ESE 3400: Medical Devices Lab

Lec 18: November 16, 2022

Quiz 2 Review

Signals and Systems

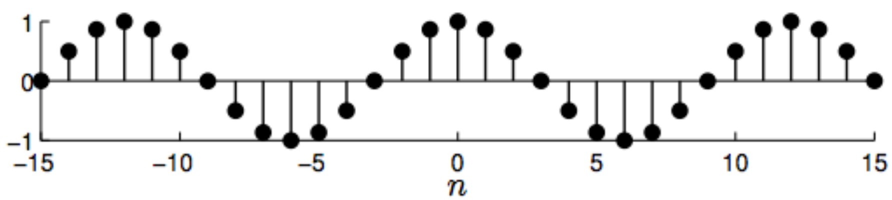
Discrete-Time Sinusoids

- Discrete-time sinusoids $e^{j(\omega n+\phi)}$ have two counterintuitive properties
- Both involve the frequency ω
- **Property #1**: Aliasing
- **Property #2**: Aperiodicity

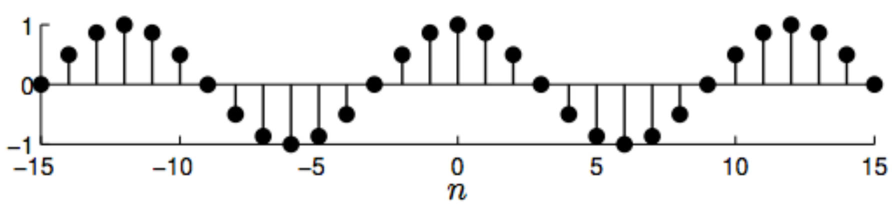
Property #1: Aliasing of Sinusoids

- The signals x_1 and x_2 have different frequencies but are **identical!**
- We say that x_1 and x_2 are aliases; this phenomenon is called aliasing

$x_1[n] = \cos\left(\frac{\pi}{6}n\right)$



$x_2[n] = \cos\left(\frac{13\pi}{6}n\right) = \cos\left(\left(\frac{\pi}{6} + 2\pi\right)n\right)$



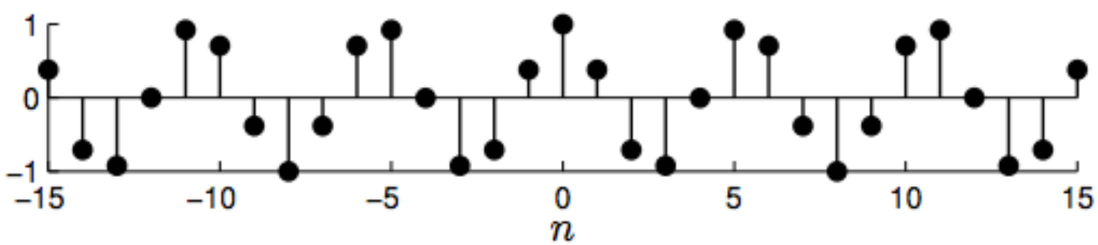
Property #2: Periodicity of Sinusoids

- Consider $x_1[n] = e^{j(\omega n+\phi)}$ with frequency $\omega = \frac{2\pi k}{N}$, $\quad k, N \in \mathbb{Z}$ (harmonic frequency)

- It is easy to show that x_1 is periodic with period N, since

$$x_1[n+N] = e^{j(\omega(n+N)+\phi)} = e^{j(\omega n+\omega N+\phi)} = e^{j(\omega n+\phi)}\, e^{j(\omega N)} = e^{j(\omega n+\phi)}\, e^{j(\frac{2\pi k}{N} N)} = x_1[n] \checkmark$$

- Ex: $x_1[n] = \cos(\frac{2\pi 3}{16} n)$, $\quad N = 16$



- Note: x_1 is periodic with the (smaller) period of $\frac{N}{k}$ when $\frac{N}{k}$ is an integer

Harmonic Sinusoids

$$e^{j(\omega n+\phi)}$$

- Semi-amazing fact: The **only** periodic discrete-time sinusoids are those with **harmonic frequencies**

$$\omega = \frac{2\pi k}{N}, \quad k, N \in \mathbb{Z}$$

- Which means that
  - **Most** discrete-time sinusoids are **not** periodic!
  - The harmonic sinusoids are somehow magical (they play a starring role later in the DFT)

Example problems:

- Which is higher in frequency?
  - $\cos(\pi n)$ or $\cos(3\pi/2 n)$?

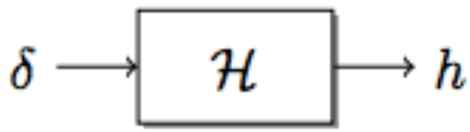
- Periodic or not?
  - $\cos(5/7\pi n)$
  - $\cos(\pi/5 n)$
  - If so, what are N and k? (I.e How many samples is one period?

LTI Systems

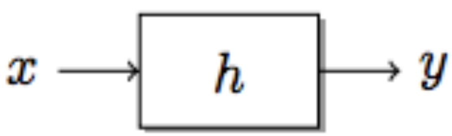
DEFINITION

A system $\mathcal{H}$ is **linear time-invariant** (LTI) if it is both linear and time-invariant

- LTI system can be completely characterized by its impulse response



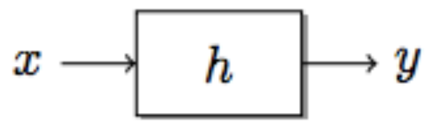
- Then the output for an arbitrary input is a sum of weighted, delay impulse responses



$$y[n] = \sum_{m=-\infty}^{\infty} h[n-m]\,x[m]$$

$$y[n] = x[n] * h[n]$$

Convolution



- Convolution formula:

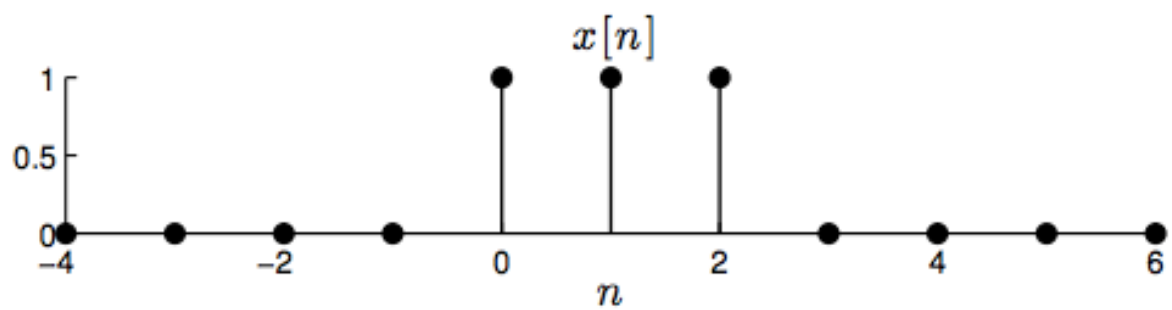
$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} h[n-m]\, x[m]$$

- Convolution method:
  - 1) Time reverse the impulse response and shift it *n* time steps to the right
  - 2) Compute the inner product between the shifted impulse response and the input vector
  - Repeat for evey *n*

