ESE 3400: Medical Devices Lab

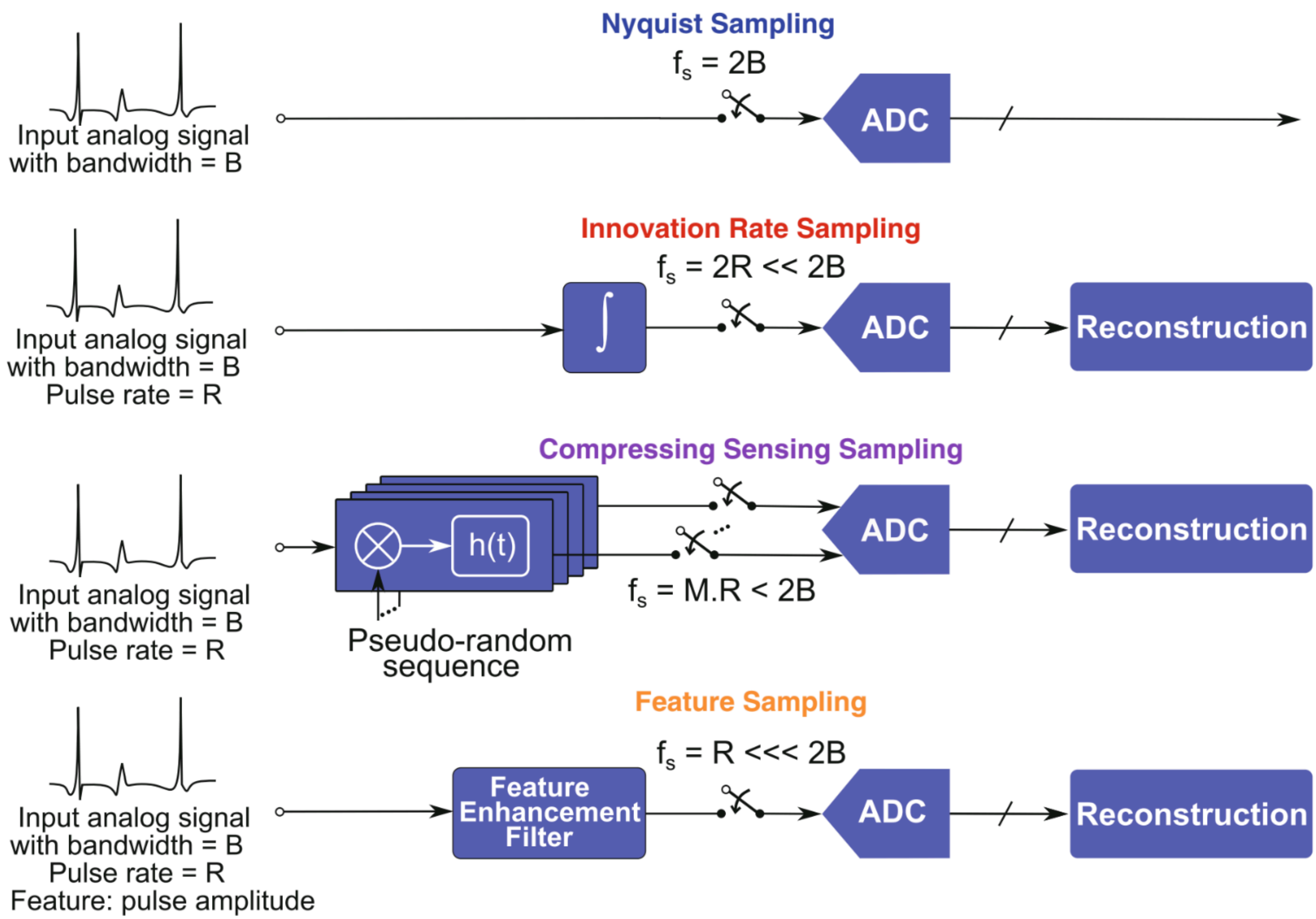
Lec 16: November 9, 2022

Compressive Sampling

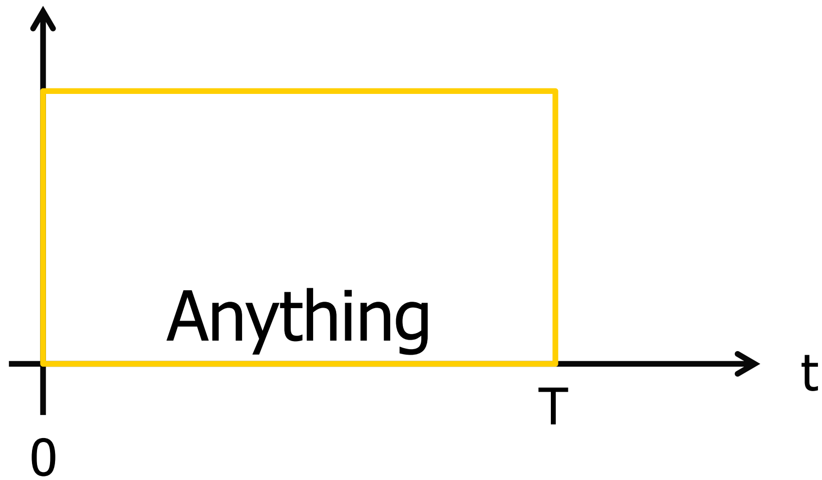
Today

- Compressive Sampling/Sensing

Sampling Architectures

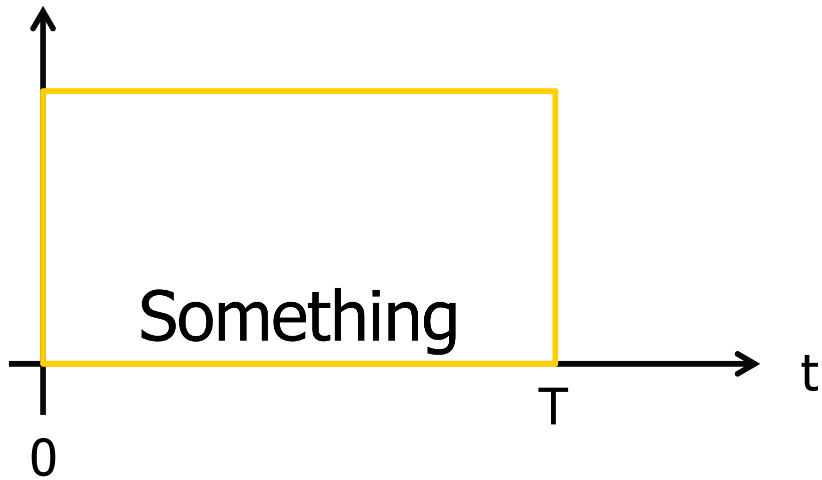


Compressive Sampling



- What is the rate you need to sample at?
  - At least Nyquist

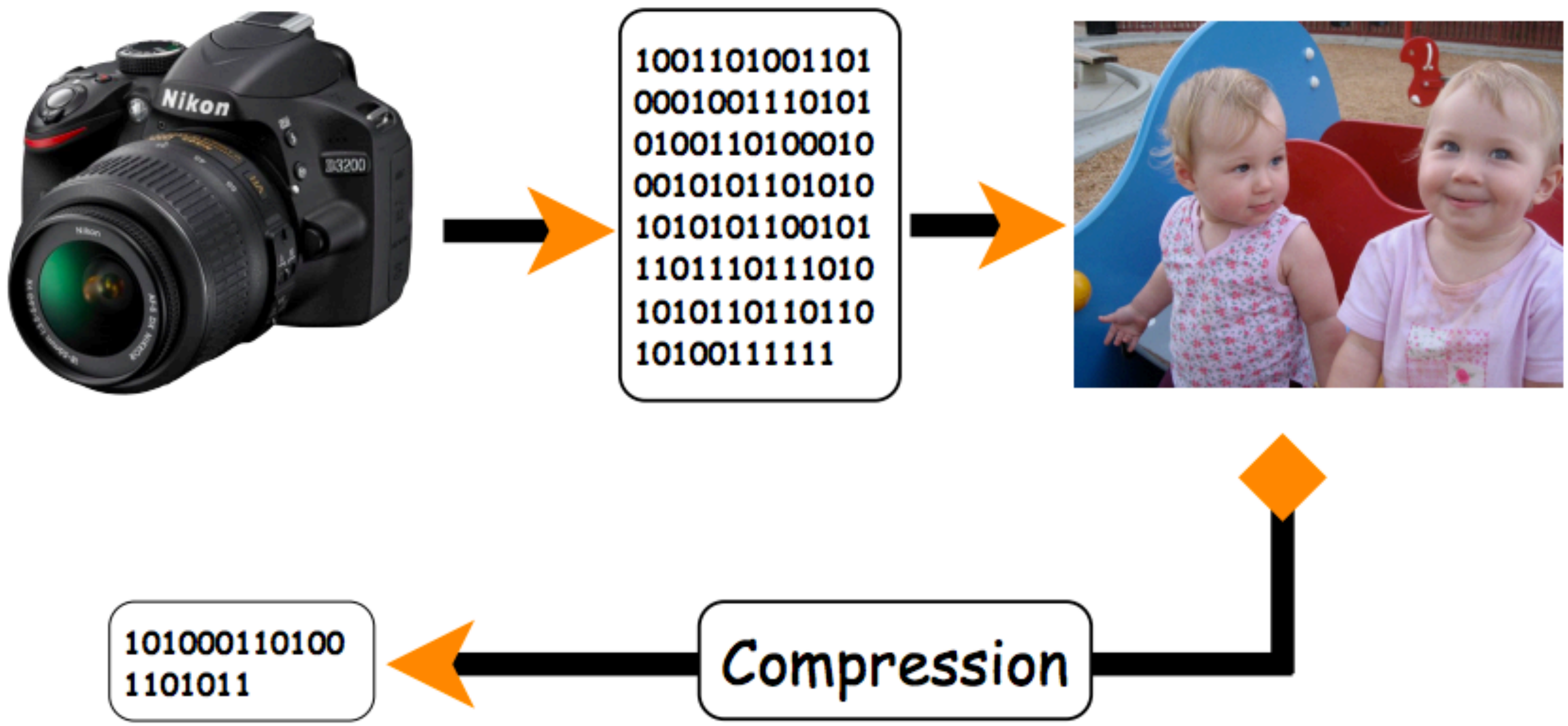
Compressive Sampling



- What is the rate you need to sample at?
  - Maybe less than Nyquist…

First: Compression

- Standard approach
  - First collect, then compress
    - Throw away unnecessary data



