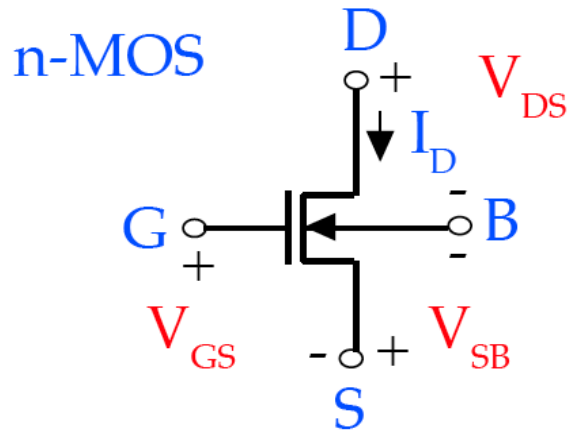


# ESE370: Circuit-Level Modeling, Design, and Optimization for Digital Systems

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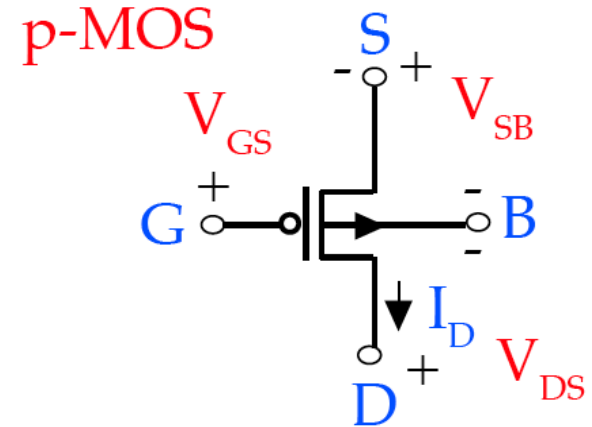
Lec 10: September 27, 2021  
MOS Transistor Variation

# Review: nMOS IV Model



$$I_D = \begin{cases} I_s \left(\frac{W}{L}\right) e^{\left(\frac{V_{GS}-V_{Tn}}{nkT/q}\right)} & V_{GS} \leq V_{Tn} & \text{Subthreshold} \\ \frac{\mu_n \cdot C_{ox}}{2} \left(\frac{W}{L}\right) (2(V_{GS} - V_{Tn})V_{DS} - V_{DS}^2) & V_{GS} > V_{Tn}, V_{DS} < V_{GS} - V_{Tn} & \text{Linear} \\ \frac{\mu_n \cdot C_{ox}}{2} \left(\frac{W}{L}\right) (V_{GS} - V_{Tn})^2 & V_{GS} > V_{Tn}, V_{DS} \geq V_{GS} - V_{Tn} & \text{Saturation} \\ \approx v_{sat} C_{ox} W \left(V_{GS} - V_{Tn} - \frac{V_{dsat}}{2}\right) & E_y > E_{cn} & \text{Velocity Saturation} \end{cases}$$