Convolution Example

$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} h[n-m]\, x[m]$$

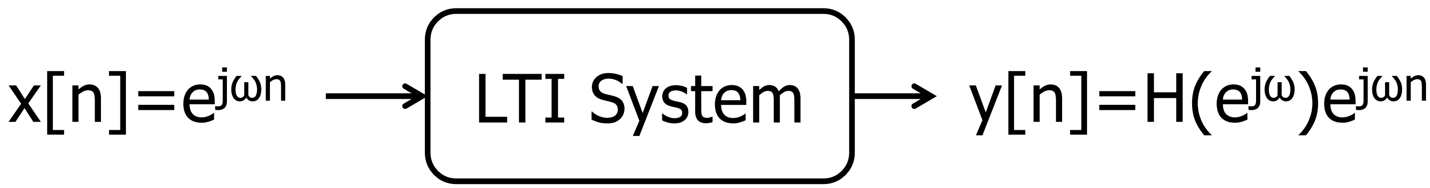
- Convolve a unit pulse with itself



- Convolve a unit pulse with a unit pulse twice the width

LTI System Frequency Response

- (DT)Fourier Transform of impulse response



$$H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} h[k] e^{-j\omega k}$$

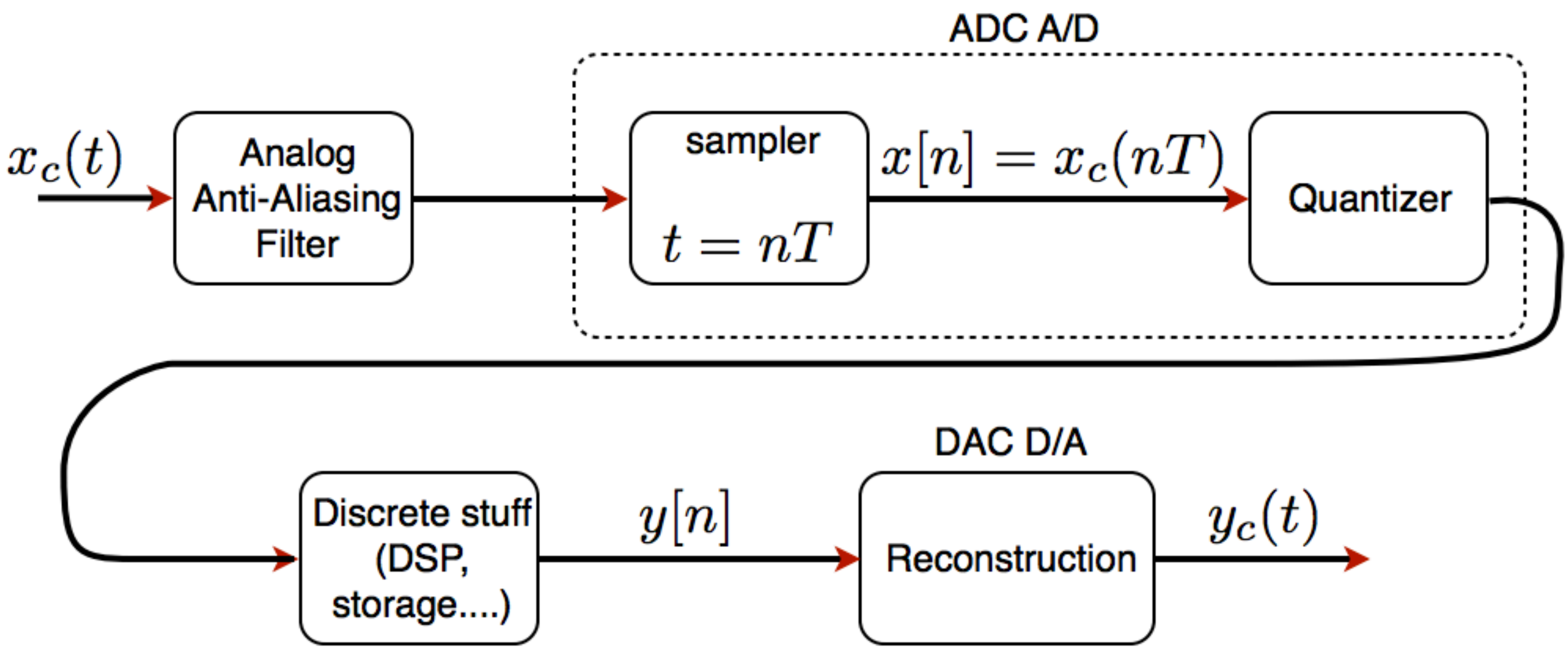
DTFT and Sampling

DTFT Definition

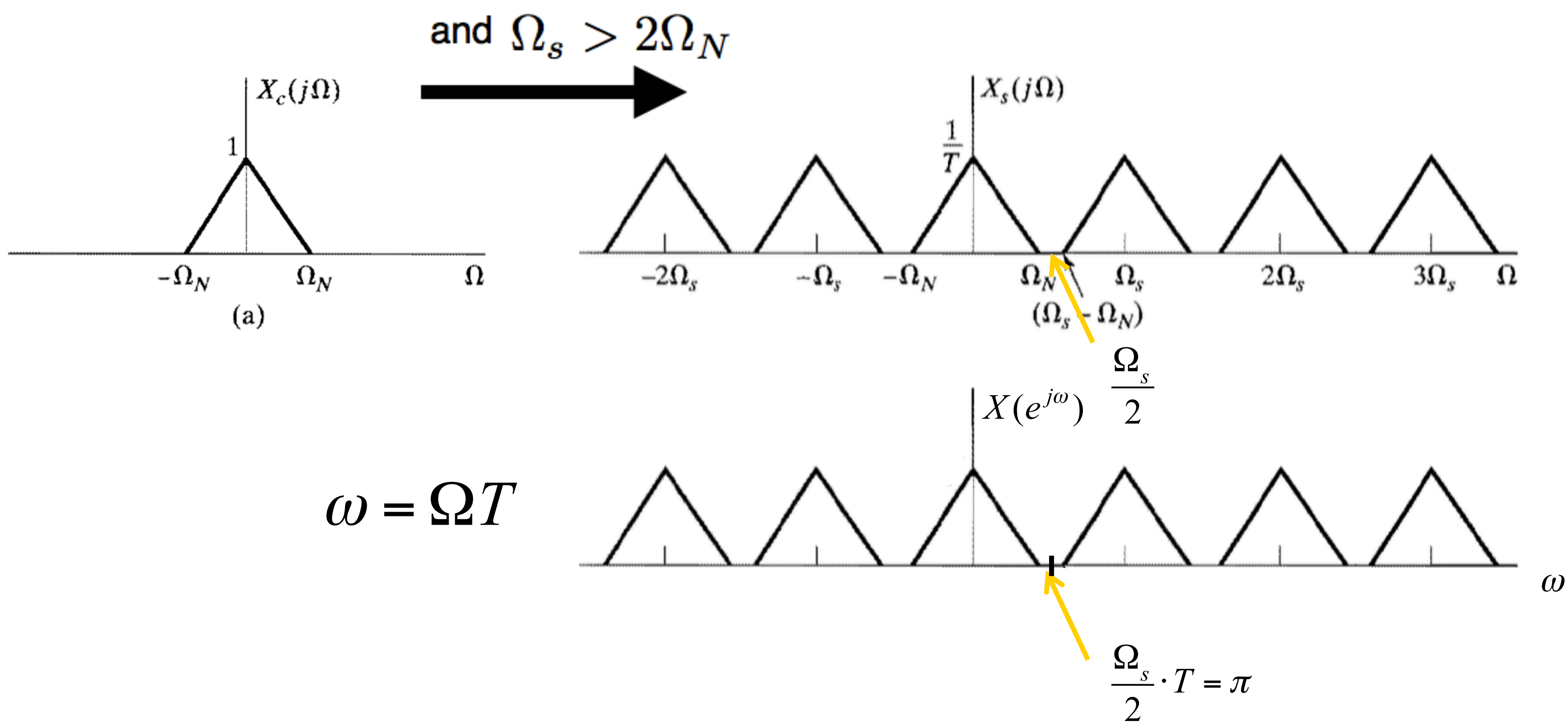
$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} x[k]e^{-j\omega k}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

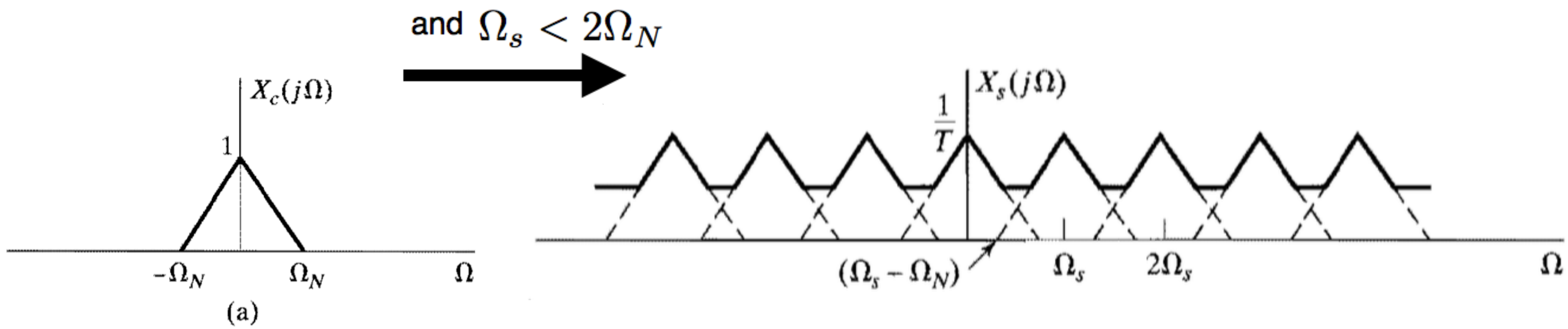
DSP System


ADC A/D
$x_c(t)$
Analog Anti-Aliasing Filter
sampler
$t = nT$
$x[n] = x_c(nT)$
Quantizer
DAC D/A
Discrete stuff (DSP, storage....)
$y[n]$
Reconstruction
$y_c(t)$