First: Compression

- Examples
  - Audio – 10x
    - Raw audio: 44.1kHz, 16bit, stereo = 1378 Kbit/sec
    - MP3: 44.1kHz, 16 bit, stereo = 128 Kbit/sec
  - Images – 22x
    - Raw image (RGB): 24bit/pixel
    - JPEG: 1280x960, normal = 1.09bit/pixel
  - Videos – 75x
    - Raw Video: (480x360)p/frame x 24b/p x 24frames/s + 44.1kHz x 16b x 2 = 98,578 Kbit/s
    - MPEG4: 1300 Kbit/s

First: Compression

- Almost all compression algorithm use transform coding
  - mp3: DCT
  - JPEG: DCT
  - JPEG2000: Wavelet
  - MPEG: DCT & time-difference

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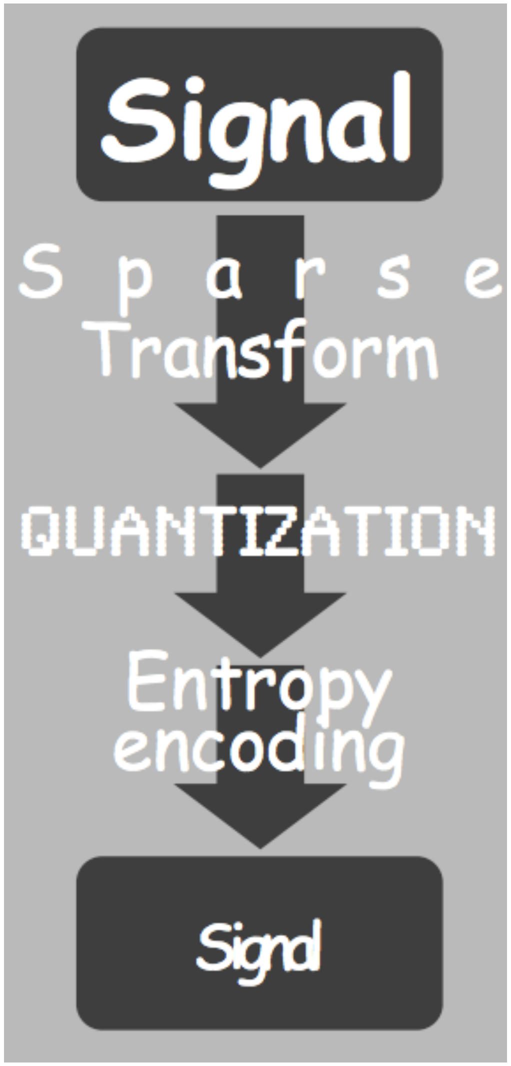
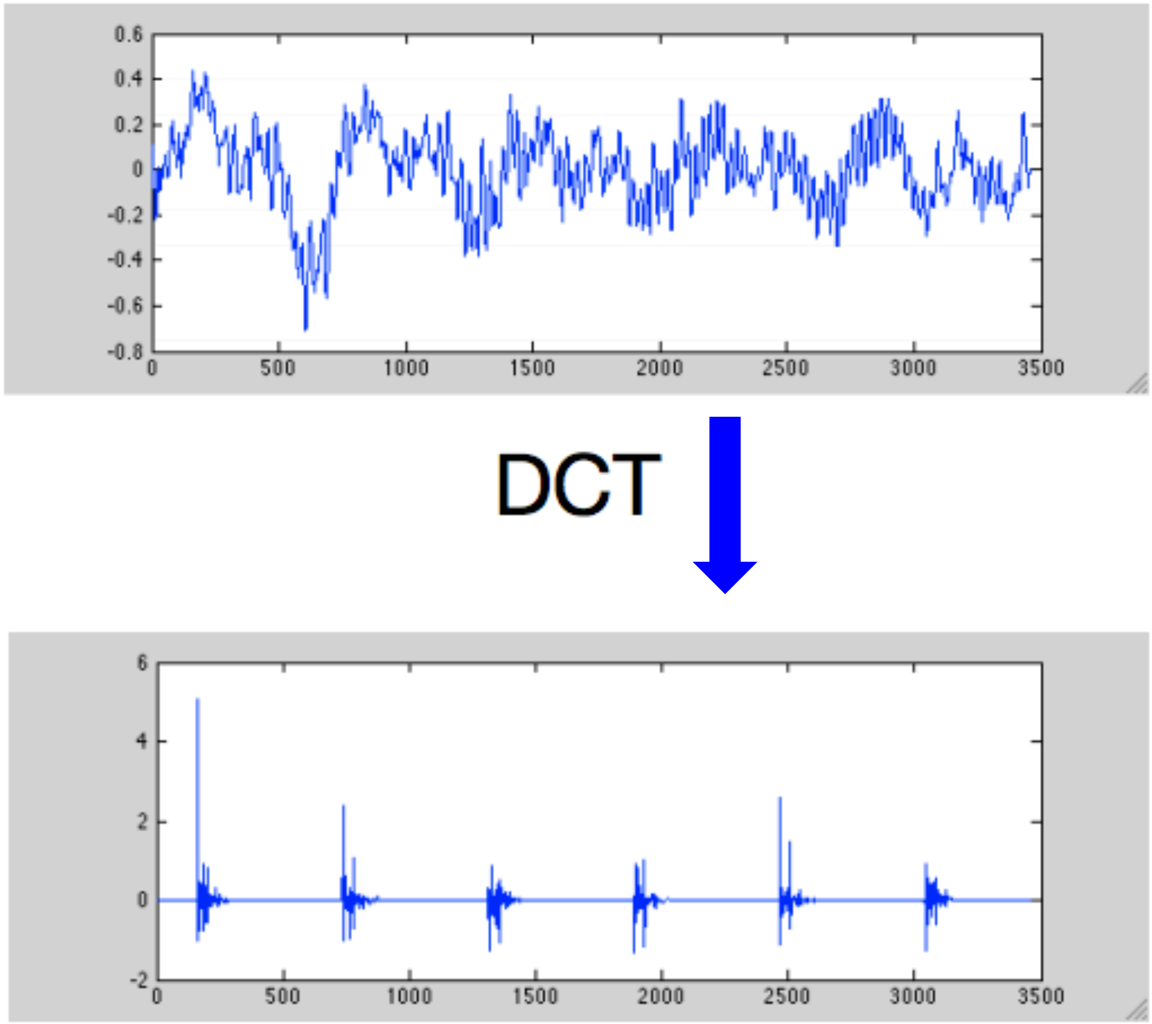


Sparse Transform



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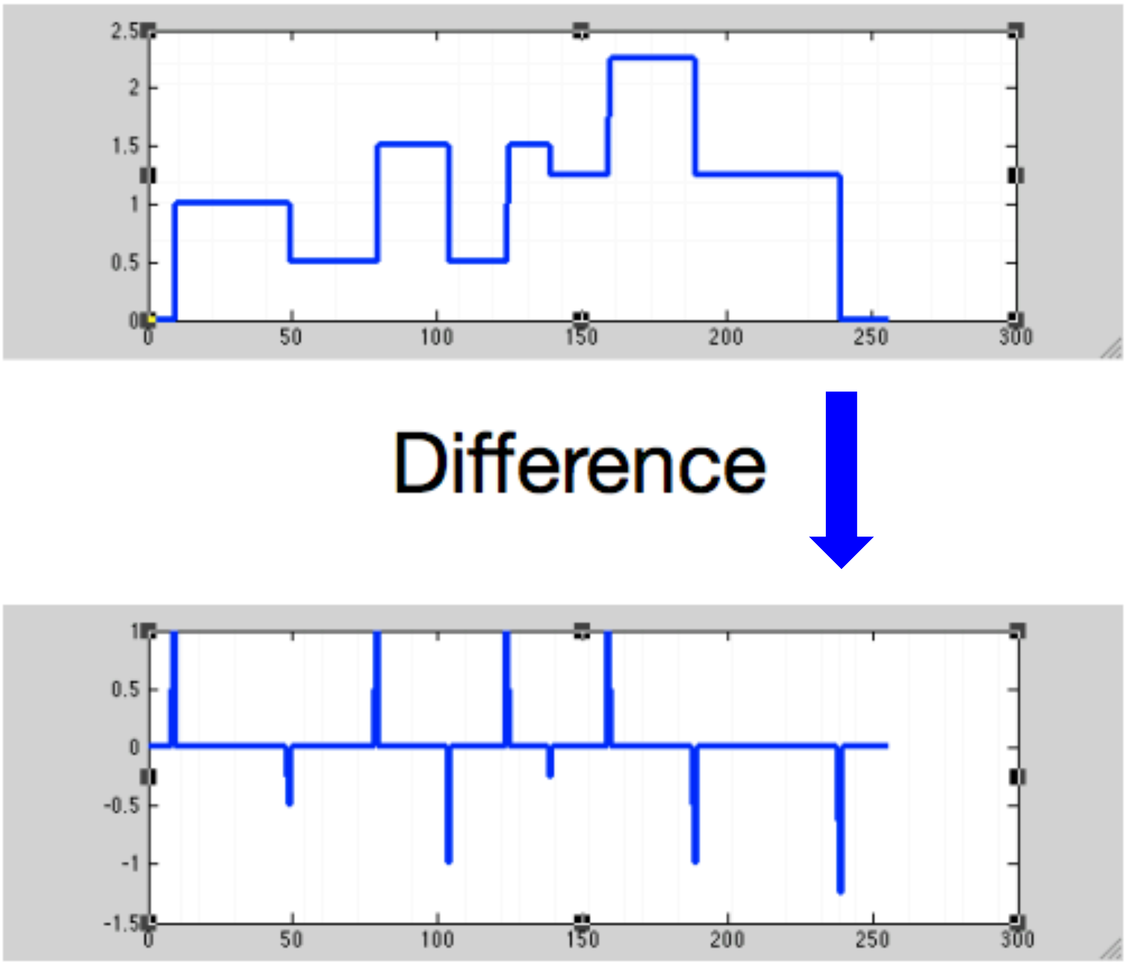