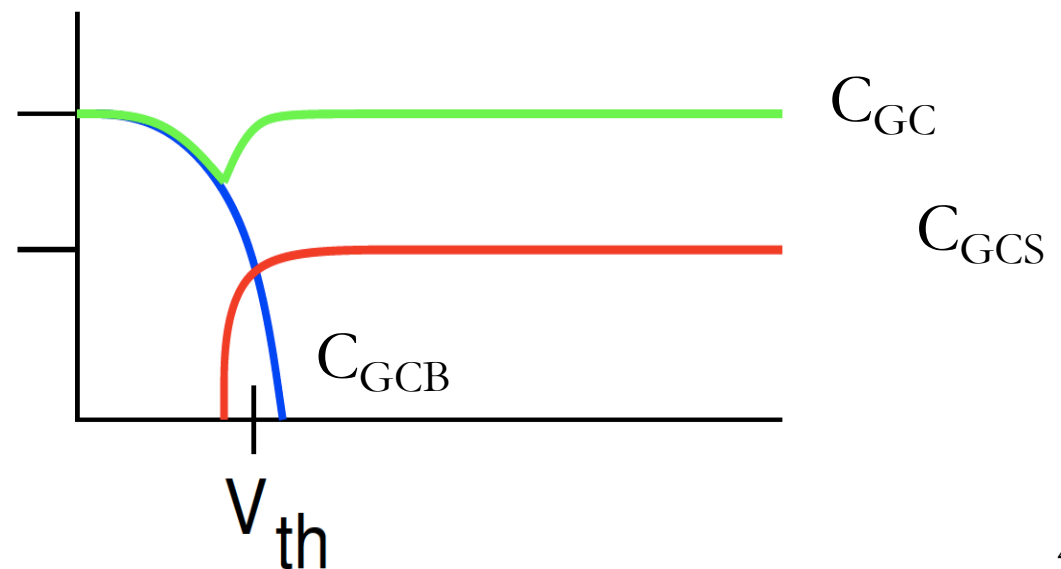
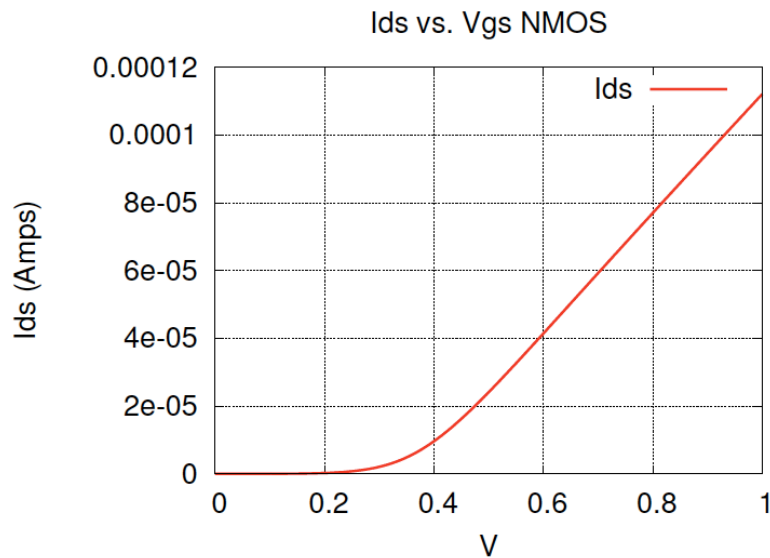
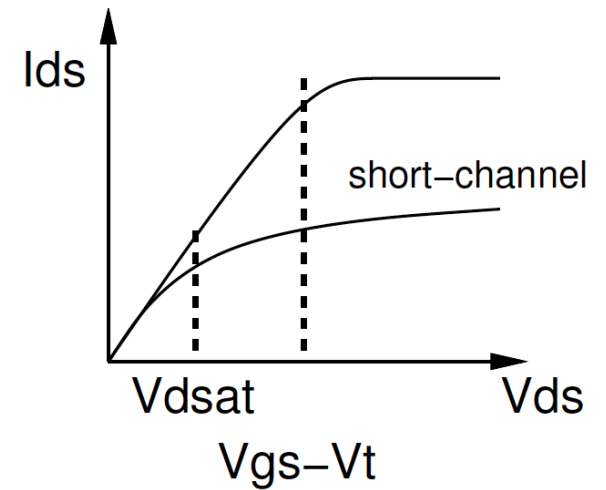
# Review: pMOS IV Model



$$I_D = \begin{cases} I_S \left(\frac{W}{L}\right) e^{\left(\frac{V_{GS}-V_{Tp}}{nkT/q}\right)} & V_{GS} \geq V_{Tp} & \text{Subthreshold} \\ \frac{\mu_p \cdot C_{ox}}{2} \left(\frac{W}{L}\right) (2(V_{GS} - V_{Tp})V_{DS} - V_{DS}^2) & V_{GS} < V_{Tp}, V_{DS} > V_{GS} - V_{Tp} & \text{Linear} \\ \frac{\mu_p \cdot C_{ox}}{2} \left(\frac{W}{L}\right) (V_{GS} - V_{Tp})^2 & V_{GS} < V_{Tp}, V_{DS} \leq V_{GS} - V_{Tp} & \text{Saturation} \\ \approx v_{sat} C_{ox} W \left(V_{GS} - V_{Tp} - \frac{V_{dsat}}{2}\right) & E_y > E_{cp} & \text{Velocity Saturation} \end{cases}$$

# Thus far...

- Understand how to model transistor behavior
- Given that we know its parameters
  - $V_{dd}$ ,  $V_{th}$ ,  $C_{OX}$ ,  $W$ ,  $L$ ,  $\mu$  ...