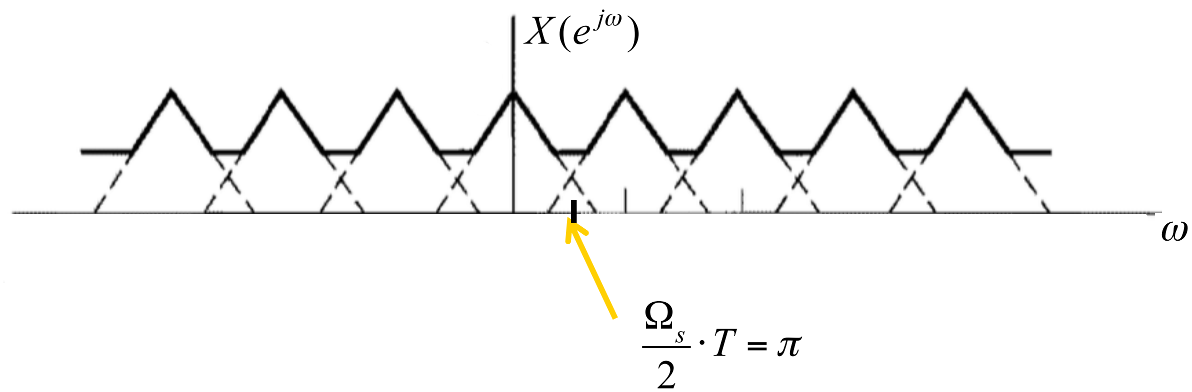
Frequency Domain Analysis



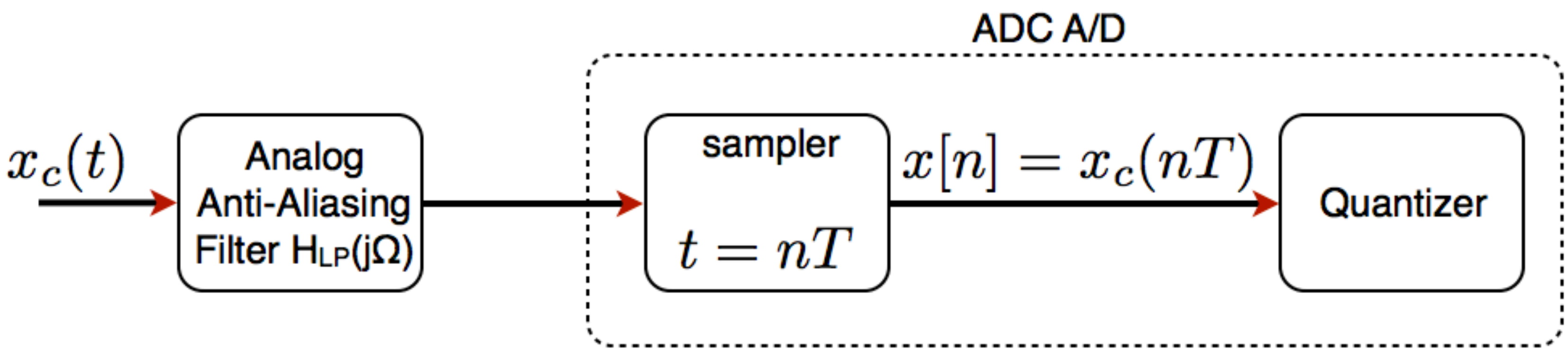
Frequency Domain Analysis w/Aliasing



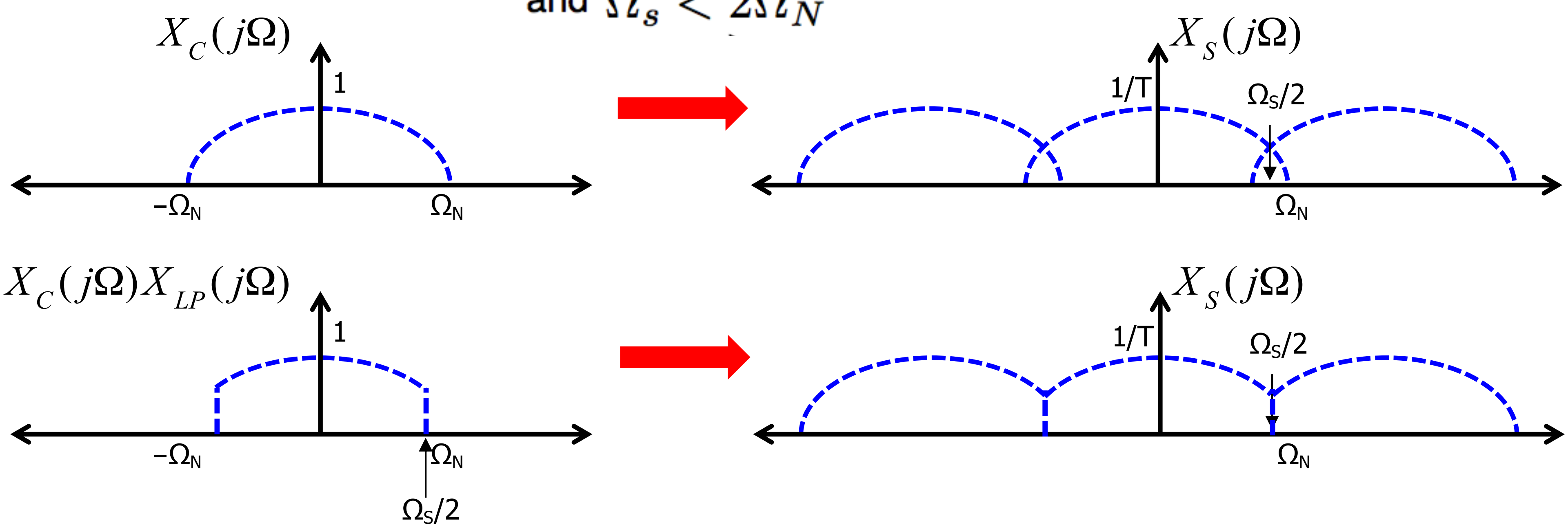
$\omega = \Omega T$



Anti-Aliasing Filter

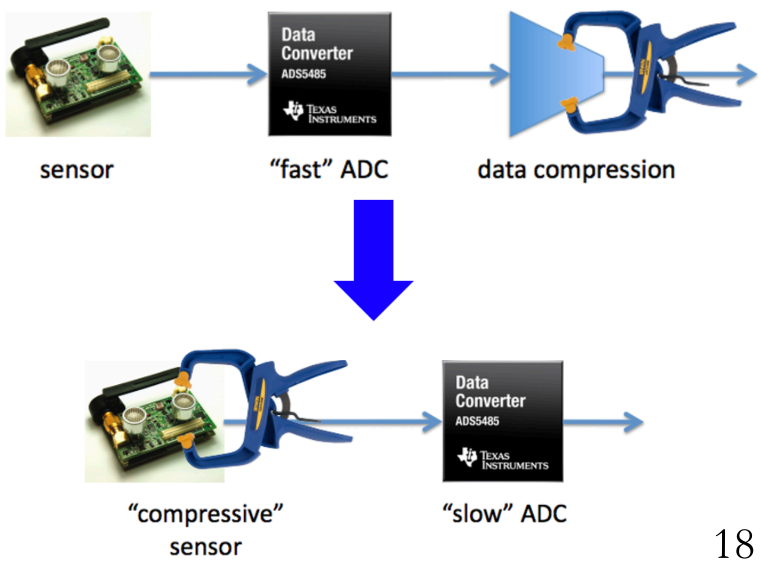


and $\Omega_s < 2\Omega_N$



Compressive Sensing

- Shannon/Nyquist theorem is pessimistic
  - 2 × bandwidth is the worst-case sampling rate — holds uniformly for any bandlimited data
  - Sparsity/compressibility is irrelevant
  - Shannon sampling based on a linear model, compression based on a nonlinear model
- Compressive sensing
  - New sampling theory that leverages compressibility
  - Two conditions needed:
    - Sparsity in some domain
    - Transformation to incoherent domain

