ESE370: Circuit-Level Modeling, Design, and Optimization for Digital Systems

Lec 16: October 14, 2015
Energy and Power Optimization

Penn ESE 370 Fall 2015 - Khanna
Previously

- Three components of power
  - Static
  - Short circuit
  - Capacitive switching

\[ P_{\text{tot}} = P_{\text{static}} + P_{\text{dyn}} + P_{\text{sc}} \]
Today

- Power Sources
  - Review
  - Static – data dependent
Power Sources

Review: \( P_{\text{tot}} = P_{\text{static}} + P_{\text{dyn}} + P_{\text{sc}} \)
CMOS circuit in steady-state

- Leaks subthreshold current “determined” by off device
  - Subthreshold device determines $V_{ds}$ and thus current
- $P_{\text{static}} = V_{dd} \times I_{\text{static}}$
Data Dependent?

- How does value the binary value of the input impact $I_{\text{static}}$?
Data Dependent?

- How does the binary value of input impact $I_{\text{static}}$?

$$I_{DS} = I_s' \left( \frac{W}{L} \right) e^{\left( \frac{V_{GS} - V_T}{n kT/q} \right)} \left( 1 - e^{-\left( \frac{V_{DS}}{kT/q} \right)} \right) \left( 1 + \lambda V_{DS} \right)$$
Switching (Dynamic)

- CMOS circuit switching output from $0 \rightarrow 1$
  - Spends energy $CV_{dd}^2$ charging load
  - $CV_{dd}^2$ energy is pulled from the energy source

\[ Q = \int I(t) \, dt = CV \]

\[ E = \int P(t) \, dt \]

\[ E = V_{dd} \int I(t) \, dt \]

\[ E = CV_{dd}^2 \]
Switching Power

- Every time output switches 0→1 pay:
  - $E = CV^2$

- $P_{\text{dyn}} = (# \ 0\rightarrow 1 \ \text{trans}) \times CV^2 / \text{time}$

- $# \ 0\rightarrow 1 \ \text{trans} = \frac{1}{2} \ # \ \text{of transitions}$

- $P_{\text{dyn}} = (# \ \text{trans}) \times \frac{1}{2}CV^2 / \text{time}$
Switching (Short Circuit)

- $P = IV$

- Short circuit current when:
  - $V_{th} < V_{in} < V_{dd} - |V_{thp}|$

- $I_{peak}$ when $V_{in} \approx V_{dd}/2$
Switching (Short Circuit)

- $P = IV$

- Short circuit current when:
  - $V_{th} < V_{in} < V_{dd} - |V_{thp}|$

- $I_{peak}$ when $V_{in} \approx V_{dd}/2$

\[
\int I(t) dt \approx I_{peak} \times t_{sc} \times \left( \frac{1}{2} \right)
\]

\[
E = V_{dd} \times I_{peak} \times t_{sc} \times \left( \frac{1}{2} \right)
\]
Switching (Short Circuit)

- $P = IV$

- Short circuit current when:
  - $V_{th} < V_{in} < V_{dd} - |V_{thp}|$

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\[
\int I(t) dt \approx I_{peak} \times t_{sc} \times \left( \frac{1}{2} \right)
\]

\[
E = V_{dd} \times I_{peak} \times t_{sc} \times \left( \frac{1}{2} \right)
\]

\[
E = V_{dd} \times Q_{SC}
\]

\[
E = V_{dd} \times (C_{SC}V_{dd}) = C_{SC}V_{dd}^2
\]

\[
P = \#\text{trans} \times C_{SC}V_{dd}^2 / \text{time}
\]
## Preclass 1

<table>
<thead>
<tr>
<th>$V_{in}$</th>
<th>$I_{static}$</th>
<th>$I_{dynamic}$</th>
<th>$I_{sc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0V</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>140mV</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>400mV</td>
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<td>500mV</td>
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<tr>
<td>600mV</td>
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<tr>
<td>860mV</td>
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<td></td>
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<tr>
<td>1V</td>
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<td></td>
<td></td>
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</tbody>
</table>
## Preclass 1

<table>
<thead>
<tr>
<th>$V_{\text{in}}$</th>
<th>$I_{\text{static}}$</th>
<th>$I_{\text{dynamic}}$</th>
<th>$I_{\text{sc}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0V</td>
<td>180pA</td>
<td>126uA</td>
<td></td>
</tr>
<tr>
<td>140mV</td>
<td>6nA</td>
<td>100uA</td>
<td></td>
</tr>
<tr>
<td>400mV</td>
<td></td>
<td>36uA</td>
<td>18uA</td>
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Switching Energy
Reminder

- Output switches 0→1 charge output from Vdd

- Output switches 1→0 discharge output to ground
  - no current from Vdd
Data Dependent Activity

- **Consider an 8b counter**
  - How often do each of the following switch?
    - Low bit?
    - High bit?

- **Assuming random inputs**
  - Activity at output of nand4?
  - Activity at output of xor4?
Gate Output Switching (random inputs)

Output states

\[ P(\text{out}_i \neq \text{out}_{i+1}) = P(\text{out}_i = 0) \times P(\text{out}_{i+1} = 1) + P(\text{out}_i = 1) \times P(\text{out}_{i+1} = 0) \]

Probablity of output switch of nand4?
Probablity of output switch of xor4?
Charging Power

- \( P_{\text{dyn}} = \text{(\# trans)} \times \frac{1}{2} CV^2 / \text{time} \)
- Often like to think about switching frequency
- Useful to consider per clock cycle
  - Frequency \( f = 1 / \text{clock-period} \)
- \( P_{\text{dyn}} = (\#\text{trans/clock}) \frac{1}{2} CV^2 f \)
Switching Power

- \( P_{\text{dyn}} = (\text{#trans/clock})^{\frac{1}{2}}CV^2 f \)
- Let \( a = \text{activity factor} \)
  \[ a = \text{average #tran/clock} \]
- \( P_{\text{dyn}} = a^{\frac{1}{2}}CV^2 f \)
- \( P_{\text{sc}} = aC_{\text{sc}}V^2 f \)
Total Power

- $P_{\text{tot}} = P_{\text{static}} + P_{\text{sc}} + P_{\text{dyn}}$

- $P_{\text{dyn}} + P_{\text{sc}} = a(\frac{1}{2}C_{\text{load}} + C_{\text{sc}})V^2f$

- $P_{\text{tot}} \approx a(\frac{1}{2}C_{\text{load}} + C_{\text{sc}})V^2f + VI_s(W/L)e^{-V_t/(nkT/q)}$

$I_{DS} = I_s' \left( \frac{W}{L} \right) e^{\left( \frac{V_{GS} - V_T}{nkT/q} \right)} \left( 1 - e^{-\left( \frac{V_{DS}}{kT/q} \right)} \right) \left( 1 + \lambda V_{DS} \right)$
Ideas

- Three components of power
  - Static
  - Dynamic
  - Short-circuit

- \[ P_{tot} = P_{static} + P_{dyn} + P_{sc} \]

- Power is data dependent and a function of our switching
Admin

- Project 1 out
  - Milestone piece due in one week
    - Turn in COMPLETE milestone early, for earlier feedback
  - Full Report in two weeks
  - That means you need to be starting on it now…and working on it all next week
    - Read assignment today

- Lecture Friday