# But...

---

- ❑ We don't know its parameters (perfectly)
  - Fabrication parameters have nominal values and error range
  - Impact on  $I_{ds}$ ?
  
- ❑ Identically drawn devices differ because of fabrication techniques (e.g. process mismatch)
- ❑ Parameters change with environment (e.g. Temperature)
- ❑ Parameters change with time (aging)



# Today

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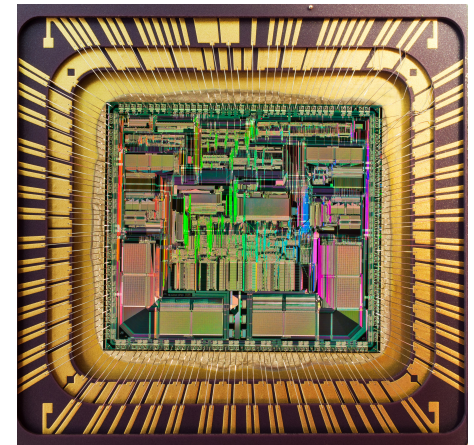
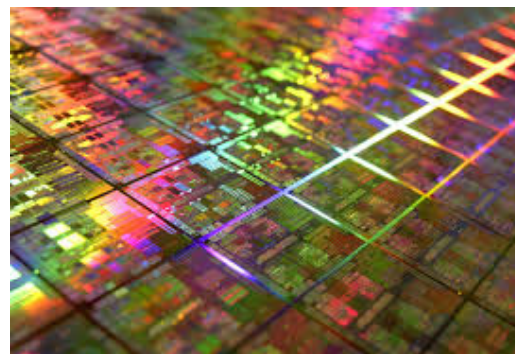
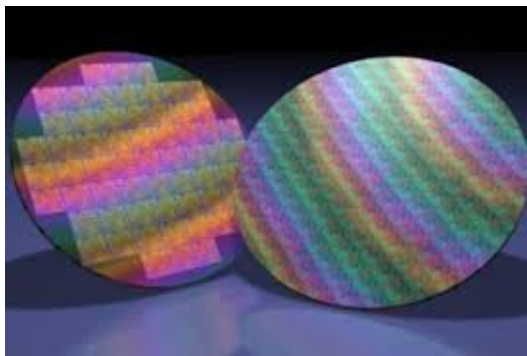
- Sources of Variation
  - Fabrication
  - Operation
  - Aging
- Designing to Account for Variation
  - Margin
  - Corners
  - Binning

# Fabrication

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# Variation Types

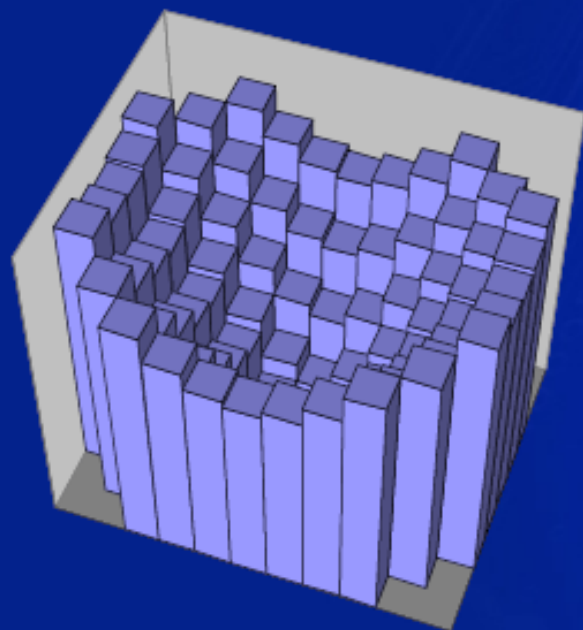
- ❑ Many reasons why variation occurs and shows up in different ways
- ❑ Scales of variation
  - Wafer-to-wafer, die-to-die, transistor-to-transistor
- ❑ Correlations of variation
  - Systematic, spatial, random (uncorrelated)





