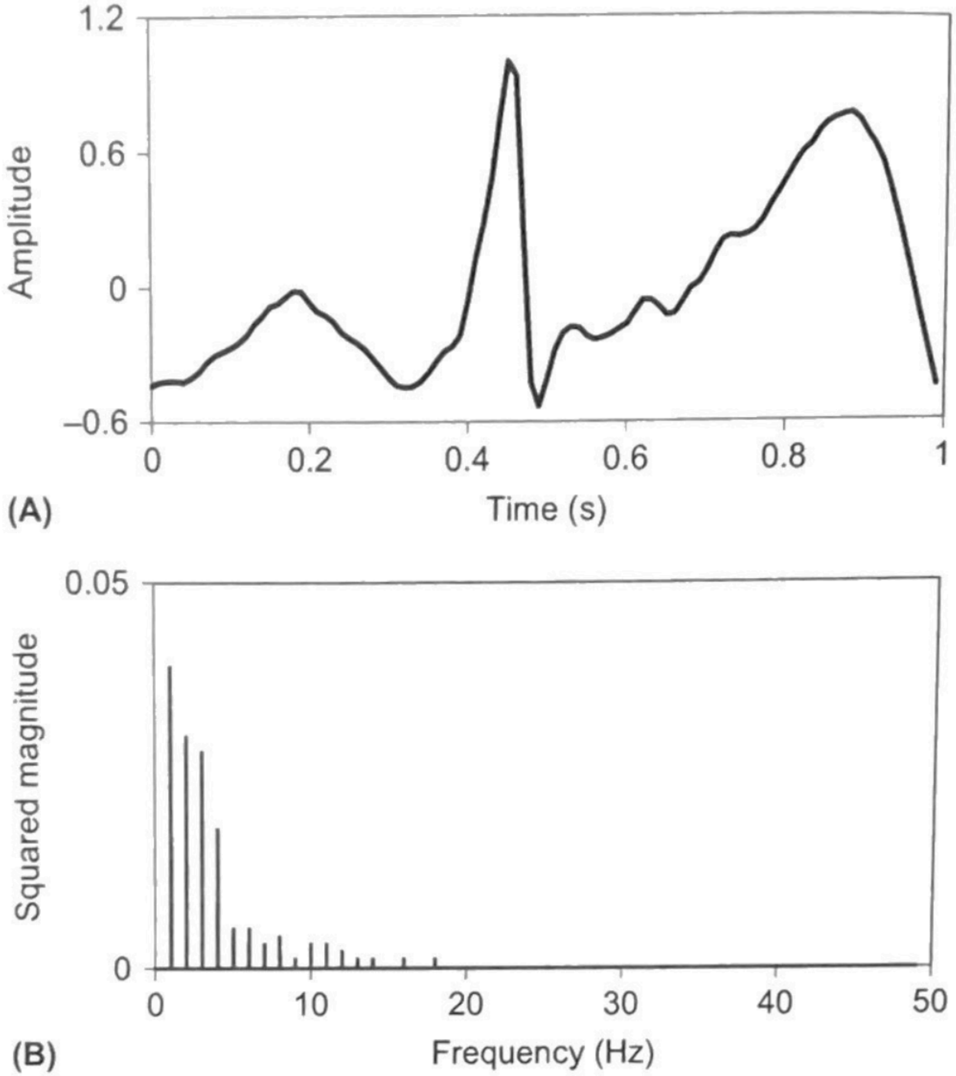
Sampling Example



Sampling Example

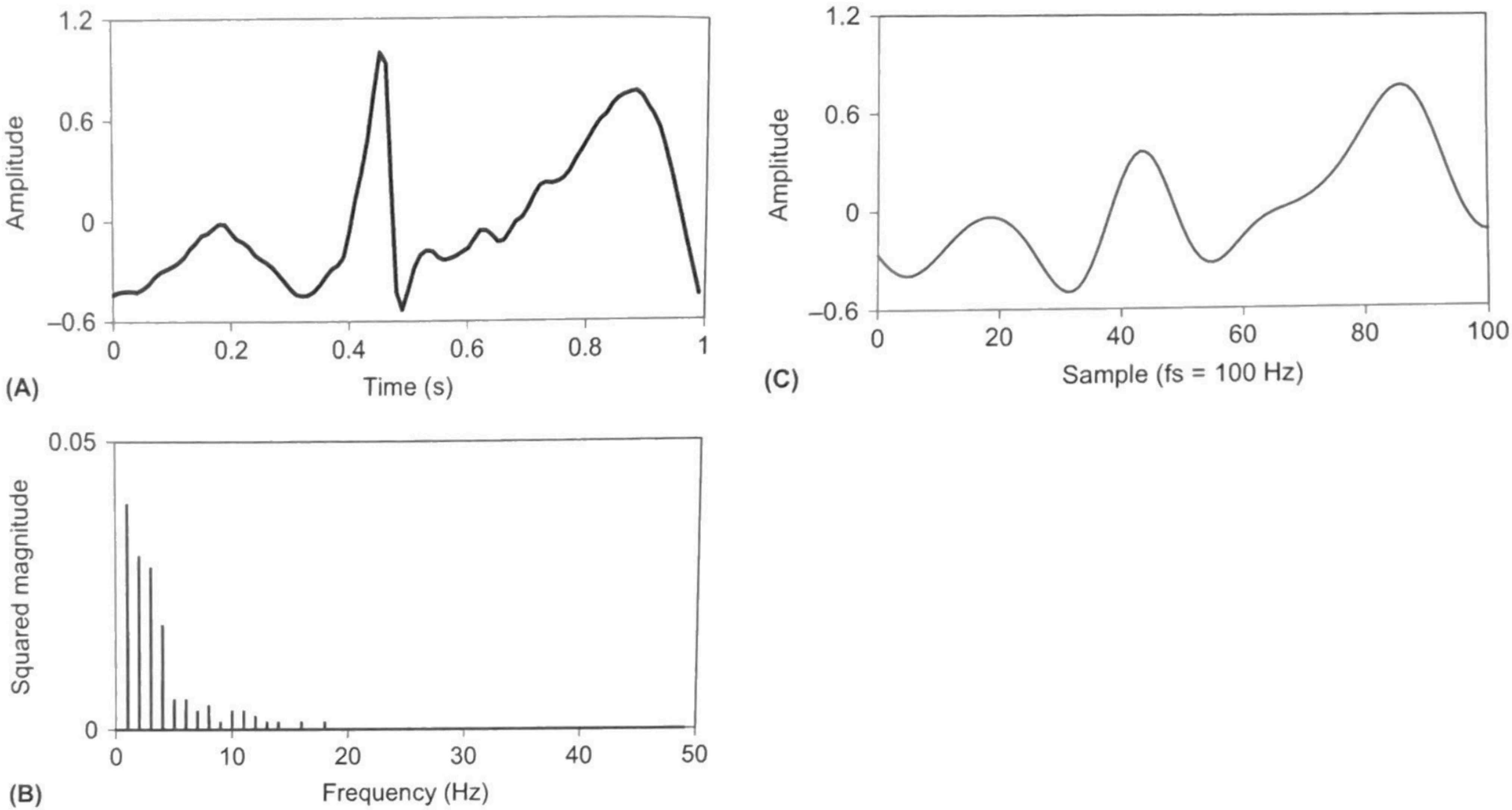


ECG Example



1.2
0.6
0
−0.6
Amplitude
0
0.2
0.4
0.6
0.8
1
Time (s)
(A)
0.05
0
Squared magnitude
0
10
20
30
40
50
Frequency (Hz)
(B)