Signal Processing Trends

- Traditional DSP → sample first, ask questions later

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- Explosion in sensor technology/ubiquity has caused two trends:
  - Physical capabilities of hardware are being stressed, increasing speed/resolution becoming expensive
    - gigahertz+ analog-to-digital conversion
    - accelerated MRI
    - industrial imaging
  - Deluge of data
    - camera arrays and networks, multi-view target databases, streaming video…

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- Compressive Sensing → sample smarter, not faster

Compressive Sensing/Sampling

- Standard approach
  - First collect, then compress
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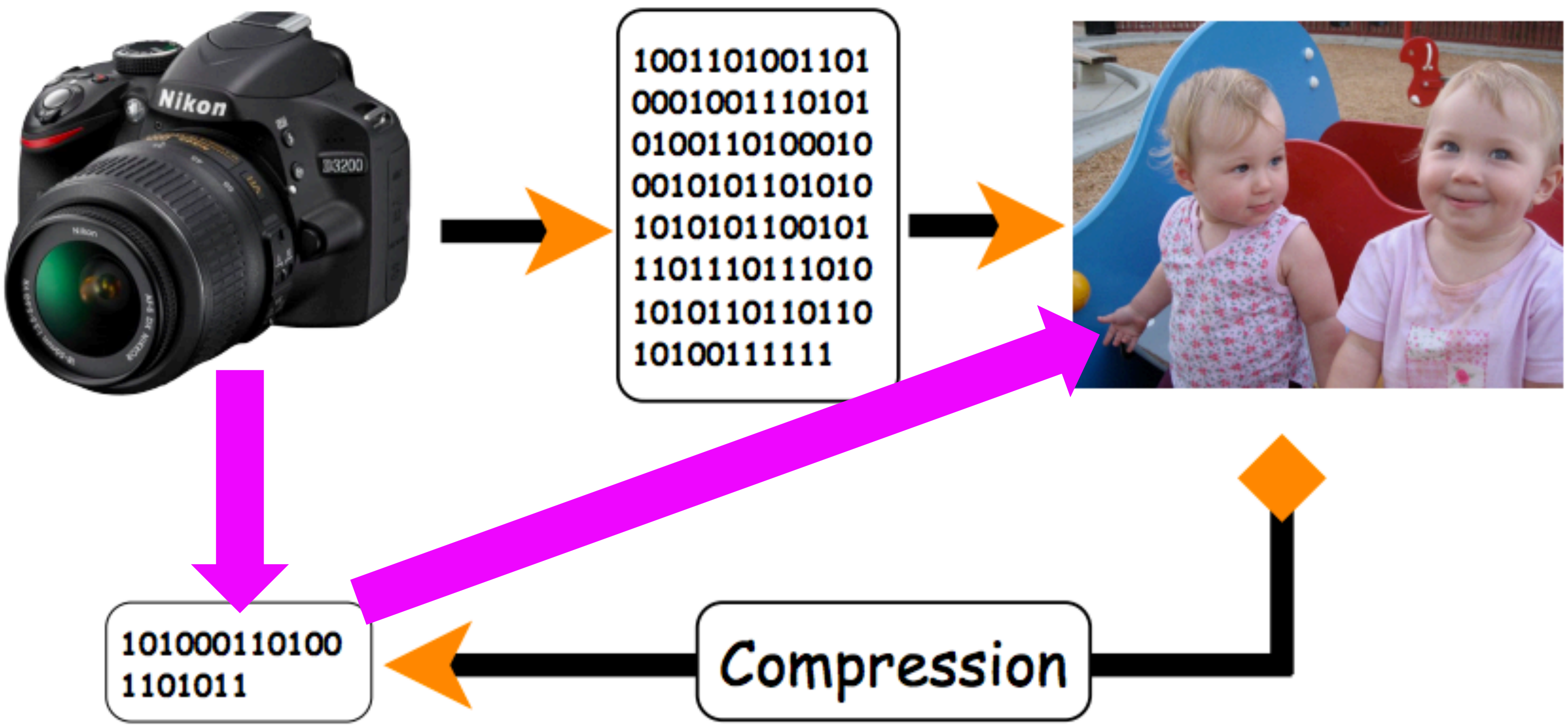
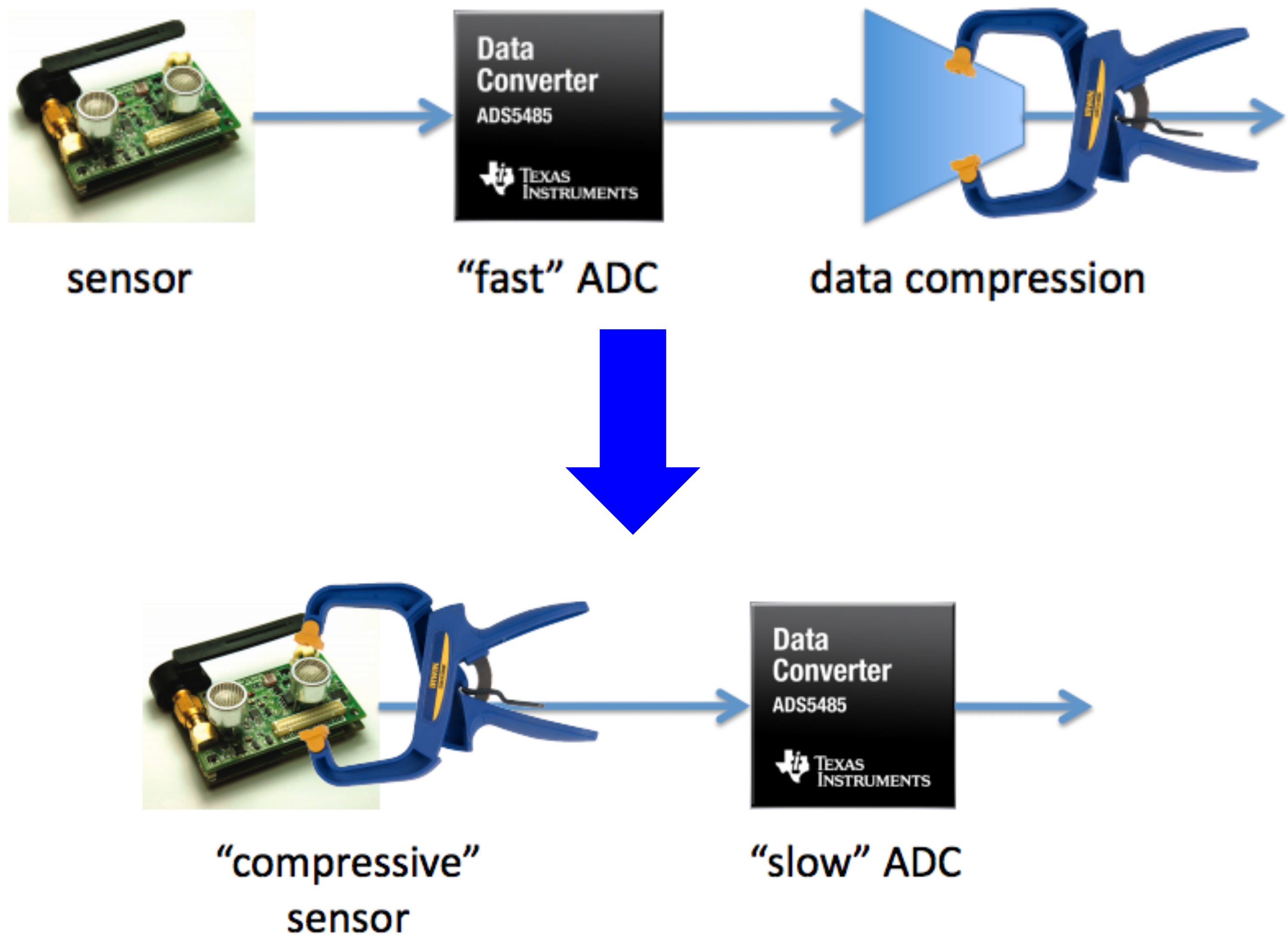


Compressive Sensing

- Shannon/Nyquist theorem is pessimistic
  - 2 × bandwidth is the worst-case sampling rate — holds uniformly for any bandlimited data
  - sparsity/compressibility is irrelevant
  - Shannon sampling based on a linear model, compression based on a nonlinear model
- Compressive sensing
  - new sampling theory that leverages compressibility
  - key roles played by new uncertainty principles and randomness

Sensing to Data



Compressive Sampling

- Sample at lower than the Nyquist rate and still accurately recover the signal, and in most cases *exactly* recover

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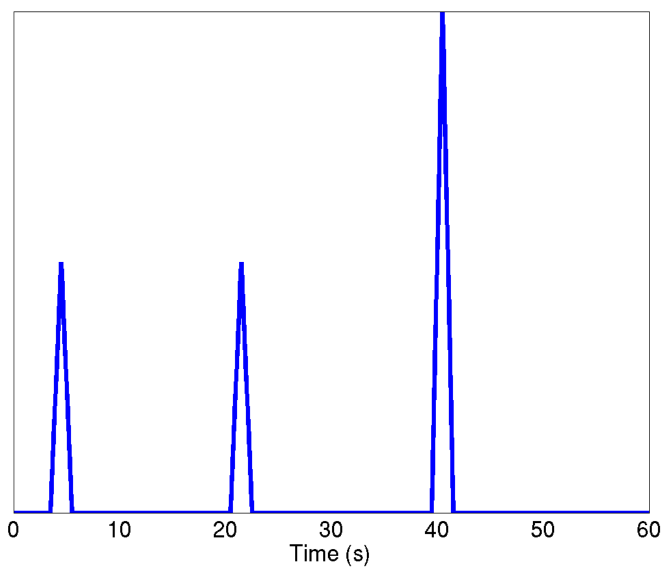
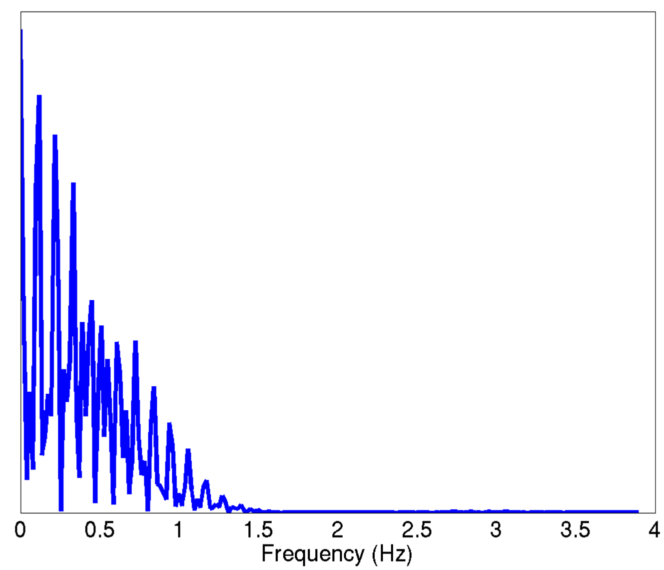
Sparse signal in time