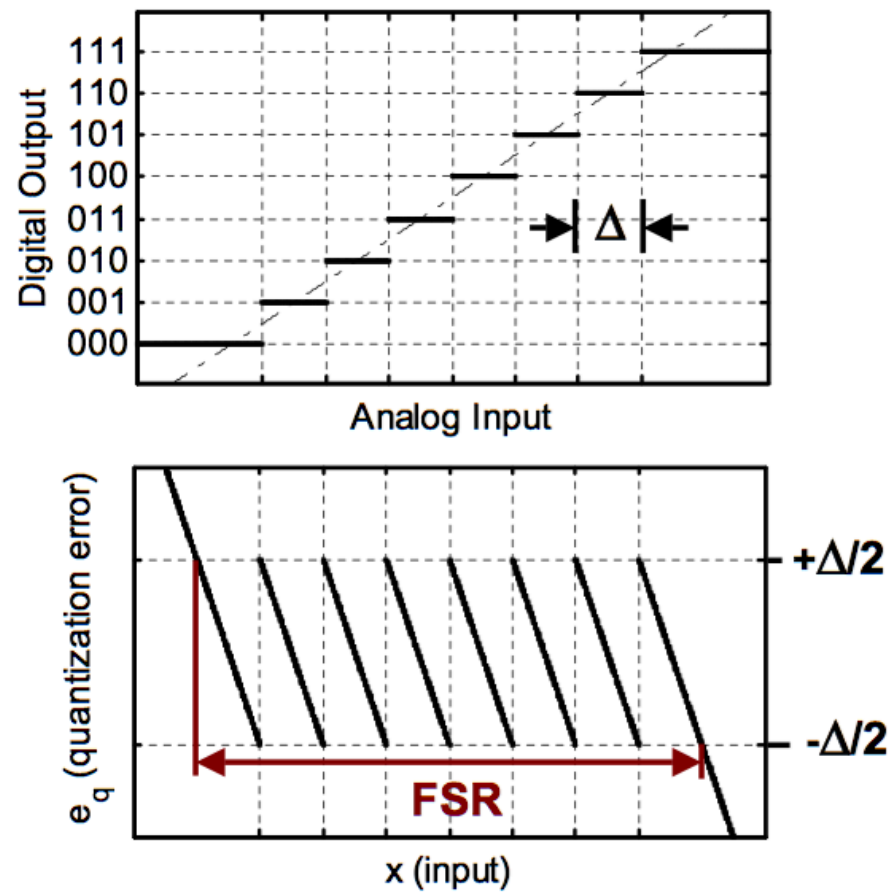


ADC

Analog to Digital Converter

Ideal B-bit Quantizer

- Practical quantizers have a limited input range and a finite set of output codes
- E.g. a 3-bit quantizer can map onto $2^3=8$ distinct output codes



- Quantization error grows out of bounds beyond code boundaries
- We define the full scale range (FSR) as the maximum input range that satisfies $|e_q| \leq \Delta/2$
  - Implies that $FSR = 2^B \cdot \Delta$

Signal-to-Quantization-Noise Ratio

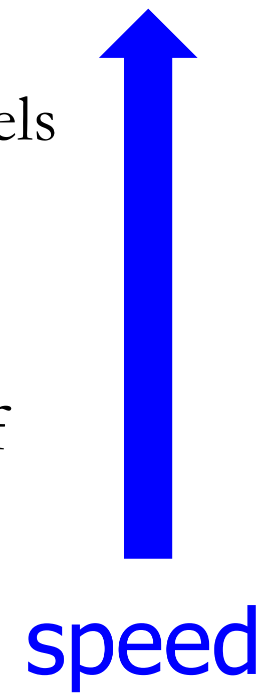
- Assuming full-scale sinusoidal input, we have

$$SNR_Q = 6.02B + 1.76 \text{ dB}$$

B (Number of Bits)	SQNR
8	50dB
12	74dB
16	98dB
20	122dB

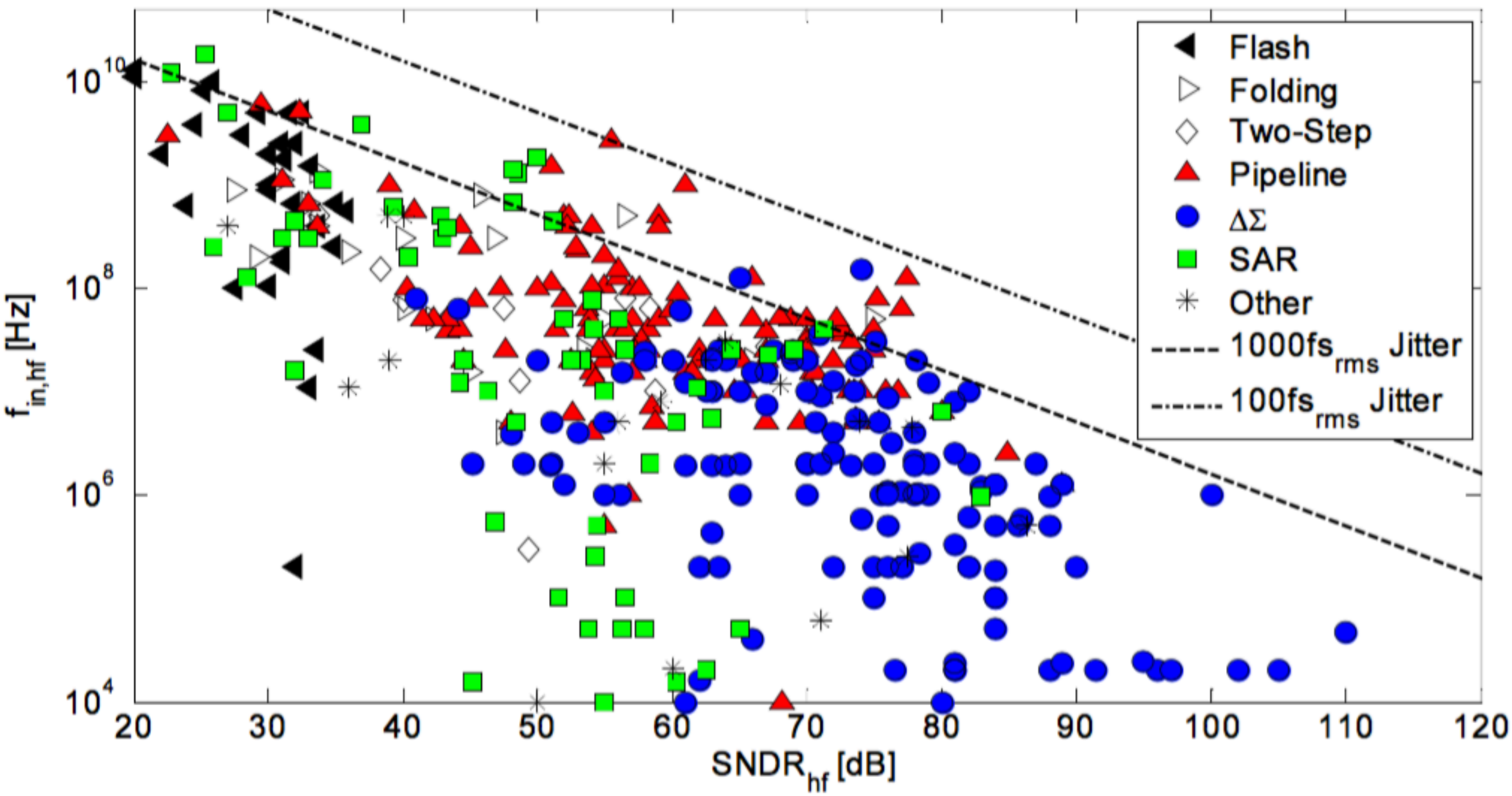
Nyquist ADC Architectures

- Word-at-a-time
  - E.g. flash ADC
  - Instantaneous comparison with 2^B-1 reference levels
- Multi-step
  - E.g. pipeline ADCs
  - Coarse conversion, followed by fine conversion of residuals
- Bit-at-a-time
  - E.g. successive approximation ADCs
  - Conversion via a binary search algorithm