ECG Example


1.2
0.6
0
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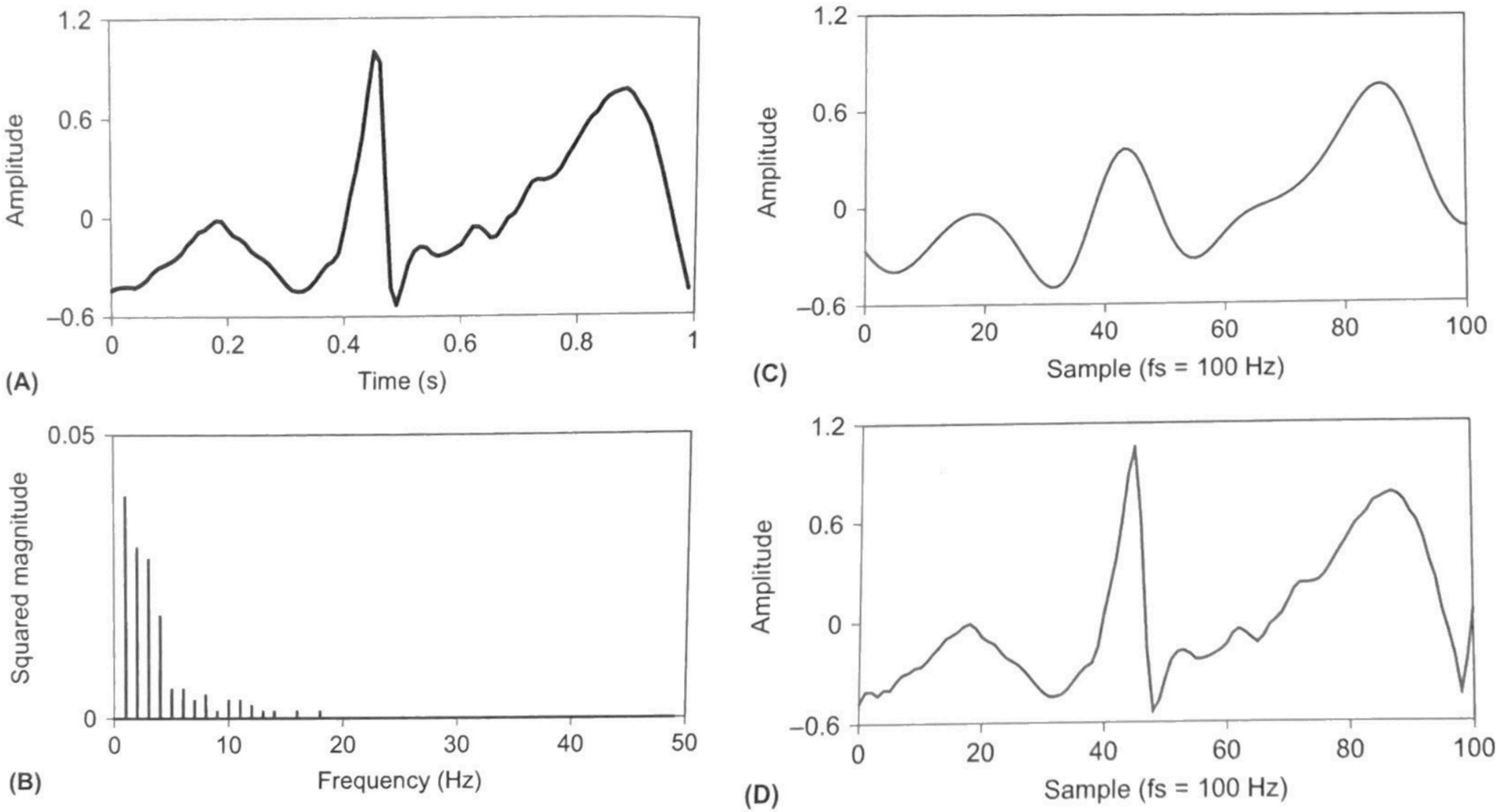
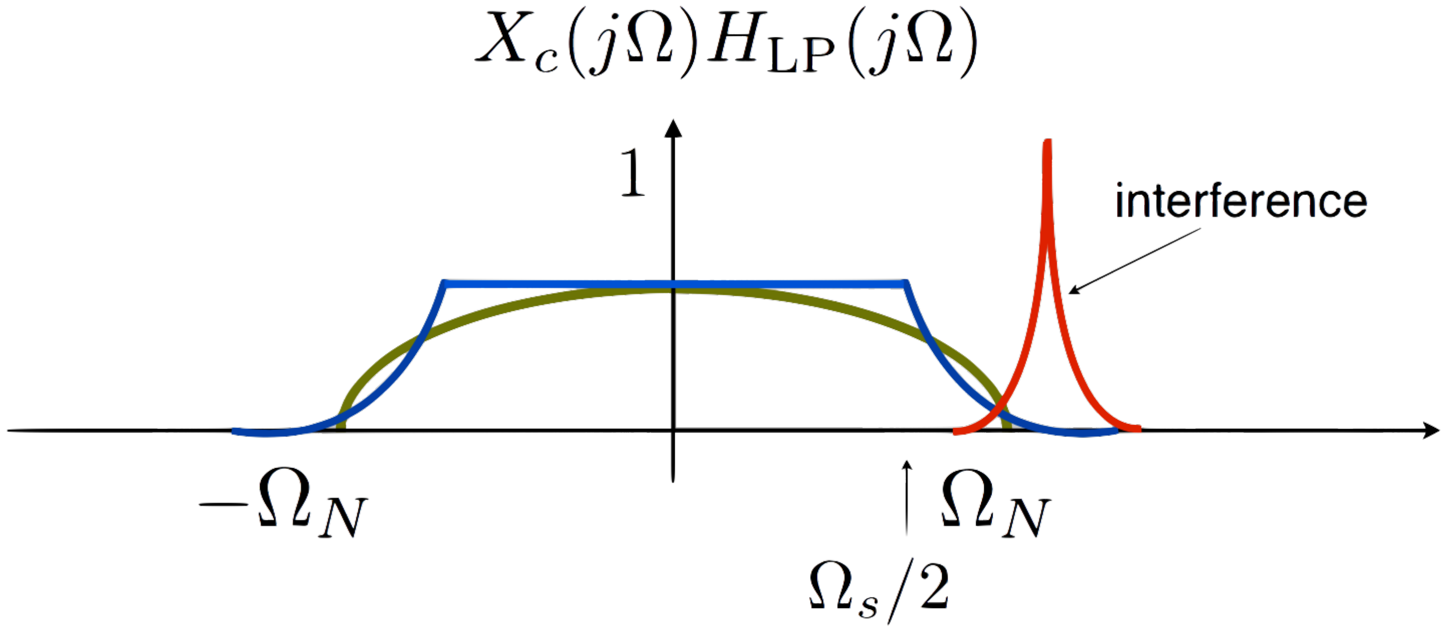
0.8
1
Time (s)
(A)
0.05
0
Squared magnitude
0
10
20
30
40
50
Frequency (Hz)
(B)
1.2
0.6
0
–0.6
Amplitude
0
20
40
60
80
100
Sample (fs = 100 Hz)
(C)

ECG Example

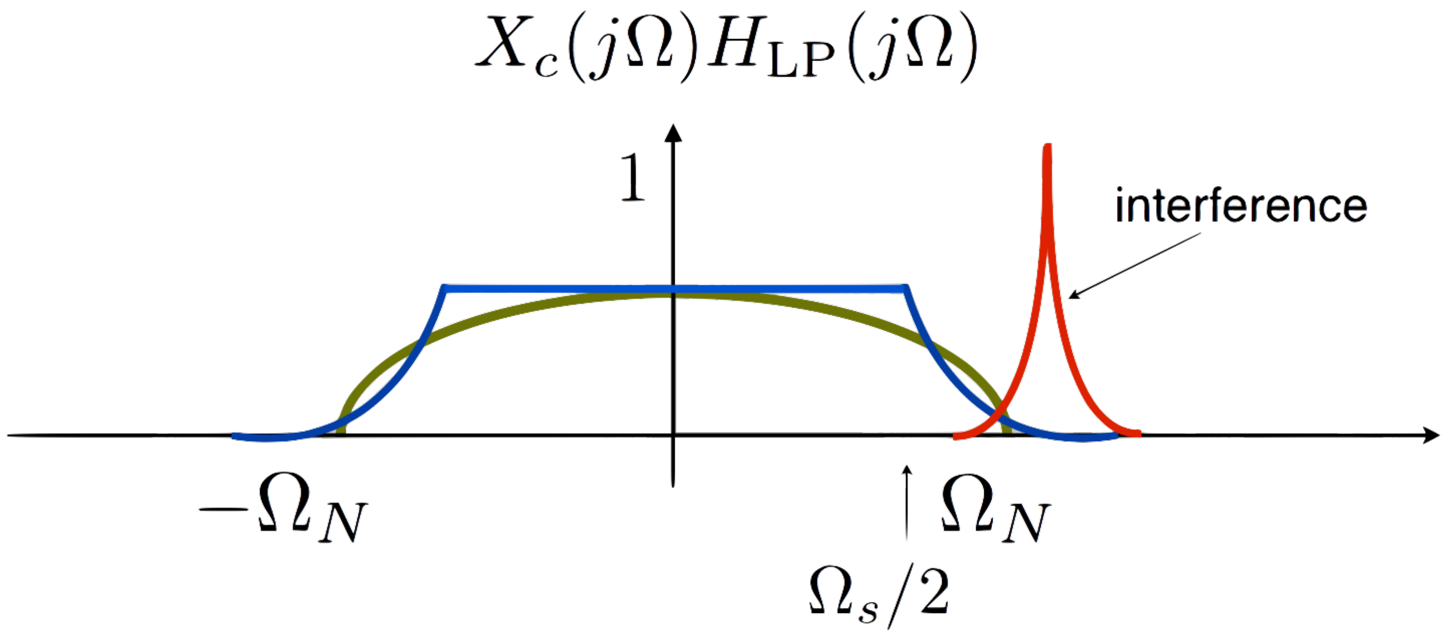


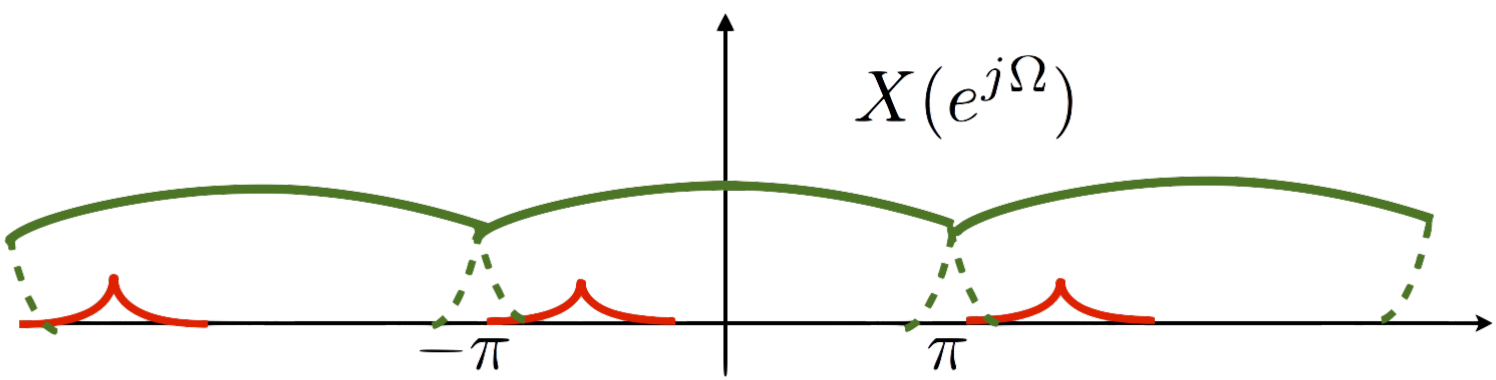
1.2
0.6
0
−0.6
Amplitude
0
0.2
0.4
0.6
0.8
1
Time (s)
(A)
0.05
0
Squared magnitude
0
10
20
30
40
50
Frequency (Hz)
(B)
1.2
0.6
0
−0.6
Amplitude
0
20
40
60
80
100
Sample (fs = 100 Hz)
(C)
1.2
0.6
0
−0.6
Amplitude
0
20
40
60
80
100
Sample (fs = 100 Hz)
(D)

