ESE 3400: Medical Devices Lab

Lec 8: November 8, 2023

Compressive Sampling, Wireless Communication

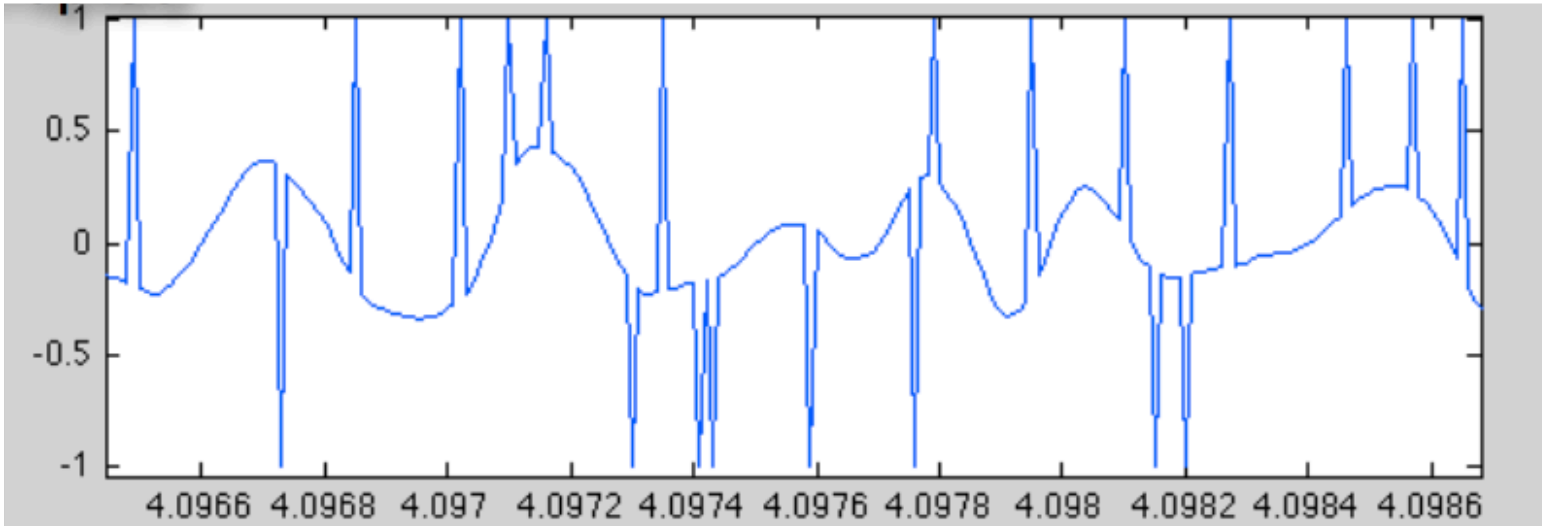
Today

- Median Filter
- Compressive Sampling/Sensing
- RF ID technology
- Wireless Communication
- Quiz 2 Review

Non-Linear System Example

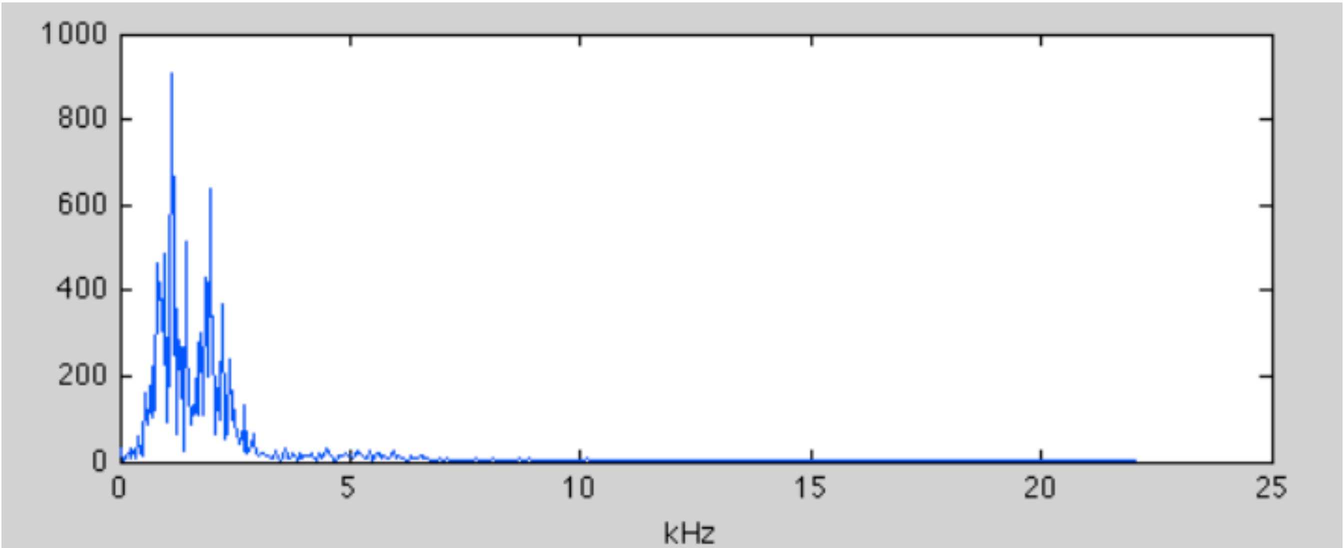
- Median Filter
  - y[n]=MED{x[n-k], …x[n+k]}