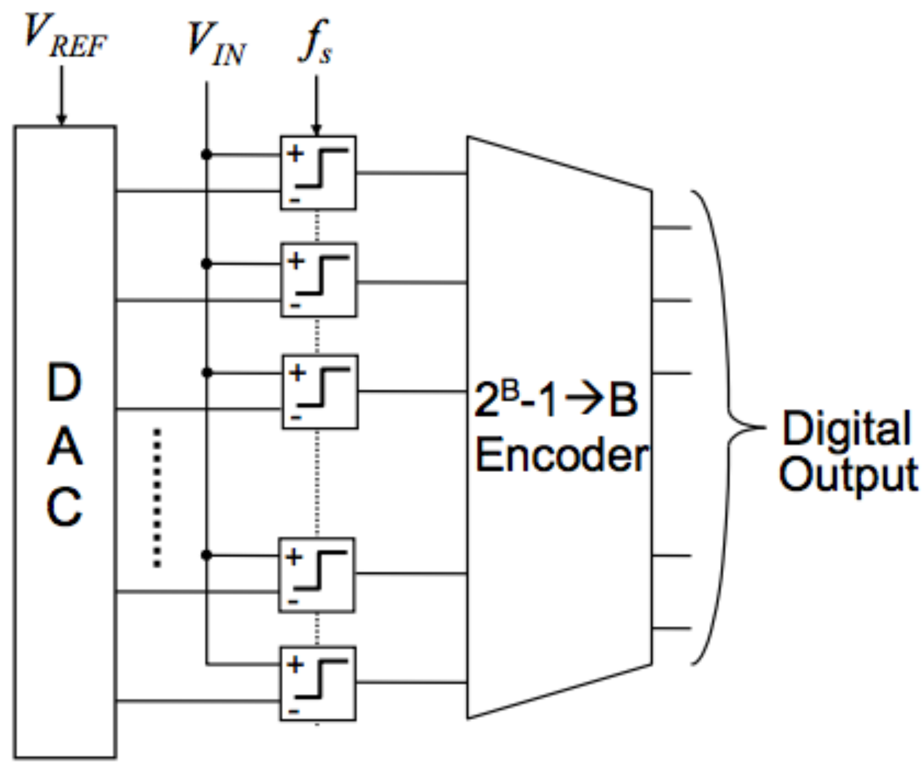
ADC Survey (ISSCC & VLSI 1997-2013)

Data: http://www.stanford.edu/~murmann/adcsurvey.html

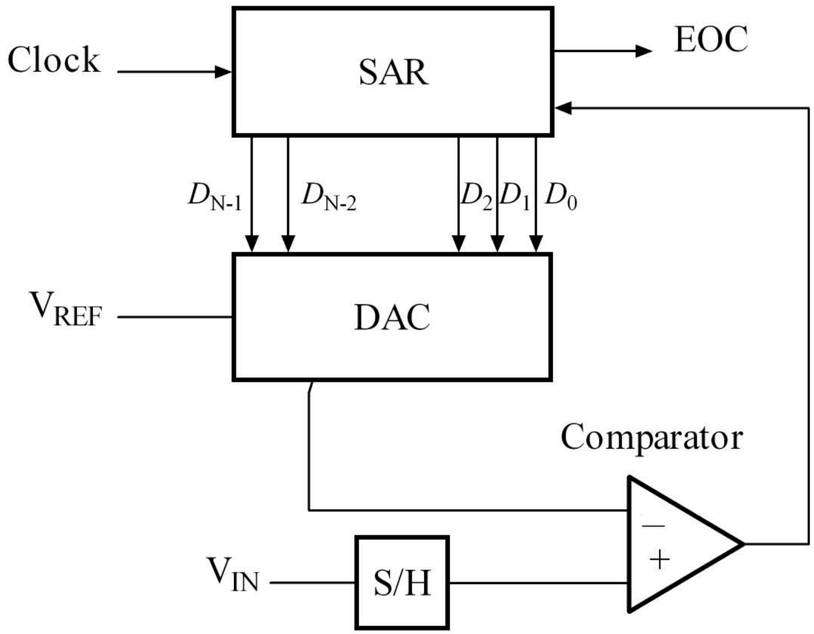


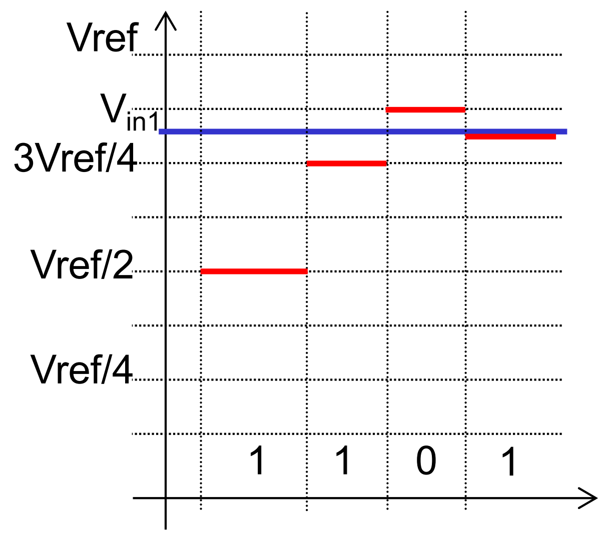
Flash ADC

- B-bit flash ADC:
  - DAC generates all possible 2^B-1 levels
  - 2^B-1 comparators compare V_{IN} to DAC outputs
  - Comparator output:
    - If $V_{DAC} < V_{IN} \rightarrow 1$
    - If $V_{DAC} > V_{IN} \rightarrow 0$
  - Comparator outputs form thermometer code
  - Encoder converts thermometer to binary code



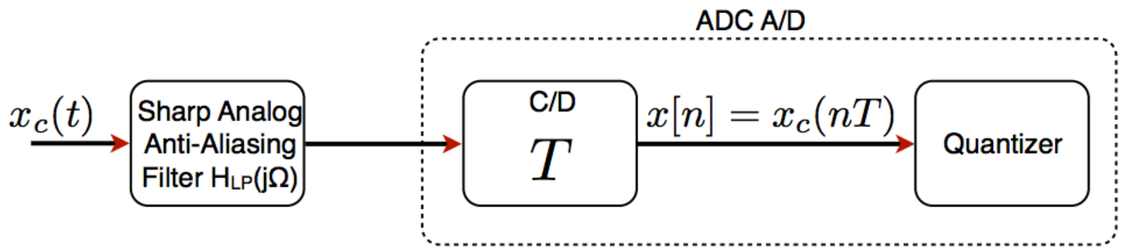
SAR ADC Block Diagram

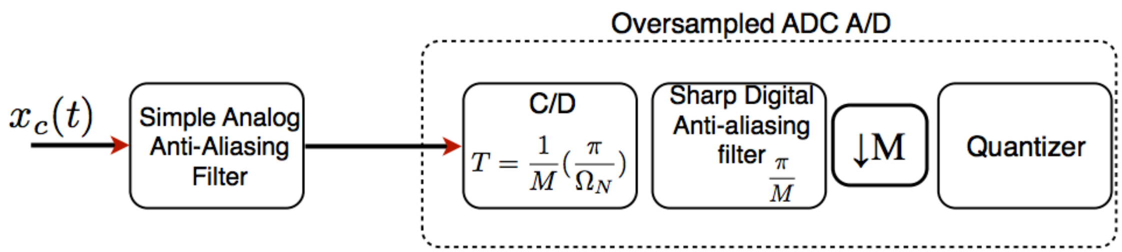




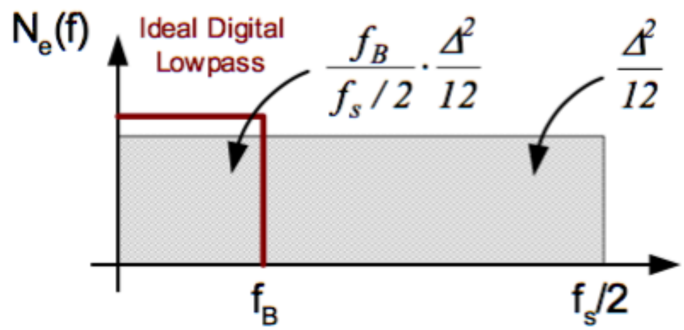
- Sampling phase: Sample input with Sample-and-Hold
- Bit-cycling: Compare with DAC output, adjusting the SAR with each clock cycle as bits are determined