Non-Ideal Anti-Aliasing Filter

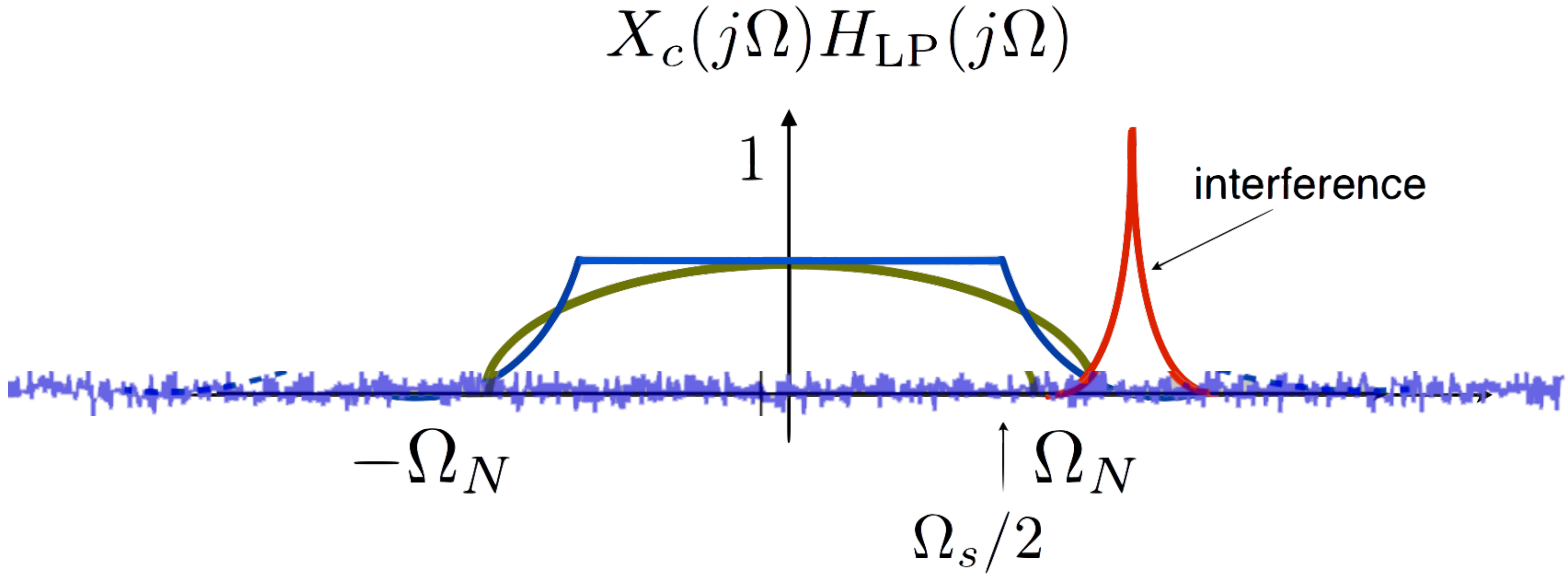


Non-Ideal Anti-Aliasing Filter



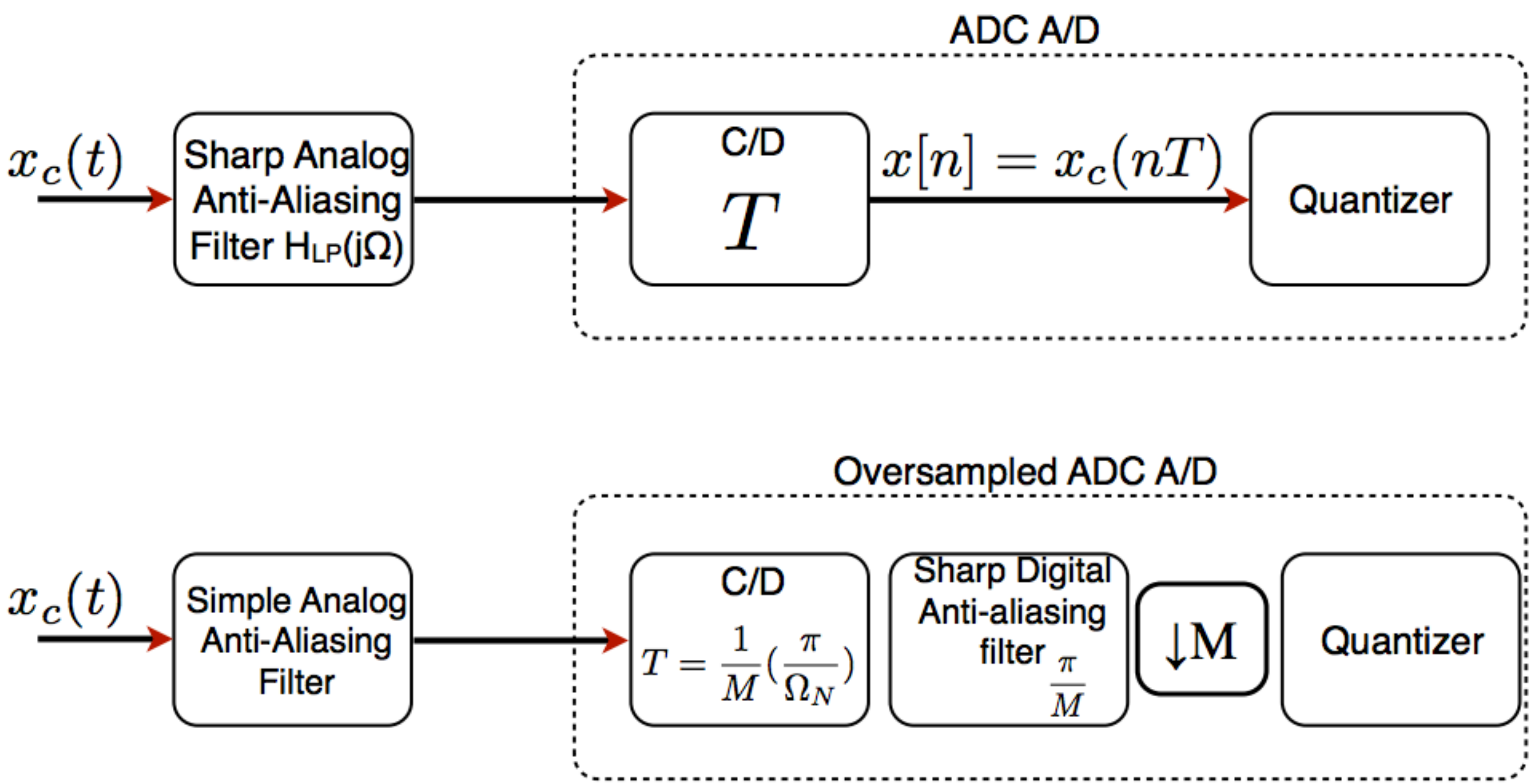


Non-Ideal Anti-Aliasing Filter

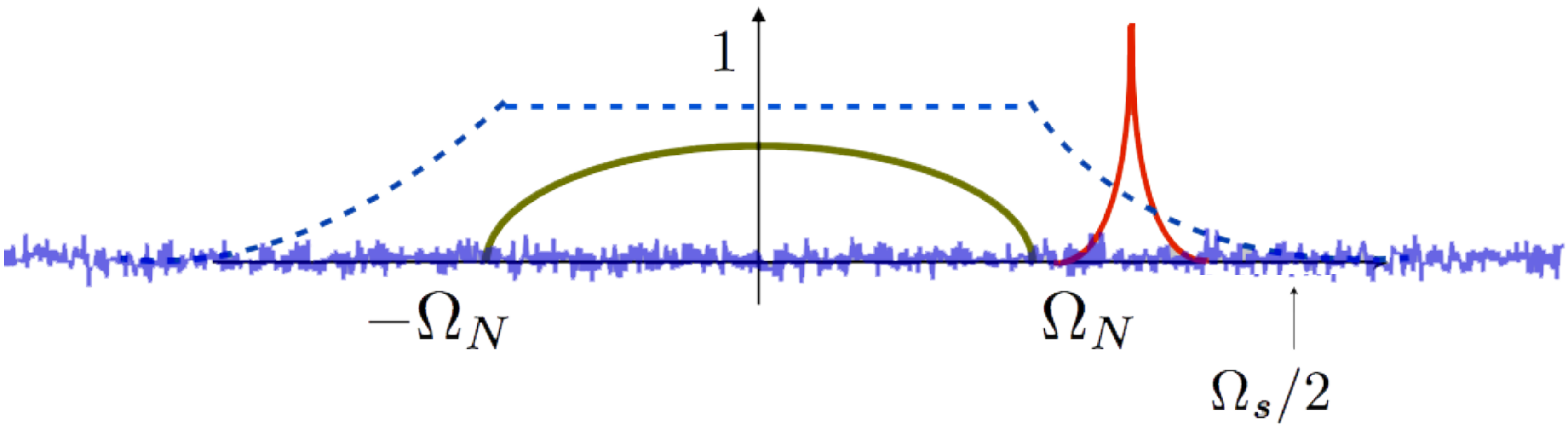


- Problem: Hard to implement sharp analog filter
- Consequence: Crop part of the signal and suffer from noise and interference