  - Let k=1
  - y[n]=MED{x[n-1], x[n], x[n+1]}

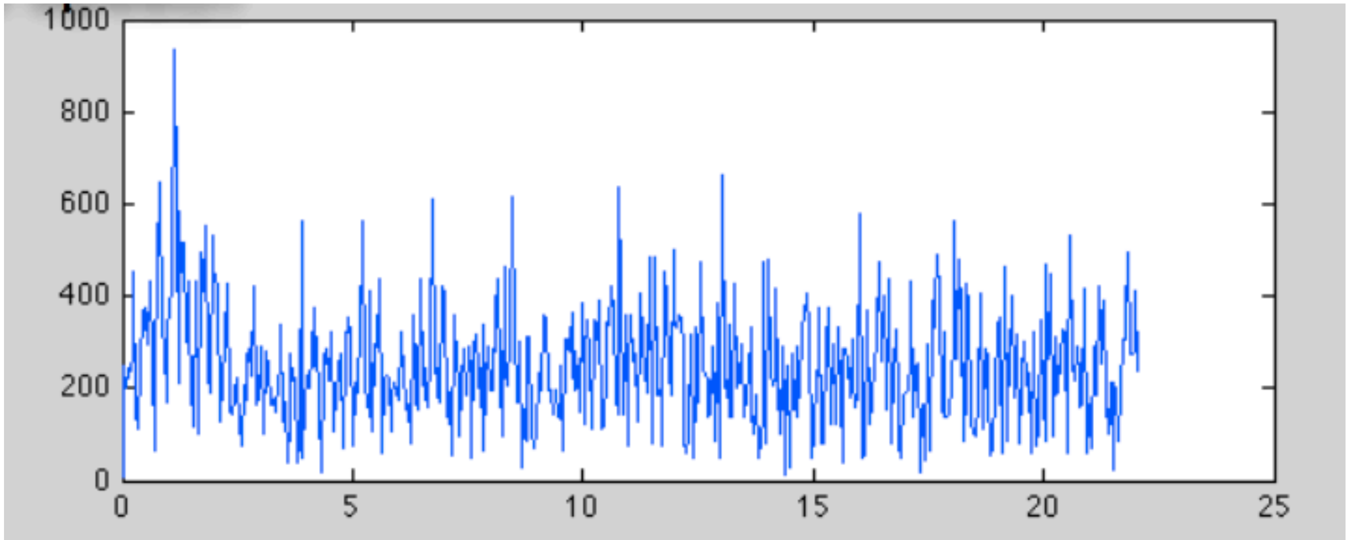


Spectrum of Speech

Speech

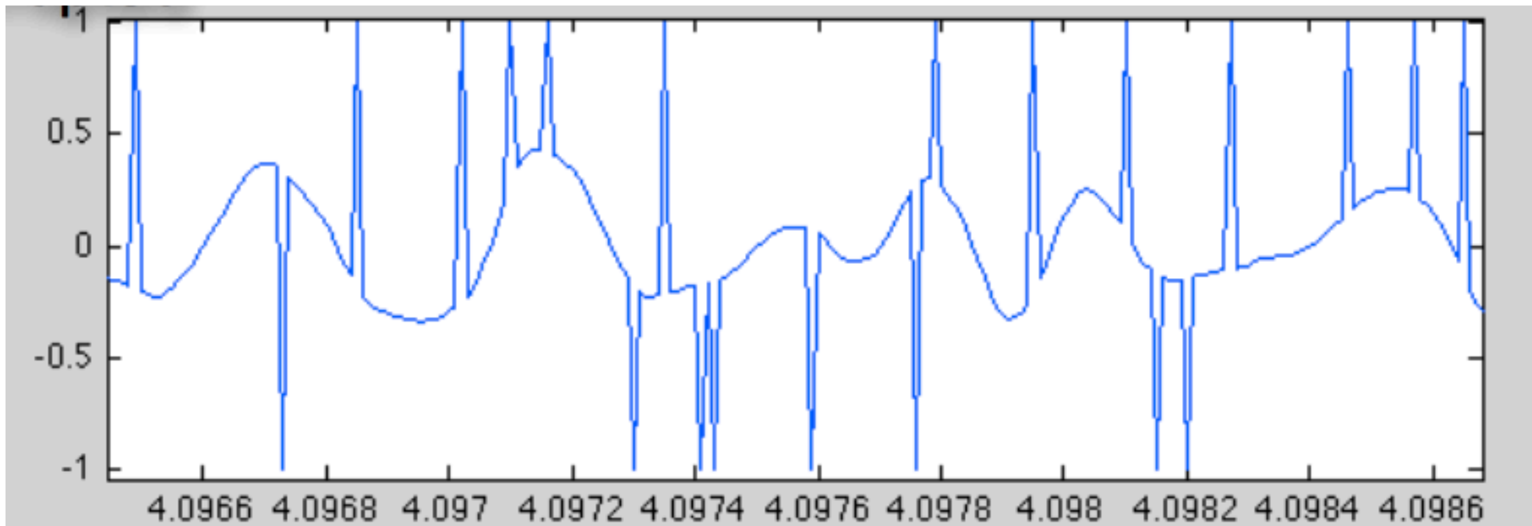


Corrupted Speech

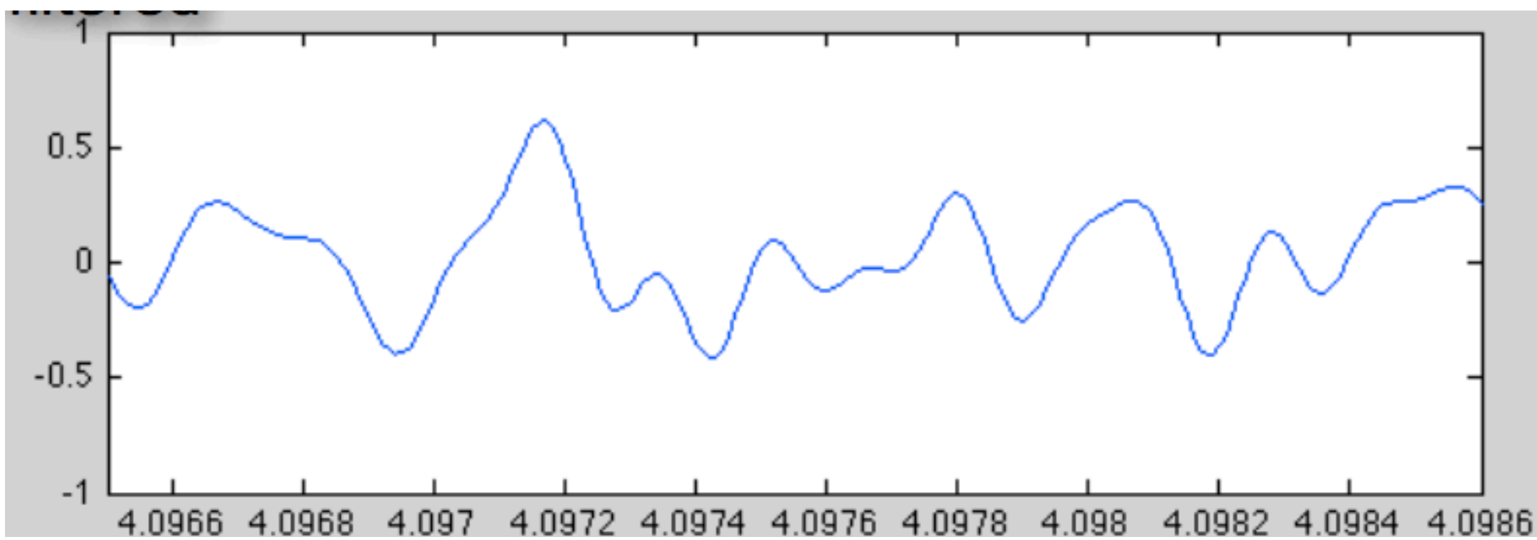


Low Pass Filtering

Corrupted Speech

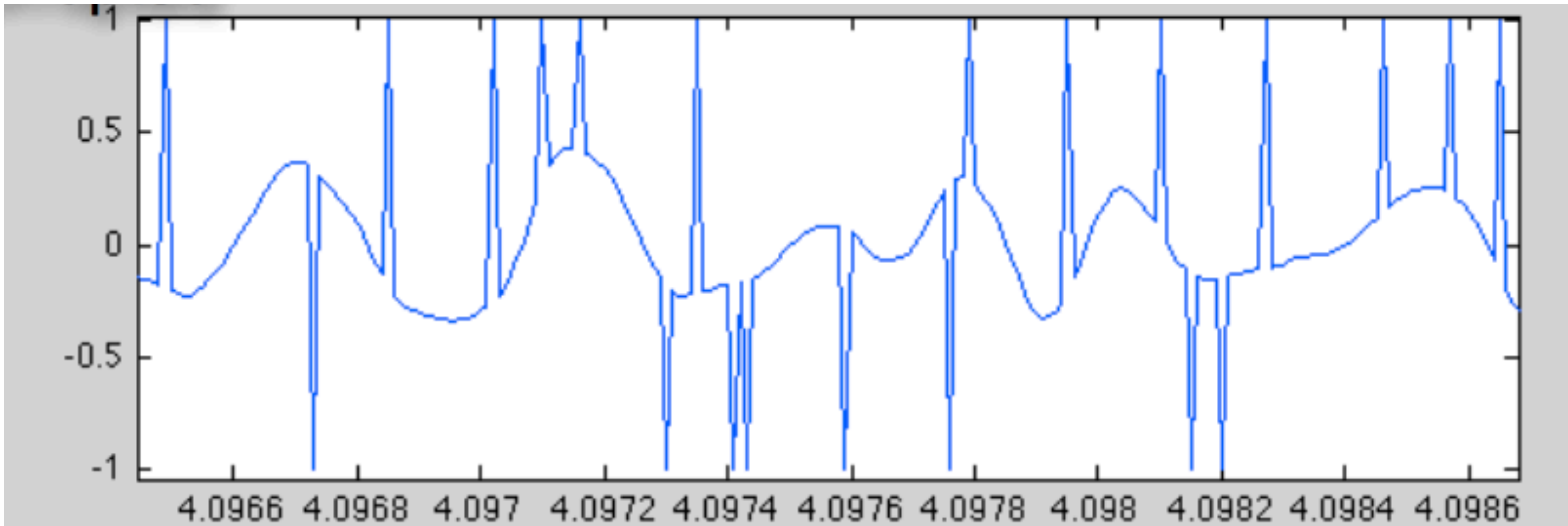


LP-Filtered Speech

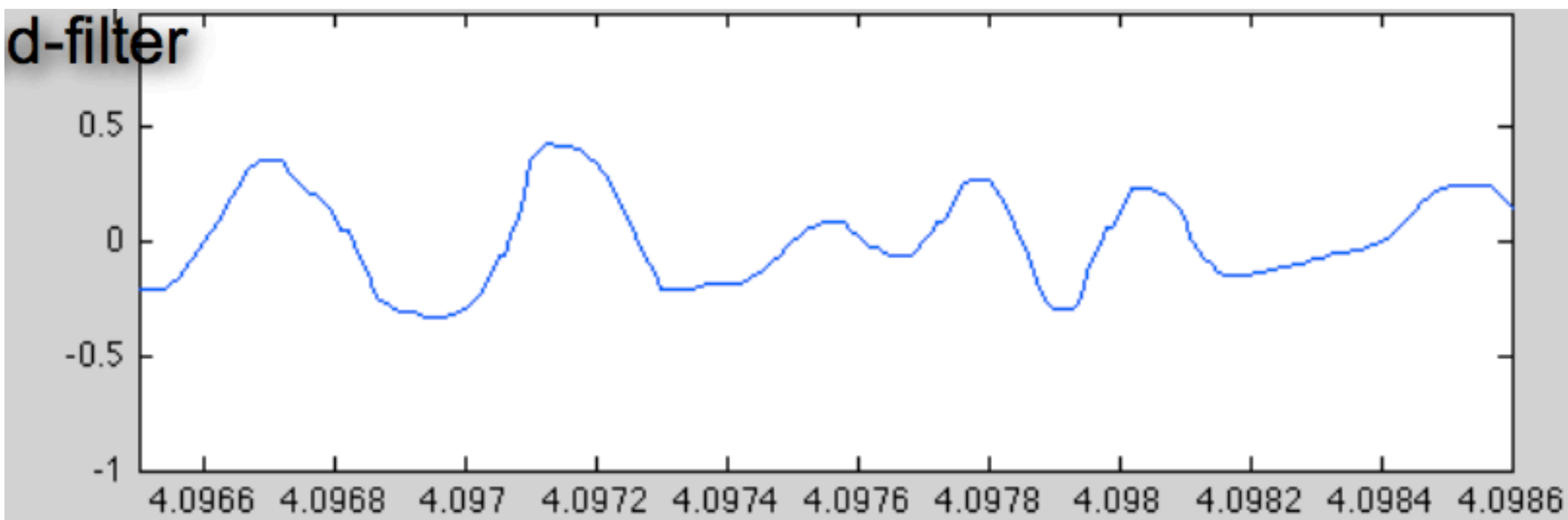


Median Filtering

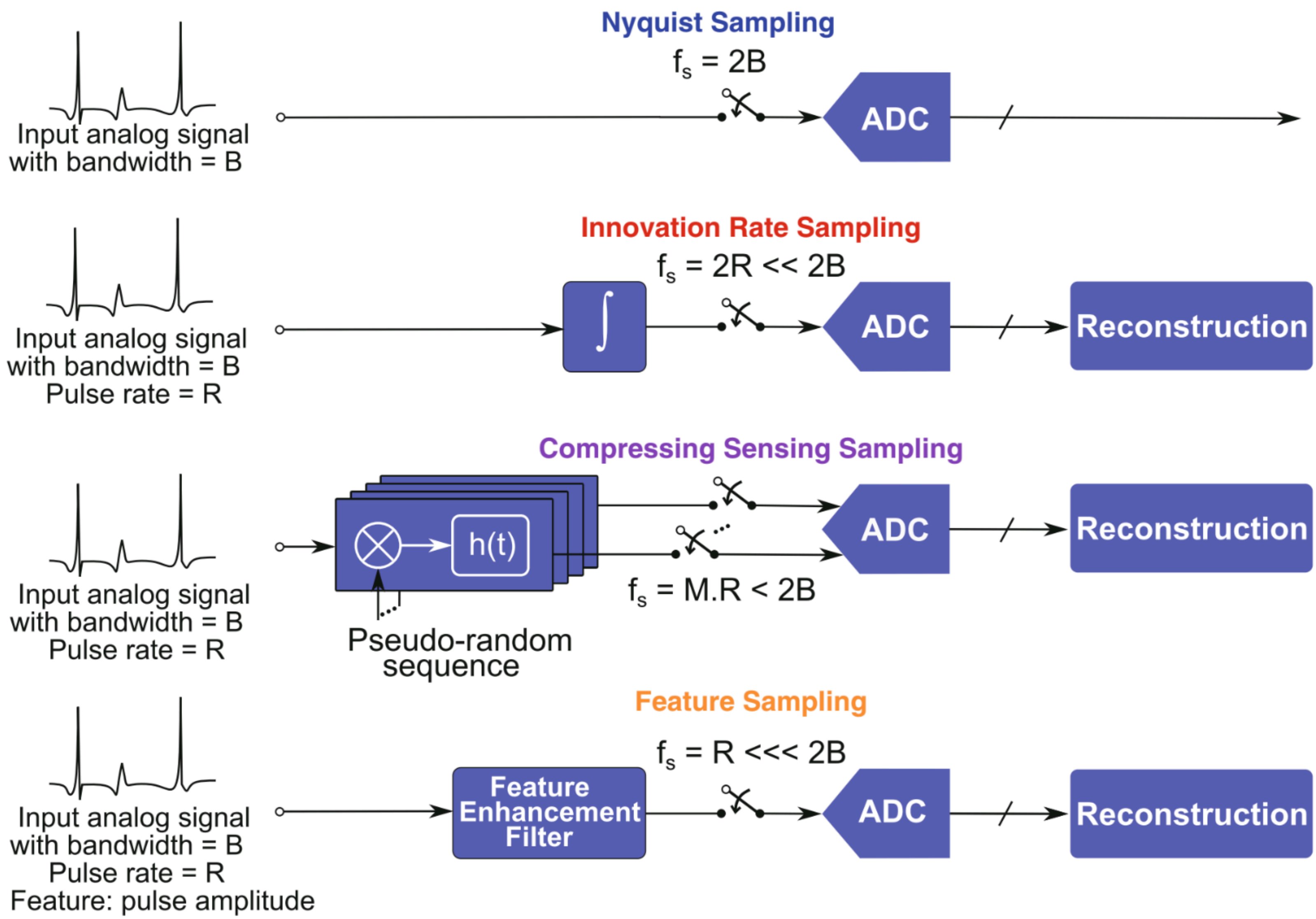
Corrupted Speech



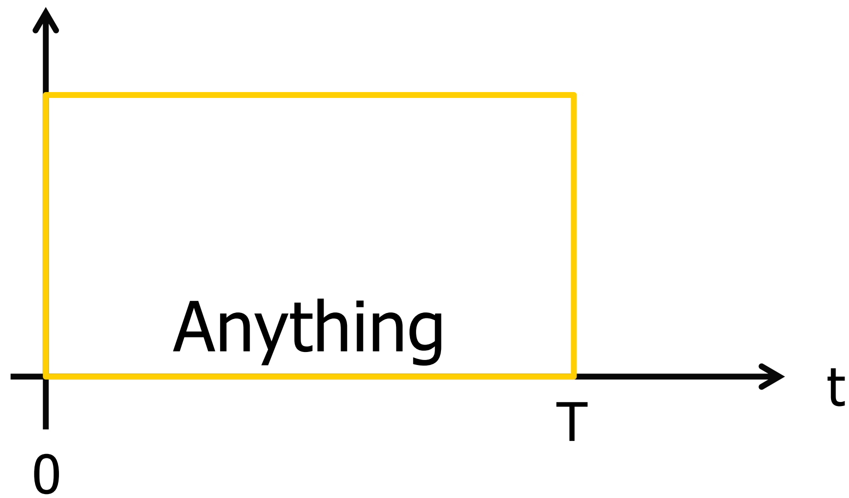
Med-Filter Speech



Sampling Architectures

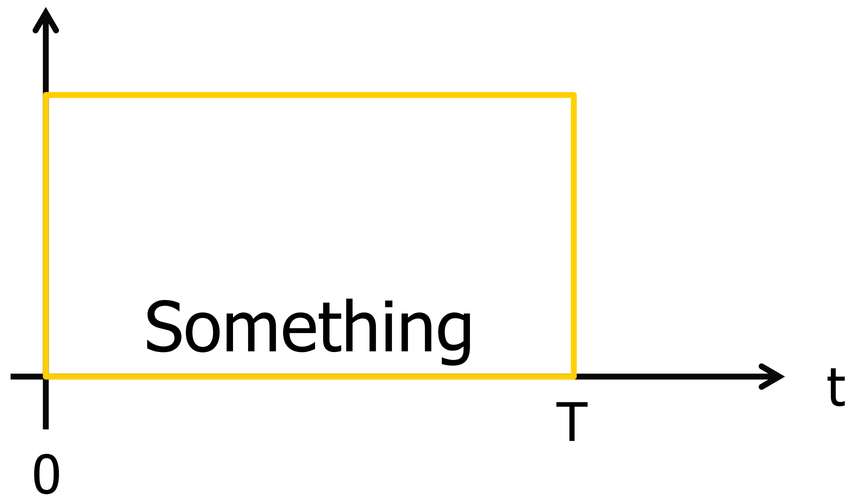


Compressive Sampling



- What is the rate you need to sample at?
  - At least Nyquist

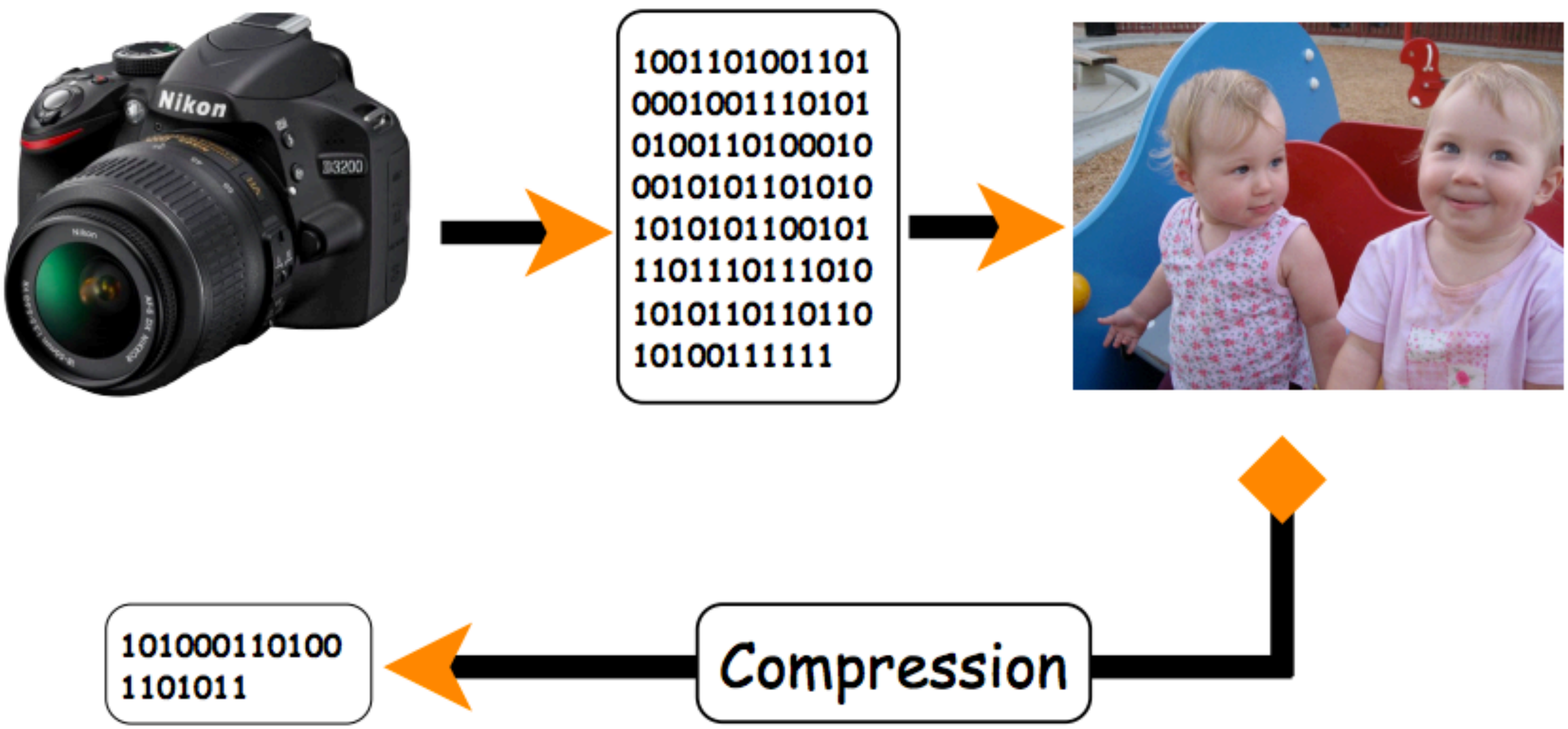
Compressive Sampling



- What is the rate you need to sample at?
  - Maybe less than Nyquist…

First: Compression

- Standard approach
  - First collect, then compress
    - Throw away unnecessary data

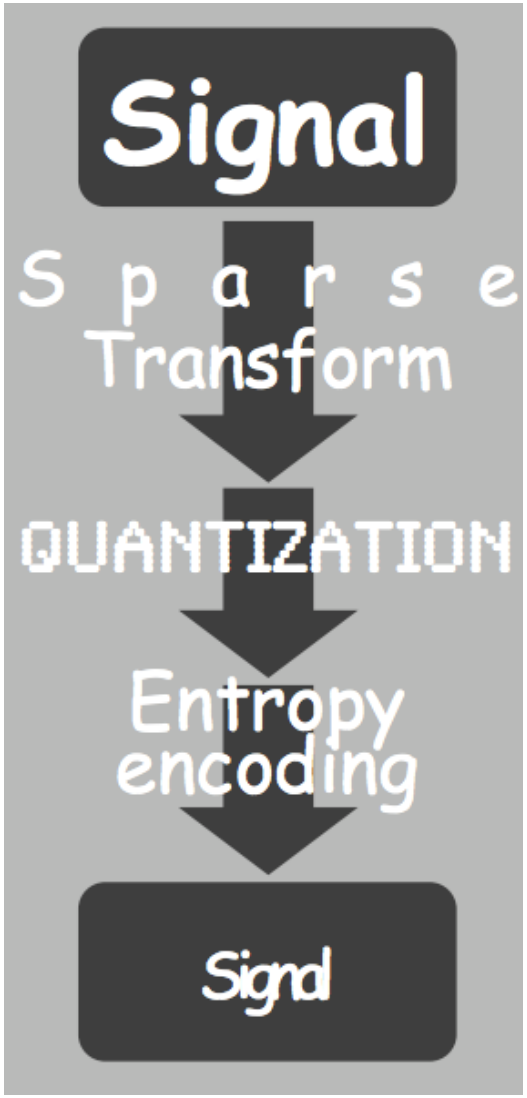


First: Compression

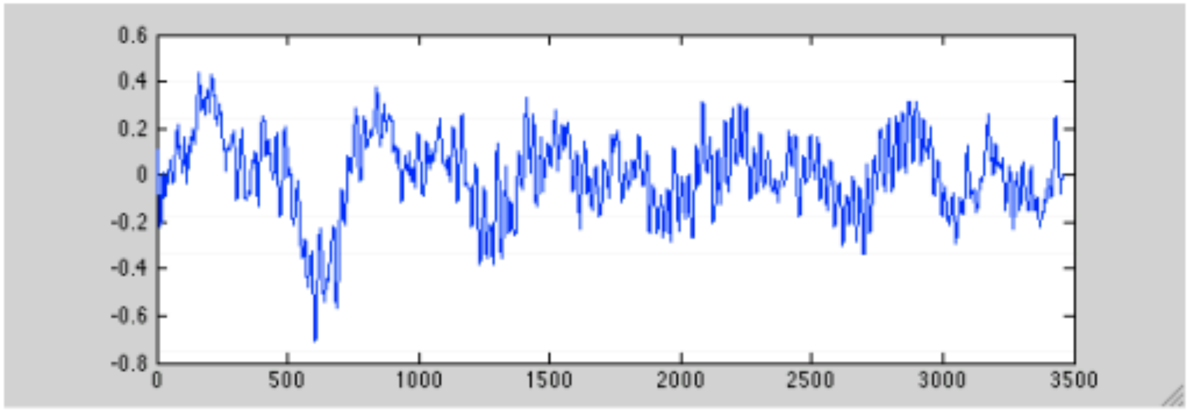
- Examples
  - Audio – 10x
    - Raw audio: 44.1kHz, 16bit, stereo = 1378 Kbit/sec
    - MP3: 44.1kHz, 16 bit, stereo = 128 Kbit/sec
  - Images – 22x
    - Raw image (RGB): 24bit/pixel
    - JPEG: 1280x960, normal = 1.09bit/pixel
  - Videos – 75x
    - Raw Video: (480x360)p/frame x 24b/p x 24frames/s + 44.1kHz x 16b x 2 = 98,578 Kbit/s
    - MPEG4: 1300 Kbit/s

First: Compression

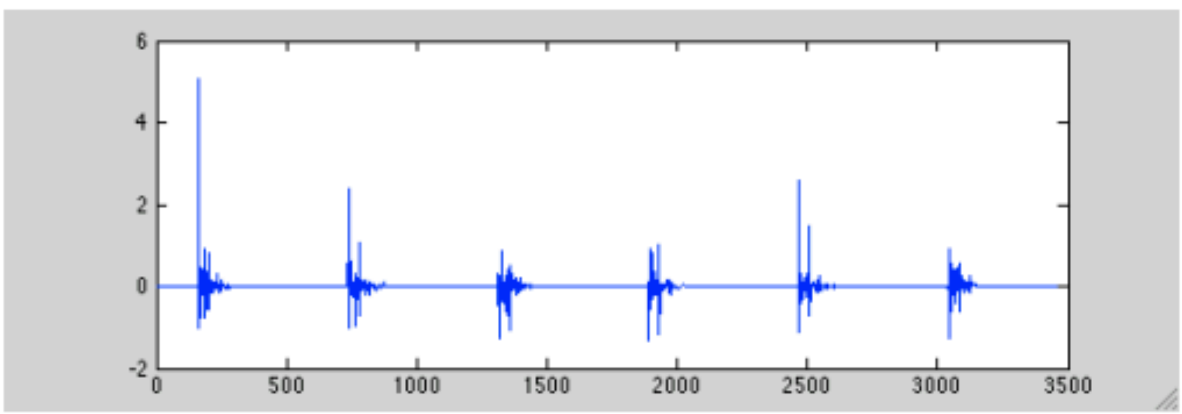
- Almost all compression algorithm use transform coding
  - mp3: DCT
  - JPEG: DCT
  - JPEG2000: Wavelet
  - MPEG: DCT & time-difference