Frequency spectrum



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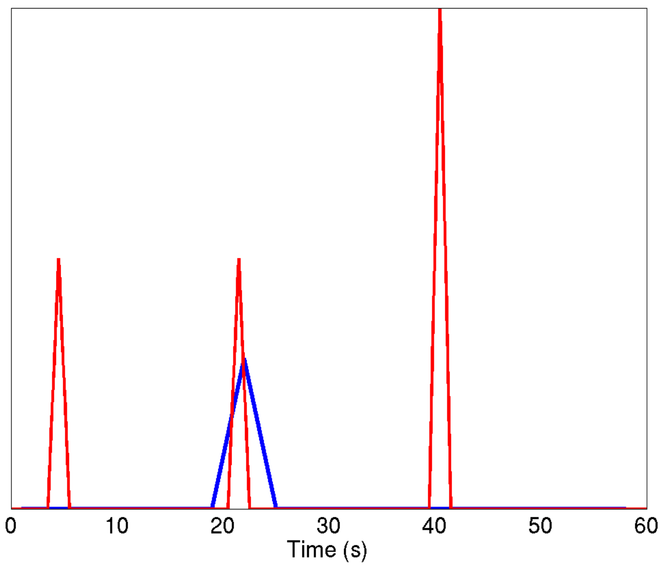
Undersampled in time



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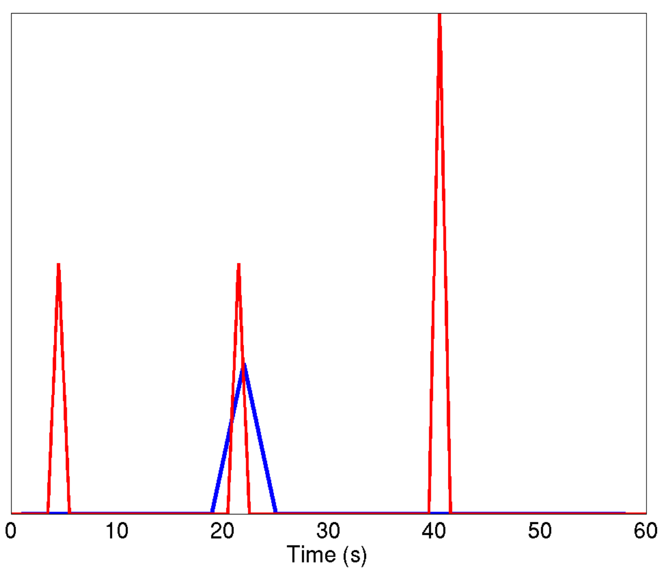

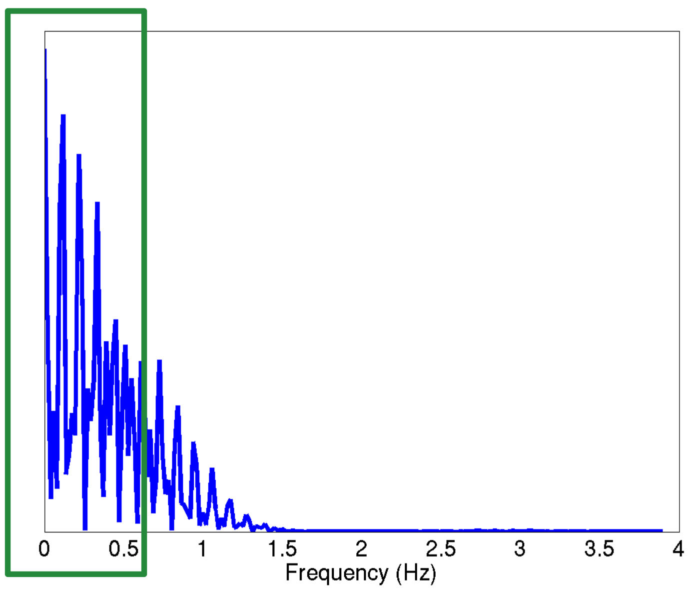

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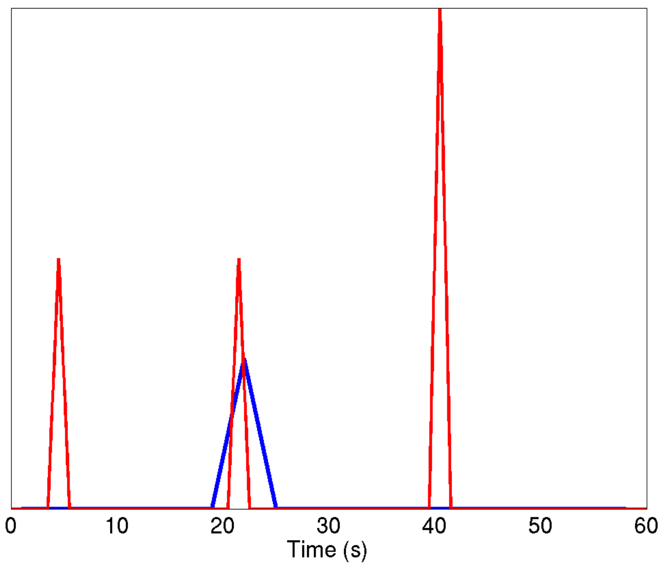
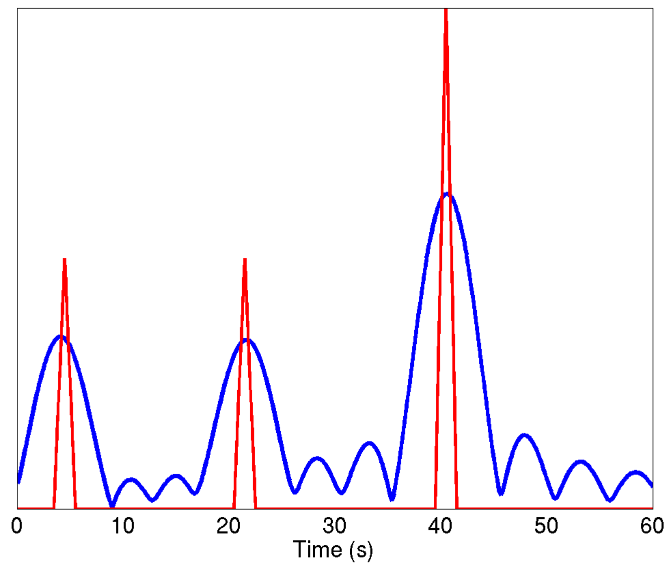


Undersampled in frequency
(reconstructed in time with IFFT)



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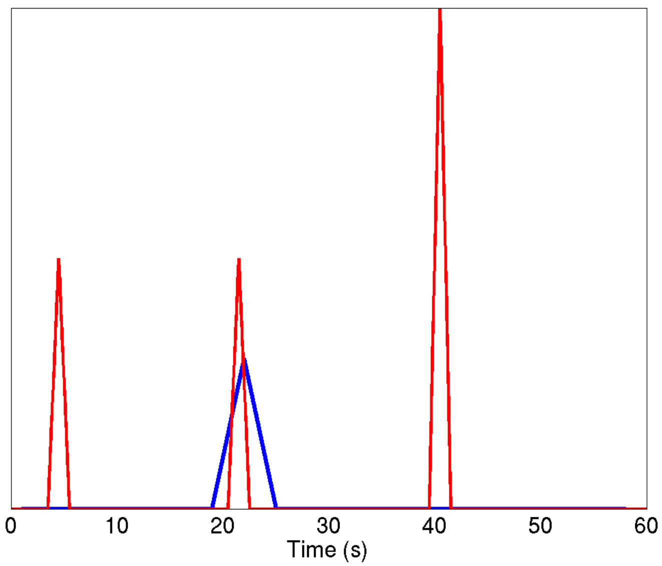