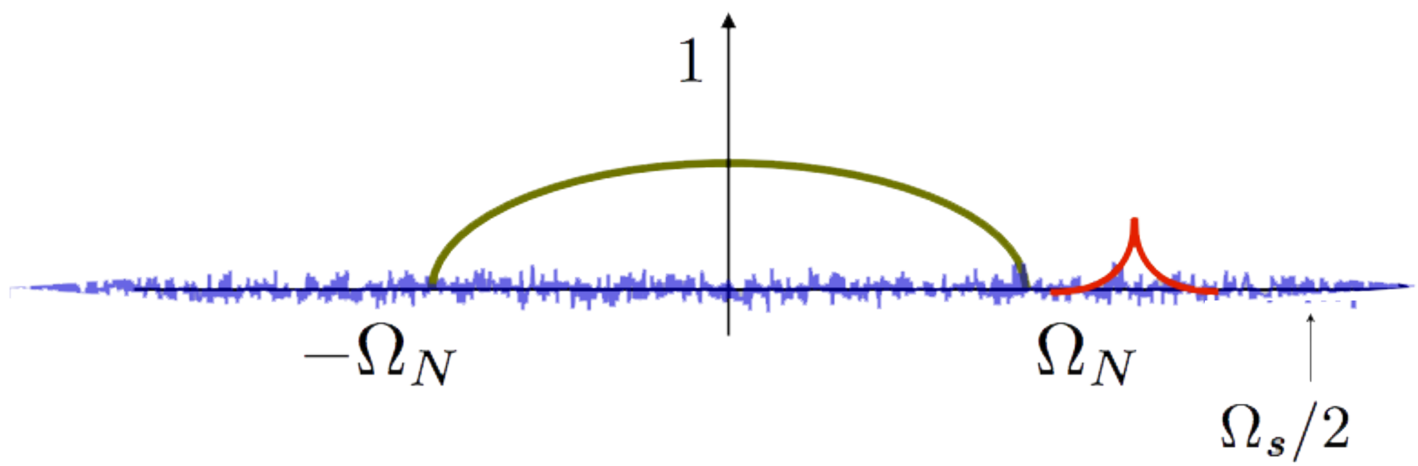
Oversampled ADC



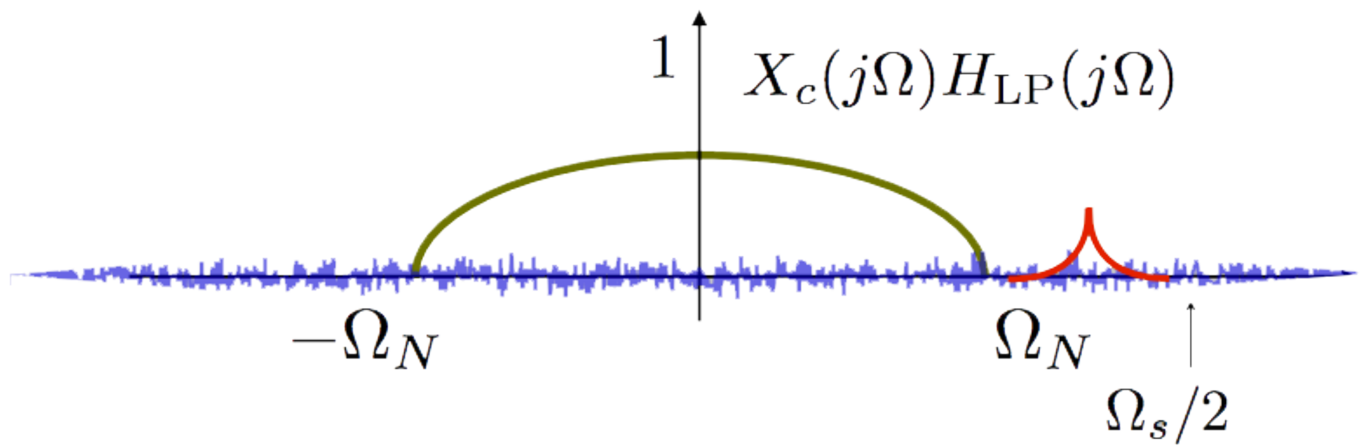
Oversampled ADC – Simple filter



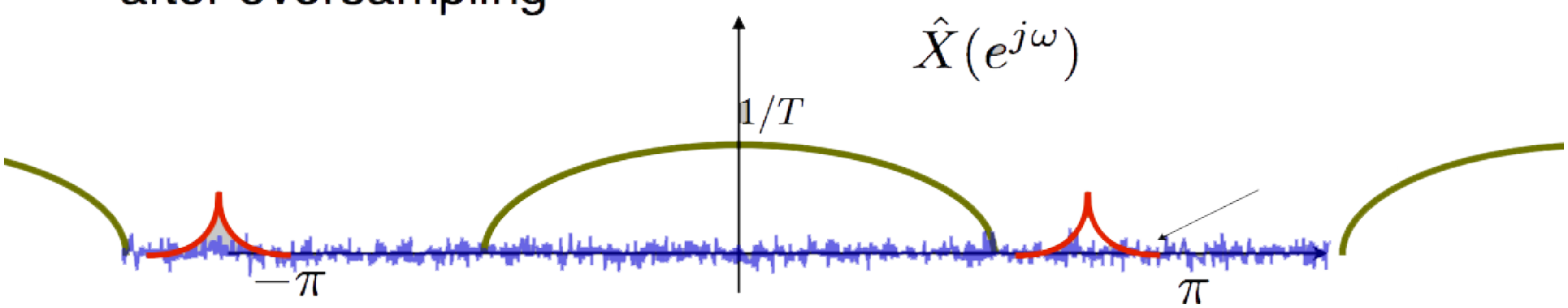
$X_c(j\Omega)H_{\mathrm{LP}}(j\Omega)$



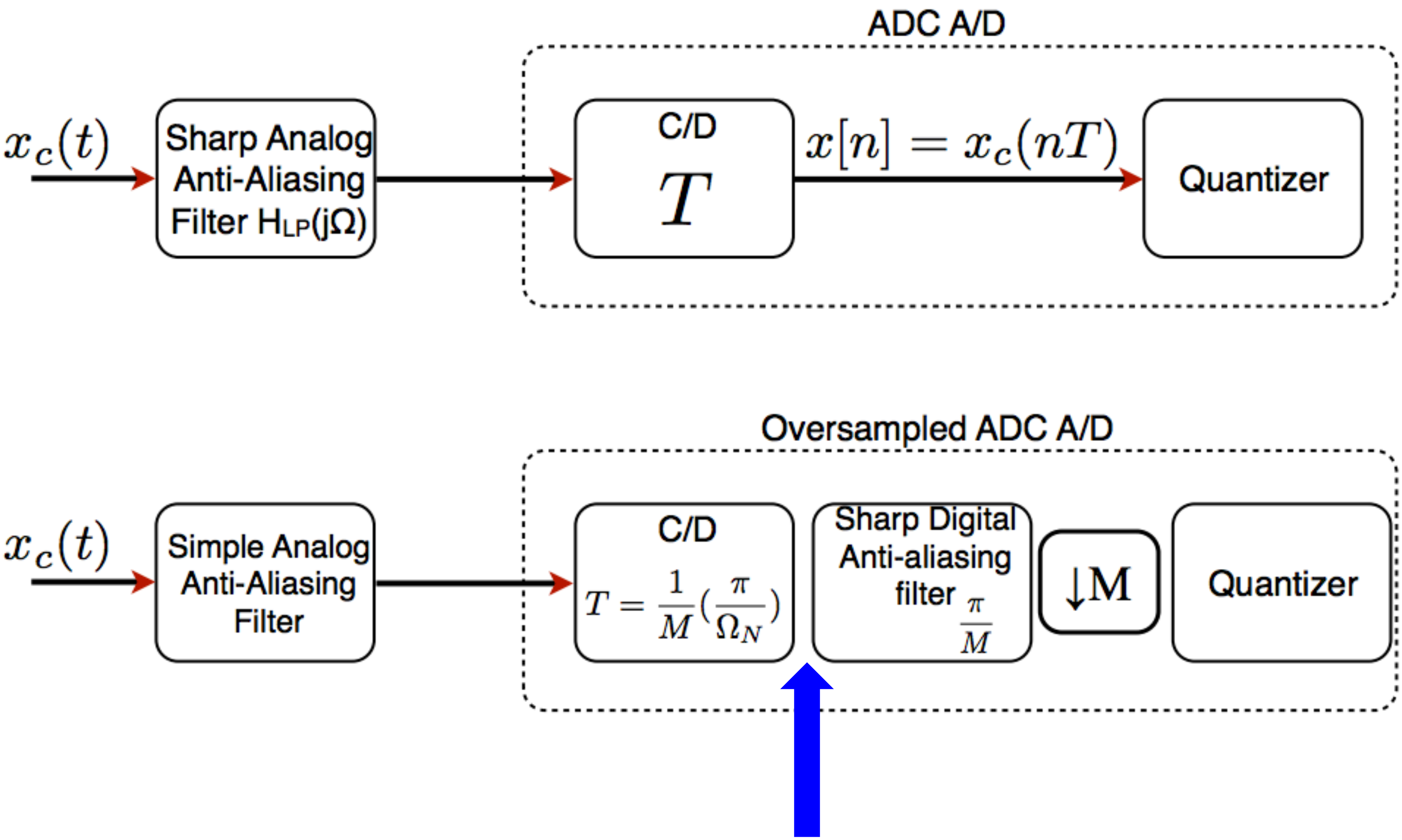
Oversampled ADC – M=2