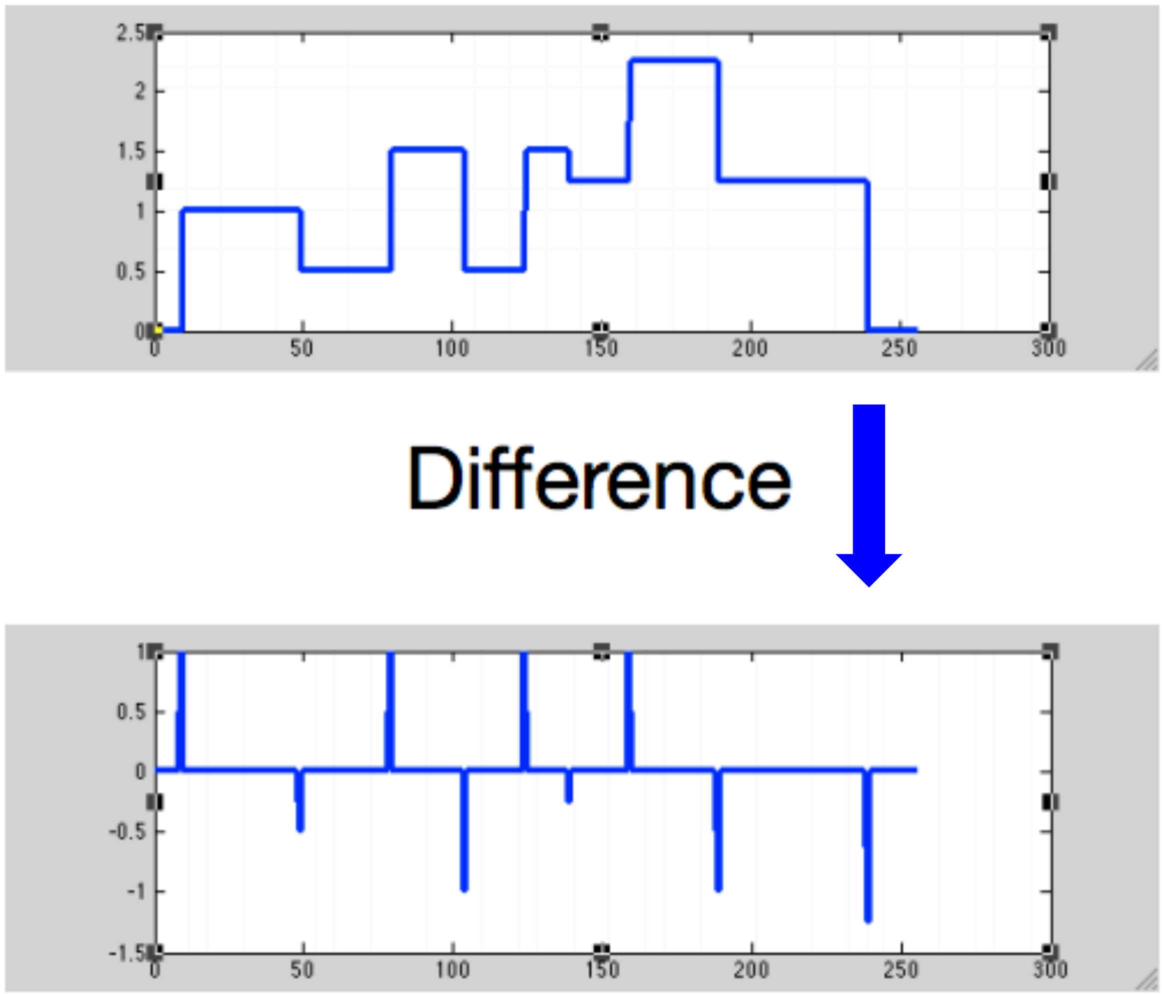
Sparse Transform



DCT



Sparse Transform

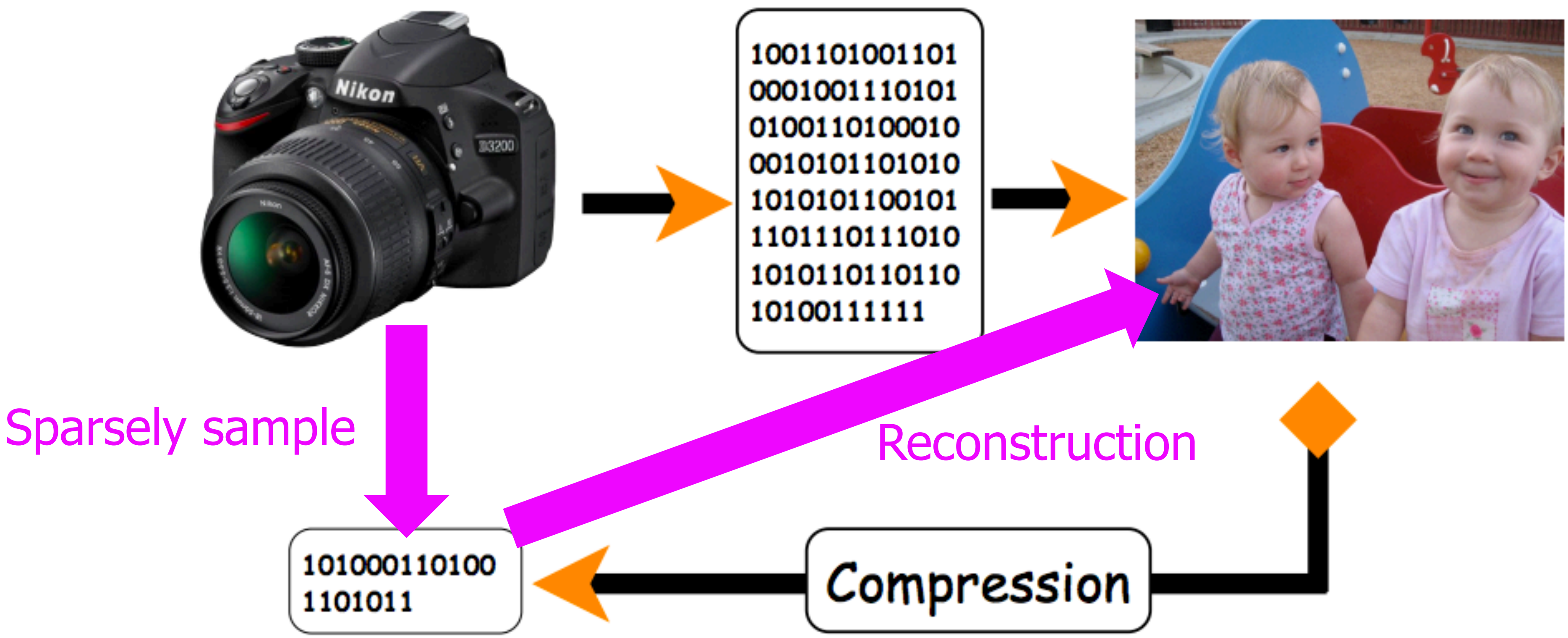


Signal Processing Trends

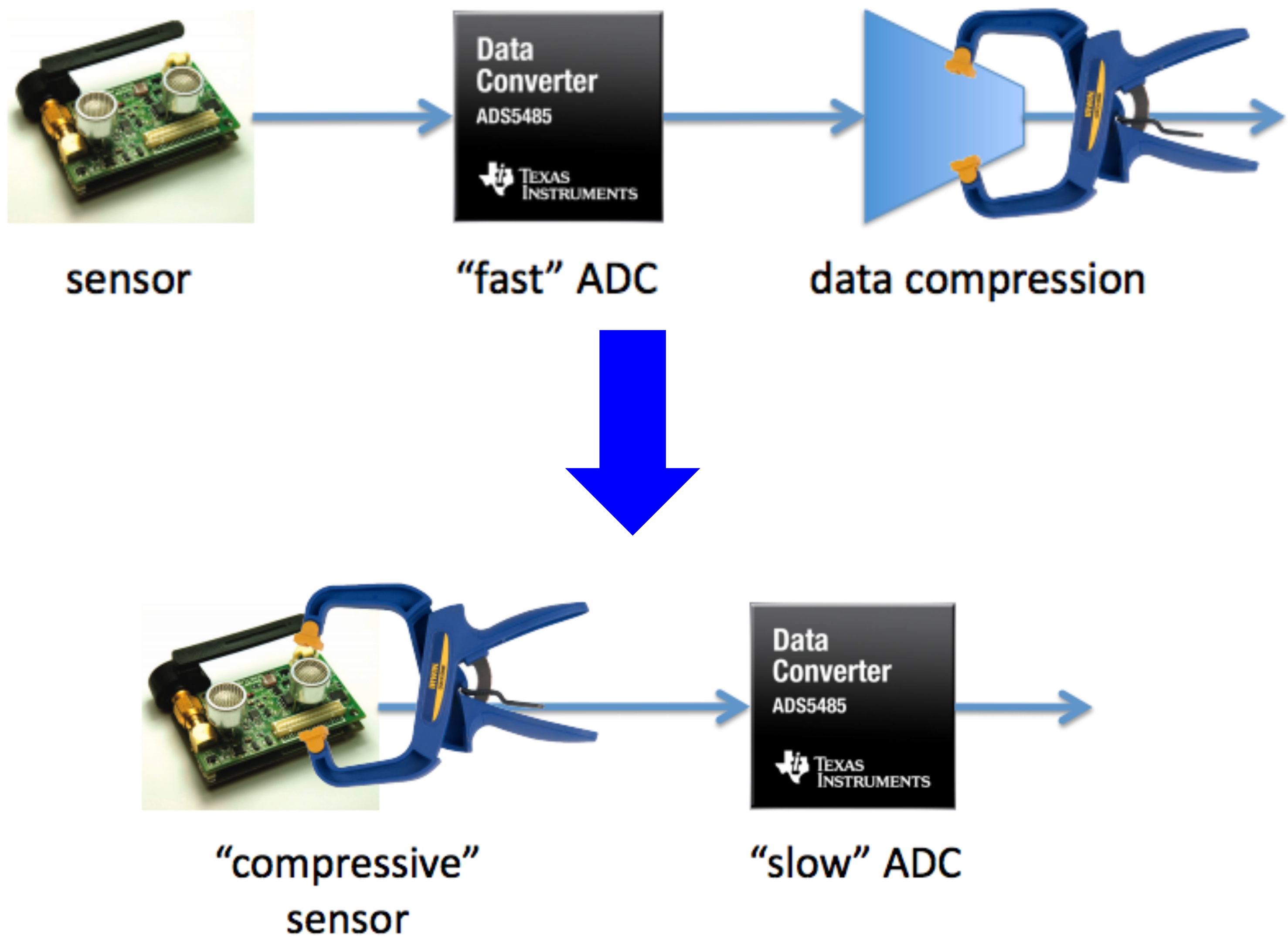
- Traditional DSP → sample first, ask questions later
- Explosion in sensor technology/ubiquity has caused two trends:
  - Physical capabilities of hardware are being stressed, increasing speed/resolution becoming expensive
    - gigahertz+ analog-to-digital conversion
    - accelerated MRI
    - industrial imaging
  - Deluge of data
    - camera arrays and networks, multi-view target databases, streaming video...
- Compressive Sensing → sample smarter, not faster

Compressive Sensing/Sampling

- Standard approach
  - First collect, then compress
    - Throw away unnecessary data

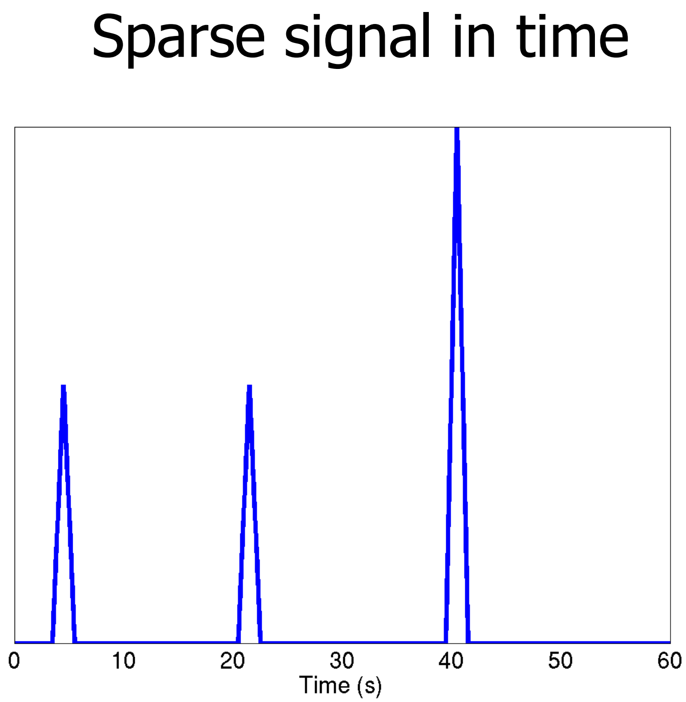


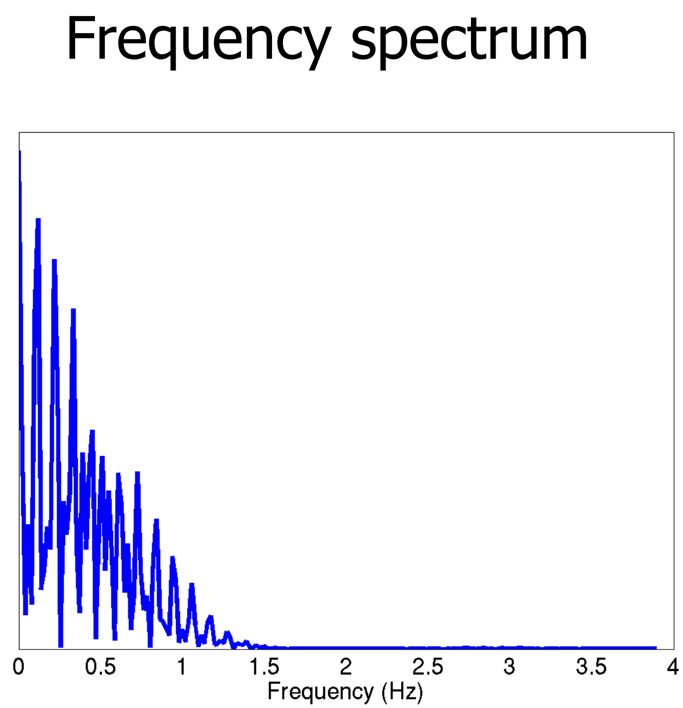
Sensing to Data



Compressive Sampling

- Sample at lower than the Nyquist rate and still accurately recover the signal, and in most cases *exactly* recover





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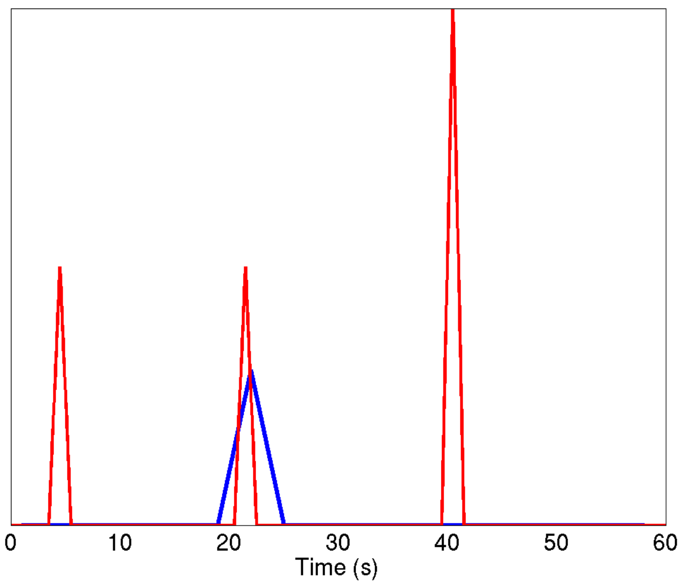
Undersampled in time



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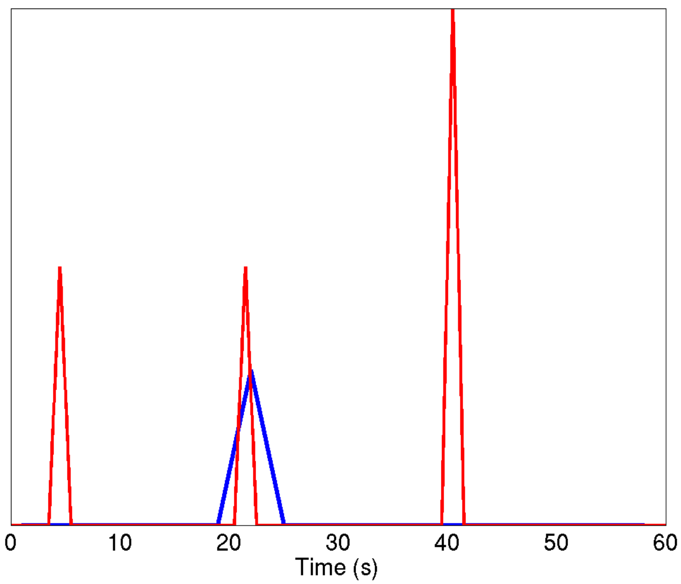

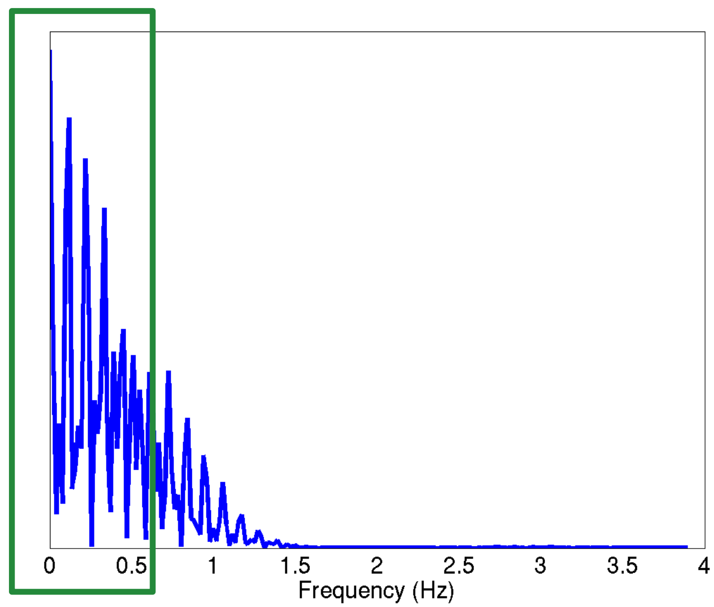

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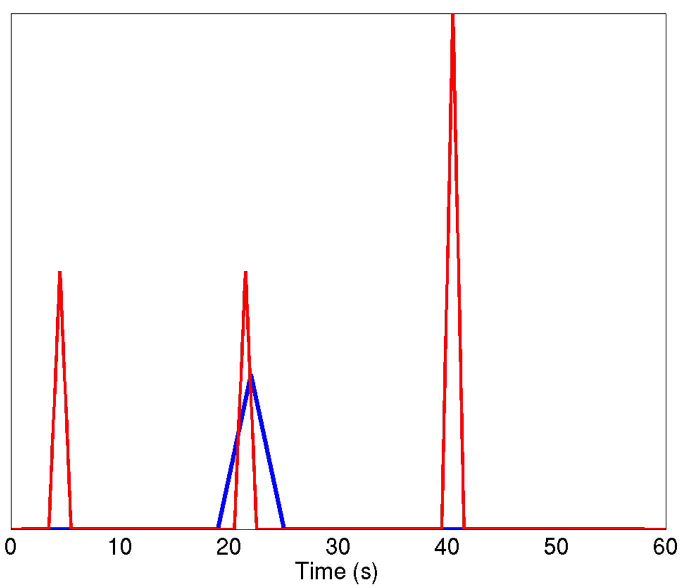
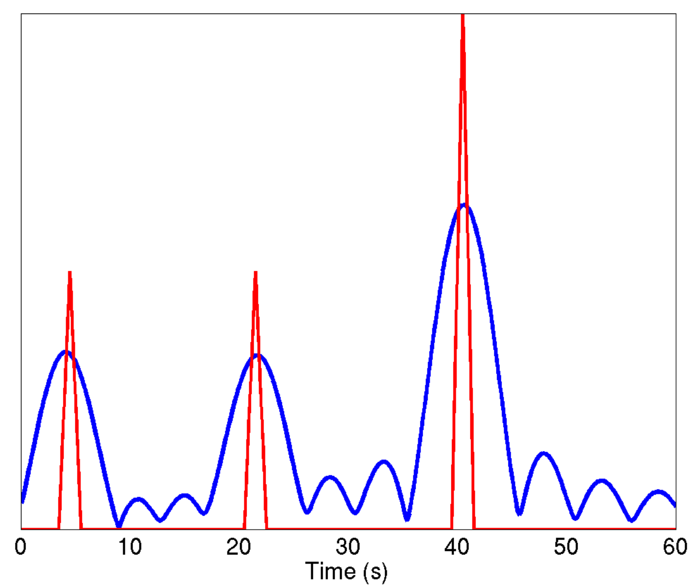


Undersampled in frequency
(reconstructed in time with IFFT)



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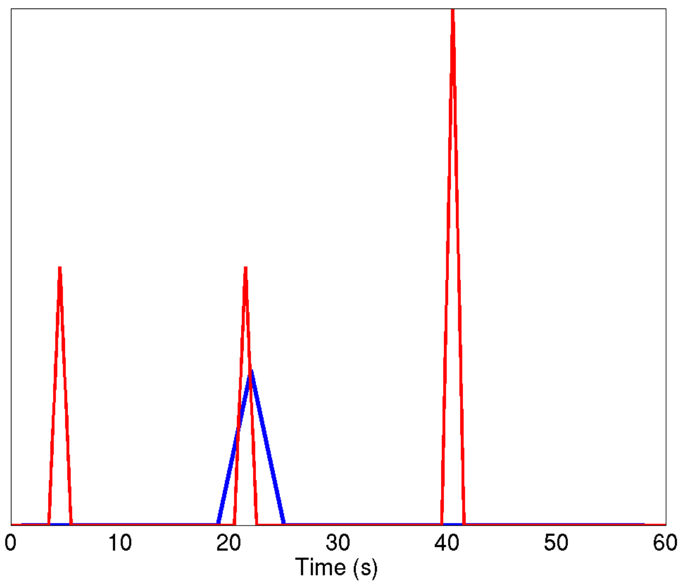

