ESE370: Circuit-Level Modeling, Design, and Optimization for Digital Systems

Lec 34: December 4, 2015
Repeaters in Wiring
Previously

- Transmission line (LC wire) wire delay scales linearly with length
- Unbuffered RC wire delay scales quadratically with length
  - delay = 0.5 $R_{\text{wire}} C_{\text{wire}} = 0.5 \text{ length}^2 R_u C_u$
Reminder: Wire Delay

- Wire N units long: 
  \[ R_{\text{wire}} = N \times R_{\text{unit}} \]
  \[ C_{\text{wire}} = N \times C_{\text{unit}} \]

  \[ = R_{\text{unit}} \times C_{\text{unit}} \times N^2 / 2 \]

- With
  - \( R_{\text{unit}} = 1k\Omega \)
  - \( C_{\text{unit}} = 1pF \)
Today: Back to RC Wire

- RC (on-chip) Interconnect Buffering
Delay of Wire

- Long Wire: 1mm
- $R_u = 60K \, \Omega$ per 1mm of wire
- $C_u = 0.16$ pF per 1mm of wire
- Driven by inverter
  - $R_0 = 25K \, \Omega$
  - $C_0 = 0.01$ fF
  - Assume velocity saturated, sized $W_p=W_n=1$
- Loaded by identical inverter
Formulate Delay

Delay of inverter driving wire?

Hint: Draw equivalent RC model of buffers and wire
Formulate Delay

Delay of inverter driving wire?

\[ R_{buf} \times (C_{self} + C_{wire} + C_{load}) + 0.5R_{wire} \times C_{wire} + R_{wire} \times C_{load} \]
Calculate Delay

\[ R_{buf} \times \left( C_{self} + C_{wire} + C_{load} \right) + 0.5R_{wire} \times C_{wire} + R_{wire} \times C_{load} \]
Calculate Delay

- $C_{\text{load}} = 2 C_0 = 0.02 \text{fF}$
- $R_{\text{buf}} = R_0 = 25 \text{K} \ \Omega$
- $C_{\text{self}} = \gamma 2 C_0 = 2 C_0 = 0.02 \text{fF}$
- $C_{\text{wire}} = L*C_u = 0.16 \text{pF}$
- $R_{\text{wire}} = L*R_u = 60 \text{K} \ \Omega$

\[
R_{\text{buf}} \times \left( C_{\text{self}} + C_{\text{wire}} + C_{\text{load}} \right) + 0.5R_{\text{wire}} \times C_{\text{wire}} + R_{\text{wire}} \times C_{\text{load}}
\]
Buffering Wire

- Complete Preclass Table
Buffering Wire: L/2

- $C_{\text{load}} =$
- $R_{\text{buf}} =$
- $C_{\text{self}} =$
- $C_{\text{wire}} =$
- $R_{\text{wire}} =$
Buffering Wire: L/2

- \( C_{\text{load}} = 2 \ C_0 = 0.02 \text{fF} \)
- \( R_{\text{buf}} = R_0 = 25 \text{K} \ \Omega \)
- \( C_{\text{self}} = 2 \ C_0 = 2 \ C_0 = 0.02 \text{fF} \)
- \( C_{\text{wire}} = \frac{L}{2} \ \times \ C_u = 0.08 \text{pF} \)
- \( R_{\text{wire}} = \frac{L}{2} \ \times \ R_u = 30 \text{K} \ \Omega \)
N Buffers

- Delay Equation for N buffers?
N Buffers

- Delay Equation for N buffers?

\[
N \left( R_{buf} \left( C_{self} + \frac{C_{wire}}{N} + C_{load} \right) \right) + 0.5 \left( \frac{R_{wire}}{N} \cdot \frac{C_{wire}}{N} \right) + \frac{R_{wire}}{N} \cdot C_{load} \\
N \cdot R_{buf} \left( C_{self} + \frac{C_{wire}}{N} + C_{load} \right) + N \cdot 0.5 \left( \frac{R_{wire}}{N} \cdot \frac{C_{wire}}{N} \right) + N \cdot \frac{R_{wire}}{N} \cdot C_{load} \\
N \cdot R_{buf} \left( C_{self} + C_{load} \right) + R_{buf} \times C_{wire} + 0.5 \left( \frac{1}{N} \right) R_{wire} C_{wire} + R_{wire} C_{load}
\]
Minimize Delay

- Minimize delay?
Minimize Delay

- Minimize delay
- Derivative with respect to N

\[ N \cdot R_{buf} \left( C_{self} + C_{load} \right) + R_{buf} \times C_{wire} + 0.5 \left( \frac{1}{N} \right) R_{wire} C_{wire} + R_{wire} C_{load} \]
Minimize Delay