# Scale of Variations

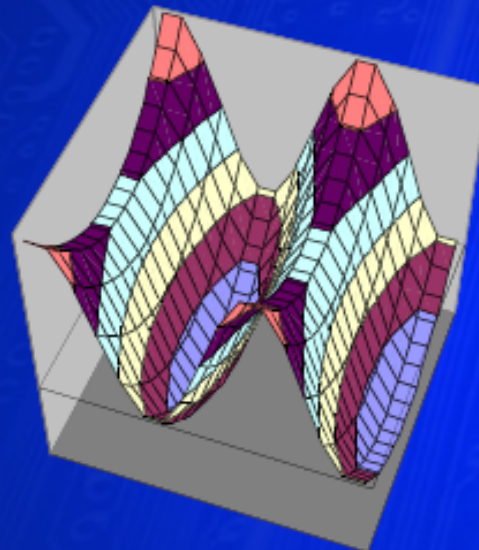
**Die-to-Die (D2D)  
Variations**



**Wafer Scale**

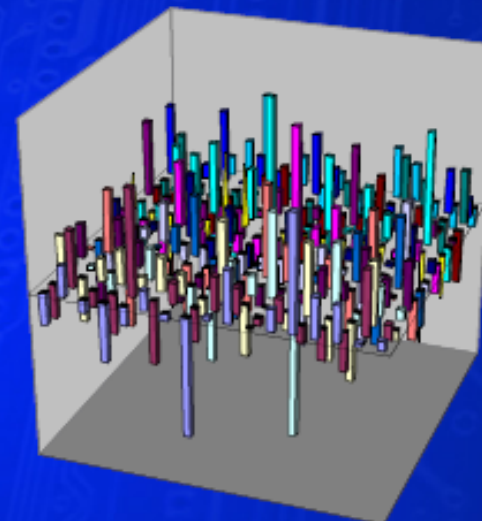
**Within-Die (WID)  
Variations**

**Systematic**



**Die Scale**

**(Uncorrelated)  
Random**

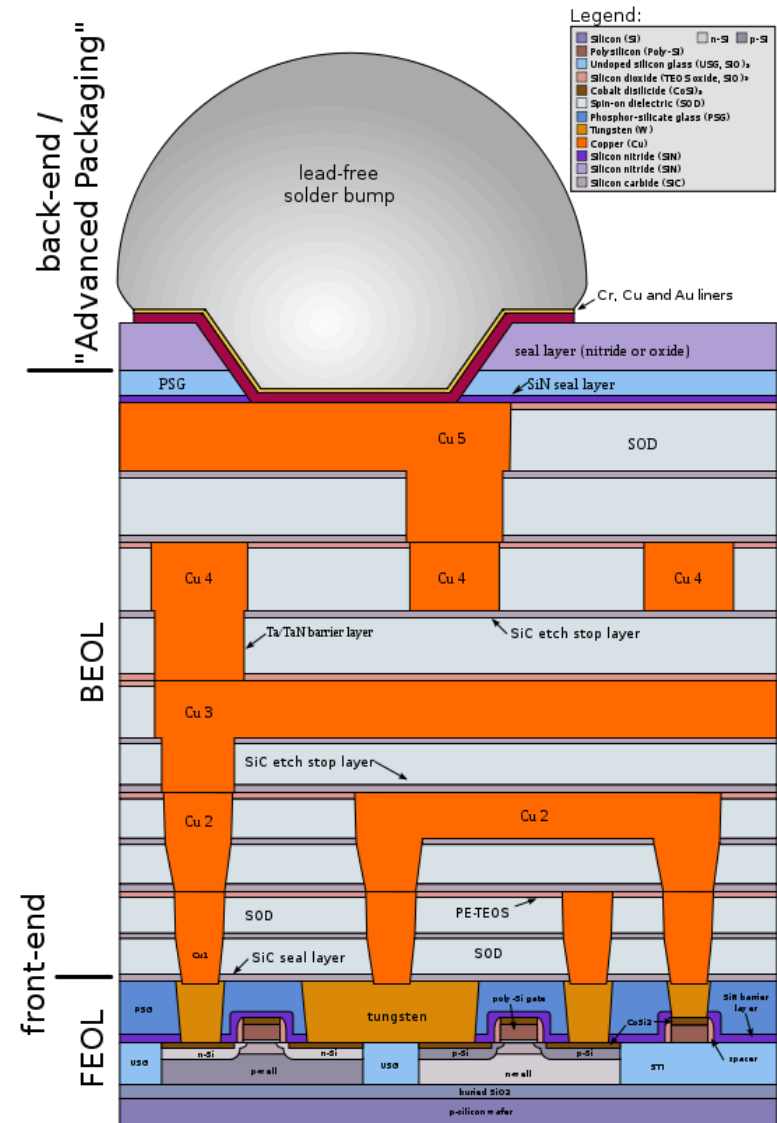


**Feature Scale**

Source: Noel Menezes, Intel ISPD2007

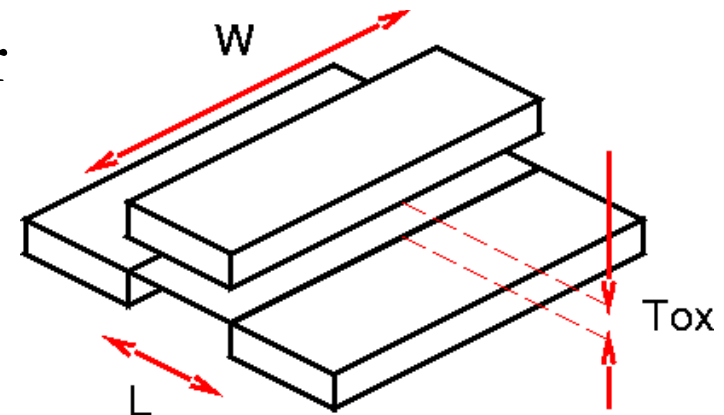
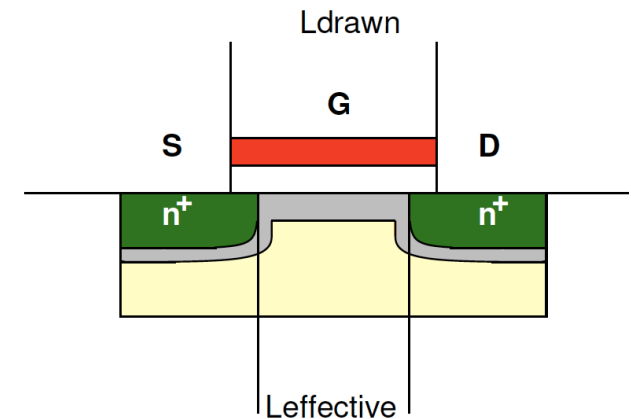
# Basic Fabrication: Two Steps

- ❑ (1) Transfer an image of the design to the wafer