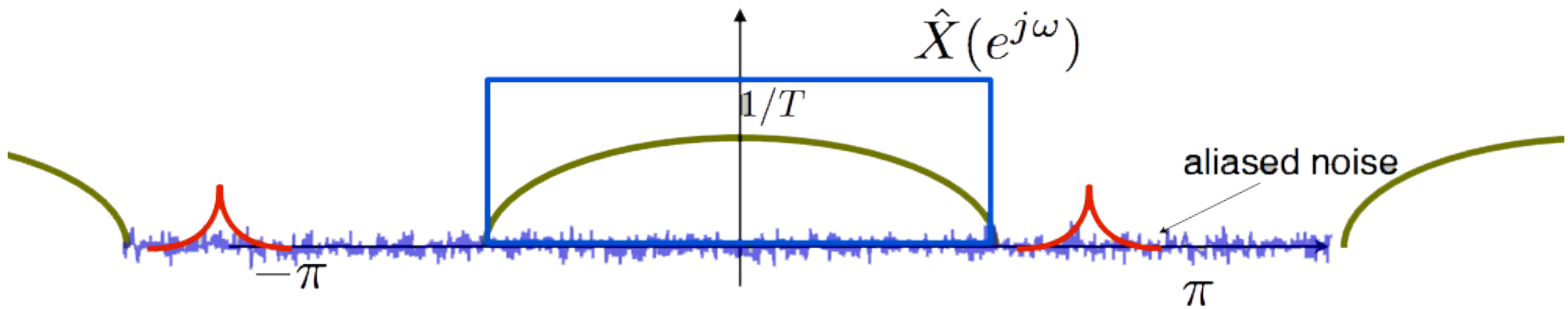
after oversampling



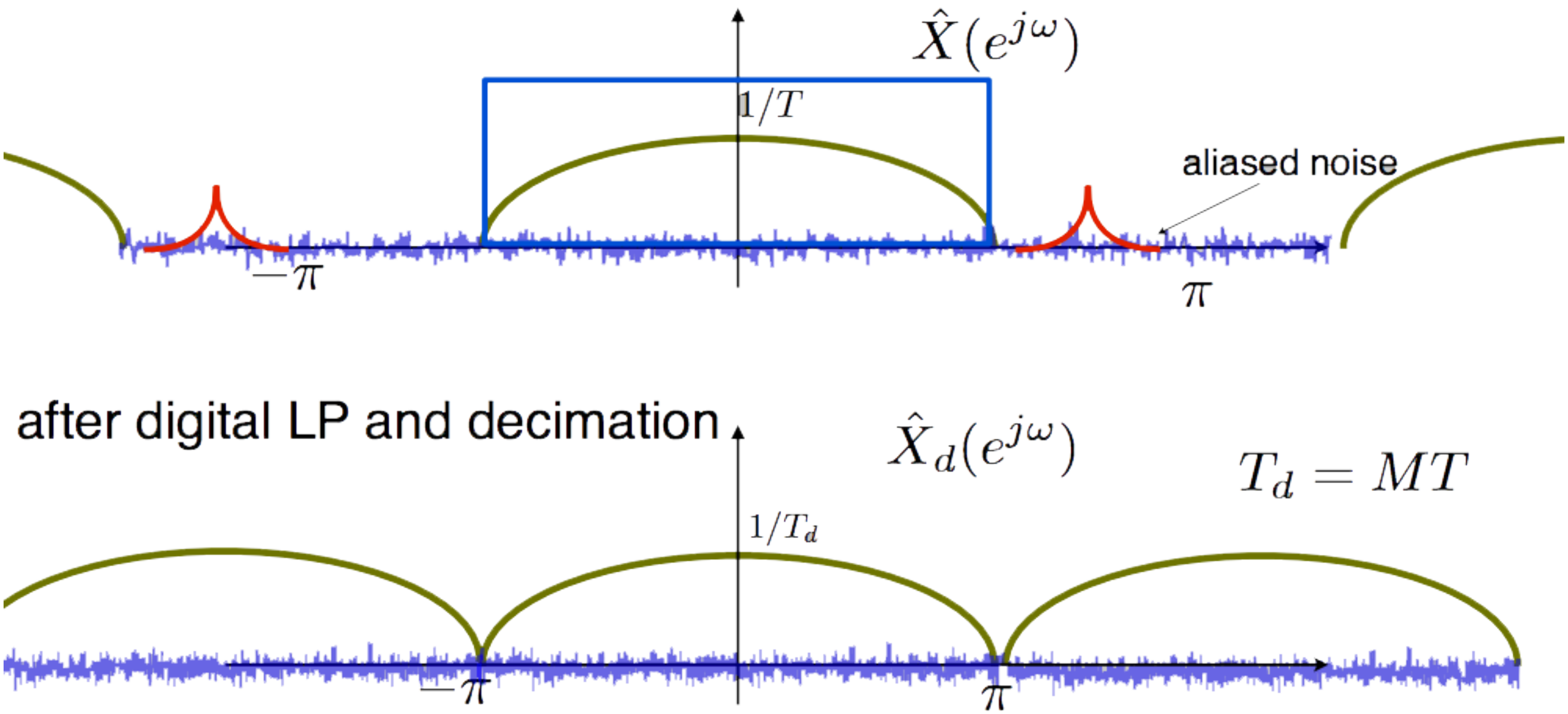
Oversampled ADC



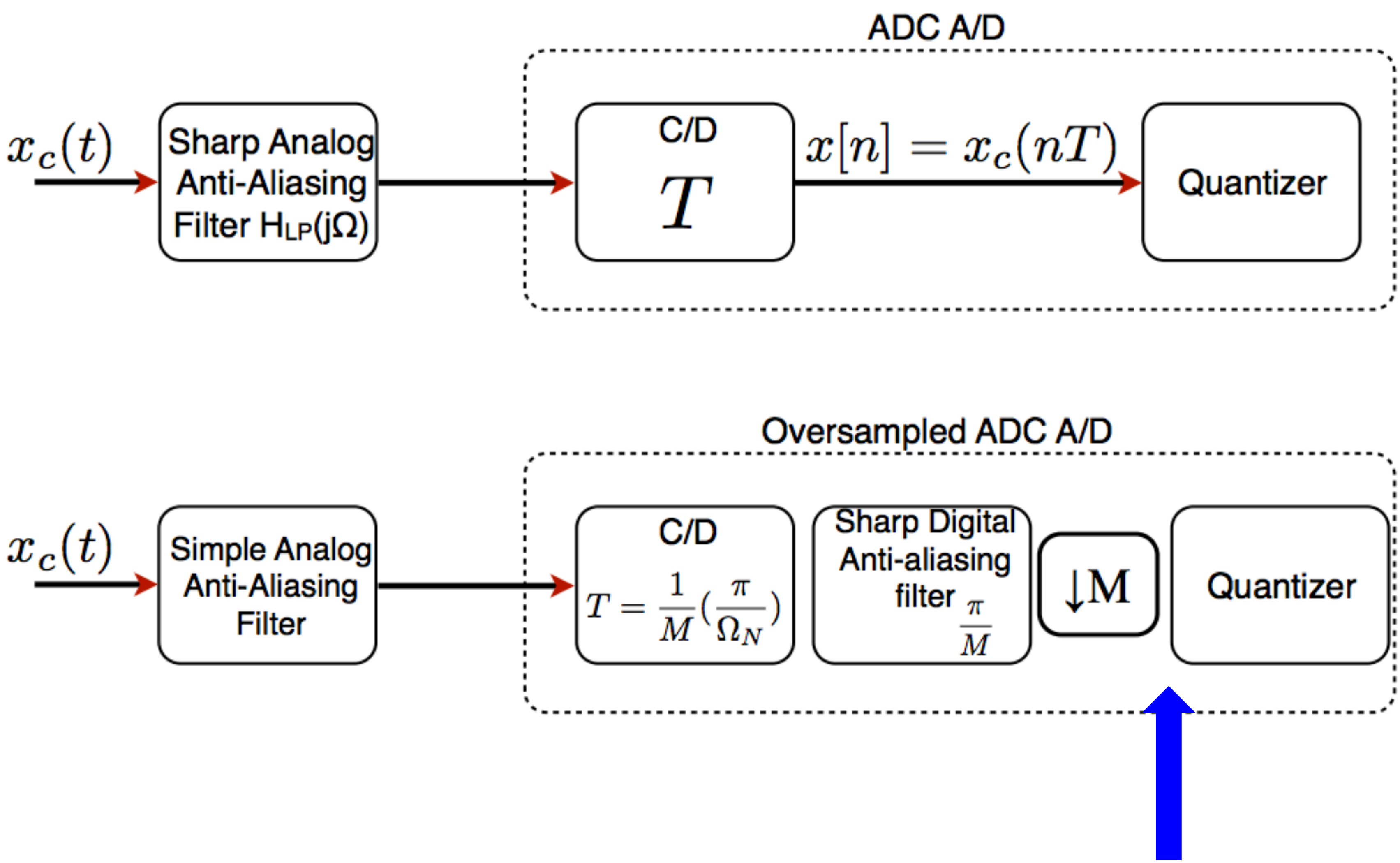
Oversampled ADC – Sharp digital filter/Downsample



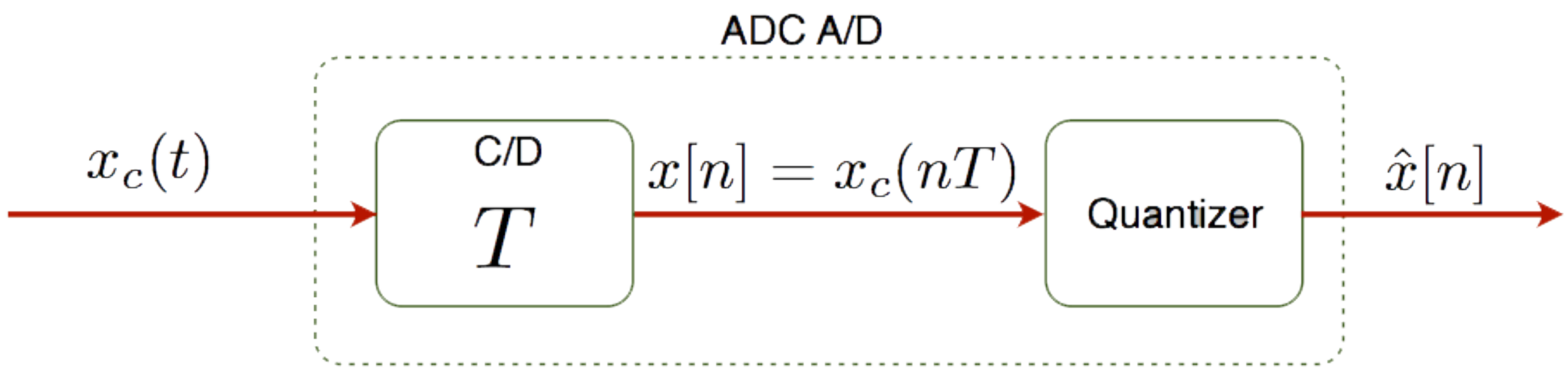
Oversampled ADC – Sharp digital filter/Downsample