Oversampled ADC

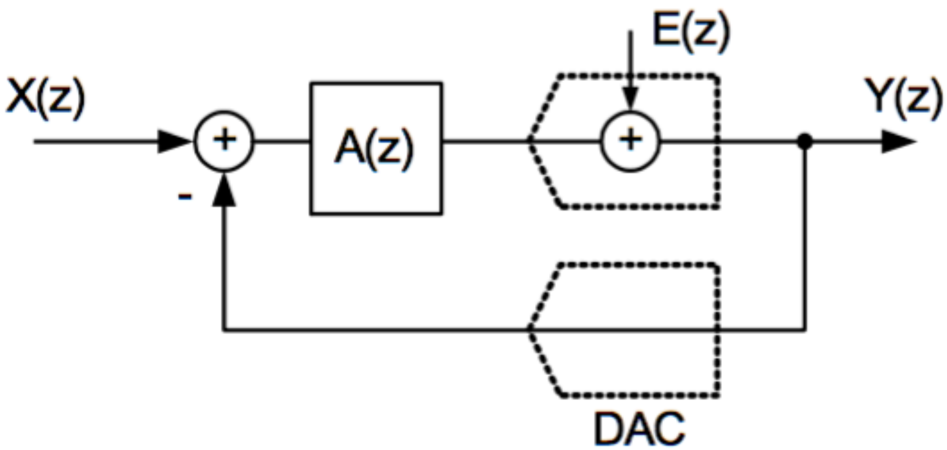




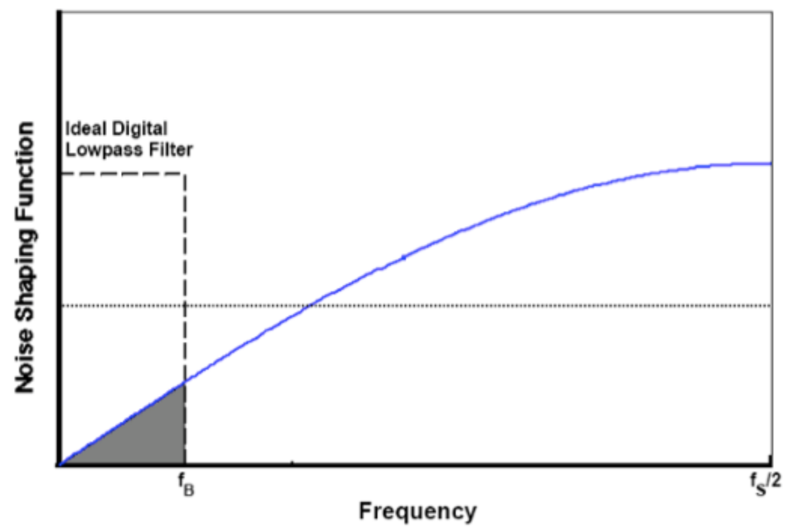
- Total quantization noise at digital output is reduced proportional to "oversampling ratio" $M=(f_S/2)/f_B$



Delta-Sigma Modulation



$$
\begin{aligned}
Y(z) &= E(z) + A(z)X(z) - A(z)Y(z) \\
&= E(z)\frac{1}{1+A(z)} + X(z)\frac{A(z)}{1+A(z)} \\
&= E(z)\underbrace{H_E(z)}_{\substack{\textit{Noise}\\ \textit{Transfer}\\ \textit{Function}}} + X(z)\underbrace{H_X(z)}_{\substack{\textit{Signal}\\ \textit{Transfer}\\ \textit{Function}}}
\end{aligned}
$$



- Noise shaping with feedback to further reduce quantization noise

ADC Big Ideas

- SQNR
  - SQNR determined by bit resolution, B
- Nyquist ADCs
  - Flash ADCs
    - Word-at-a-time for high speed, low resolution applications
  - SAR ADCs
    - Bit-at-a-time for low speed, low power applications
    - Highly suited for medical devices
- Oversampling
  - Enables reduction in quantization noise with digital filter
  - Sigma-Delta ADCs
    - Use integrator in feedback to shape noise and achieve high resolution
    - Usually for low speed, low power applications

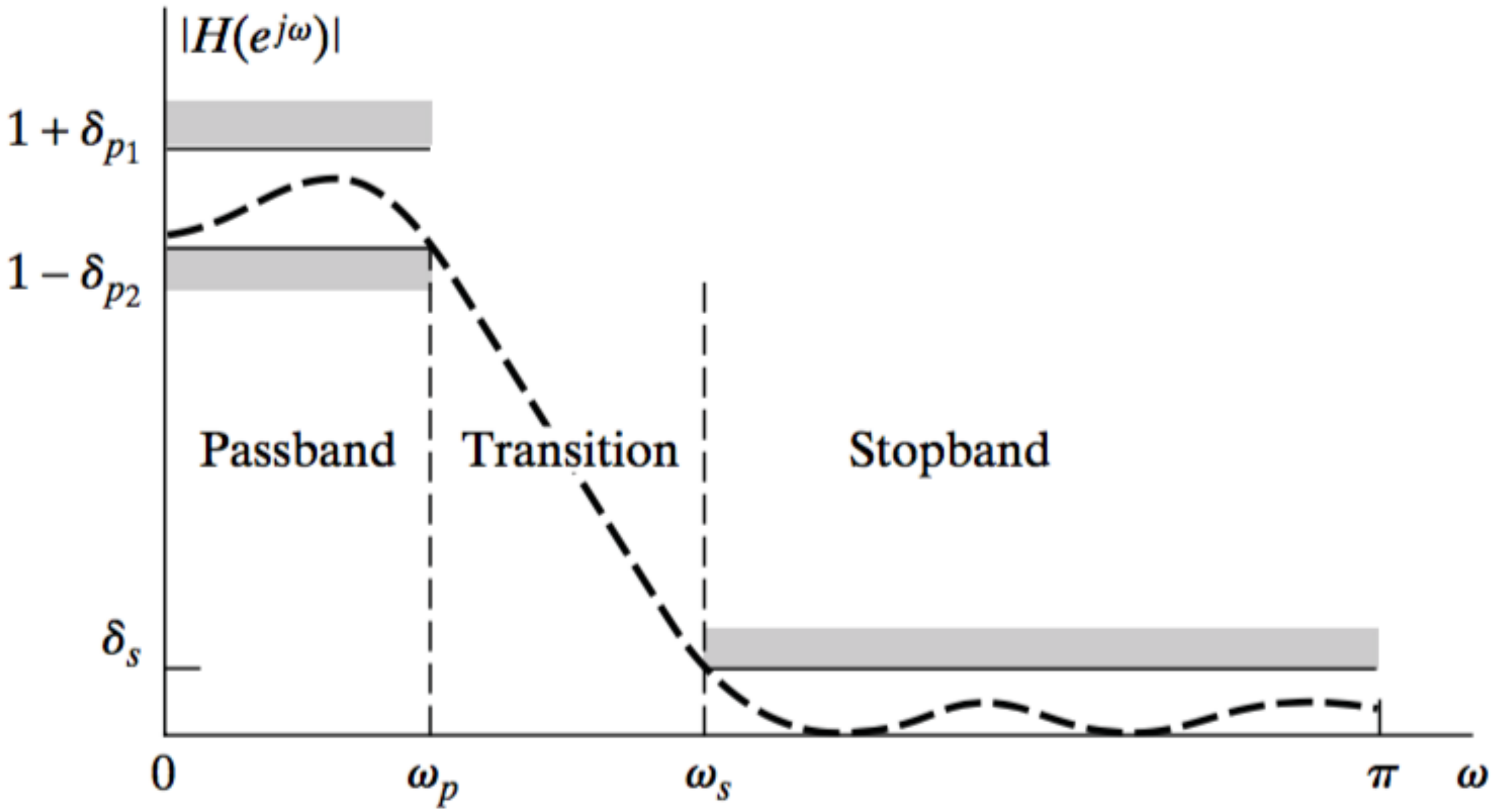
Example Problems

- Which ADC topology would you choose and why?
  - A 6b ADC with sampling frequency of 1Mhz?
  - A 14b ADC with sampling frequency of 10 khz?
  - A 8b ADC with sampling frequency of 100 khz?

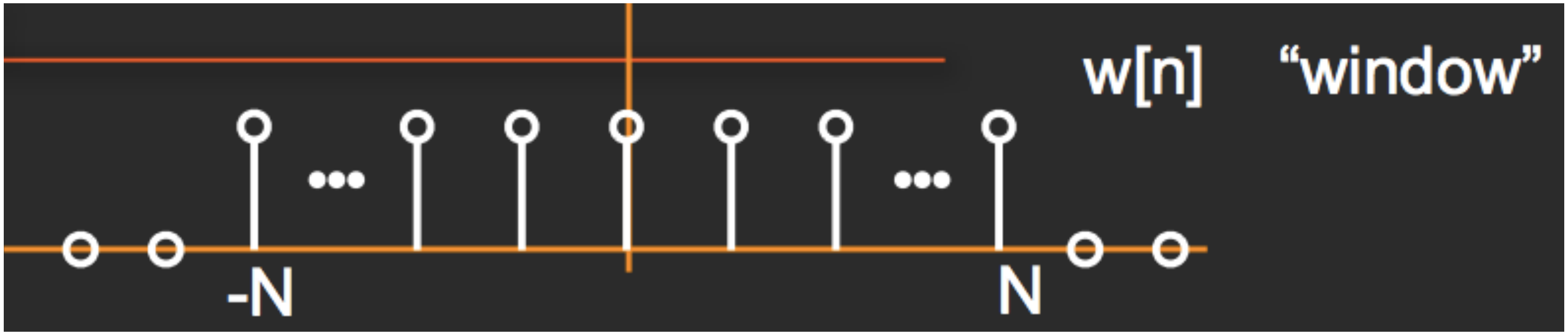
- What is the SQNR of each design?