- Minimize delay
- Derivative with respect to $N$

\[ N \cdot R_{buf} \left( C_{self} + C_{load} \right) + R_{buf} \times C_{wire} + 0.5 \left( \frac{1}{N} \right) R_{wire} C_{wire} + R_{wire} C_{load} \]

\[ 0 = R_{buf} \left( C_{self} + C_{load} \right) - 0.5 \left( \frac{1}{N^2} \right) R_{wire} C_{wire} \]
Solve for \(N\)

\[
0 = R_{buf} \left( C_{self} + C_{load} \right) - 0.5 \left( \frac{1}{N^2} \right) R_{wire} C_{wire}
\]

\[
R_{buf} \left( C_{self} + C_{load} \right) = 0.5 \left( \frac{1}{N^2} \right) R_{wire} C_{wire}
\]

\[
N^2 = \frac{0.5 R_{wire} C_{wire}}{R_{buf} \left( C_{self} + C_{load} \right)}
\]

\[
N = \sqrt{\frac{0.5 R_{wire} C_{wire}}{R_{buf} \left( C_{self} + C_{load} \right)}}
\]
Minimize Delay

Equalizes delay in buffer and wire

\[ N = \sqrt{\frac{0.5 R_{\text{wire}} C_{\text{wire}}}{R_{\text{buf}} (C_{\text{self}} + C_{\text{load}})}} \]

\[ N \cdot R_{\text{buf}} (C_{\text{self}} + C_{\text{load}}) + R_{\text{buf}} C_{\text{wire}} + 0.5 \left( \frac{1}{N} \right) R_{\text{wire}} C_{\text{wire}} + R_{\text{wire}} C_{\text{load}} \]
Delay with Optimal N

\[
\sqrt{\frac{0.5R_{\text{wire}}C_{\text{wire}}}{R_{\text{buf}}(C_{\text{self}} + C_{\text{load}})}} \cdot R_{\text{buf}}(C_{\text{self}} + C_{\text{load}}) + R_{\text{buf}}C_{\text{wire}} + 0.5 \left( \sqrt{\frac{R_{\text{buf}}(C_{\text{self}} + C_{\text{load}})}{0.5R_{\text{wire}}C_{\text{wire}}}} \right)R_{\text{wire}}C_{\text{wire}} + R_{\text{wire}}C_{\text{load}}
\]

\[
\sqrt{0.5R_{\text{wire}}C_{\text{wire}} \left( R_{\text{buf}} \left( C_{\text{self}} + C_{\text{load}} \right) \right) + R_{\text{buf}}C_{\text{wire}} + \sqrt{0.5R_{\text{wire}}C_{\text{wire}} \left( R_{\text{buf}} \left( C_{\text{self}} + C_{\text{load}} \right) \right) + R_{\text{wire}}C_{\text{load}}}
\]

\[
2\sqrt{0.5R_{\text{wire}}C_{\text{wire}} \left( R_{\text{buf}} \left( C_{\text{self}} + C_{\text{load}} \right) \right) + R_{\text{buf}}C_{\text{wire}} + R_{\text{wire}}C_{\text{load}}}
\]
Calculate: Delay at Optimum Stages

- $R_u = 60K \ \Omega \ \text{per 1mm of wire}$
- $C_u = 0.16 \ \text{pF per 1mm of wire}$
- $R_{buf} = R_0 = 25K \ \Omega$
- $C_{self} = C_{load} = 2(C_0 = 0.01 \ \text{fF}) = 0.02 \text{fF}$

\[
N = \sqrt{\frac{0.5 R_{\text{wire}} C_{\text{wire}}}{R_{buf} \left( C_{\text{self}} + C_{\text{load}} \right)}}
\]
\[
2 \sqrt{0.5 R_{\text{wire}} C_{\text{wire}} \left( R_{buf} \left( C_{\text{self}} + C_{\text{load}} \right) \right)} + R_{buf} C_{\text{wire}} + R_{\text{wire}} C_{\text{load}}
\]
Segment Length

- \( R_{\text{wire}} = L \times R_{\text{unit}} \)
- \( C_{\text{wire}} = L \times C_{\text{unit}} \)

\[
L^*_{\text{seg}} = \frac{L}{N}
\]

\[
N = \sqrt{\frac{0.5 \left( \frac{R_{\text{wire}} \times C_{\text{wire}}}{R_{\text{buf}} \times (C_{\text{self}} + C_{\text{load}})} \right)}{R_{\text{buf}} \times (C_{\text{self}} + C_{\text{load}})}}
\]

\[
N = L \sqrt{0.5 \left( \frac{R_u \times C_u}{R_{\text{buf}} \times (C_{\text{self}} + C_{\text{load}})} \right)}
\]

\[
L^*_{\text{seg}} = \frac{L}{N} = \sqrt{2 \left( \frac{R_{\text{buf}} \times (C_{\text{self}} + C_{\text{load}})}{R_u \times C_u} \right)}
\]
Optimal Segment Length

- Delay scales linearly with distance once optimally buffered

\[ L_{seg}^* = \frac{L}{N} = \sqrt{2 \left( \frac{R_{buf} \times \left( C_{self} + C_{load} \right)}{R_u \times C_u} \right)} \]

\[ N = L \sqrt{0.5 \left( \frac{R_u \times C_u}{R_{buf} \times \left( C_{self} + C_{load} \right)} \right)} \]
Buffer Size?