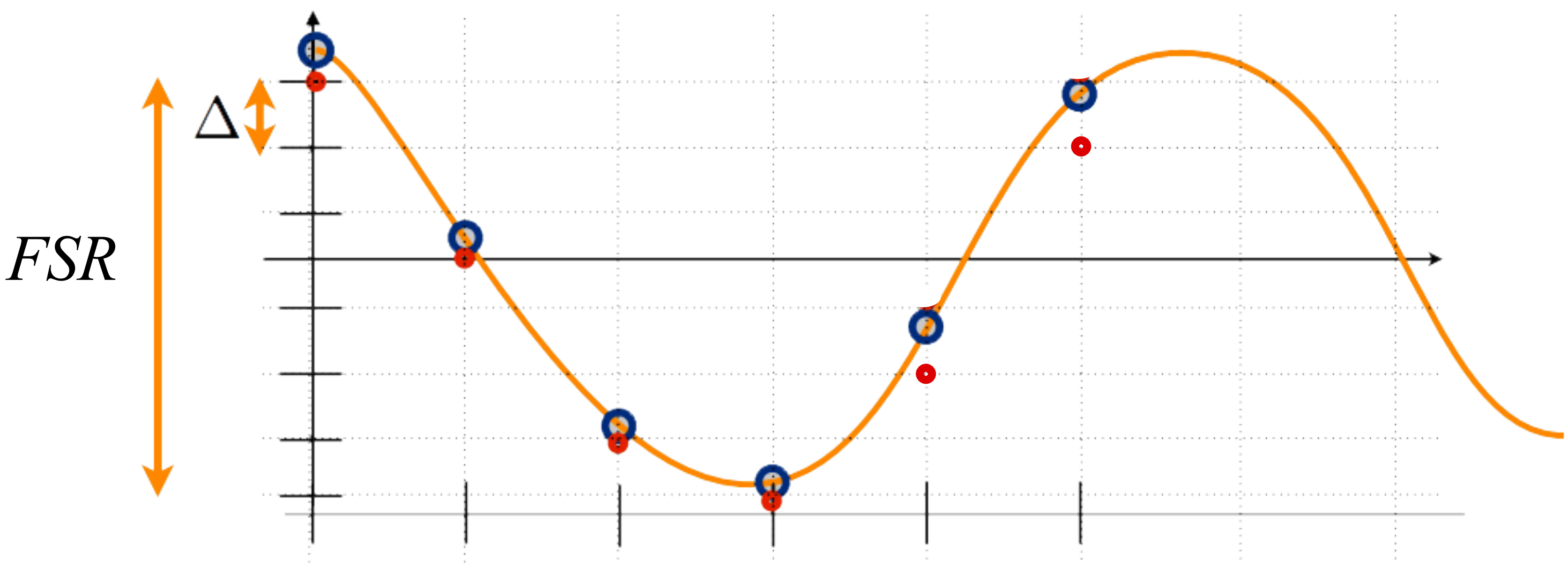


Oversampled ADC



Sampling and Quantization

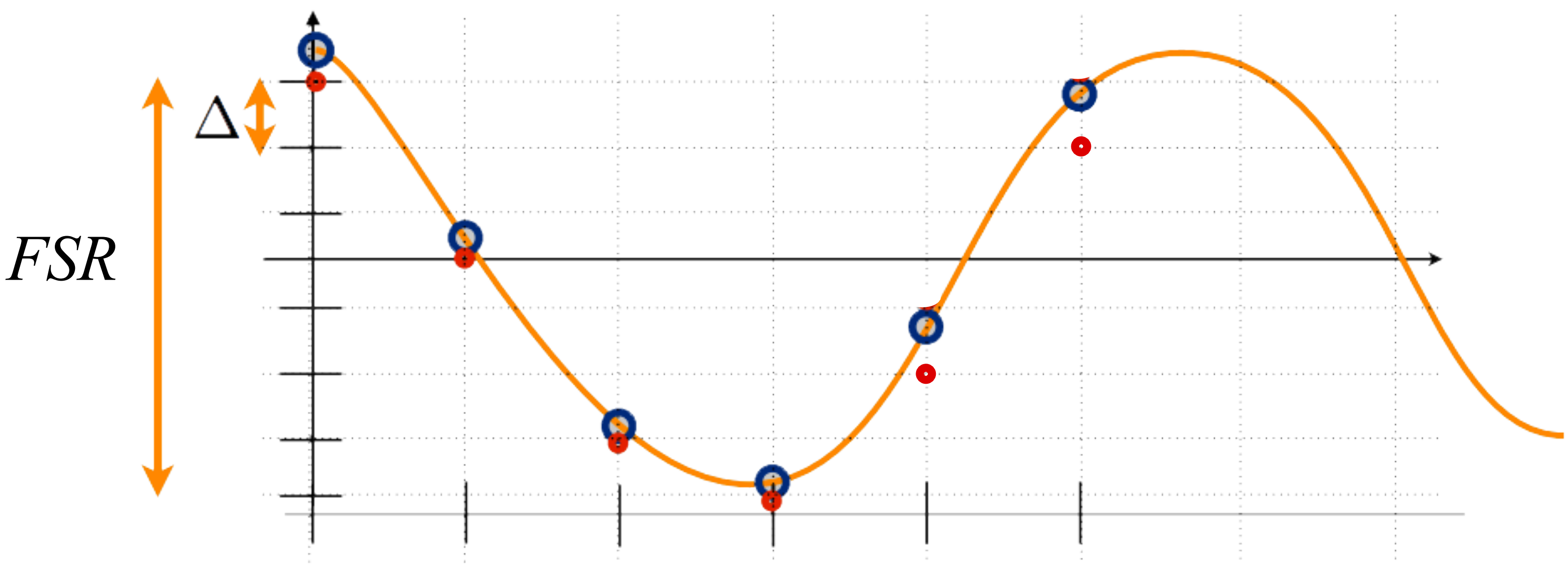




Sampling and Quantization

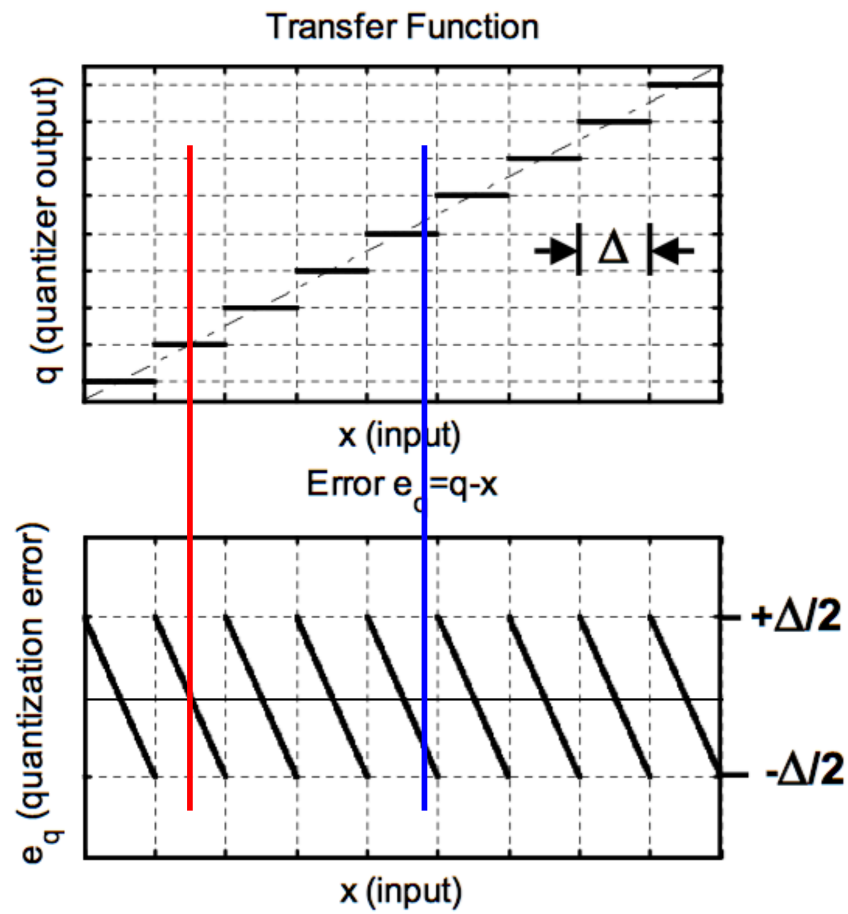
- For an input signal with V_{pp}=FSR with B bits

$$\Delta = \frac{FSR}{2^B}$$



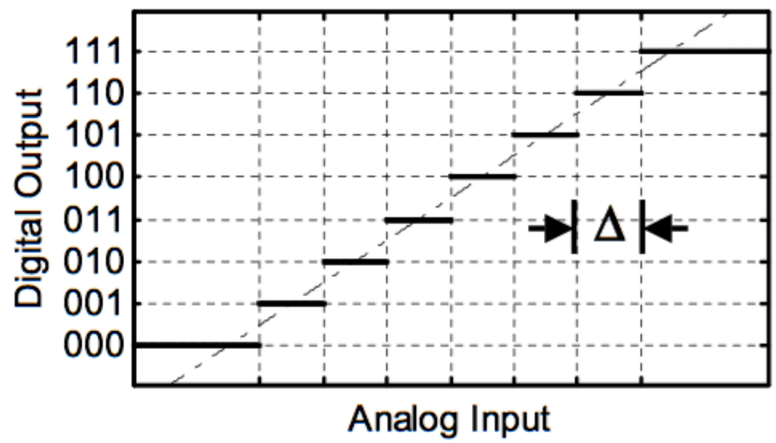
Ideal Quantizer

- Quantization step Δ
- Quantization error has sawtooth shape
  - Bounded by –Δ/2, +Δ/2
- Ideally infinite input range and infinite number of quantization levels

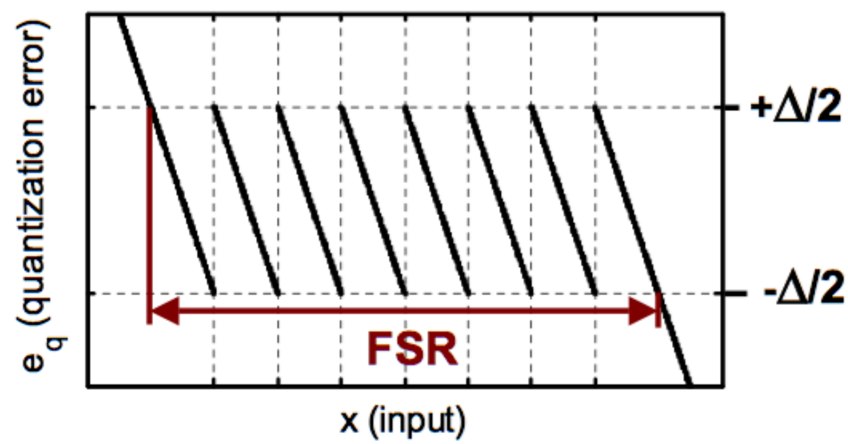


Ideal B-bit Quantizer

- Practical quantizers have a limited input range and a finite set of output codes
- E.g. a 3-bit quantizer can map onto $2^3=8$ distinct output codes



- Quantization error grows out of bounds beyond code boundaries
- We define the full scale range (FSR) as the maximum input range that satisfies $|e_q| \leq \Delta/2$
  - Implies that $FSR = 2^B \cdot \Delta$

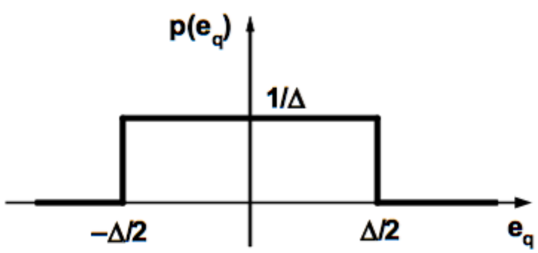


Effect of Quantization Error on Signal

- Quantization error is a deterministic function of the signal
  - Consequently, the effect of quantization strongly depends on the signal itself
- Unless, we consider fairly trivial signals, a deterministic analysis is usually impractical
  - More common to look at errors from a statistical perspective
  - "Quantization noise”

Quantization Error Statistics

- Crude assumption: $e_q(x)$ has uniform probability density
- This approximation holds reasonably well in practice when
  - Signal spans large number of quantization steps
  - Signal is "sufficiently active"
  - Quantizer does not overload

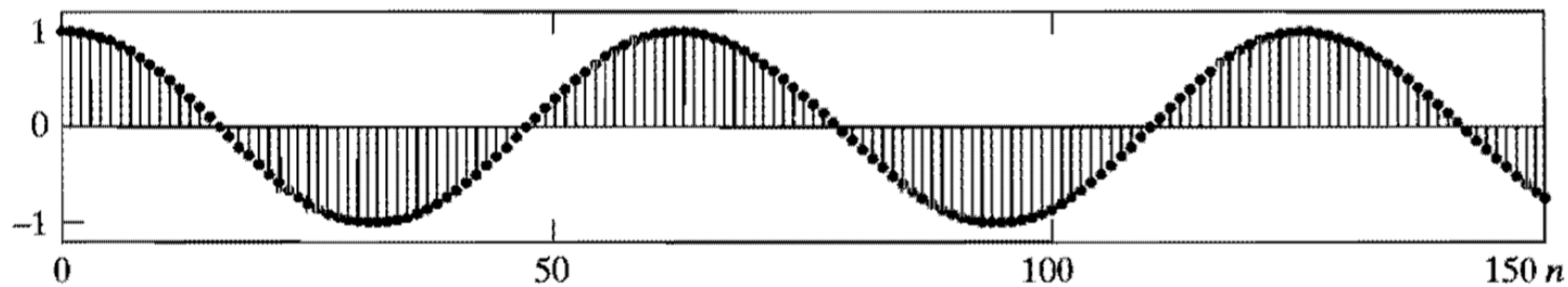


Mean $\bar{e} = \int_{-\Delta/2}^{+\Delta/2} \frac{e}{\Delta} de = 0$

Variance $\overline{e^2} = \int_{-\Delta/2}^{+\Delta/2} \frac{e^2}{\Delta} de = \frac{\Delta^2}{12}$

Quantization Noise

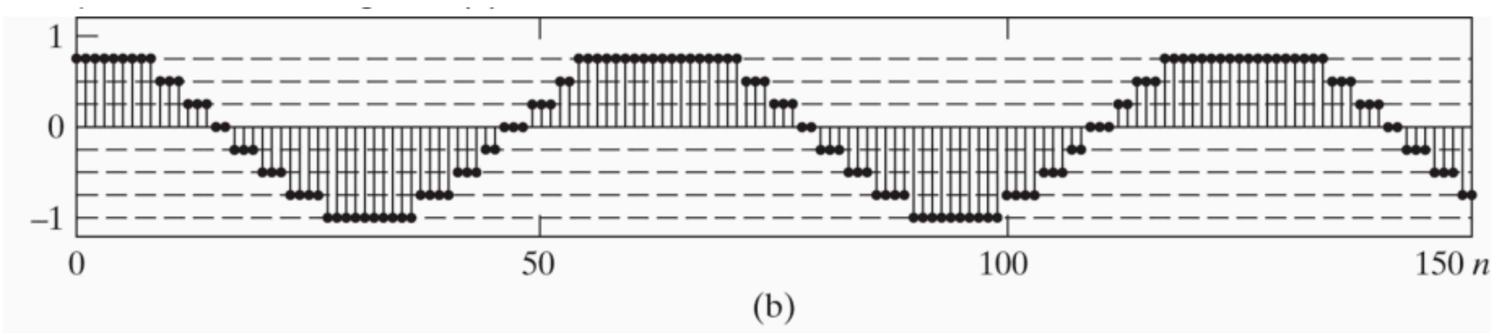
- **Figure 4.57** Example of quantization noise. (a) Unquantized samples of the signal x[n] = 0.99cos(n/10).