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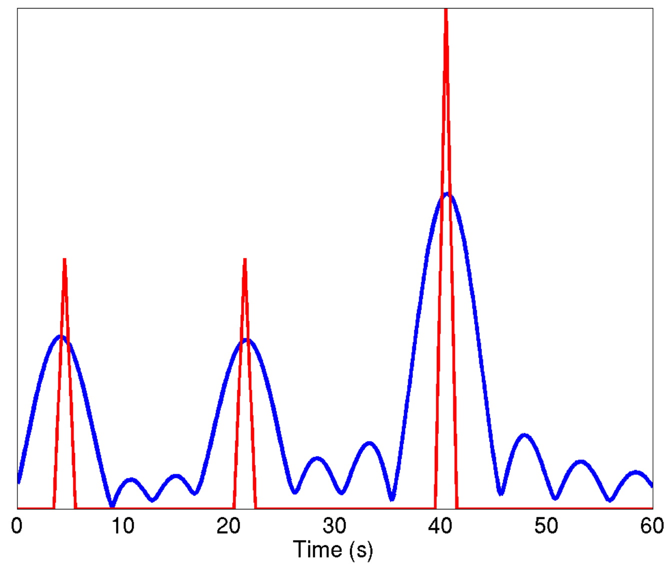
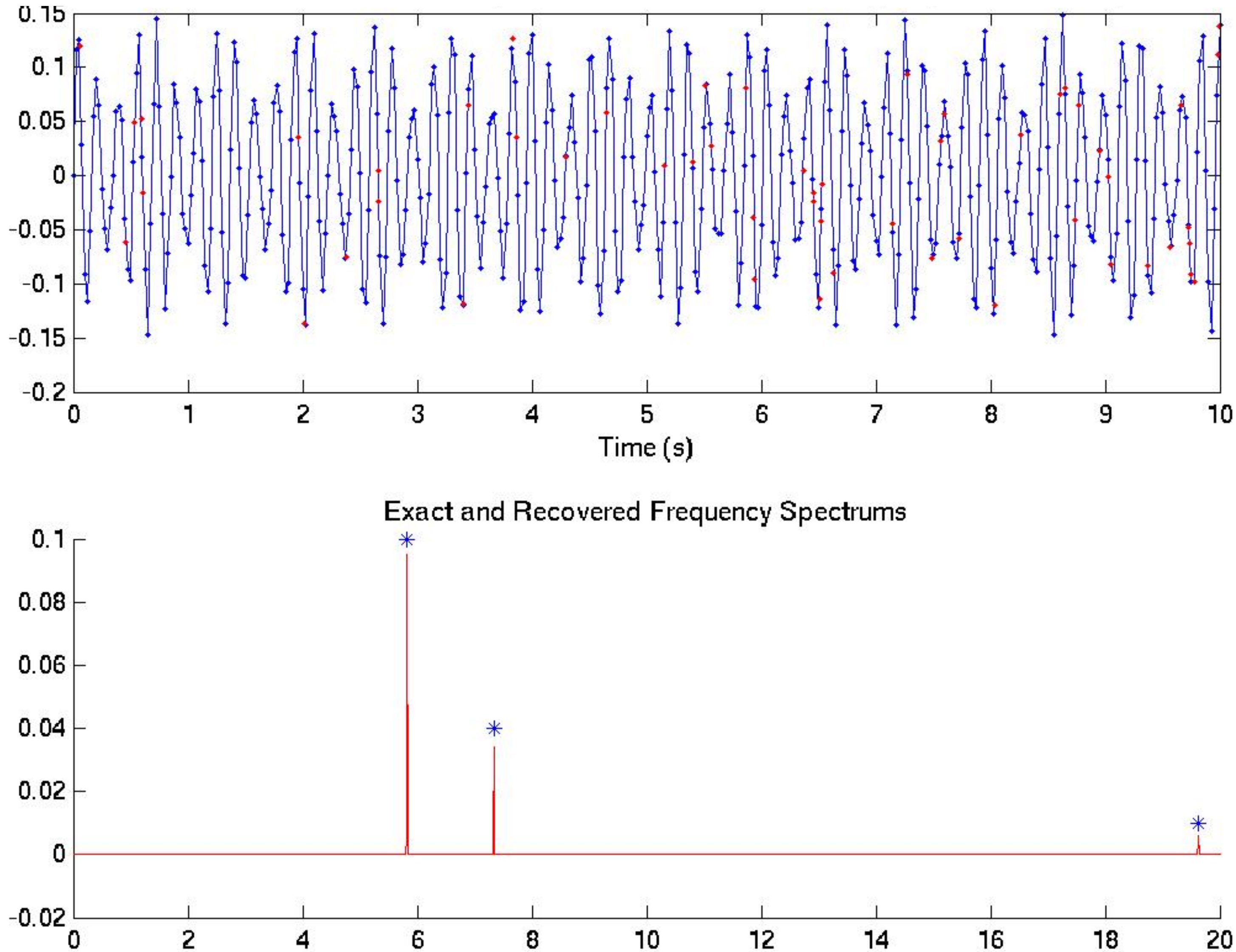
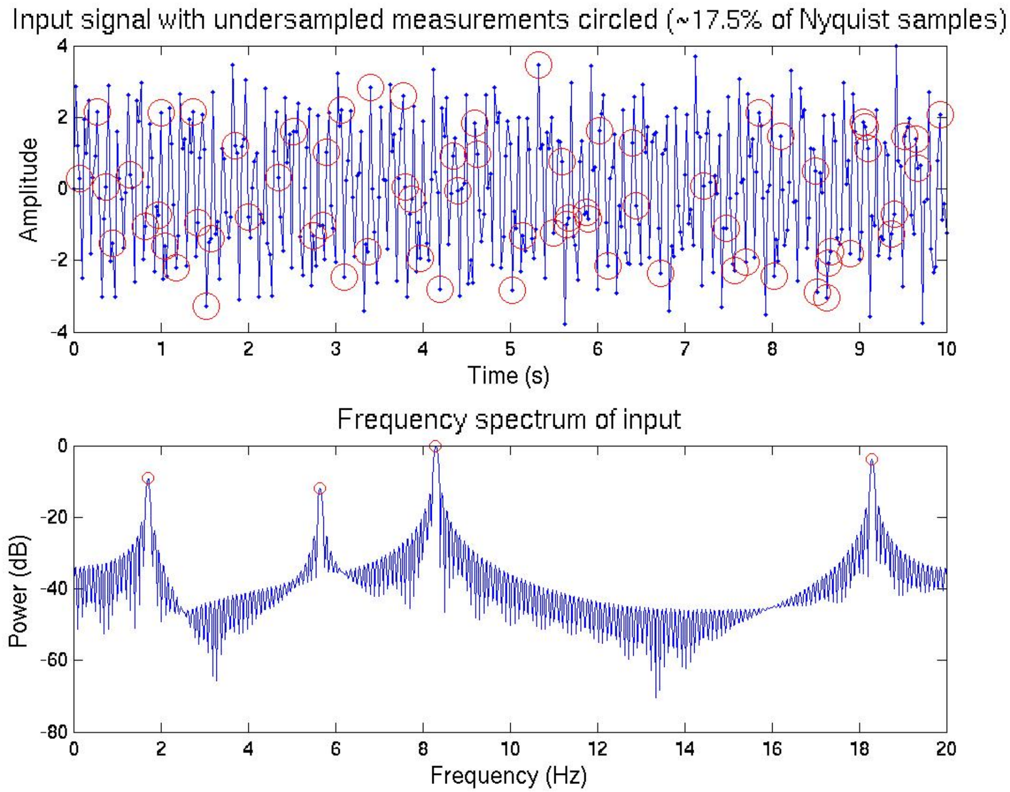


Requires sparsity and incoherent sampling

Compressive Sampling: Simple Example



Compressive Sampling

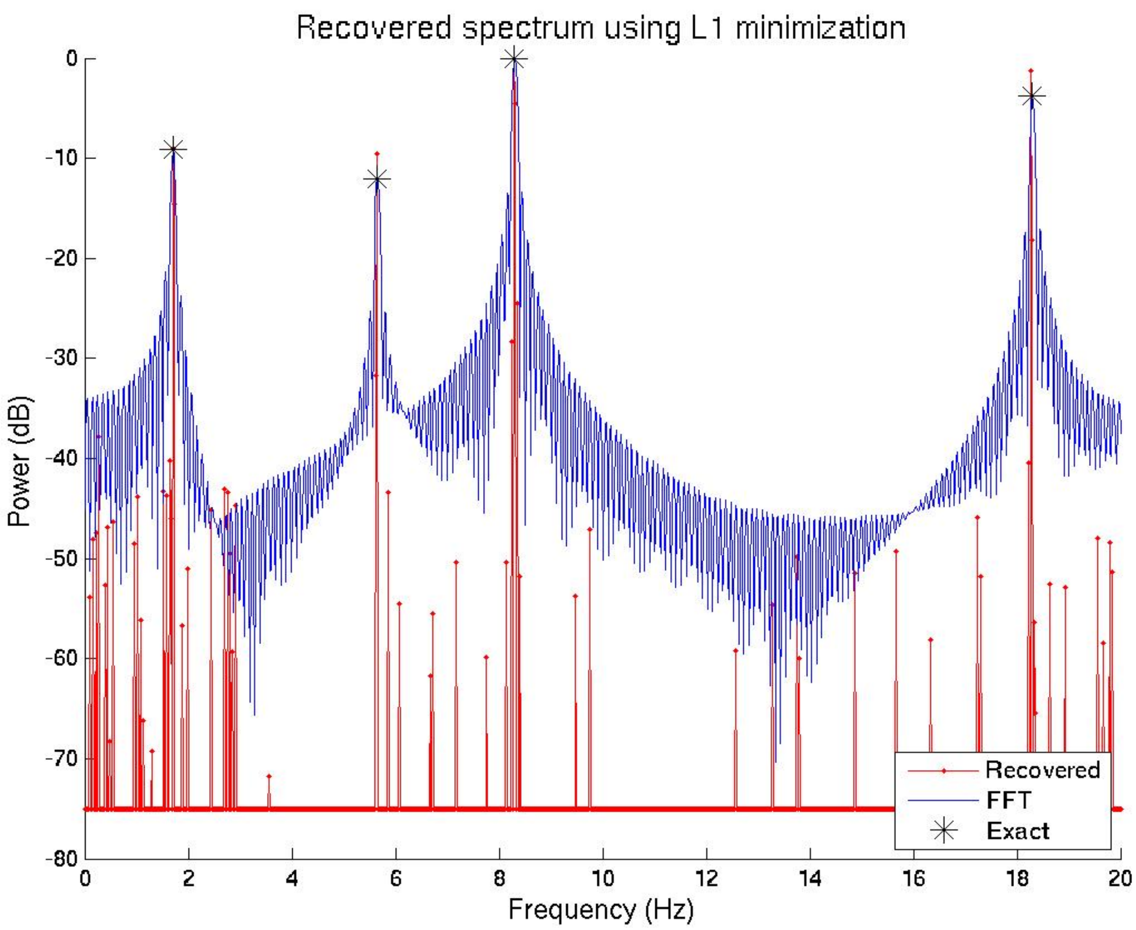


$$\hat{f}(\omega) = \sum_{i=1}^{K} \alpha_i \delta(\omega_i - \omega) \overset{\mathcal{F}}{\Leftrightarrow} f(t) = \sum_{i=1}^{K} \alpha_i e^{i\omega_i t}$$