- How big should buffer be?
  - $R_{buf} = \frac{R_0}{W}$
  - $C_{load} = 2W C_0$ (assuming velocity saturation)
  - $C_{self} = \gamma 2W C_0$
Buffer Size?

- How big should buffer be?
  - \( R_{\text{buf}} = \frac{R_0}{W} \)
  - \( C_{\text{load}} = 2W C_0 \) (assuming velocity saturation)
  - \( C_{\text{self}} = \gamma 2W C_0 \)

\[
2 \sqrt{0.5 R_{\text{wire}} C_{\text{wire}} \left( R_{\text{buf}} \left( C_{\text{self}} + C_{\text{load}} \right) \right) + R_{\text{buf}} C_{\text{wire}} + R_{\text{wire}} C_{\text{load}}}
\]

\[
2 \sqrt{0.5 R_{\text{wire}} C_{\text{wire}} \left( \frac{R_0}{W} \left( 2WC_0 \left( 1 + \gamma \right) \right) \right) + \frac{R_0}{W} C_{\text{wire}} + R_{\text{wire}} \cdot 2WC_0}
\]

\[
2 \sqrt{0.5 R_{\text{wire}} C_{\text{wire}} \left( 2R_0 C_0 \left( 1 + \gamma \right) \right) + \frac{R_0}{W} C_{\text{wire}} + R_{\text{wire}} \cdot 2WC_0}
\]
Implication W

- $R_{\text{wire}} = L \times R_{\text{unit}}$
- $C_{\text{wire}} = L \times C_{\text{unit}}$
- $\Rightarrow$ W independent of Length
  - Depends on technology

$$
2\sqrt{0.5 R_{\text{wire}} C_{\text{wire}} \left(2 R_0 C_0 (1+\gamma)\right)} + \frac{R_0}{W} C_{\text{wire}} + R_{\text{wire}} \cdot 2 WC_0
$$

$$
0 = 2R_{\text{wire}} C_0 - R_0 C_{\text{wire}} \frac{1}{W^2}
$$

$$
W = \sqrt{\frac{R_0 C_{\text{wire}}}{2 R_{\text{wire}} C_0}} = \sqrt{\frac{R_0 C_{\text{unit}}}{2 R_{\text{unit}} C_0}}
$$
Delay at Optimum W

\[
2\sqrt{0.5R_{\text{wire}}C_{\text{wire}} \left(2R_0C_0 \left(1 + \gamma\right)\right)} + \frac{R_0}{W} C_{\text{wire}} + R_{\text{wire}} \cdot 2WC_0
\]

\[
2\sqrt{0.5R_{\text{wire}}C_{\text{wire}} \left(2R_0C_0 \left(1 + \gamma\right)\right)} + \frac{R_0}{\sqrt{\frac{R_0C_{\text{wire}}}{2R_{\text{wire}}C_0}} C_{\text{wire}} + R_{\text{wire}} \cdot 2\sqrt{\frac{R_0C_{\text{wire}}}{2R_{\text{wire}}C_0}} C_0
\]

\[
2\sqrt{0.5R_{\text{wire}}C_{\text{wire}} \left(2R_0C_0 \left(1 + \gamma\right)\right)} + \sqrt{R_0C_{\text{wire}} \cdot 2R_{\text{wire}}C_0} + \sqrt{R_0C_{\text{wire}} \cdot 2R_{\text{wire}}C_0}
\]

\[
2\sqrt{0.5R_{\text{wire}}C_{\text{wire}} \left(2R_0C_0 \left(1 + \gamma\right)\right)} + 2\sqrt{2R_0C_0C_{\text{wire}}R_{\text{wire}}}
\]
Delay at Optimum W

- If $\gamma=1$

\[
2\sqrt{R_{\text{wire}}C_{\text{wire}} \left( R_0C_0(1+1) \right)} + 2\sqrt{2R_0C_0C_{\text{wire}}R_{\text{wire}}}
\]

\[
2\sqrt{2R_0C_0C_{\text{wire}}R_{\text{wire}}} + 2\sqrt{2R_0C_0C_{\text{wire}}R_{\text{wire}}}
\]

\[
4\sqrt{2R_0C_0C_{\text{wire}}R_{\text{wire}}}
\]

- Optimal design equalizes all delays!
Ideas

- Wire delay linear once buffered optimally
- Optimal buffering equalizes delays
  - Buffer delay
  - Delay on wire between buffers
  - Delay of wire driving buffer
Final

- Cumulative and will cover everything including today
  - Lec1-28: Quantitative understanding
    - Know material covered at a quantitative level
    - Be able to apply analysis and design methods and techniques to new circuit/system
  - Lec29-34: Qualitative understanding
    - Know material covered in slides at a qualitative level
    - Will not be expected to apply to something not seen before
  - Expect comprehensive memory problem
  - Big Idea Presentation for topics covered

  --2011 many “small” problems – good coverage
Admin

- PROJECT DUE TODAY @ 11:59pm in Canvas
  - Don’t turn it in late! NO EXCEPTIONS
- Hans Review Monday (12/7)
- Final (12/15) noon Towne 311