(a)

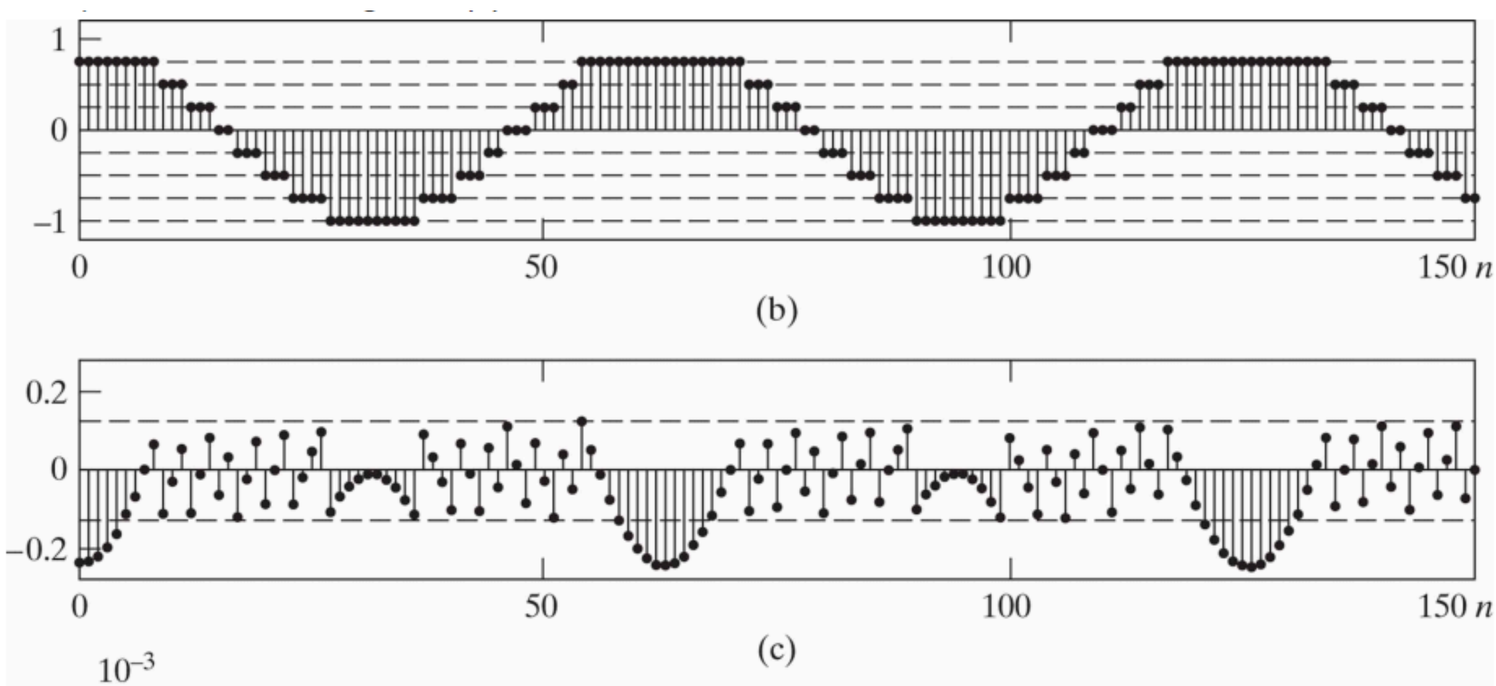
Quantization Noise

- **Figure 4.57**(continued) (b) Quantized samples of the cosine waveform in part (a) with a 3-bit quantizer.



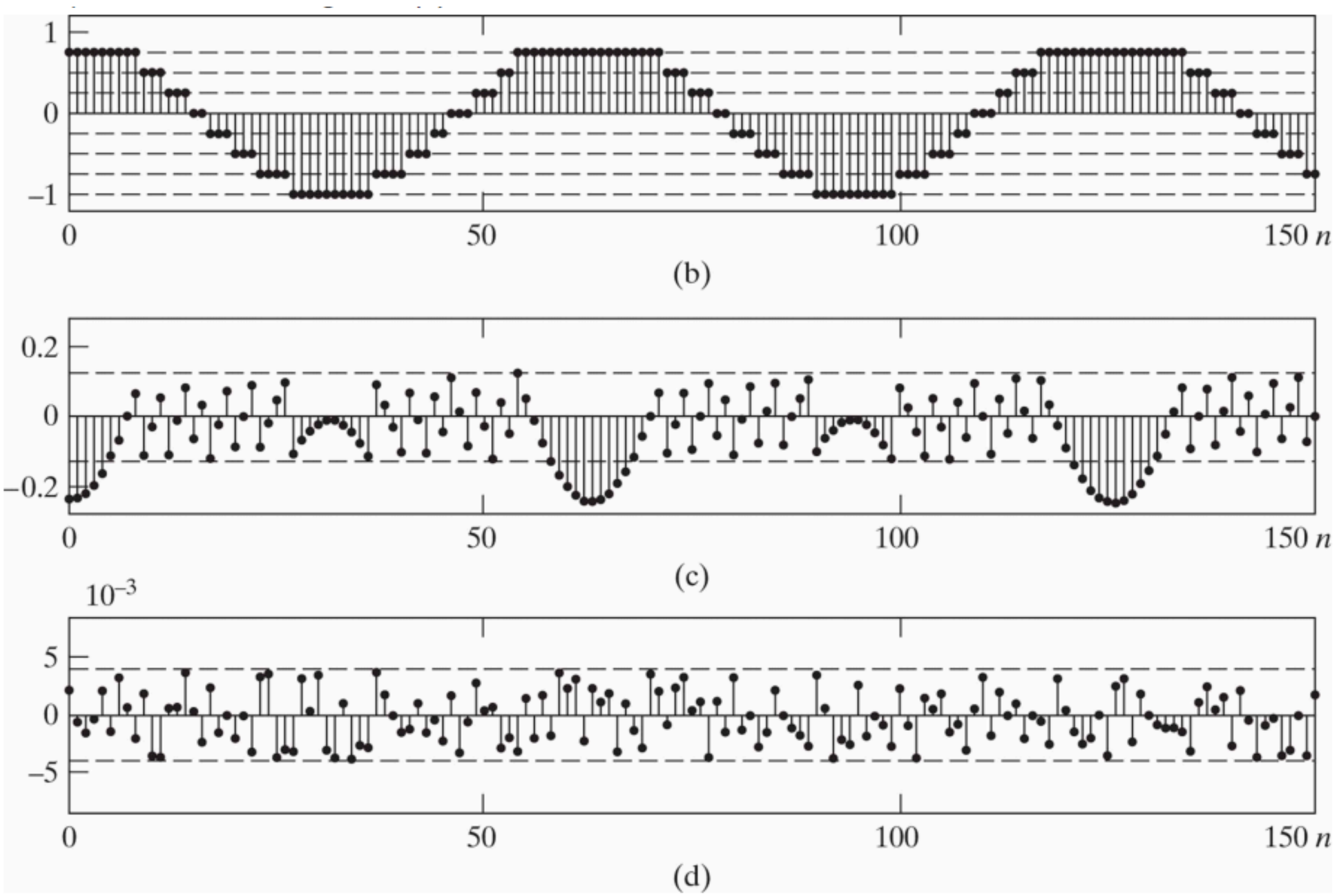
Quantization Noise

- **Figure 4.57**(continued) (b) Quantized samples of the cosine waveform in part (a) with a 3-bit quantizer. (c) Quantization error sequence for 3-bit quantization of the signal in (a).



Quantization Noise

- **Figure 4.57**(continued) (b) Quantized samples of the cosine waveform in part (a) with a 3-bit quantizer. (c) Quantization error sequence for 3-bit quantization of the signal in (a). (d) Quantization error sequence for 8-bit quantization of the signal in (a).