- Sense signal M times
- Recover with linear program

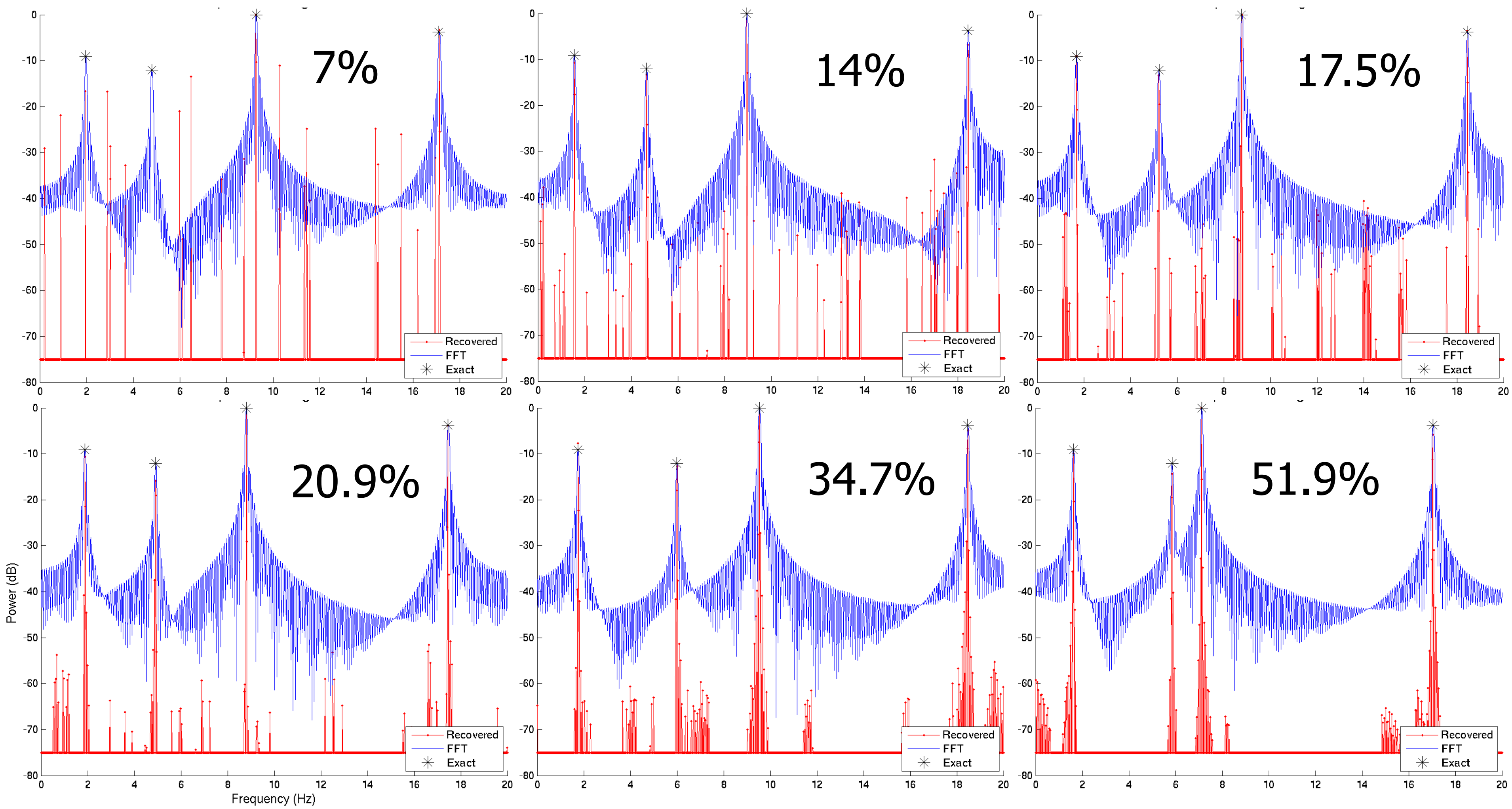
$$\min \sum_{\omega} |\hat{g}(\omega)| \quad \text{subject to} \quad g(t_m) = f(t_m), \quad m = 1, \dots, M$$

Example: Sum of Sinusoids

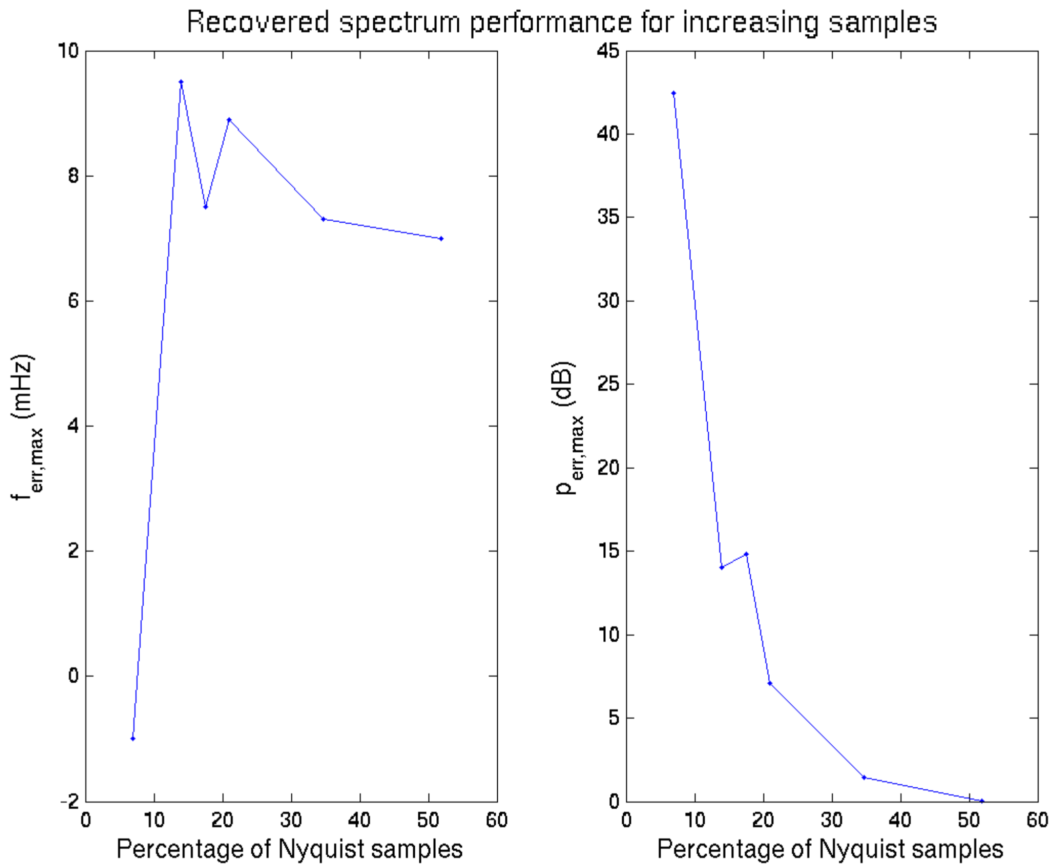


- Two relevant "knobs"
  - percentage of Nyquist samples as altered by adjusting number of samples, *M*
  - input signal duration, *T*
    - Data block size