Digital Filters

Filter Specifications



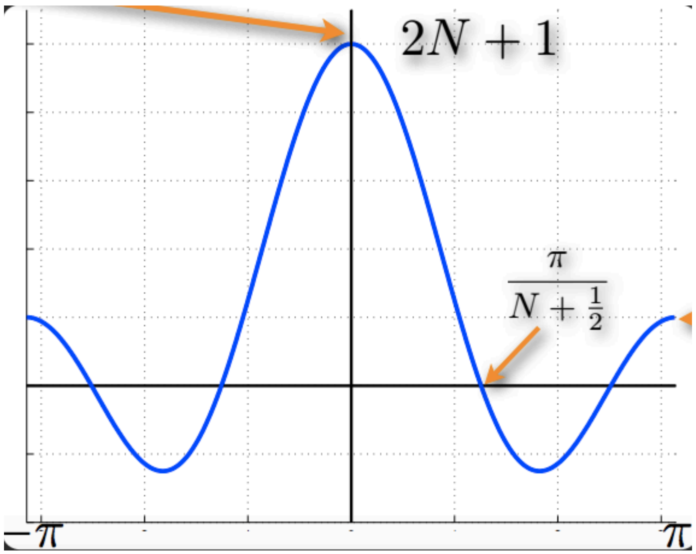
Example: Window DTFT



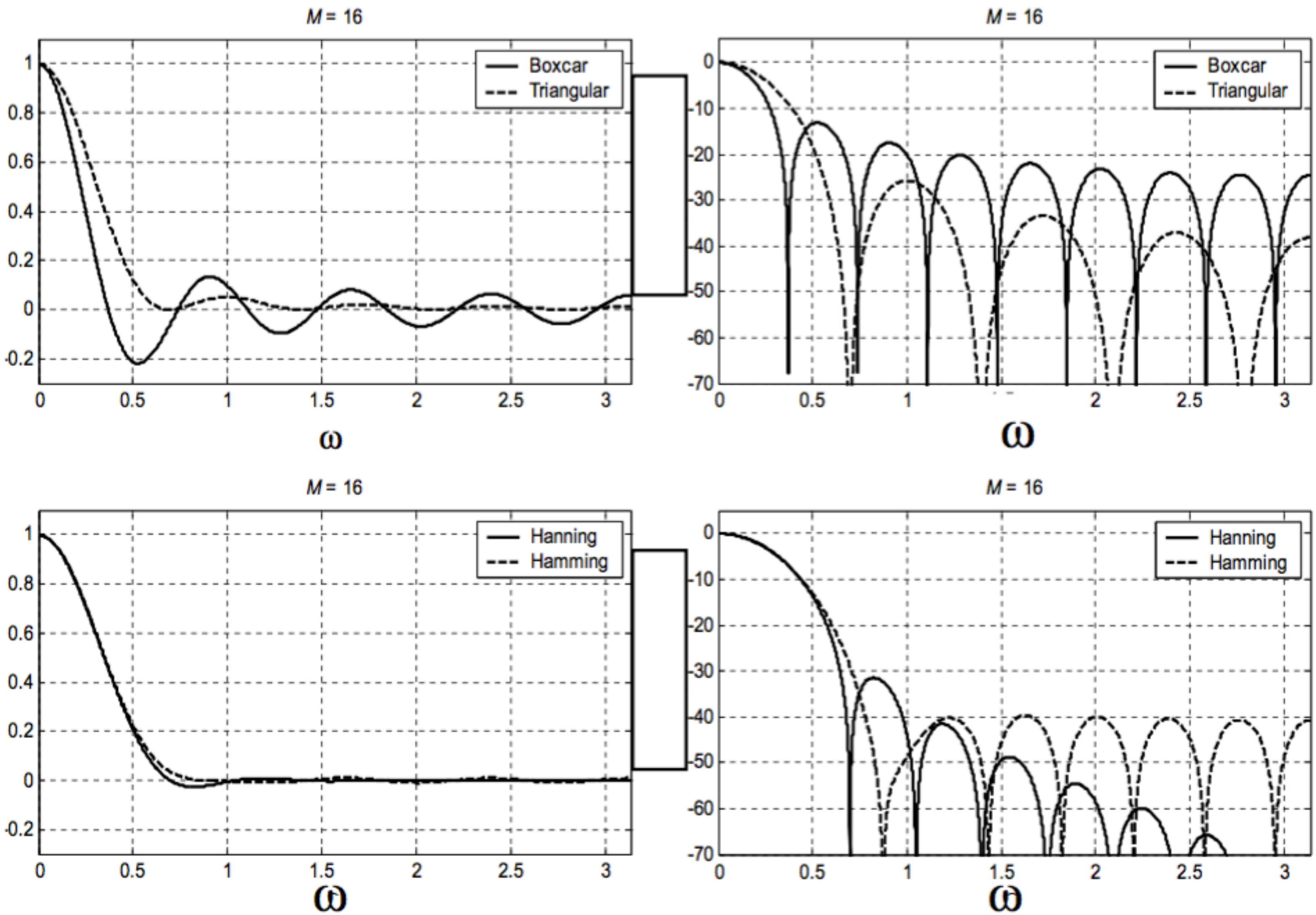
$$W(e^{j\omega}) = \sum_{k=-\infty}^{\infty} w[k] e^{-j\omega k}$$

$$= \sum_{k=-N}^{N} e^{-j\omega k}$$

$$W(e^{j\omega}) = \frac{\sin\left((N+1/2)\omega\right)}{\sin\left(\omega/2\right)}$$



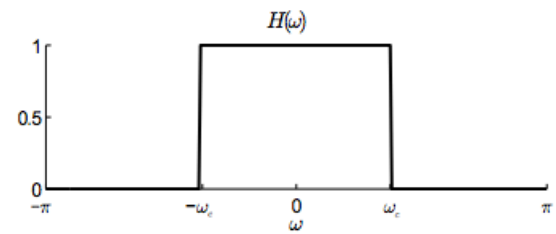
Tradeoff – Ripple vs. Transition Width



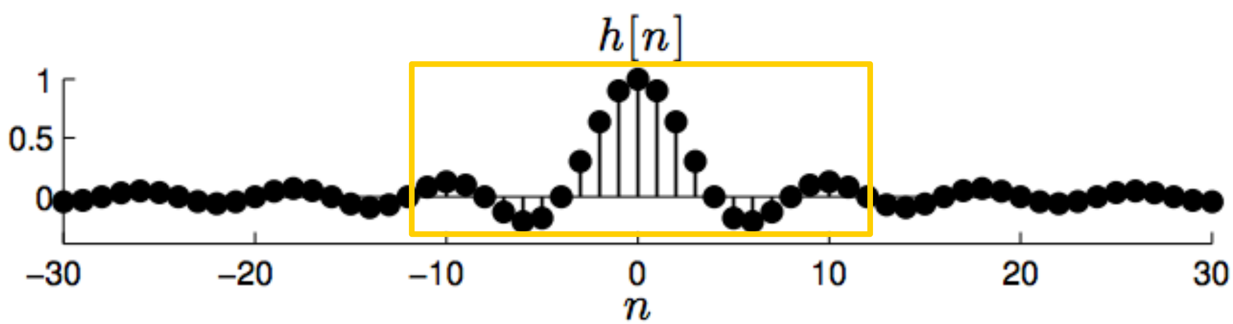
Example: Ideal Low-Pass Filter

- The frequency response $H(\omega)$ of the ideal low-pass filter passes low frequencies (near $\omega = 0$) but blocks high frequencies (near $\omega = \pm\pi$)

$$H(\omega) = \begin{cases} 1 & -\omega_c \leq |\omega| \leq \omega_c \\ 0 & \text{otherwise} \end{cases}$$