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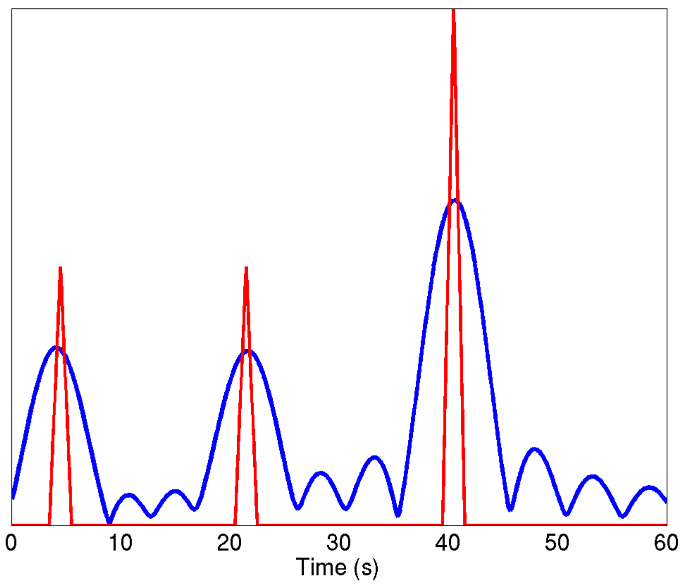
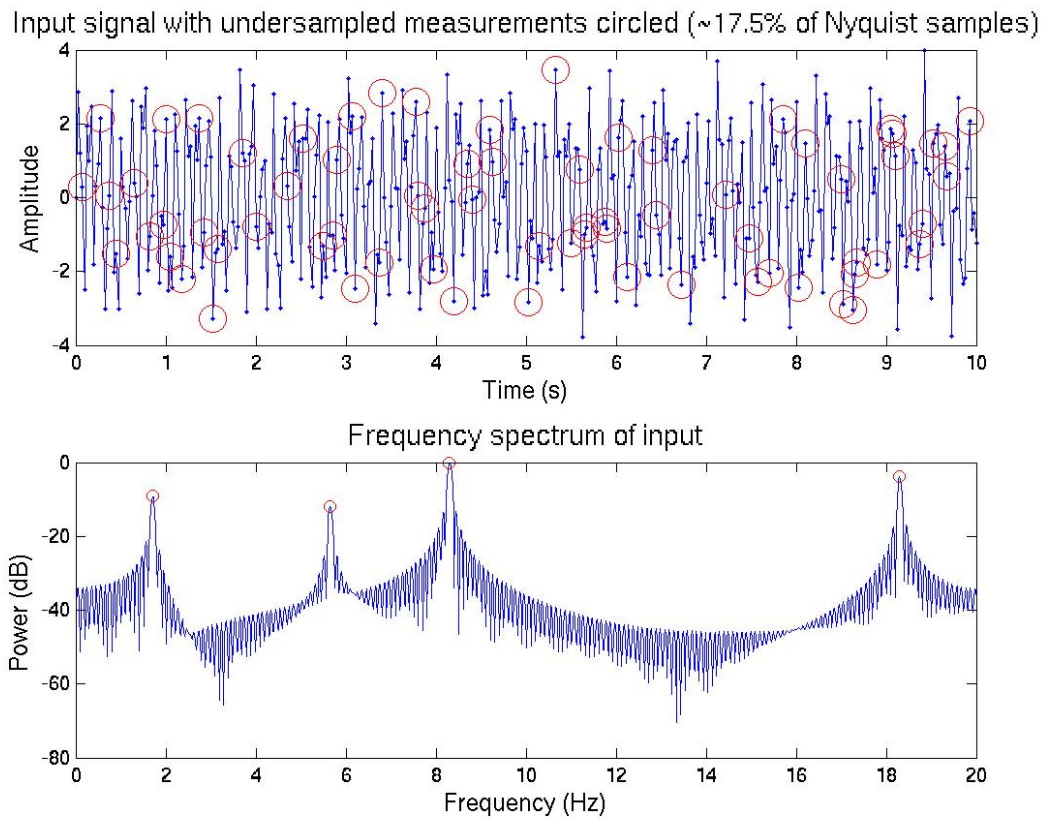


Requires sparsity and incoherent sampling

Compressive Sampling

$$\hat{f}(\omega) = \sum_{i=1}^{K} \alpha_i \delta(\omega_i - \omega) \overset{\mathcal{F}}{\Leftrightarrow} f(t) = \sum_{i=1}^{K} \alpha_i e^{i\omega_i t}$$

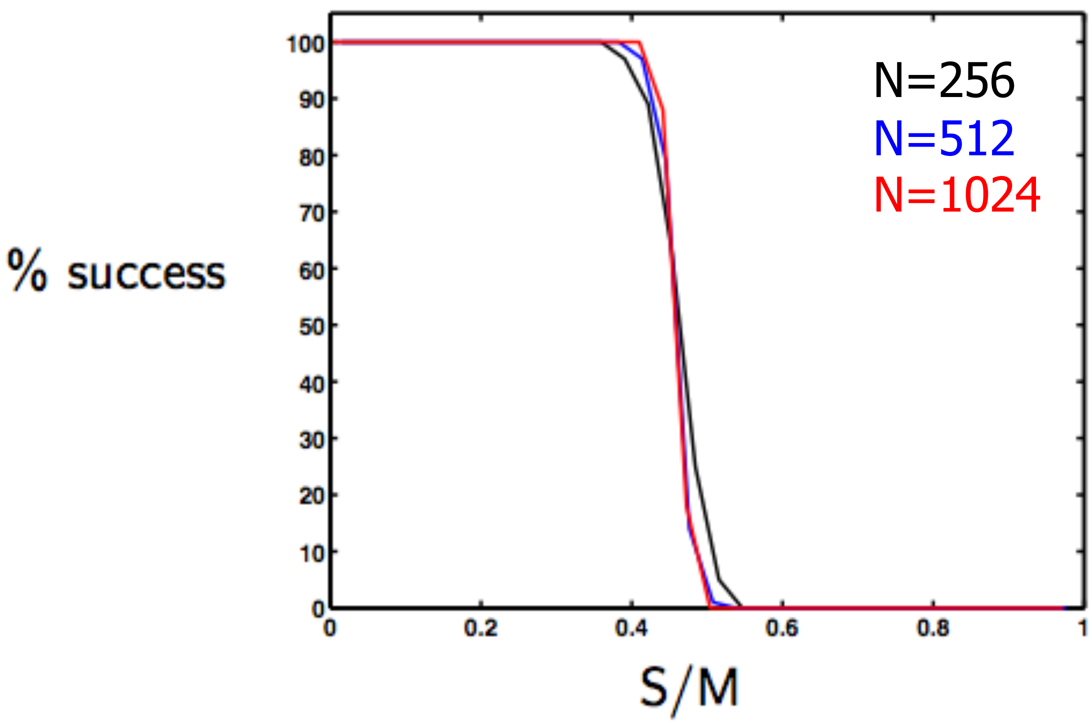


- Sense signal M times
- Recover with linear program

$$\min \sum_{\omega} |\hat{g}(\omega)| \quad \text{subject to} \quad g(t_m) = f(t_m)\,, \quad m = 1, \ldots, M$$

Numerical Recovery Curves

- Sense S-sparse signal of length N randomly M times



  - In practice, perfect recovery occurs when $M \approx 2S$ for $N \approx 1000$

A Non-Linear Sampling Theorem

- Exact Recovery Theorem (Candès, R, Tao, 2004):
  - Select M sample locations $\{t_m\}$ "at random" with

$$M \geq \mathrm{Const} \cdot S \log N$$

- Take time-domain samples (measurements)

$$y_m = x_0(t_m)$$

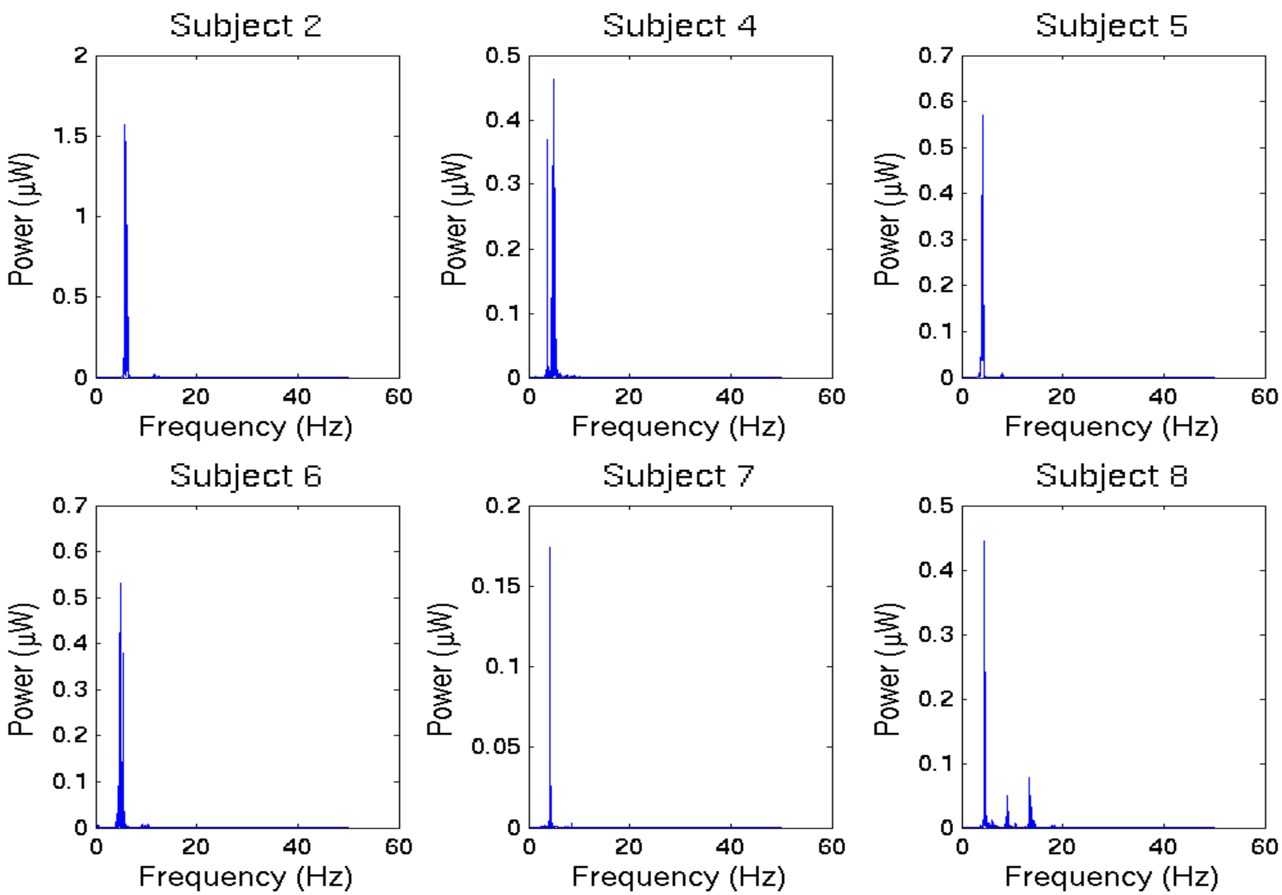
- Solve

$$\min_x \|\hat{x}\|_{\ell_1} \quad \textbf{subject to} \quad x(t_m) = y_m, \quad m = 1, \ldots, M$$

- Solution is exactly recovered signal with extremely high probability

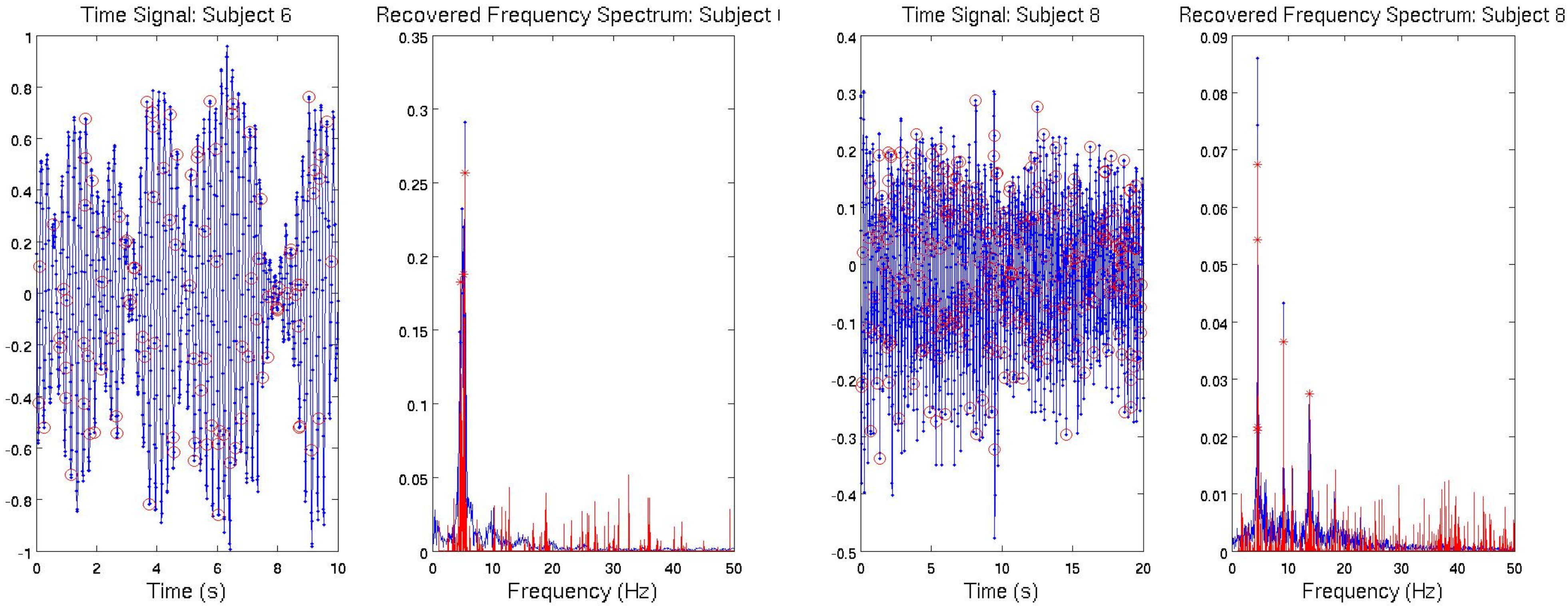
$$\boxed{M > C \cdot \mu^2(\Phi, \Psi) \cdot S \cdot \log N}$$

Biometric Example: Parkinson's Tremors

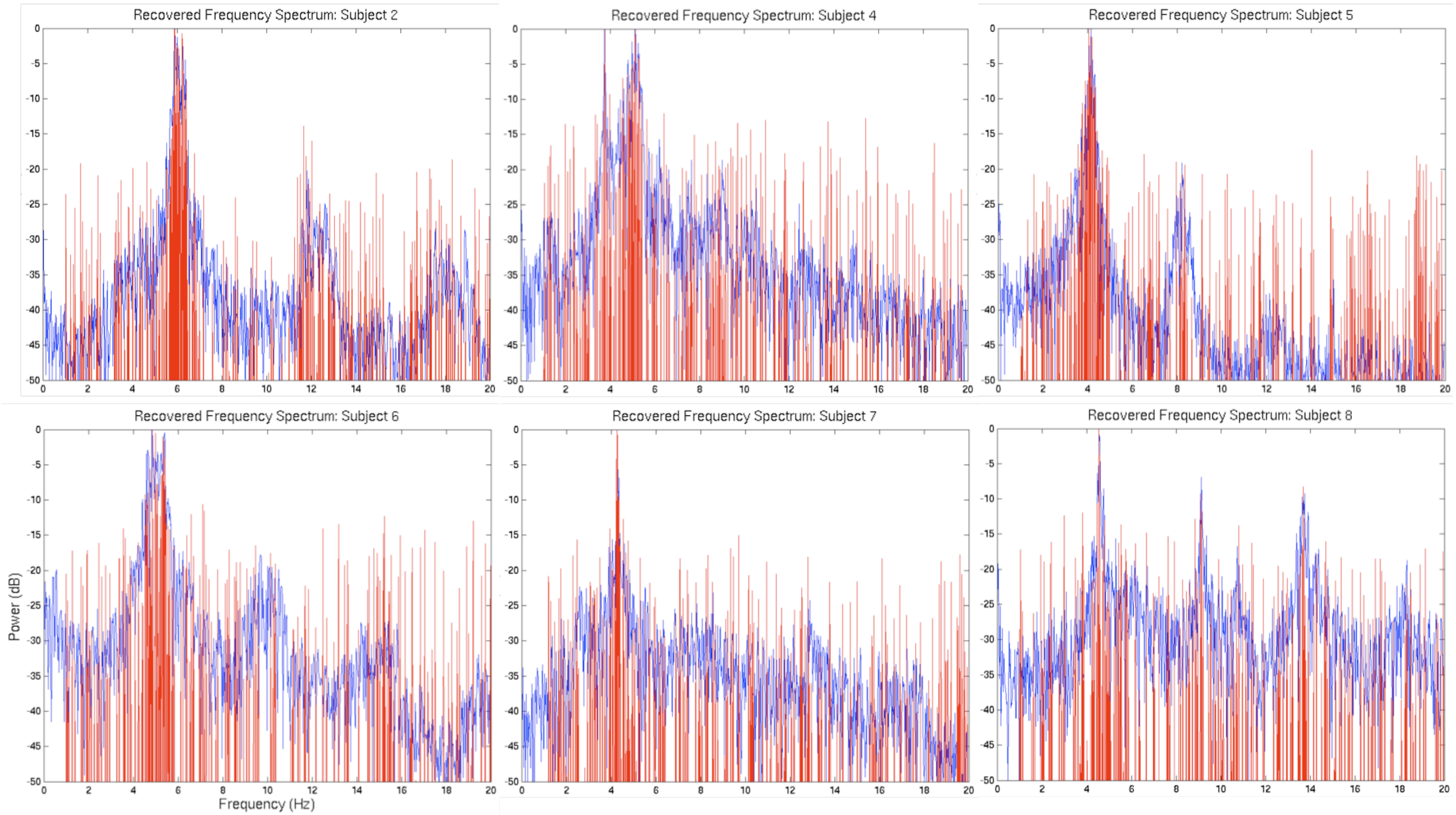


- 6 Subjects of real tremor data
  - collected using low intensity velocity-transducing laser recording aimed at reflective tape attached to the subjects' finger recording the finger velocity
  - All show Parkinson's tremor in the 4-6 Hz range.
  - Subject 8 shows activity at two higher frequencies
  - Subject 4 appears to have two tremors very close to each other in frequency