Example: Increasing M

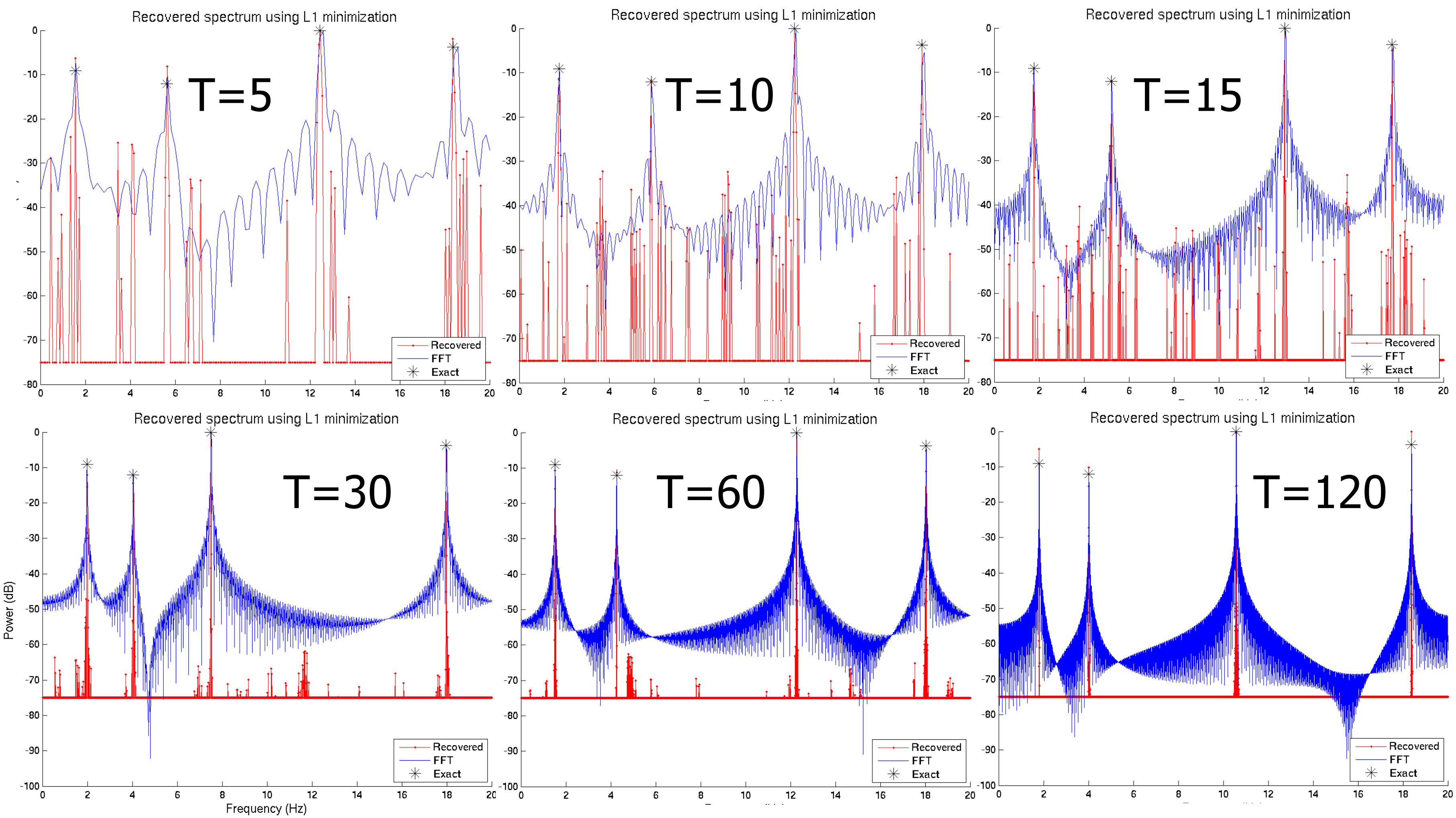


Example: Increasing M

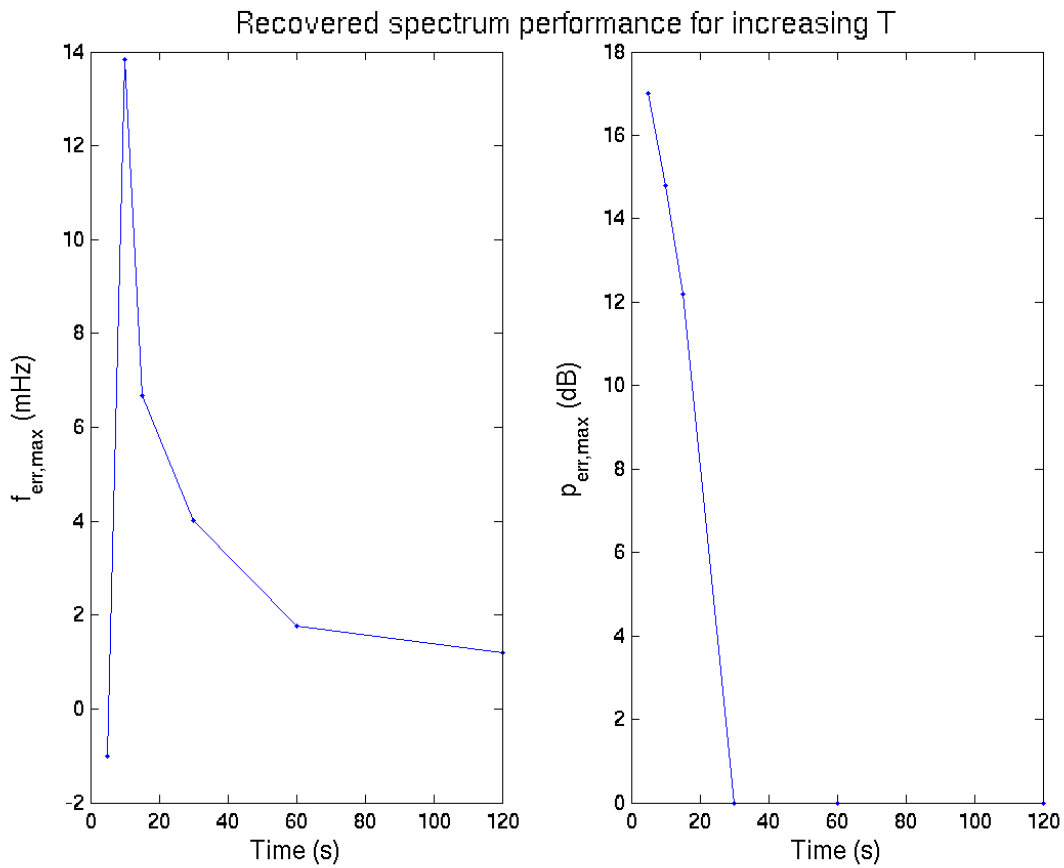


- $f_{err,max}$ **within 10 mHz**
- $p_{err,max}$ **decreasing**

Example: Increasing T



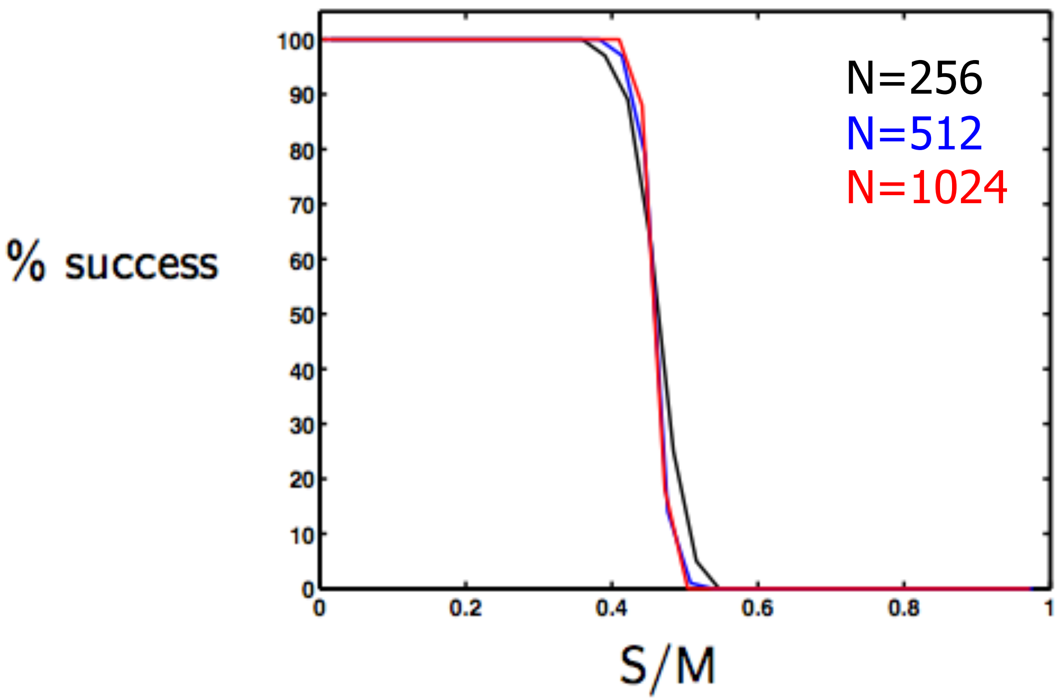
Example: Increasing T



- $f_{err,max}$ **decreasing**
- $p_{err,max}$ **decreasing**

Numerical Recovery Curves

- Sense S-sparse signal of length N randomly M times



- In practice, perfect recovery occurs when $M \approx 2S$ for $N \approx 1000$

A Non-Linear Sampling Theorem

- Exact Recovery Theorem (Candès, R, Tao, 2004):
  - Select M sample locations $\{t_m\}$ "at random" with

$$M \geq \text{Const} \cdot S \log N$$

- Take time-domain samples (measurements)

$$y_m = x_0(t_m)$$

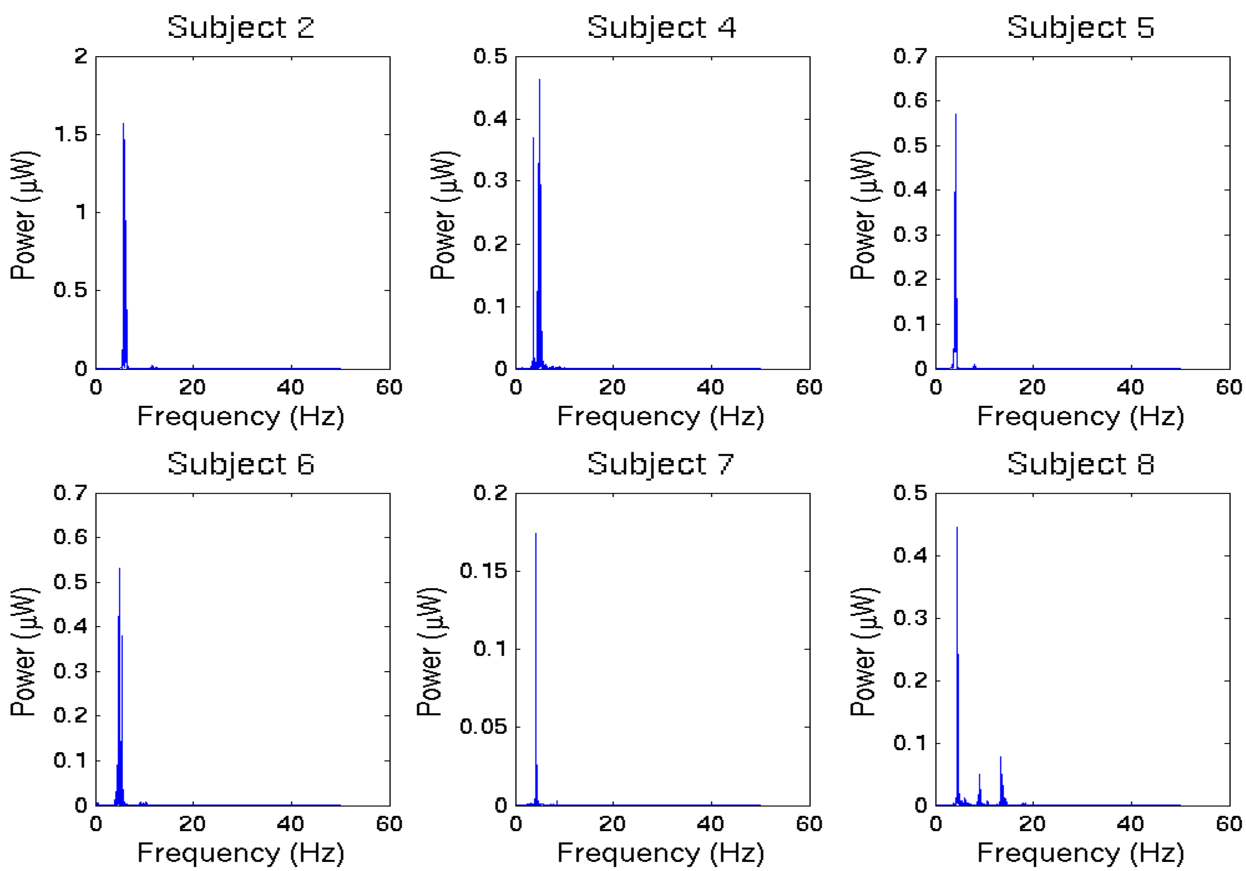
- Solve

$$\min_x \|\hat{x}\|_{\ell_1} \quad \textsf{subject to} \quad x(t_m) = y_m, \quad m = 1, \ldots, M$$