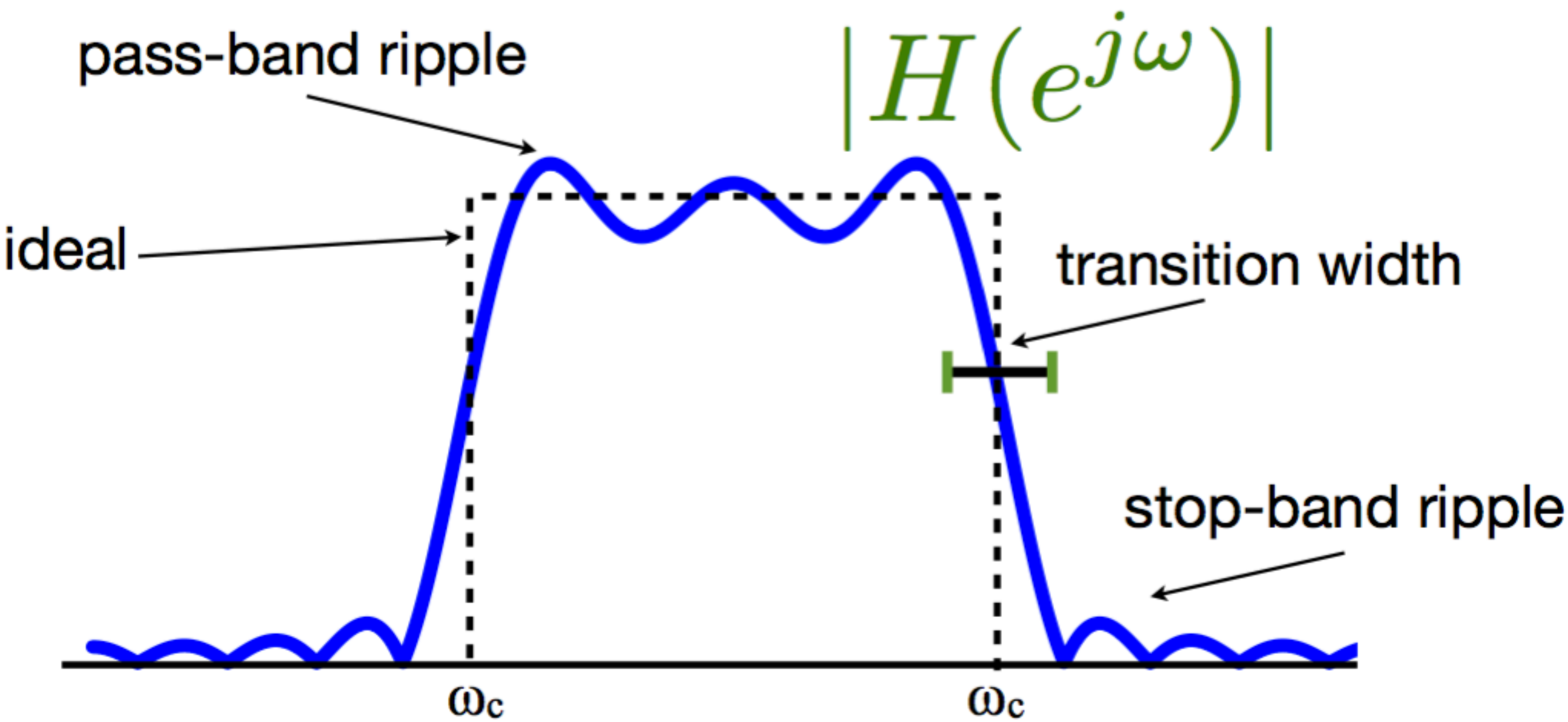
$$h[n] \;=\; 2\omega_c \frac{\sin(\omega_c n)}{\omega_c n}$$



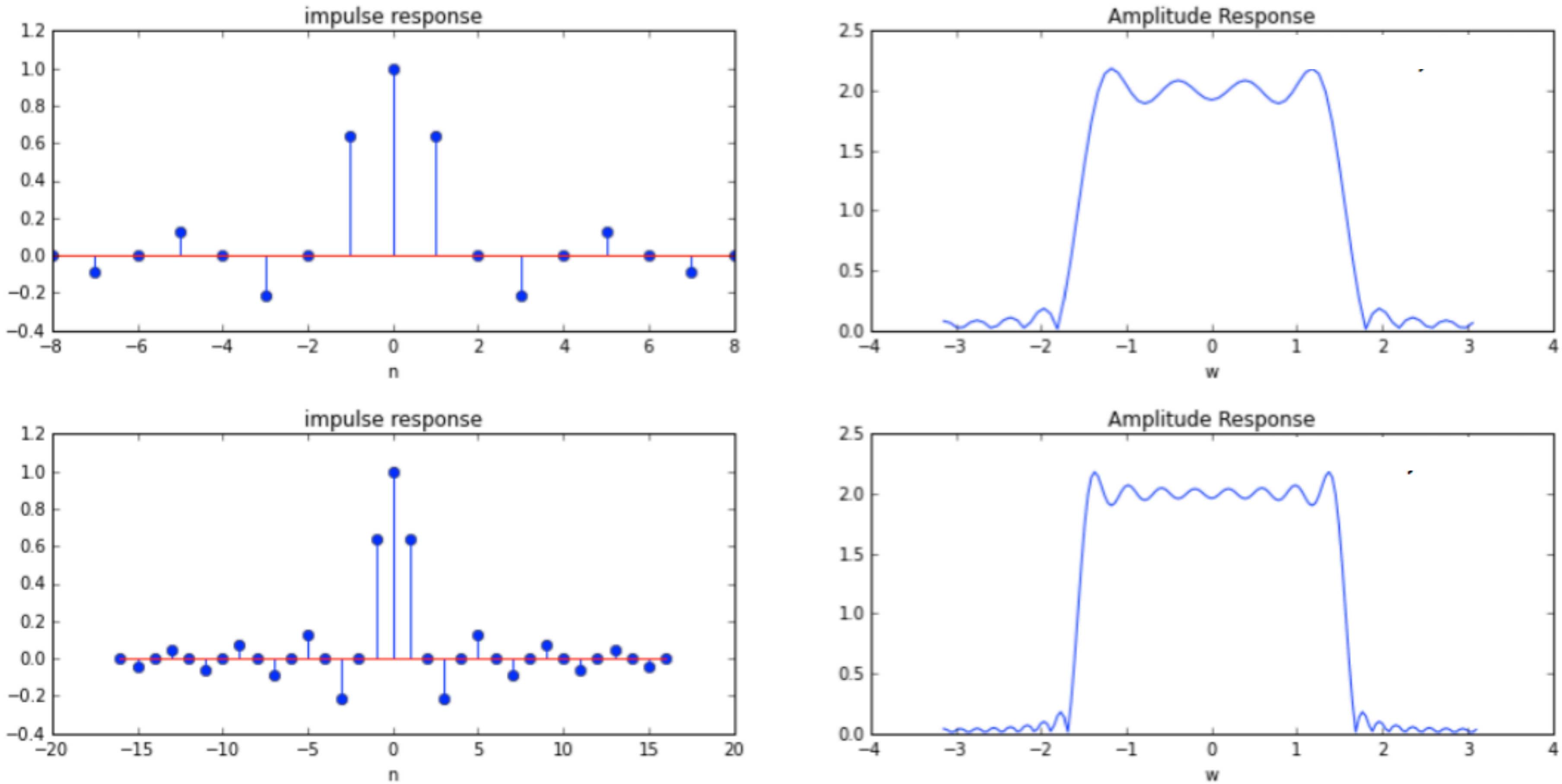
Truncate and shift

$$h_{LP}[n] = w_N[n-N] \cdot h[n-N]$$

FIR Design by Windowing



Time Bandwidth Product



Example Problems

- Compute the DTFT of a simple(ish) window function

- Given a filter frequency response, what is the transition bandwidth? Pass ripple? Stop ripple?

- Create a `freq` and `gain` array for a given frequency spectrum requirement.

- Review Lab 8 and 9! Lots of problems there

Admin

- Quiz 2
  - In class Monday 11/21
  - 60 minutes, start at exactly 5:15pm
  - Calculators allowed (non-cell phone)
  - 8.5x11 cheat sheet allowed
  - Cumulative, but will focus on lectures 9-16 and labs 6, 8 and 9 (ADC and filter labs)
- No lecture or lab next week TW because of Tgiving
- M 11/28 lecture cancelled, OH instead
- T 11/29 Free lab time for project
- Guest Lectures - I expect your attendance