Compressive Sampling: Real Data

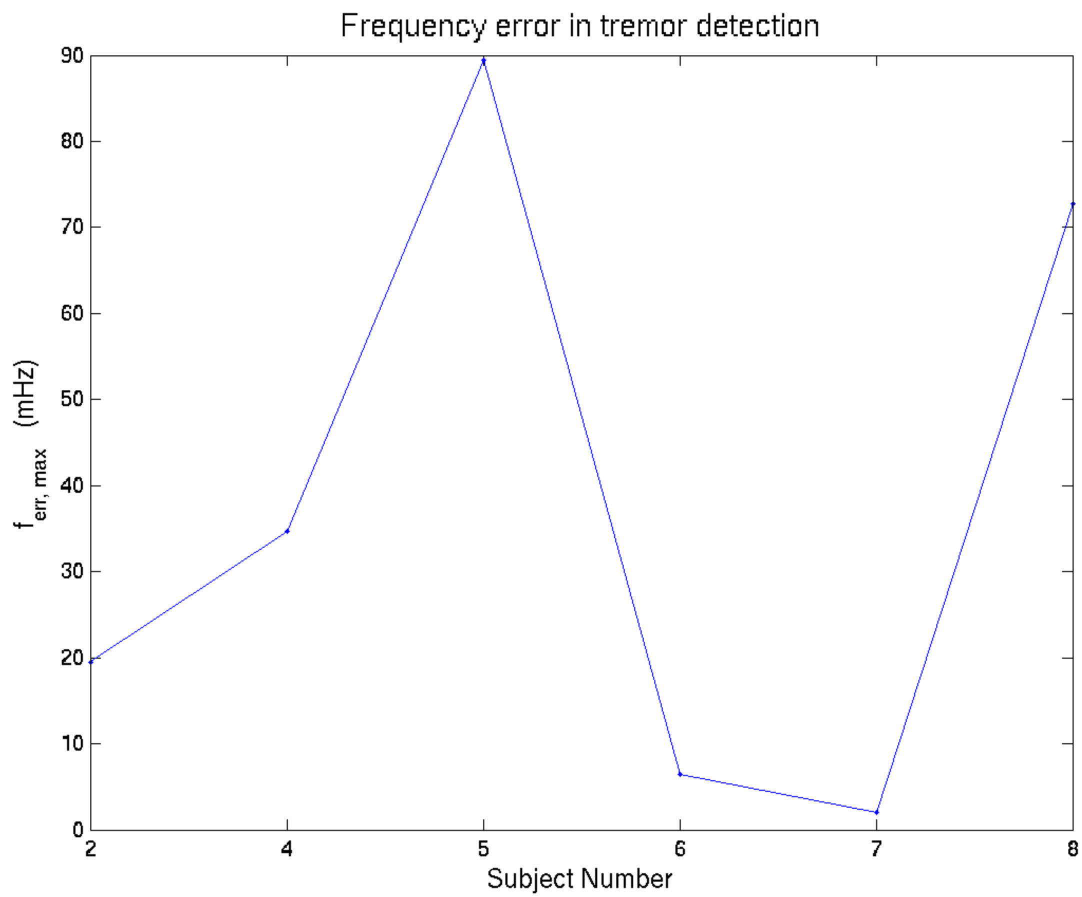


Biometric Example: Parkinson's Tremors



- **C=10.5, T=30**
  - 20% Nyquist required samples

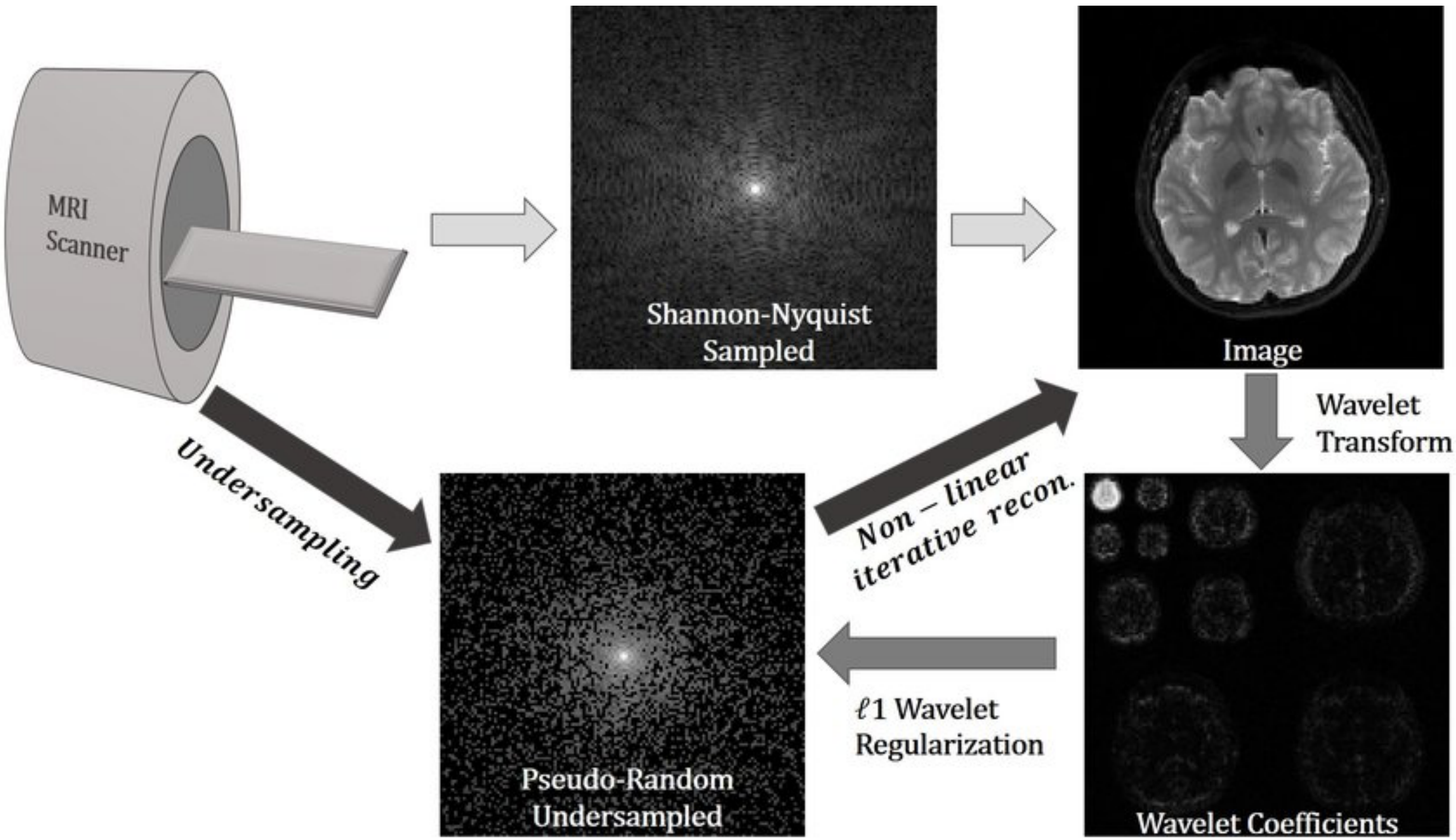
Biometric Example: Parkinson's Tremors



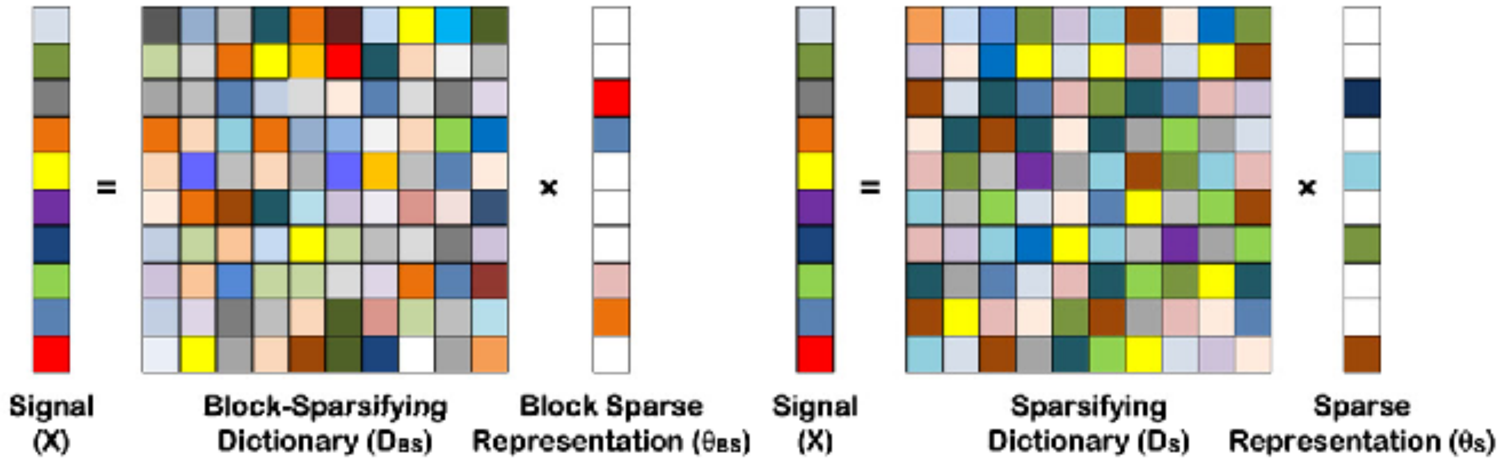
- **Tremors detected within 100 mHz**
- **randomly sample 20% of the Nyquist required samples**

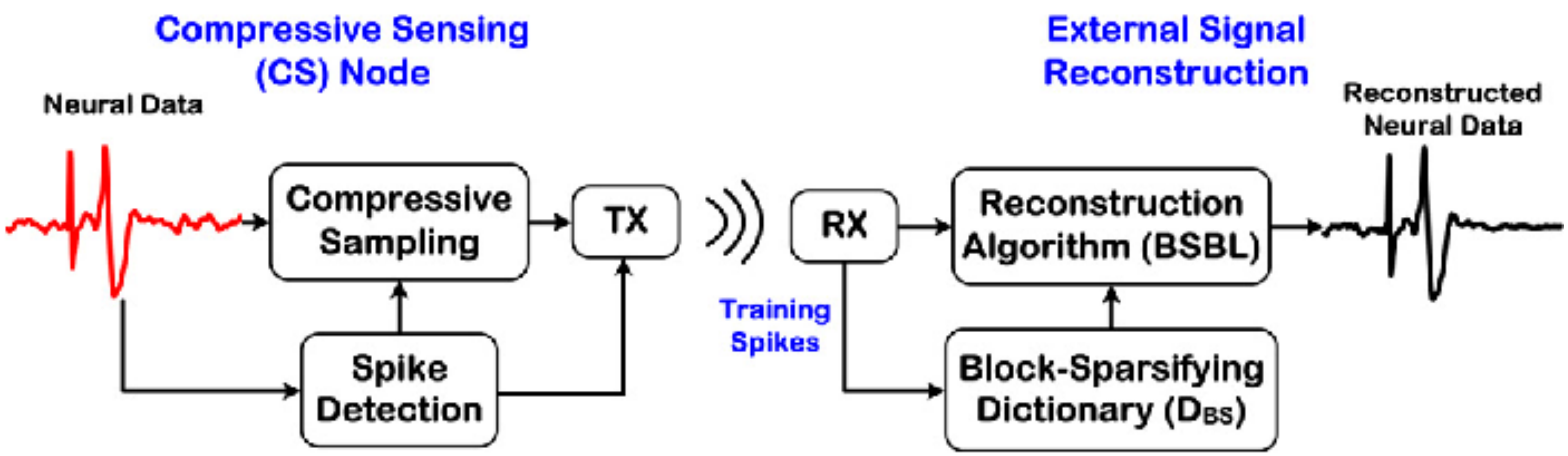
Requires post processing to randomly sample!

MRI Compressed Sampling



Neural Spike Detection





RF ID

RF ID

- Radio Frequency Identification (RFID), a wireless technology primarily known from the field of logistics, has become a focal point in hospitals and similar areas
- RFID makes it possible to manage hospital beds from a central location or track the whereabouts of surgical instruments

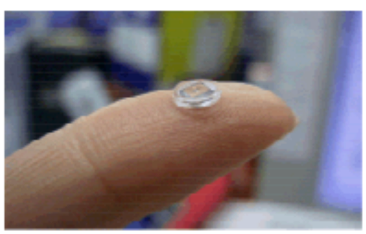
Optimization of Clinical Use

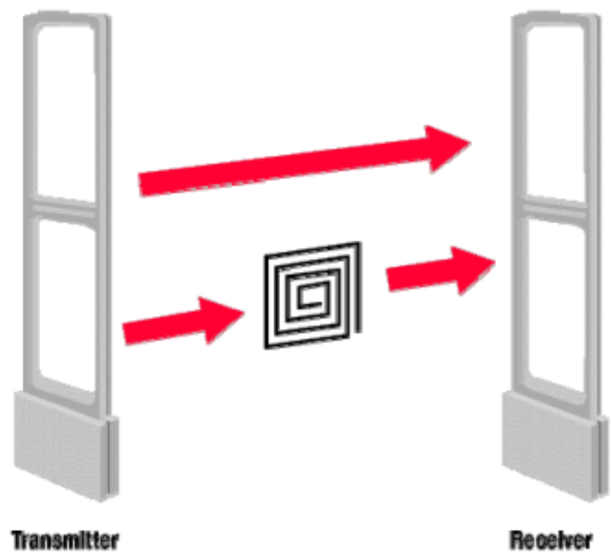
- Medical accessories now provides new possibilities in the area of intensive care
- For example, hospital staff can be relieved of routine activities when a signal indicates that a water trap must be replaced or a ventilator automatically adjusts settings of a connected accessory such as a ventilation hose
- This enables the optimization of clinical workflows.

RFID simplifies the connection and monitoring of medical accessory components

What is RFID

- Radio Frequency Identification
- Reader queries using RF
- Tag/Fob sends its ID using RF





RFID Tags

- Tag = Antenna, Radio receiver, radio modulator, control logic, memory and a power system
- Power Source:
  - Passive Tags: Powered by incoming RF. Smaller, cheaper, long-life. Approx range 5m.
  - Active Tags: Battery powered. Can be read 100 ft away. More reliable reading.
  - Semi-Passive tags: Transmit using 'Backscatter' of readers' RF power. Battery for logic. Range like passive. Reliability like active.

RFID Readers

- Sends a pulse of radio energy and listens for tags response
- Readers may be always on, e.g., toll collection system or turned on by an event, e.g., animal tracking
- Postage stamps size readers for embedding in cell phones Larger readers are size of desktop computers
- Most RFID systems use License-exempt spectrum
- Trend towards high-frequency

Band	Frequency	λ	Classical Use
LF	125-134.2 kHz	2,400 m	Animal tagging and keyless entry
HF	13.56 MHz	22 m	
UHF	865.5-867.6 MHz (Europe) 915 MHz (USA) 950-956 MHz (Japan)	32.8 cm	Smart cards, logistics, and item management
ISM	2.4 GHz	12.5 cm	Item Management

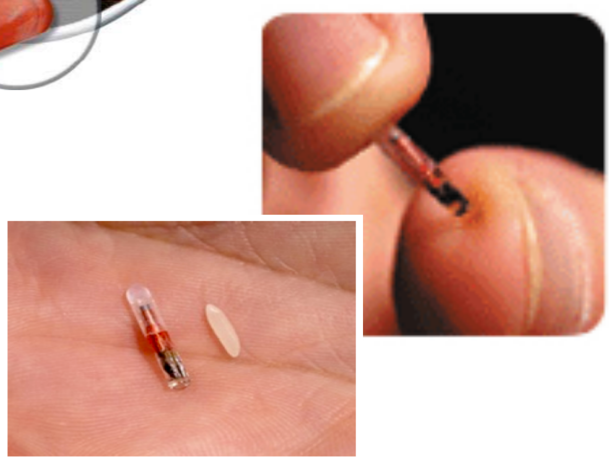
Microchip Implants

- Microchip implants are generally shaped like cylinders.
- They contain a small microchip, a bio-safe epoxy resin, and a copper antenna wire coil encased in glass.
- Microchips used for both animals and humans are field powered and have no battery or power source.
- Therefore, they are inert until they come within the field produced by a reader device, which implants communicate with over a magnetic field.

Commercial Products

Respironics

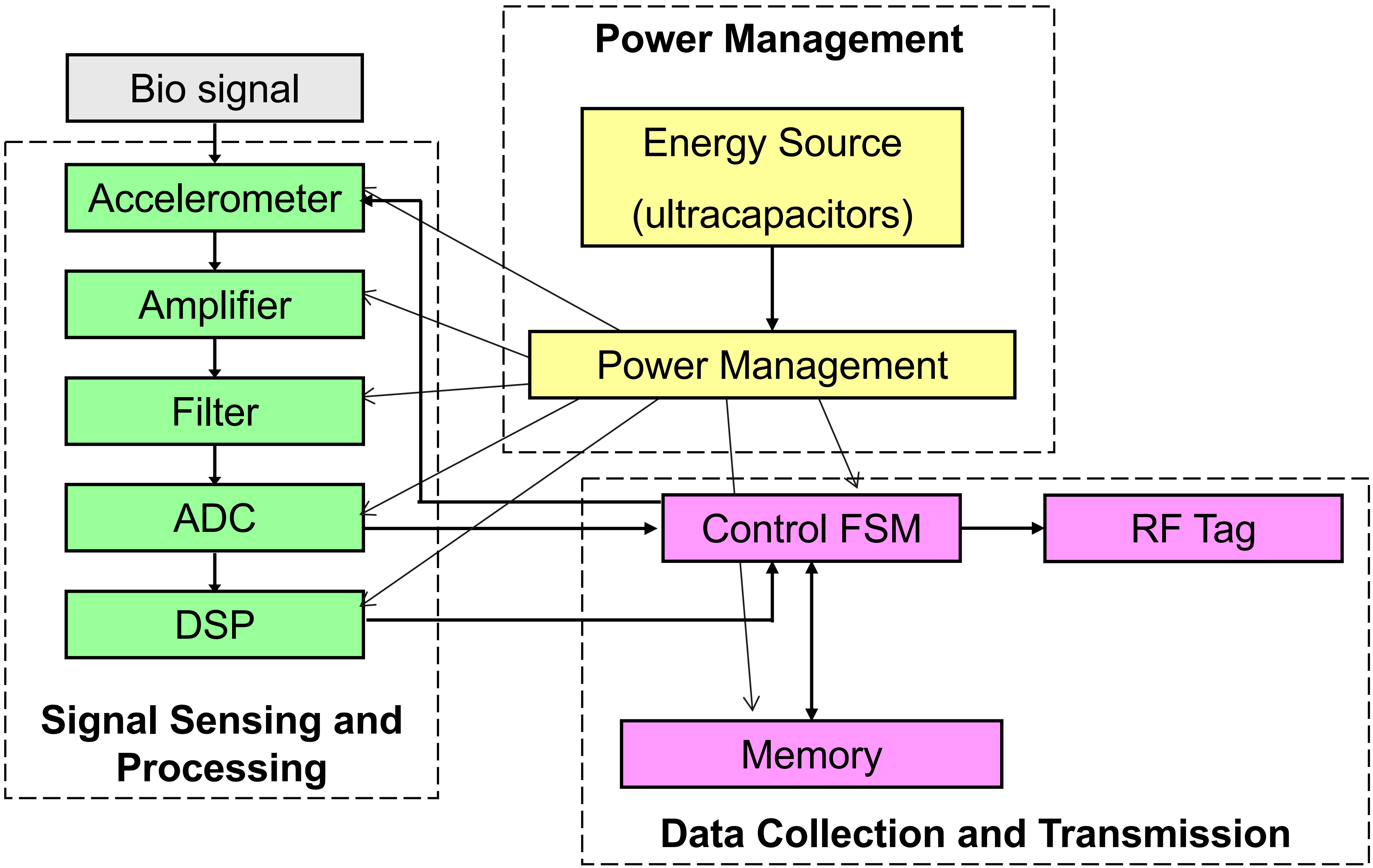
<2mA from 1.05-1.5V battery



VeriChip™

- For the desired functionality, current products are:
  - Too power hungry
  - Too big
- Proposed design meets desired functionality with
  - A new power management scheme to eliminate a battery
  - System design that includes application as a system tradeoff to optimize circuits
    - Ex. tradeoff speed for power

System Diagram



Bluetooth LE

Motivation

- With micro-sized, ultra-thin, flexible, and biocompatible electronic systems
- giving way to wearable and implantable devices that can achieve the same functionality at greatly reduced patient discomfort
- In addition, wireless medical solutions are often much more affordable for patients and lower cost for healthcare providers

Bluetooth Low Energy

- Bluetooth technologies epitomize recent advances in wireless technologies that allow for the remote operation of mobile medical devices
- In 2010, Bluetooth released its latest wireless platform: Bluetooth Low Energy (BLE), aimed at creating wireless applications in numerous fields including healthcare
  - provides devices with wireless communications at aggressive power metrics and low costs without sacrificing performance relative to other wireless standards.

Bluetooth Low Energy

Parameter	Value	Unit
Open Field Transmission Range	150	m
Output Power	10	dBm
Max Current Draw	15	mA
Sleep Current	1.0	µA
Carrier Frequency	2.4	GHz
Data Throughput	1.0	Mbps

Bluetooth Low Energy Specifications. Source: Bluetooth 4.0: Low Energy (2010, p. 8).