- Solution is exactly recovered signal with extremely high probability

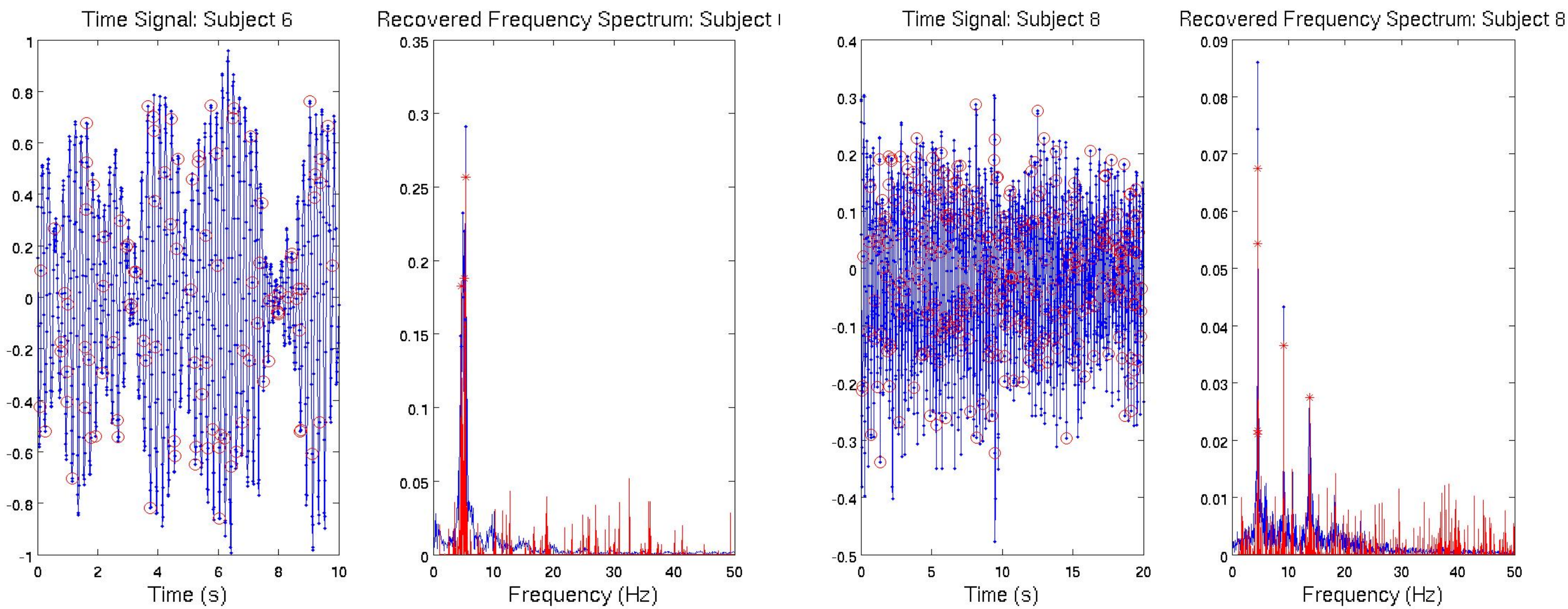
$M > C \cdot \mu^2(\Phi,\Psi) \cdot S \cdot \log N$

Biometric Example: Parkinson's Tremors



- 6 Subjects of real tremor data
  - collected using low intensity velocity-transducing laser recording aimed at reflective tape attached to the subjects' finger recording the finger velocity
  - All show Parkinson's tremor in the 4-6 Hz range.
  - Subject 8 shows activity at two higher frequencies
  - Subject 4 appears to have two tremors very close to each other in frequency

Compressive Sampling: Real Data



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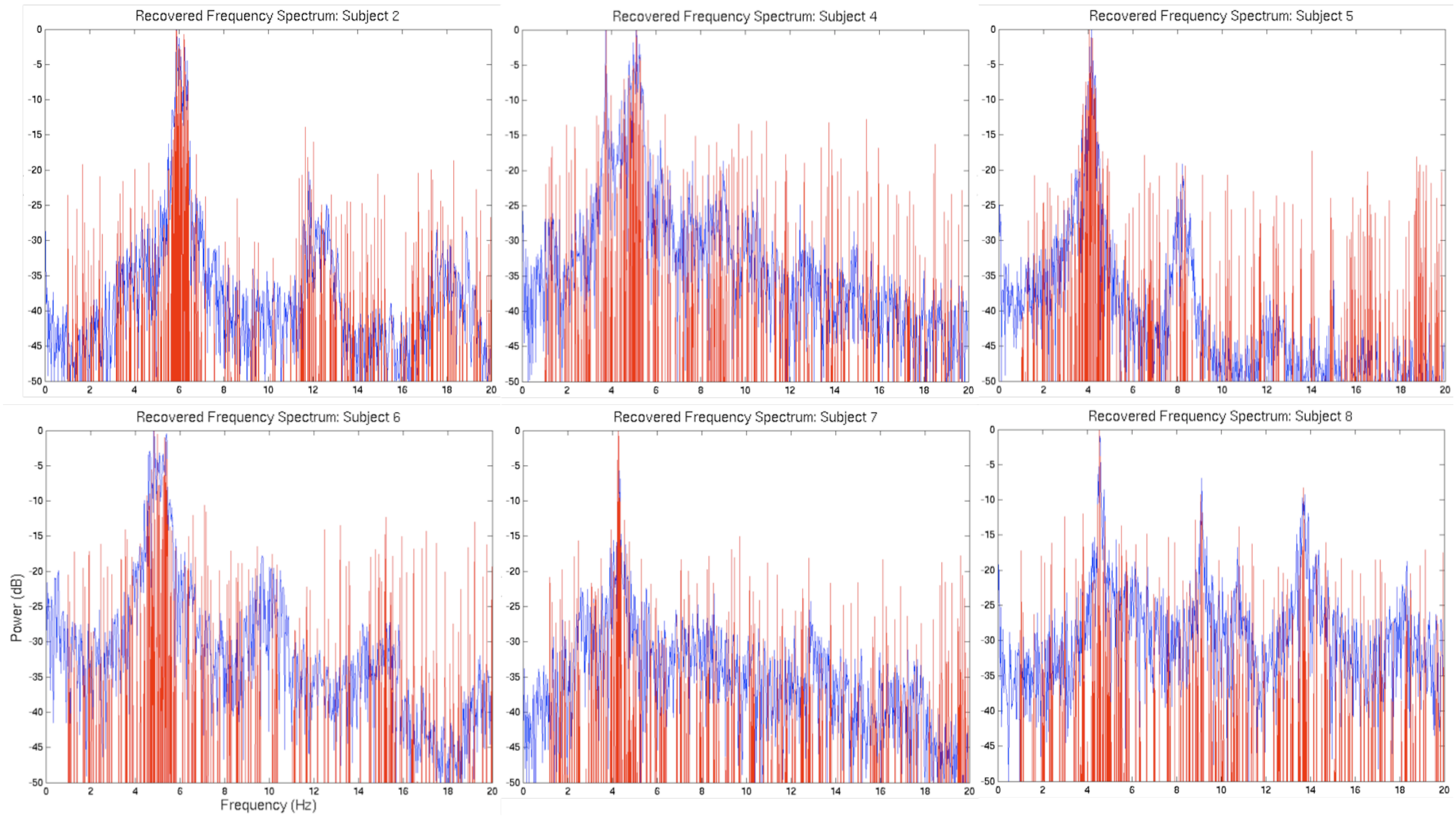
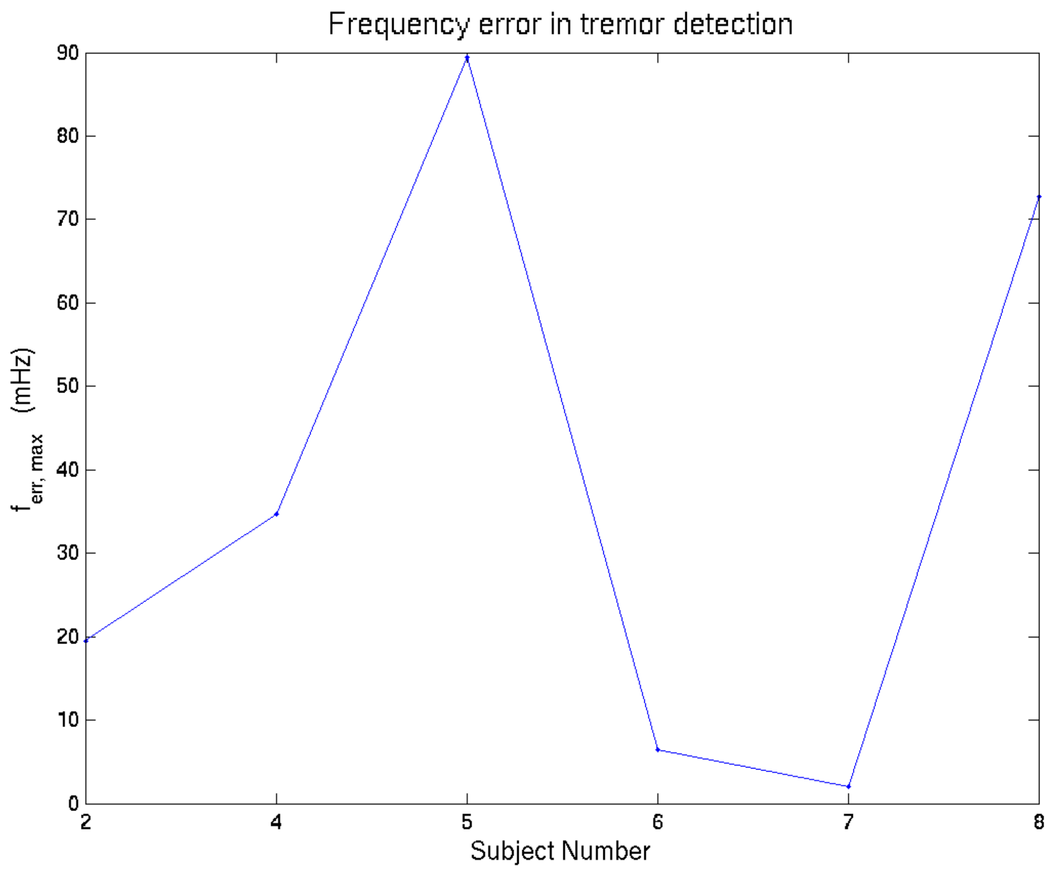


- **C=10.5, T=30**
  - 20% Nyquist required samples

Biometric Example: Parkinson's Tremors



- **Tremors detected within 100 mHz**
- **randomly sample 20% of the Nyquist required samples**

Requires post processing to randomly sample!

Admin

- Finish Lab 9 by next week
  - Submit Google Colab PDF in Canvas
  - Keep filled out Google Colab doc in drive
    - You each have your own drive