Signal-to-Quantization-Noise Ratio

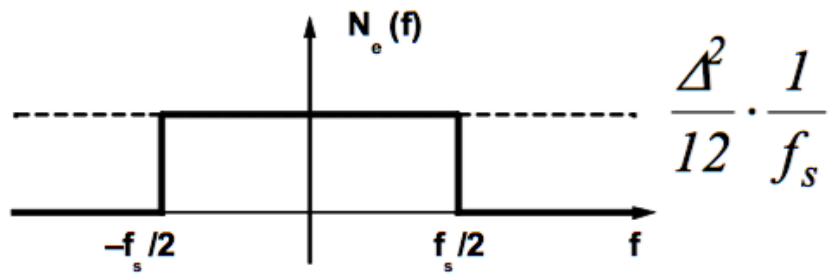
- Assuming full-scale sinusoidal input, we have

$$SNR_Q = 6.02B + 1.76 \text{ dB}$$

B (Number of Bits)	SQNR
8	50dB
12	74dB
16	98dB
20	122dB

Quantization Noise Spectrum

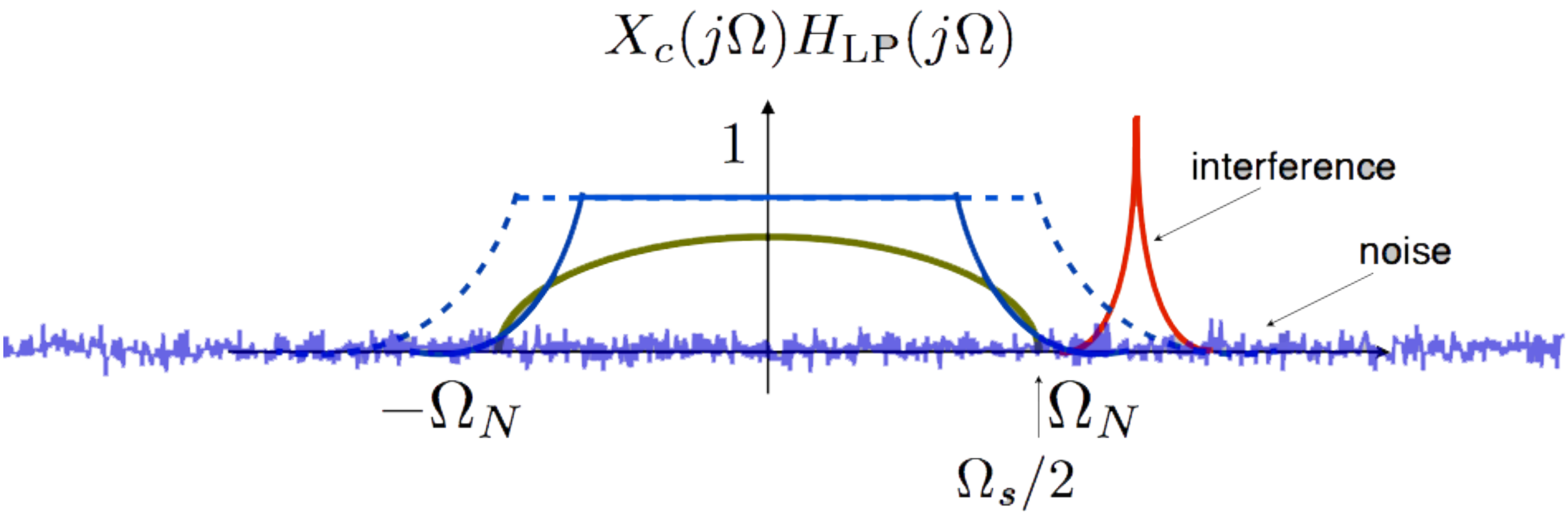
- If the quantization error is "sufficiently random", it also follows that the noise power is uniformly distributed in frequency



- References

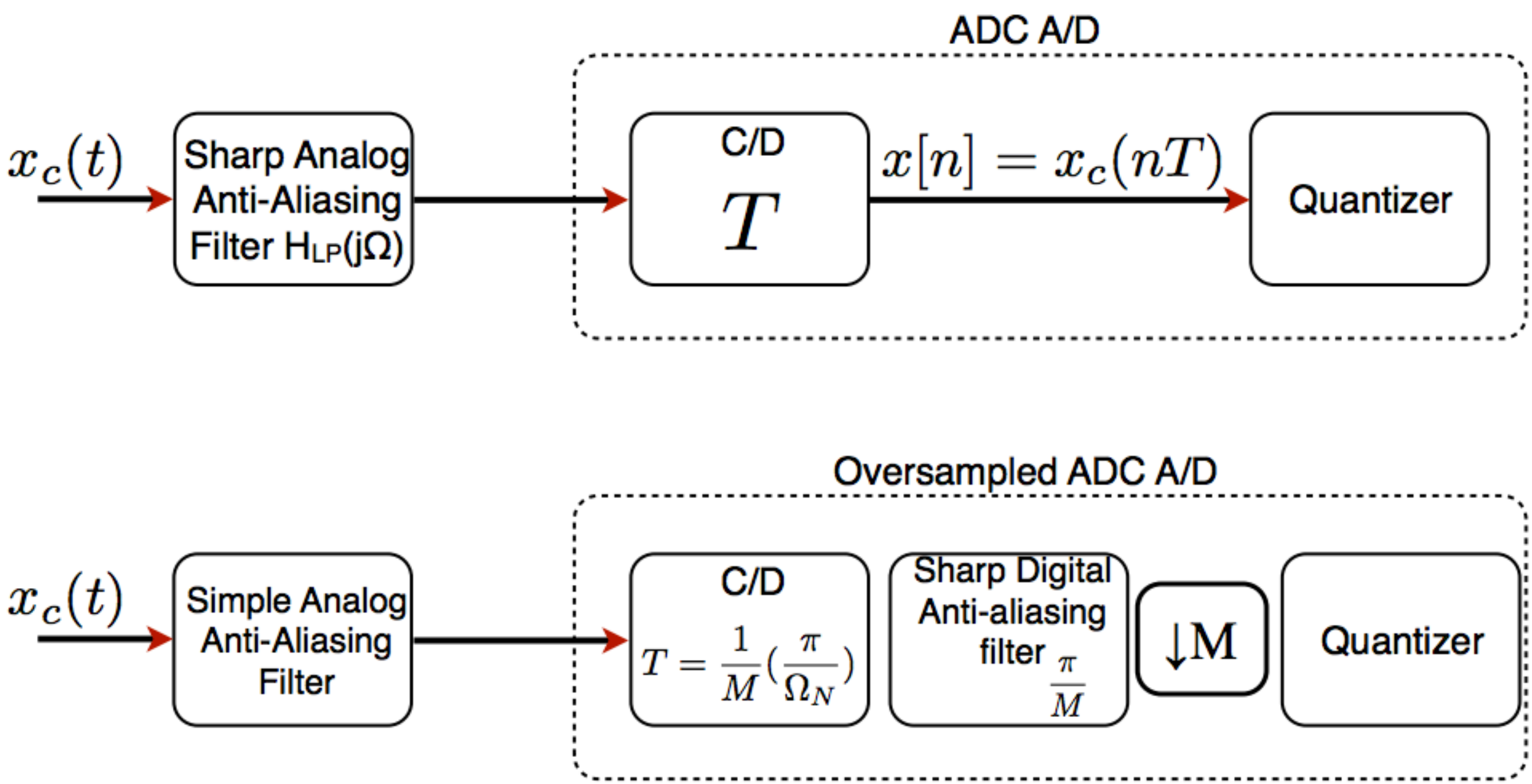
  - W. R. Bennett, "Spectra of quantized signals," Bell Syst. Tech. J., pp. 446-72, July 1988.
  - B. Widrow, "A study of rough amplitude quantization by means of Nyquist sampling theory," IRE Trans. Circuit Theory, vol. CT-3, pp. 266-76, 1956.

Non-Ideal Anti-Aliasing Filter

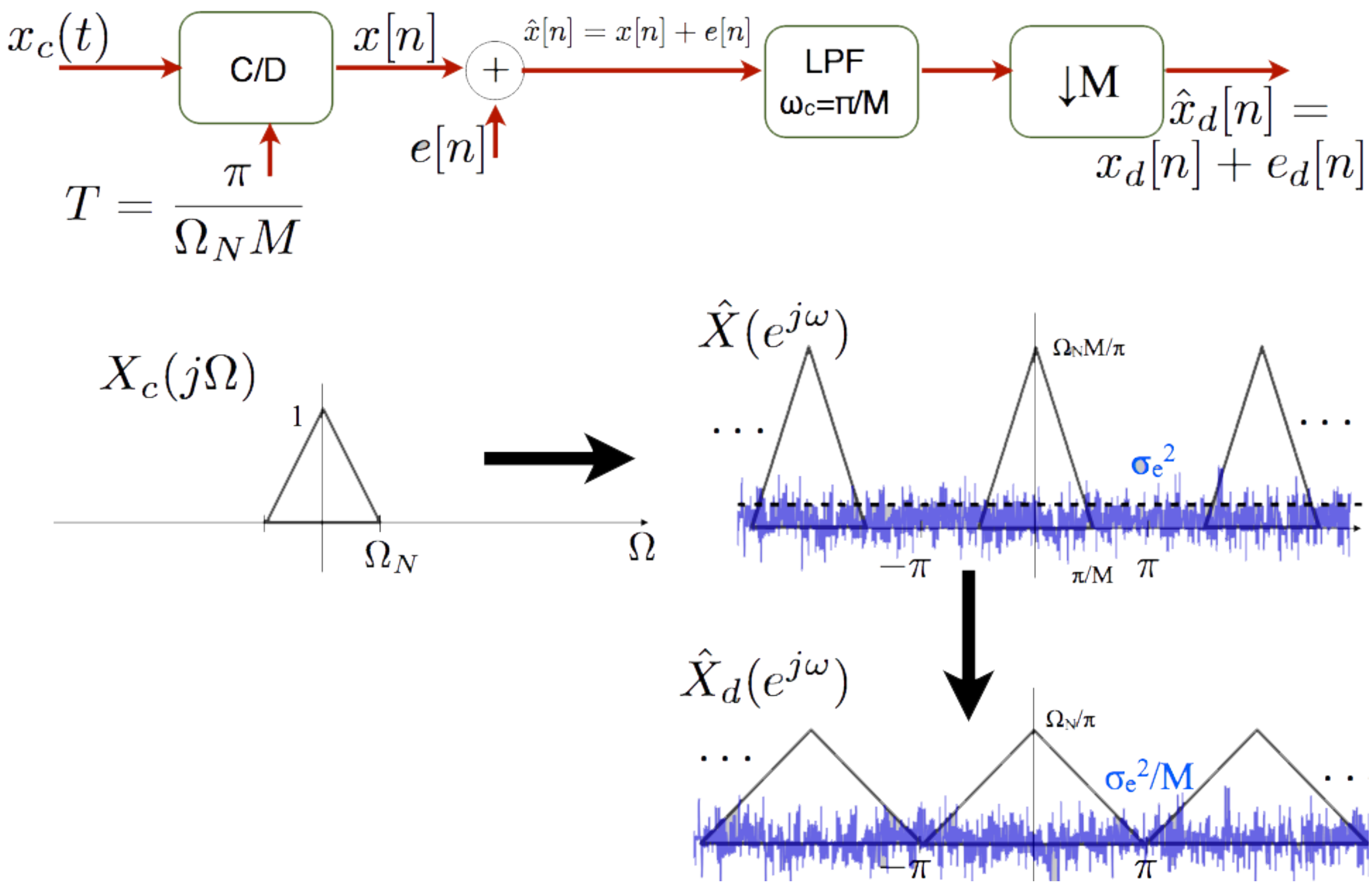


- Problem: Hard to implement sharp analog filter
- Consequence: Crop part of the signal and suffer from noise and interference

Oversampled ADC



Quantization Noise with Oversampling



Big Ideas

- DTFT vs DFT
  - The DFT characterizes the spectral content of the desired signals
    - Fundamental and harmonics
- Quantization noise
  - Limits achievable bit resolution of ADCs
- Oversampling
  - Enables for more accurate signal capture
  - Interference reduction
  - Lowered quantization noise
    - More on this later…

Admin

- Lab next week
  - More PCB population and testing