Other Protocols

	ANT	Bluetooth	Bluetooth LE	ZigBee
Standardisation	Proprietary	Standard	Standard	Standard
Topologies	Point-to-point, star, tree, mesh[3]	Point-to-point, scatternet	Point-to-point, star, mesh	Mesh
Band	2.4 GHz	2.4 GHz	2.4 GHz	2.4 GHz (+ sub-GHz for ZigBee PRO)
Range	30 metres at 0 dBm[8]	1–100 metres	10–600 metres in air (Bluetooth 5)	10–100 metres
Max data rate	Broadcast/Ack - 200 Hz[9] × 8 bytes × 8 bits = 12.8 kbit/s Burst - 20 kbit/s[9] Advanced Burst - 60kbit/s[9]	1-3 Mbit/s[8]	125 kbit/sec, 250 kbit/sec, 500 kbit/sec, 1 Mbit/s,[8] 2 Mbit/s (Bluetooth 5 PHY speeds)	250 kbit/s (at 2.4 GHz)
Application throughput	0.5 Hz to 200 Hz (8 bytes data)[9]	0.7-2.1 Mbit/s[8]	305 kbit/s[8] (Bluetooth 4.0)	
Max nodes in piconet	65533 per shared channel (8 shared channels) [8]	1 sink and 7 active sensors, 200+ inactive[8]	1 sink and 7 sensors (but scatternet unlimited),[8] mesh - 32767[10]	star - 65536[8]
Security	AES-128 and 64-bit key	56-128 bit key	AES-128	AES-128
Modulation	GFSK	GFSK	GFSK	OQPSK

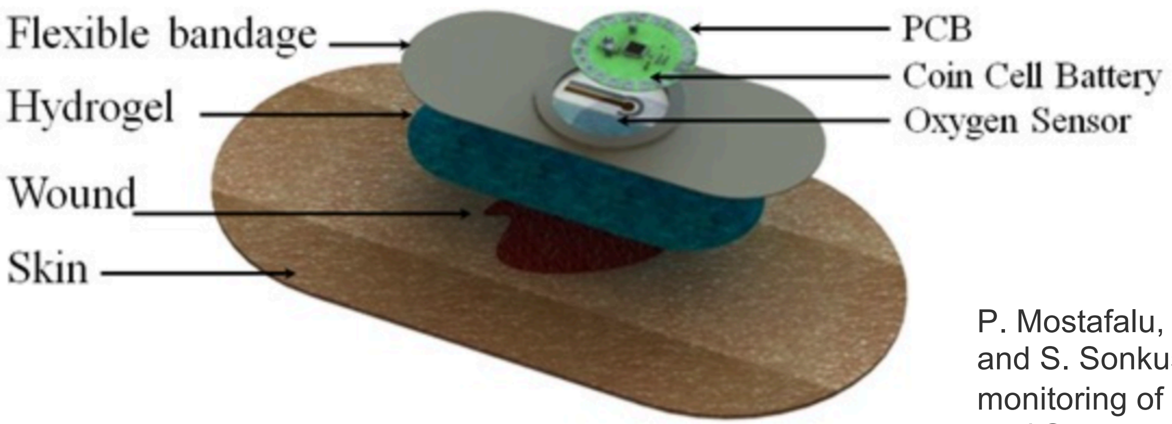
Power Comparison

EXPERIMENTAL RESULTS USING 3.3 V SUPPLY

	BLE	ZigBee	ANT
Time of one connection ±SD*	1150 ms ±260 ms	250 ms ±9.1 ms	930 ms ±230 ms
Sleep current	0.78 uA	4.18 uA	3.1 uA
Awake current	4.5 mA	9.3 mA	2.9 mA
Min current (at 120 sec interval)	10.1 uA	15.7 uA	28.2 uA
Optimal sleep interval	10.0 s	14.3 s	15.3 s
*SD: standard deviation			

A. Dementyev, S. Hodges, S. Taylor and J. Smith, "Power consumption analysis of Bluetooth Low Energy, ZigBee and ANT sensor nodes in a cyclic sleep scenario," *2013 IEEE International Wireless Symposium (IWS)*, 2013, pp. 1-4, doi: 10.1109/IEEE-IWS.2013.6616827.

Example: Smart Wound Dressing



P. Mostafalu, W. Lenk, M. Dokmeci, B. Ziaie, A. Khademhosseini and S. Sonkusale, "Wireless flexible smart bandage for continuous monitoring of wound oxygenation," *2014 IEEE Biomedical Circuits and Systems Conference (BioCAS) Proceedings*, 2014, pp. 456-459, doi: 10.1109/BioCAS.2014.6981761.

- A flexible, galvanic oxygen sensor on the order of 100 μm in diameter
- The oxygen sensor is interfaced via a flexible conductor to an analog-front-end circuit for amplification
- The output of the analog-front-end is read into a microcontroller through an analog-to-digital converter
- Data is converted back to a voltage value and wirelessly transmitted to a nearby computer or smartphone via a Bluetooth Low Energy

Security Risks of Wireless Communication

- A typical mobile medical device will have a low-power wireless communications system, such as a BLE or ZigBee radio.
- The use of low power radios requires an intermediate base station in close proximity to the user (e.g. 150 meters maximum for BLE) where data can be dumped and subsequently uploaded to a "secure" server
- The transmission of data across a wireless network presents a glaring security vulnerability if malicious hackers can penetrate the network security and gain access to confidential patient information.
- Furthermore, if the medical device itself can directly be accessed or programmed from a remote location, such as the previously discussed smart wound dressing, malicious hackers could actually hijack operation of the device to steal private information or cause device malfunction.

Security Breaches

- Three categories (Rushanan, 2014):
  - Telemetry Interface Breaches
    - Passive – eavesdropping breaching patient confidentiality
    - Active – jam, modify or forge the information exchange
  - Software Threats
  - Hardware/Sensor Threats
    - Eg. Rowhammer attack

M. Rushanan, A. D. Rubin, D. F. Kune and C. M. Swanson, "SoK: Security and Privacy in Implantable Medical Devices and Body Area Networks," *2014 IEEE Symposium on Security and Privacy*, 2014, pp. 524-539, doi: 10.1109/SP.2014.40.

Example: Insulin Pump

- Hacking Medical Devices for Fun and Insulin: Breaking the Human SCADA System
  - Jerome Radcliffe
- Used a relatively cheap microcontroller and available details on wireless communication command codes
- Can potentially alter readings or dosages

- https://cs.uno.edu/~dbilar/BH-US-2011/materials/Radcliffe/BH_US_11_Radcliffe_Hacking_Medical_Devices_WP.pdf

Security Solutions

- Advances in electrical engineering and related fields such as computer science can certainly mitigate these risks as well
- Researches are investigating highly advanced data encryption methods, security protocols, and trust models to help secure wireless medical instruments

Example: Trustworthy Data Collection

- Public-key cryptography standard (IEEE 1363) with a complex, probabilistic trust model to demonstrate highly trustworthy data collection
- Data is scrambled and two "keys" are required to unscramble
  - Keys are mathematically related but computationally infeasible to generate private from public
- Trust model no longer binary but continuous between 0 and 1

Hu F, Hao Q, Lukowiak M, Sun Q, Wilhelm K, Radziszowski S, Wu Y. Trustworthy data collection from implantable medical devices via high-speed security implementation based on IEEE 1363. IEEE Trans Inf Technol Biomed. 2010 Nov;14(6):1397-404. doi: 10.1109/TITB.2010.2049204. Epub 2010 Apr 26. PMID: 20423808.

Big Ideas

- Compressive Sampling
  - Sample at less than Nyquist using sparsity
- RF ID used to automate and optimize clinical systems
  - Tags hold information and transmit data to reader
  - Mostly near field use
- Wireless communication
  - Needs to be low energy
    - BLE is taking over as industry standard
  - Poses security risk
    - Need trustworthy security protocols

Admin

- Finish Lab 9 by Monday
  - Submit Google Colab PDF in Canvas
- Lab 10 Monday
  - CircuitPython and BLE
- Quiz 2 Wednesday
- Project details posted on Wednesday

Quiz Admin

- Quiz 2
  - In Towne 305 Wednesday 11/15
  - 90 minutes, start at exactly 10:15am
  - Calculators allowed (non-cell phone)
  - 8.5x11 cheat sheet allowed
  - Cumulative, but will focus on lectures 5-8 and labs 6, 8 and 9 (ADC and filter labs)

Quiz 2 Review

Signals and Systems

Discrete-Time Sinusoids

- Discrete-time sinusoids $e^{j(\omega n+\phi)}$ have two counterintuitive properties

- Both involve the frequency ω

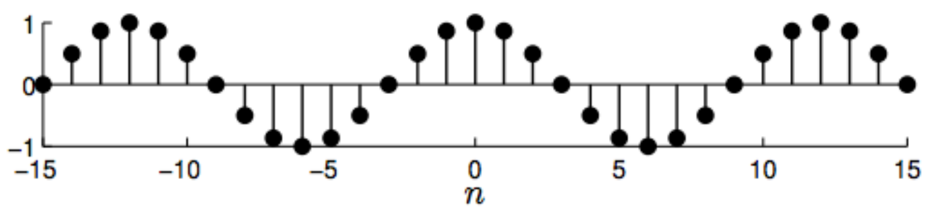
- **Property #1**: Aliasing

- **Property #2**: Aperiodicity

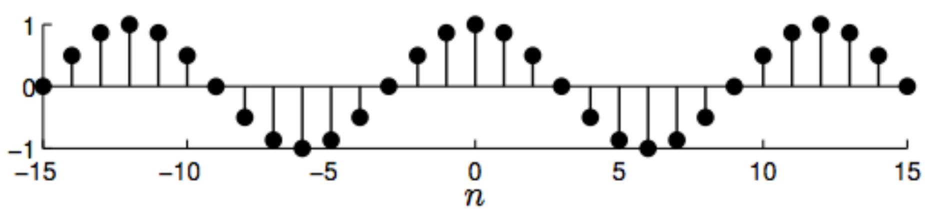
Property #1: Aliasing of Sinusoids

- The signals x_1 and x_2 have different frequencies but are **identical!**
- We say that x_1 and x_2 are aliases; this phenomenon is called aliasing

$x_1[n] = \cos\left(\frac{\pi}{6}n\right)$



$x_2[n] = \cos\left(\frac{13\pi}{6}n\right) = \cos\left(\left(\frac{\pi}{6} + 2\pi\right)n\right)$



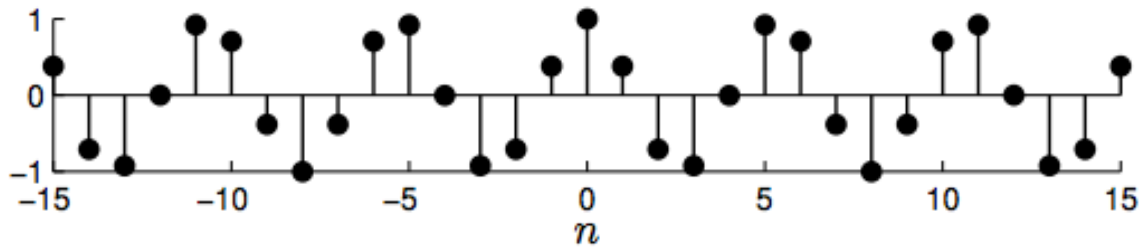
Property #2: Periodicity of Sinusoids

- Consider $x_1[n] = e^{j(\omega n+\phi)}$ with frequency $\omega = \frac{2\pi k}{N}$, $\quad k, N \in \mathbb{Z}$ (harmonic frequency)

- It is easy to show that x_1 is periodic with period N, since

$$x_1[n+N] = e^{j(\omega(n+N)+\phi)} = e^{j(\omega n+\omega N+\phi)} = e^{j(\omega n+\phi)}\, e^{j(\omega N)} = e^{j(\omega n+\phi)}\, e^{j(\frac{2\pi k}{N} N)} = x_1[n] \checkmark$$

- Ex: $x_1[n] = \cos(\frac{2\pi 3}{16} n)$, $\quad N = 16$



- Note: x_1 is periodic with the (smaller) period of $\frac{N}{k}$ when $\frac{N}{k}$ is an integer

Harmonic Sinusoids

$$e^{j(\omega n+\phi)}$$

- Semi-amazing fact: The **only** periodic discrete-time sinusoids are those with **harmonic frequencies**

$$\omega = \frac{2\pi k}{N}, \quad k, N \in \mathbb{Z}$$

- Which means that
  - **Most** discrete-time sinusoids are **not** periodic!
  - The harmonic sinusoids are somehow magical (they play a starring role later in the DFT)

Example problems:

- Which is higher in frequency?
  - $\cos(\pi n)$ or $\cos(3\pi/2 n)$?

- Periodic or not?
  - $\cos(5/7\pi n)$
  - $\cos(\pi/5 n)$
  - If so, what are N and k? (I.e How many samples is one period?

LTI Systems

> **DEFINITION**
>
> A system $\mathcal{H}$ is **linear time-invariant** (LTI) if it is both linear and time-invariant

- LTI system can be completely characterized by its impulse response

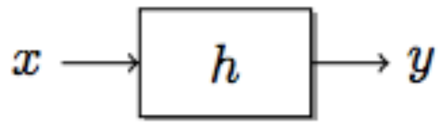
$$\delta \longrightarrow \boxed{\mathcal{H}} \longrightarrow h$$

- Then the output for an arbitrary input is a sum of weighted, delay impulse responses

$$x \longrightarrow \boxed{h} \longrightarrow y \qquad\qquad y[n] \;=\; \sum_{m=-\infty}^{\infty} h[n-m]\,x[m]$$

$$y[n] = x[n] * h[n]$$

Convolution



- Convolution formula:

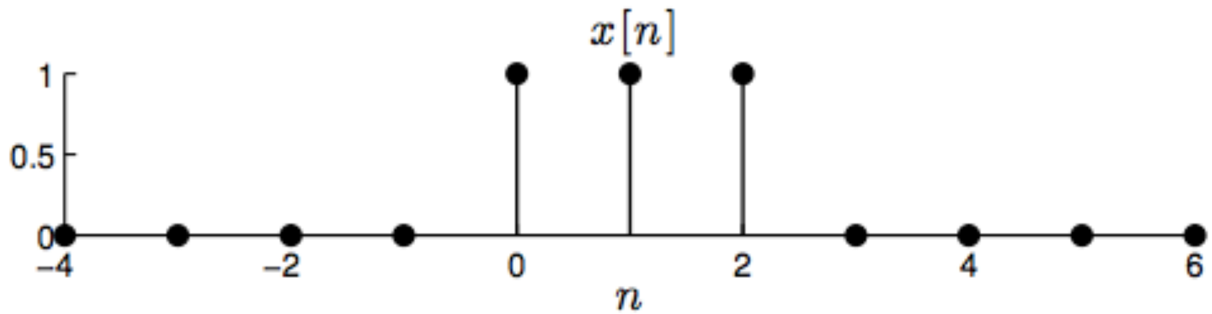
$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} h[n-m]\, x[m]$$

- Convolution method:
  - 1) Time reverse the impulse response and shift it *n* time steps to the right
  - 2) Compute the inner product between the shifted impulse response and the input vector
  - Repeat for every *n* to get the output

Convolution Example

$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} h[n-m]\, x[m]$$

- Convolve a unit pulse with itself



- Convolve a unit pulse with a unit pulse twice the width

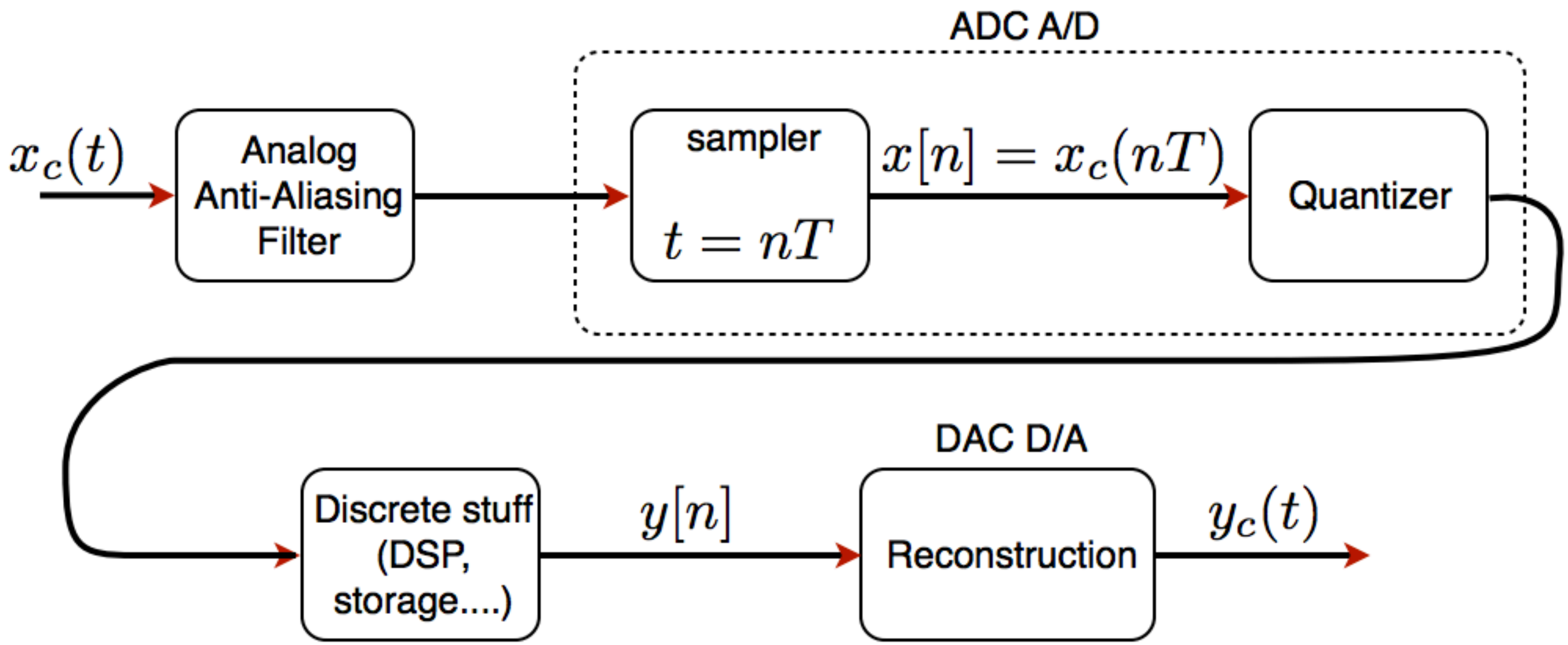
DTFT and Sampling

DTFT Definition

$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} x[k]e^{-j\omega k}$$

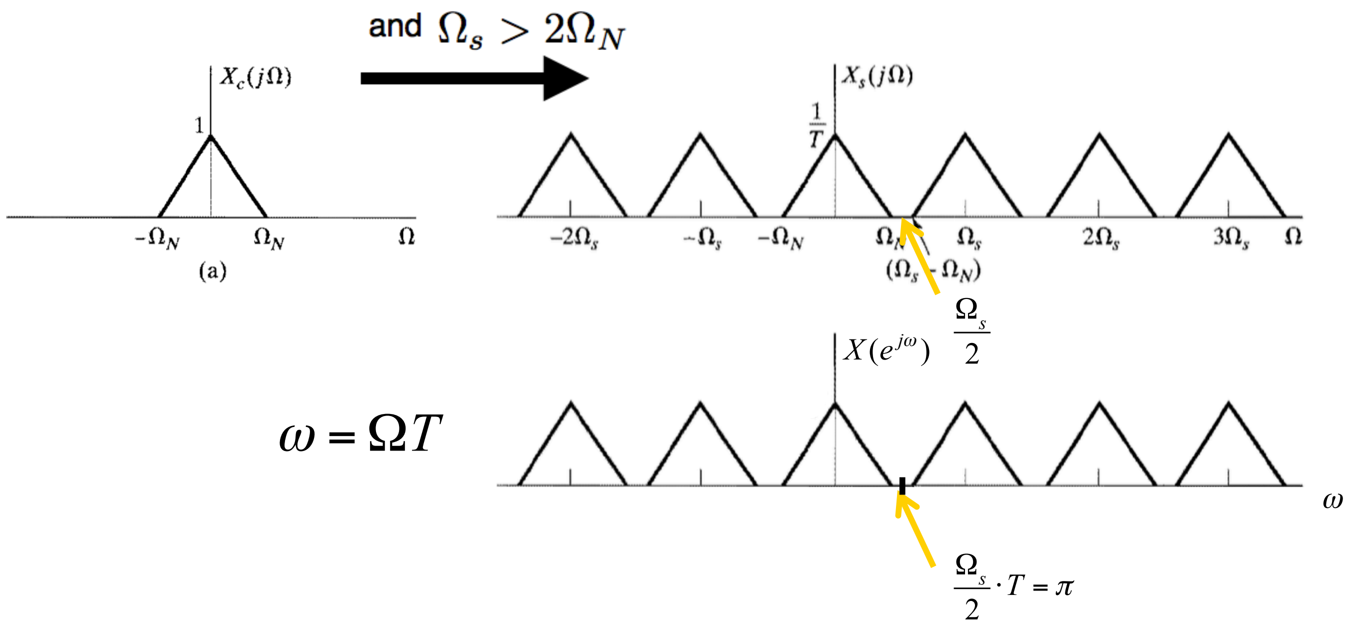
$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

