Next few Lectures
- Saw in action in lab (last Friday)
- Where transmission lines arise?
- General wire formulation
- Lossless Transmission Line
- End of Transmission Line?
- Termination
- Discuss Lossy
- Implications

Where Transmission Lines Arise
- Cable: coaxial
- PCB
  - Strip line
  - Microstrip line
- Twisted Pair (Cat5)

Transmission Lines
- How did the coaxial cables behave in lab on Friday?
- How does this differ from
  - Ideal equipotential?
  - RC-wire on chip?

Transmission Lines
- This is what wires/cables look like
  - Aren't an ideal equipotential
  - Signals do take time to propagate
  - Maintain shape of input signal
    - Within limits
  - Shape and topology of wiring effects how signals propagate
Transmission Lines

- Need theory/model to support design
  - Reason about behavior
  - Understand what can cause noise
  - Engineer high performance/speed communication

Wires

- In general, our “wires” have distributed R, L, C components

RC Wire

- When R dominates L
  - We have the distributed RC Wires
  - Typical of on-chip wires in ICs
  - What is RC response to step?

Transmission Line

- When resistance is negligible
  - Have LC wire = Lossless Transmission Line
  - No energy dissipation (loss) through R’s
  - More typical of Printed Circuit Board wires and bond wires

Build Intuition from LC

- What did one LC do?
- What will chain do?
Build Intuition from LC

- What did one LC do?
- What will chain do?

Intuitive: Lossless

- Pulses travel as waves without distortion
  - (up to a characteristic frequency)

SPICE Simulation

Step Response SPICE

Pulse Response SPICE

Contrast RC Wire
Contrast

Model

- Now voltage is a function of time and position
  - Position along wire – distance from source
- Want to get $V(x,t)$
  - And $I(x,t)$

Setup Relations

- $i$ is a node, $x$ is distance from source, $\Delta x$ is distance between nodes
- Position: $x=i \times \Delta x$
- So $V_j = V(x=i \Delta x)$

Setup Relations

- KCL @ $V_i$: $I_i - I_{i+1} = 0$
- $I_c = 0$
- $V_i \cdot V_{i+1} = 0$

Visualization

- RC Pulse:
- RLC Pulse:
**Setup Relations**

- **KCL @ \( V_i - l_{i-1} \):**
  \[ I_i - I_{i-1} = C \frac{dV_i}{dt} \]

- **\( V_1 \):**
  \[ V_1 = V_{i-1} - I_{i-1} \frac{dl_i}{dt} \]

- **\( V_{i-1} \):**
  \[ V_{i-1} = -I_i \frac{dl_i}{dt} \]

**i** is a node, but has spatial dimension along the line.

\( V_i \) changes at different positions.

**Reduce to Single Equation**

- **Eliminate \( I_i \), \( I_{i-1} \):**
  \[ \frac{dl_i}{dt} - \frac{dl_{i-1}}{dt} = C \frac{dV_i}{dt} \]

- **Eliminate \( I_i \):**
  \[ \frac{dl_i}{dt} = C \frac{dV_i}{dt} \]

- **Take derivative with respect to time:**
  \[ \frac{d^2V_i}{dt^2} = \frac{d^2V_i}{dt^2} \]

**Eliminate \( I_{i-1} \), \( V_{i-1} \):**

\[ \frac{dV_i}{dt} = L \frac{dl_i}{dt} \]

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- **Eliminate \( I_i \):**
  \[ \frac{dl_i}{dt} = L \frac{dV_i}{dt} \]

- **Eliminate \( I_{i-1} \):**
  \[ \frac{dl_{i-1}}{dt} = L \frac{dV_i}{dt} \]
**Implication**

- Wave equation:
  \[
  \frac{\partial^2 V}{\partial x^2} = LC \frac{\partial^2 V}{\partial t^2}
  \]
- Solution:
  \[
  V(x, t) = A + Be^{x-wt}
  \]
- What is \( w \)?
  \[
  Be^{x-wt} = LCw^2Be^{x-wt}
  \]
  \[
  w = \frac{1}{\sqrt{LC}}
  \]

**Light Cycle Example**

- What is the position of a cycle at time \( t \)
  - Start at \( x=0 \)
  - Travel at speed \( v \)

- Light Cycle is step function at \( x=0 \)
  - Cycle creates trail of height 1
  - \( F(0, t=0)=1, F(x>0, t=0)=0 \Rightarrow F(x,t)=1-u(x) \)

- Light Cycle is step function at \( x=0 \)
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**Propagation Rate in Example**

- \( L=1uH \)
- \( C=1pF \)
- What is \( w \)?
  \[
  w = \frac{1}{\sqrt{LC}}
  \]

**TRON: Light Cycle Context**

- [http://www.youtube.com/watch?v=GNfs6v71eY](http://www.youtube.com/watch?v=GNfs6v71eY)
Signal Propagation

Delay linear in length

Contrast RC Wire

RC wire delay quadratic in length

Propogation

- Solution:
  \[ V(x,t) = A + Be^{x-wt} \]
  - Rate of propagation, \( w \):
    \[ w = \frac{1}{\sqrt{LC}} \]
  - Previously:
    \[ c^2 = \frac{1}{\varepsilon_0\mu_0} \Rightarrow \]
    \[ w = \frac{c}{\sqrt{\varepsilon_0\mu_0}} \]

Idea

- Signal propagates as wave down transmission line
  - Delay linear in wire length, if resistance negligible
  - Rate of propagation
    \[ w = \frac{1}{\sqrt{LC}} = \frac{c}{\sqrt{\varepsilon_0\mu_0}} \]

Admin

- Back here on Monday
- Project 2 due Friday next week
- Happy Thanksgiving!