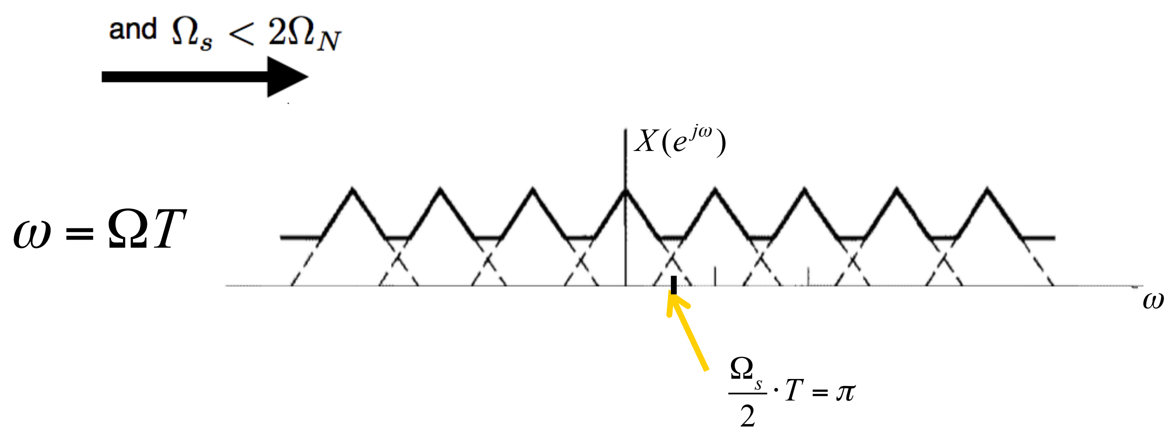
DSP System



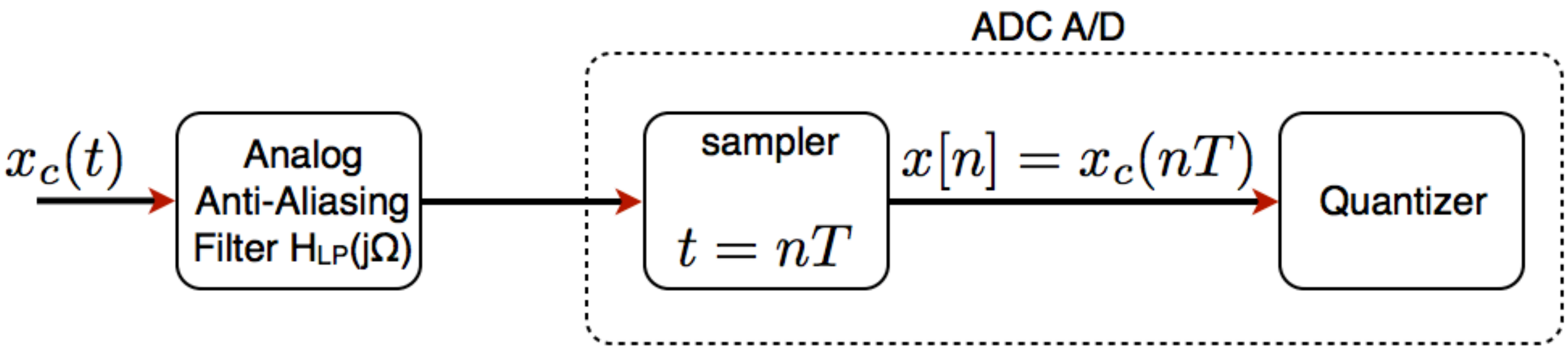
ADC A/D
$x_c(t)$
Analog Anti-Aliasing Filter
sampler
$t = nT$
$x[n] = x_c(nT)$
Quantizer
DAC D/A
Discrete stuff (DSP, storage....)
$y[n]$
Reconstruction
$y_c(t)$

Frequency Domain Analysis

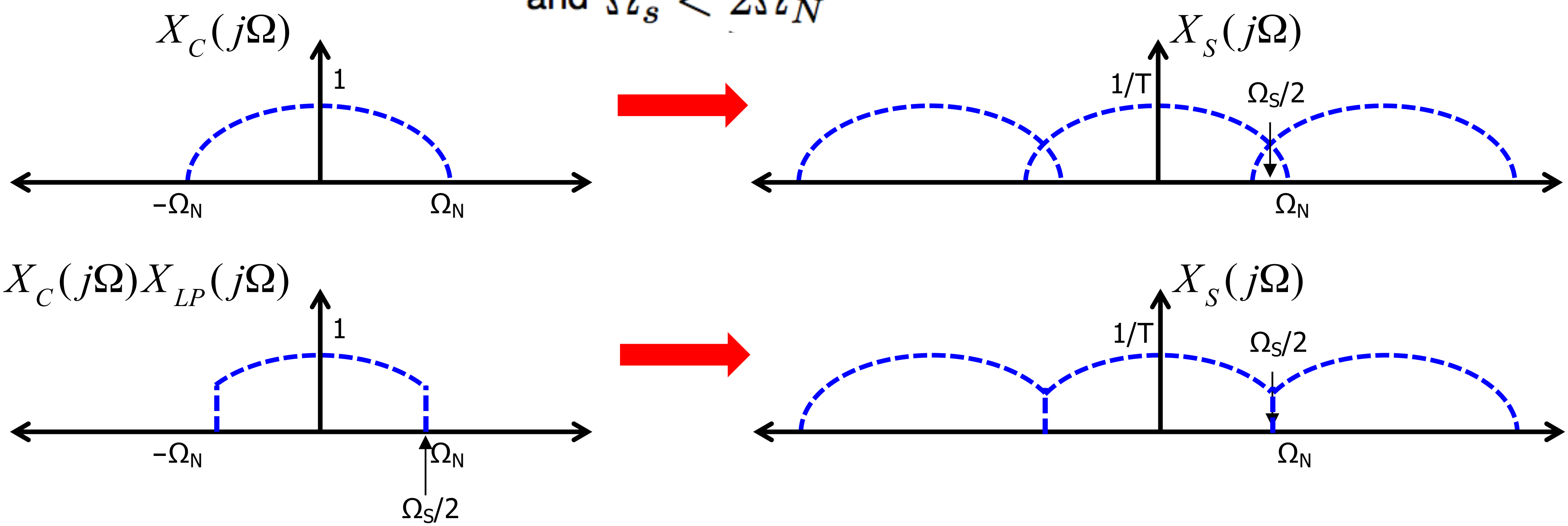




Anti-Aliasing Filter



and $\Omega_s < 2\Omega_N$



DTFT Vs. DFT

DTFT:

$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} x[k]e^{-j\omega k}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

DFT:

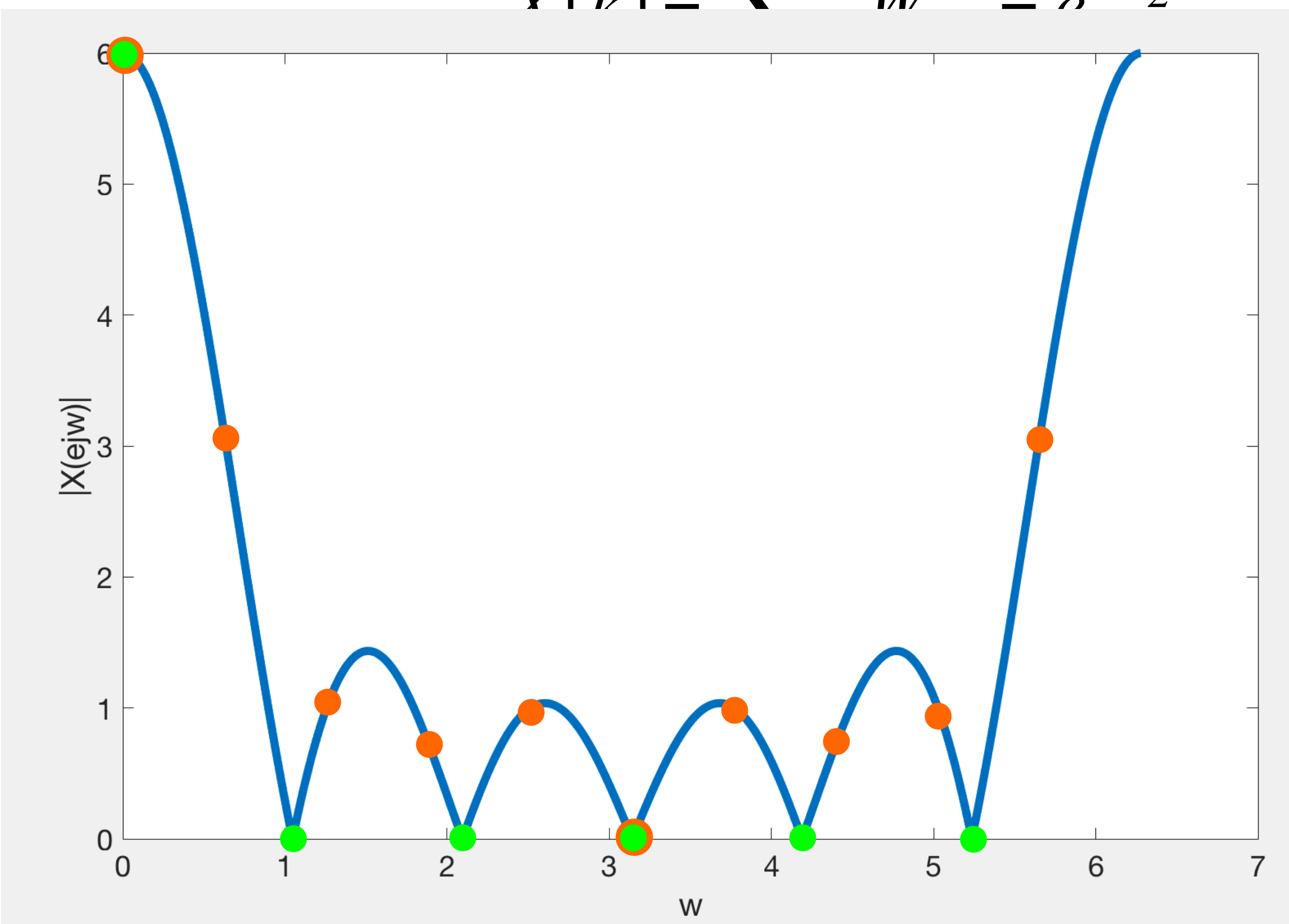
$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]W_N^{-kn}$$

$$X[k] = \sum_{n=0}^{N-1} x[n]W_N^{kn}$$

DFT vs DTFT

- Back to example

$$X[k]=\sum^{5} W^{nk} = e^{-j\frac{\pi}{2}k}\frac{\sin\left(\frac{3\pi}{5}k\right)}{\sin\left(\frac{\pi}{10}k\right)}$$



"6-point" DFT

"10-point" DFT

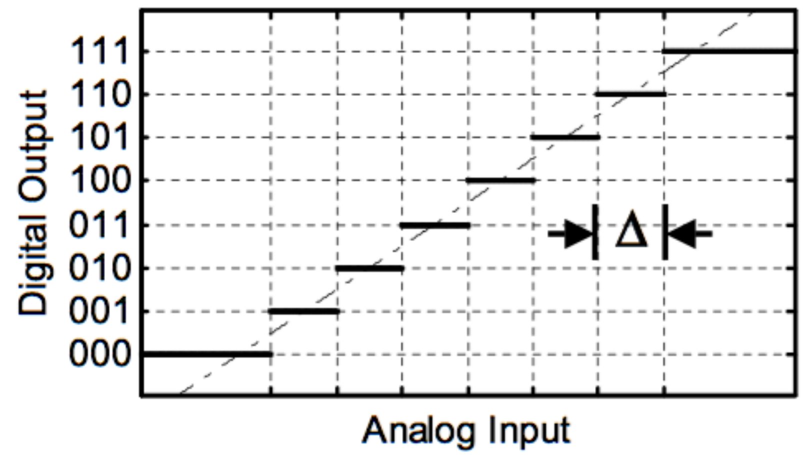
Use `fftshift` to center around dc

ADC

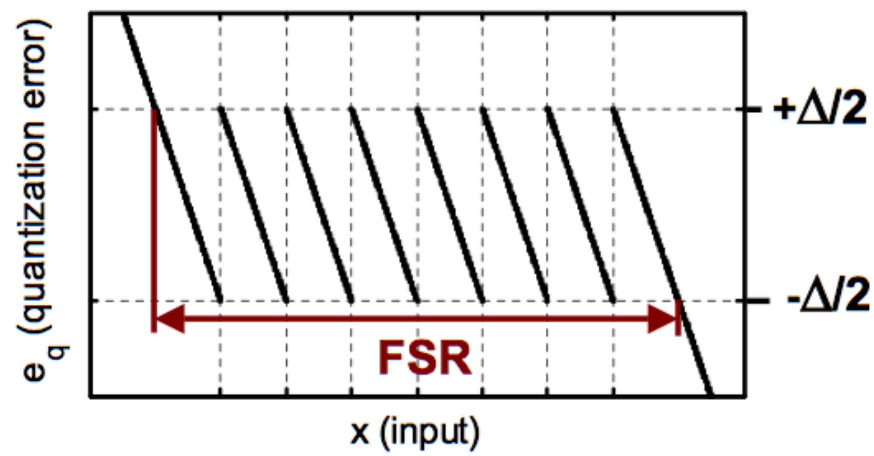
Analog to Digital Converter

Ideal B-bit Quantizer

- Practical quantizers have a limited input range and a finite set of output codes
- E.g. a 3-bit quantizer can map onto $2^3=8$ distinct output codes



- Quantization error grows out of bounds beyond code boundaries
- We define the full scale range (FSR) as the maximum input range that satisfies $|e_q|\leq\Delta/2$
  - Implies that $FSR = 2^B \cdot \Delta$



Signal-to-Quantization-Noise Ratio

- Assuming full-scale sinusoidal input, we have

$$SNR_Q = 6.02B + 1.76 \text{ dB}$$

B (Number of Bits)	SQNR
8	50dB
12	74dB
16	98dB
20	122dB

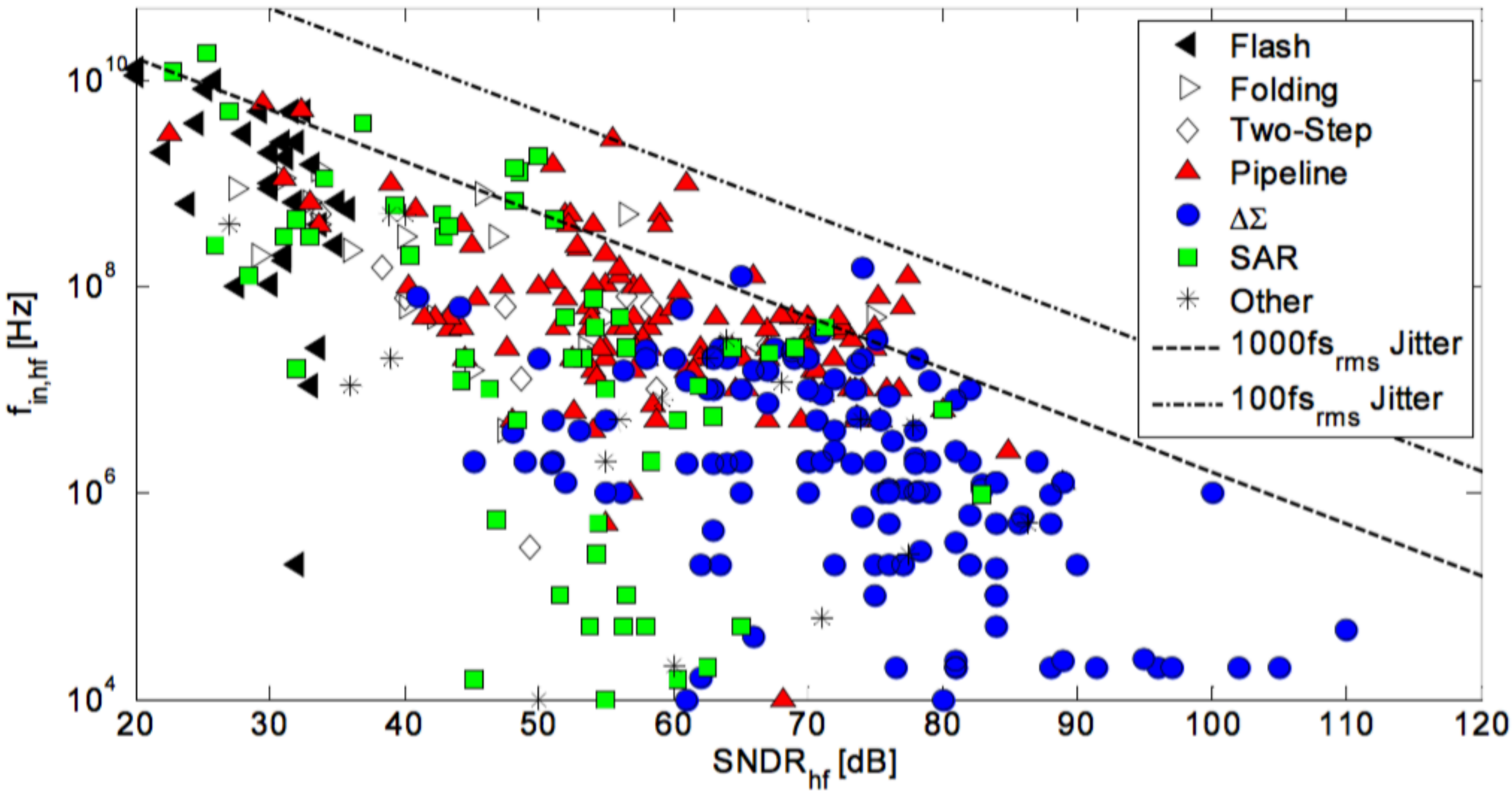
Nyquist ADC Architectures

- Word-at-a-time
  - E.g. flash ADC
  - Instantaneous comparison with 2^B-1 reference levels
- Multi-step
  - E.g. pipeline ADCs
  - Coarse conversion, followed by fine conversion of residuals
- Bit-at-a-time
  - E.g. successive approximation ADCs
  - Conversion via a binary search algorithm

speed

ADC Survey (ISSCC & VLSI 1997-2013)

Data: http://www.stanford.edu/~murmann/adcsurvey.html



ADC Big Ideas

- SQNR
  - SQNR determined by bit resolution, B
- Nyquist ADCs
  - Flash ADCs
    - Word-at-a-time for high speed, low resolution applications
  - SAR ADCs
    - Bit-at-a-time for low speed, low power applications
    - Highly suited for medical devices
- Oversampling
  - Enables reduction in quantization noise with digital filter
  - Sigma-Delta ADCs
    - Use integrator in feedback to shape noise and achieve high resolution
    - Usually for low speed, low power applications

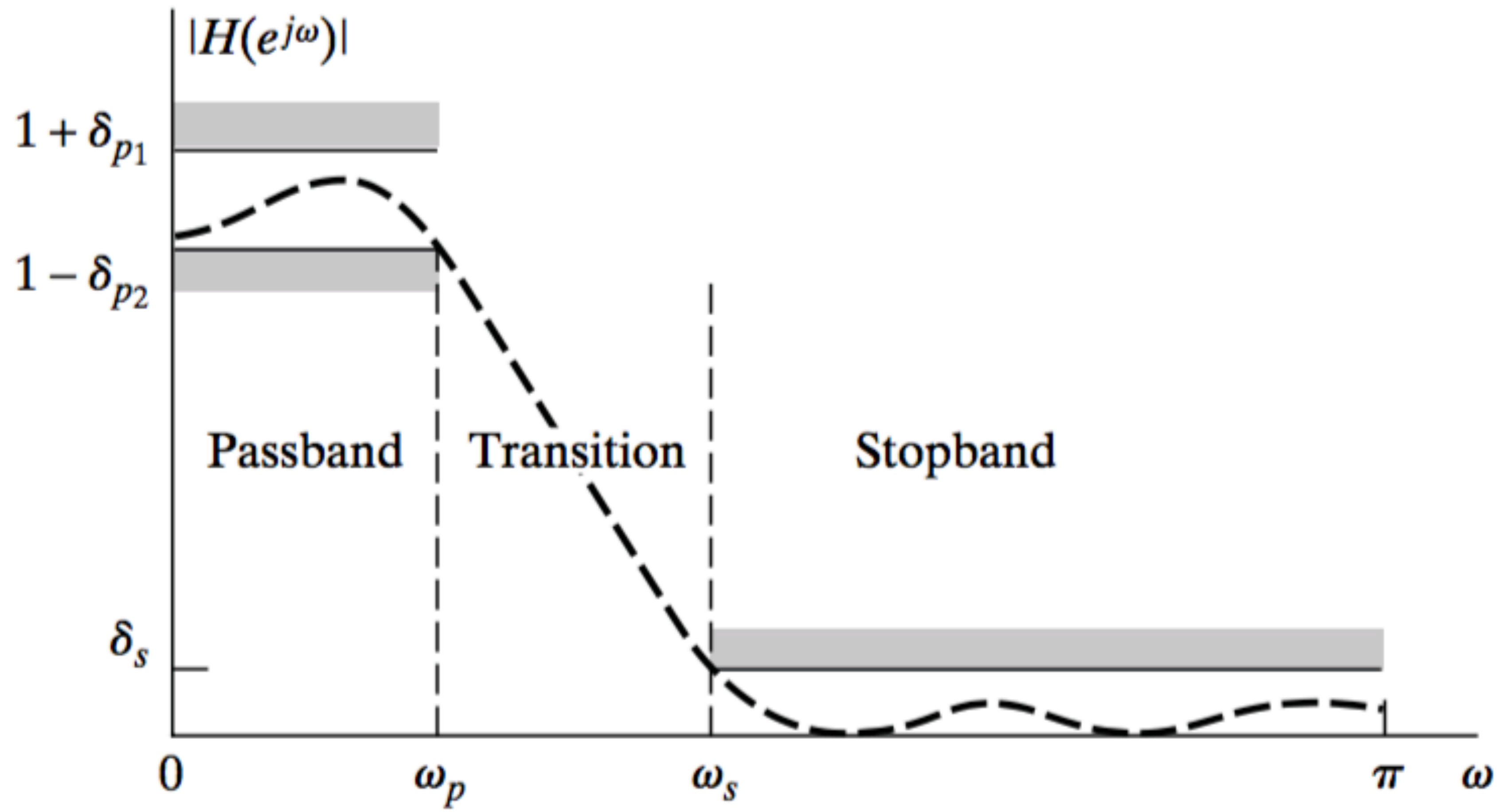
Example Problems

- Which ADC topology would you choose and why?
  - A 6b ADC with sampling frequency of 1Mhz?
  - A 14b ADC with sampling frequency of 10 khz?
  - A 8b ADC with sampling frequency of 100 khz?

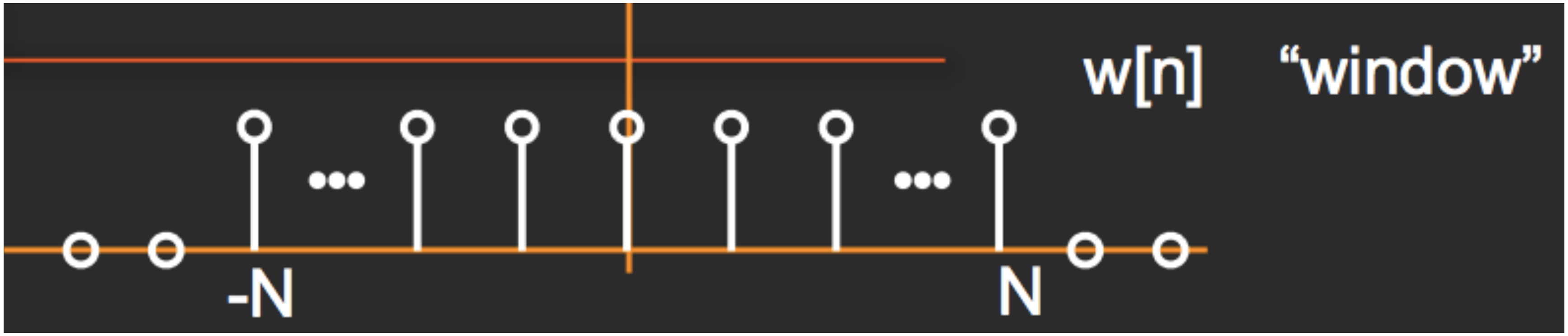
- What is the SQNR of each design?

Digital Filters

Filter Specifications



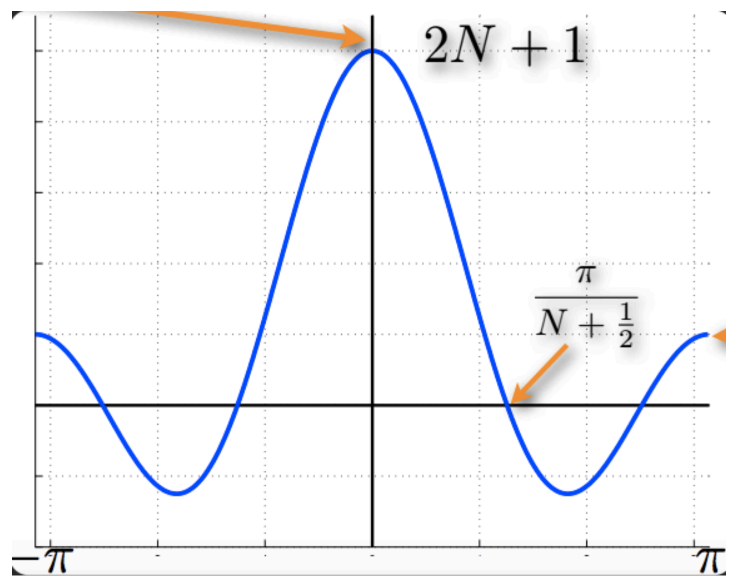
Example: Window DTFT



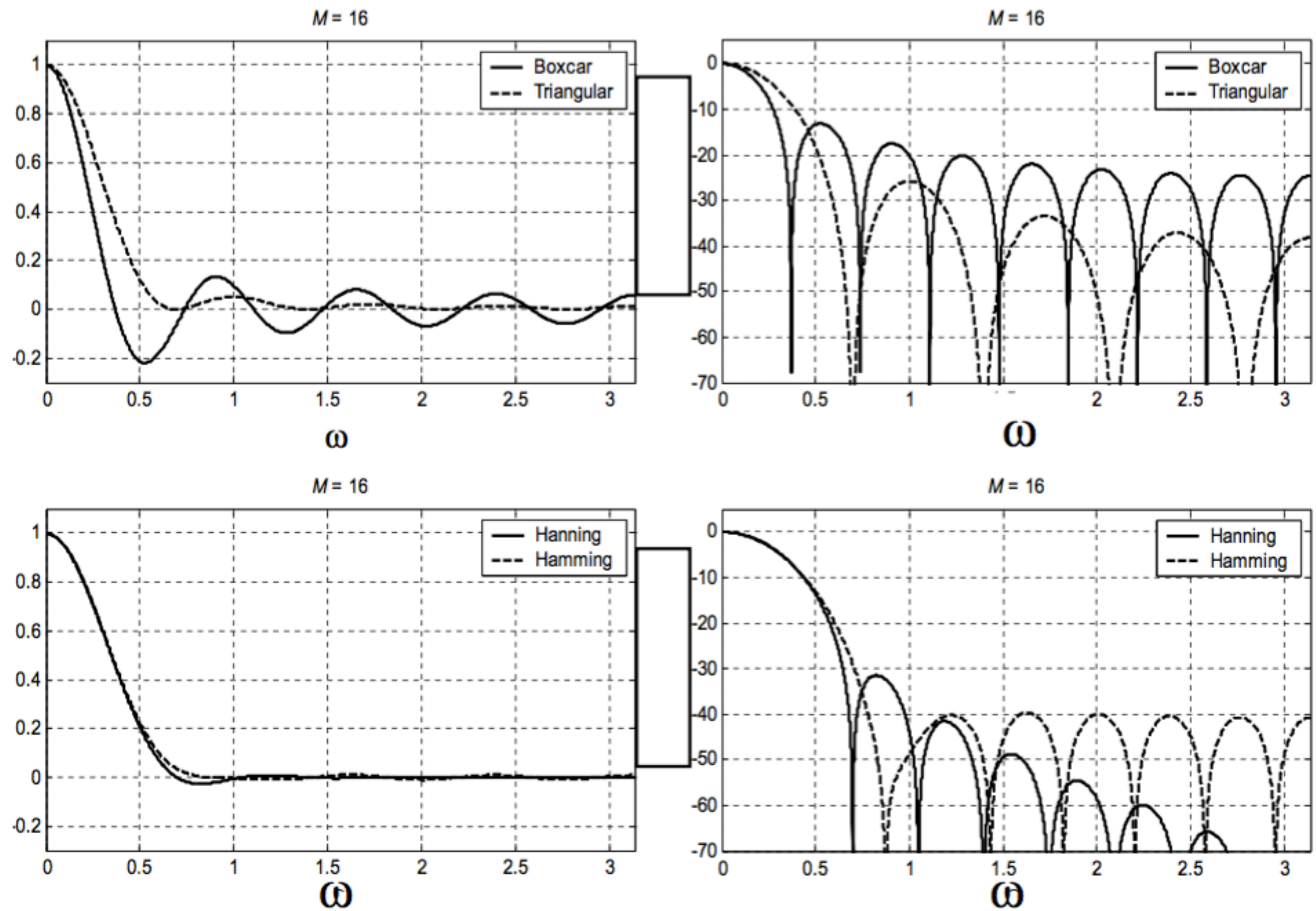
$$W(e^{j\omega}) = \sum_{k=-\infty}^{\infty} w[k] e^{-j\omega k}$$

$$= \sum_{k=-N}^{N} e^{-j\omega k}$$

$$W(e^{j\omega}) = \frac{\sin\left((N+1/2)\omega\right)}{\sin\left(\omega/2\right)}$$



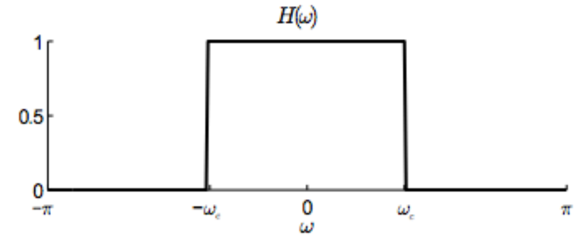
Tradeoff – Ripple vs. Transition Width



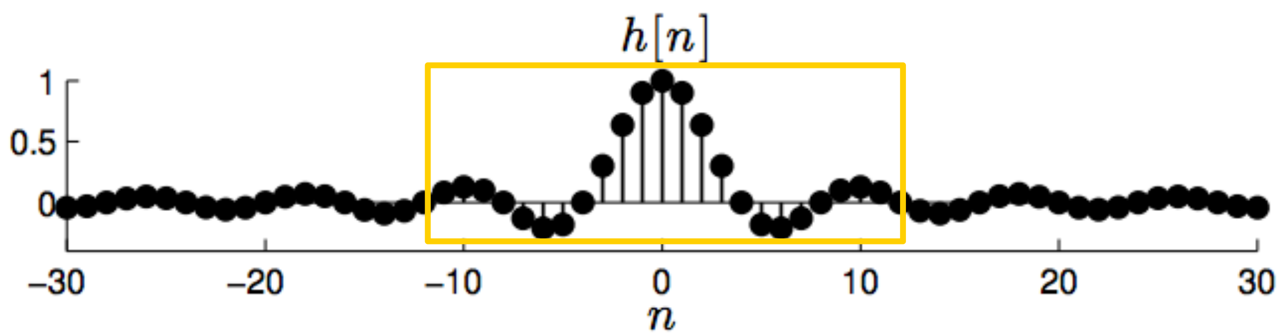
Example: Ideal Low-Pass Filter

- The frequency response $H(\omega)$ of the ideal low-pass filter passes low frequencies (near $\omega = 0$) but blocks high frequencies (near $\omega = \pm\pi$)

$$H(\omega) = \begin{cases} 1 & -\omega_c \leq |\omega| \leq \omega_c \\ 0 & \text{otherwise} \end{cases}$$



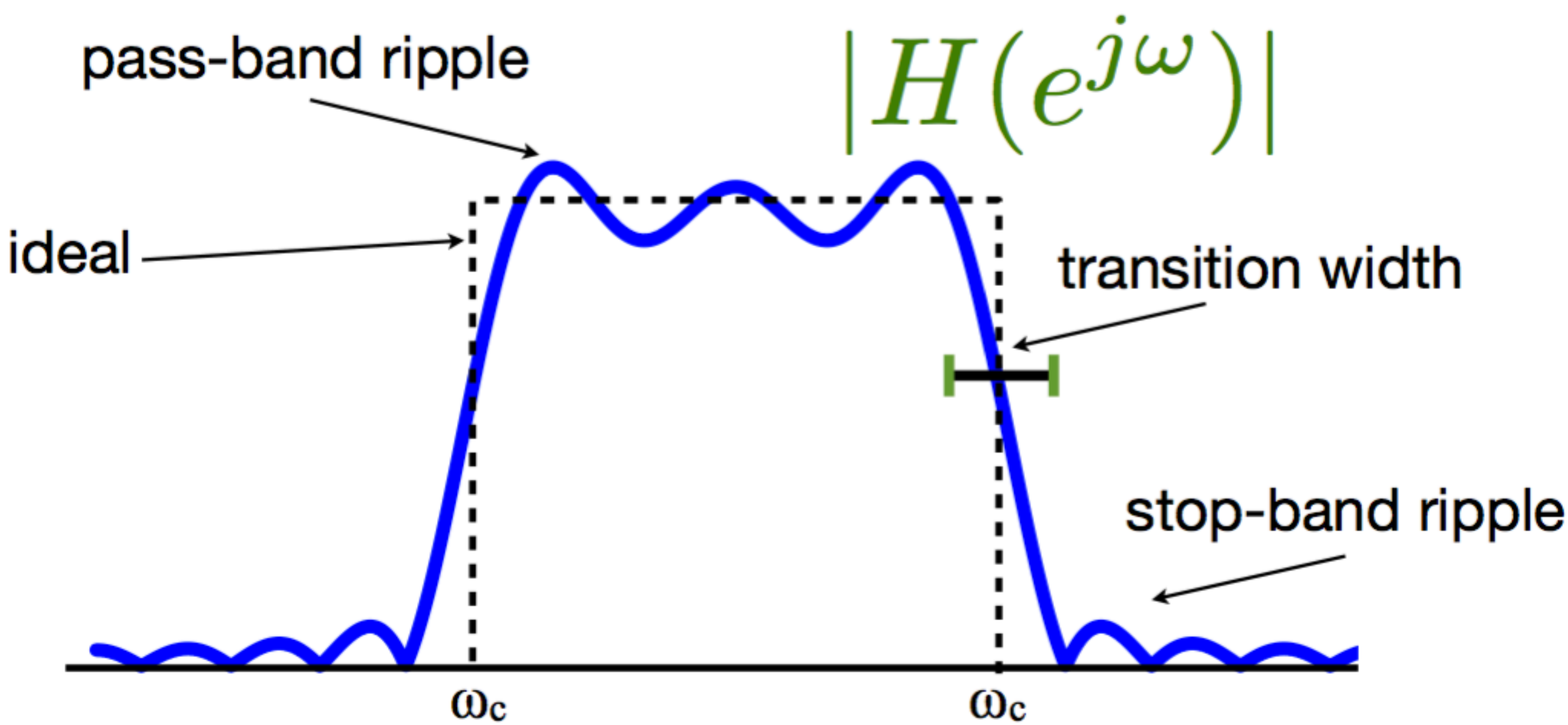
$$h[n] \;=\; 2\omega_c \frac{\sin(\omega_c n)}{\omega_c n}$$



Truncate and shift

$$h_{LP}[n] = w_N[n-N] \cdot h[n-N]$$

FIR Design by Windowing



Example Problems

- Compute the DTFT of a simple(ish) window function
- Given a filter frequency response, what is the transition bandwidth? Pass ripple? Stop ripple?
- Create a `freq` and `gain` array for a given frequency spectrum requirement.
- Review Lab 8 and 9! Lots of problems there

Frequency Analysis with DFT

- Length of window determines spectral resolution
- Type of window determines side-lobe amplitude/main-lobe width (spectral leakage/spreading)
  - Some windows have better tradeoff between resolution and side-lobe height
- Zero-padding approximates the DTFT better (spectral sampling). Does not introduce new information!