- ❑ (2) Using that image (mask) as a guide, create the desired layers on silicon
  - Diffusion (add dopants to the silicon)
  - Oxide (create an insulating layer)
  - Metal (create a wire layer)



# Wafer Scale: Process Shift

- ❑ Oxide thickness
- ❑ Doping level
- ❑ Layer alignment
- ❑ Growth and Etch rates and times
  - Depend on chemical concentrations
    - How precisely can we control those?
- ❑ Vary machine-to-machine, day-to-day
- ❑ Impact all transistors on wafer



# Systematic Spatial Variation

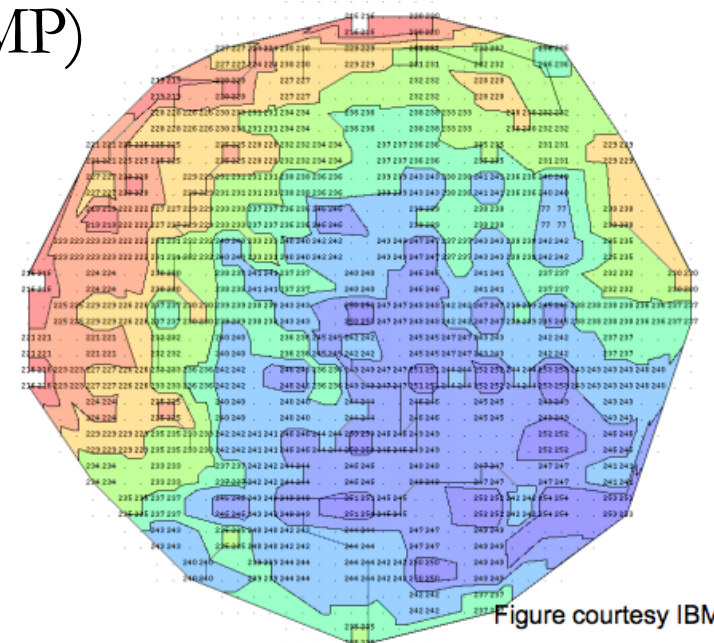
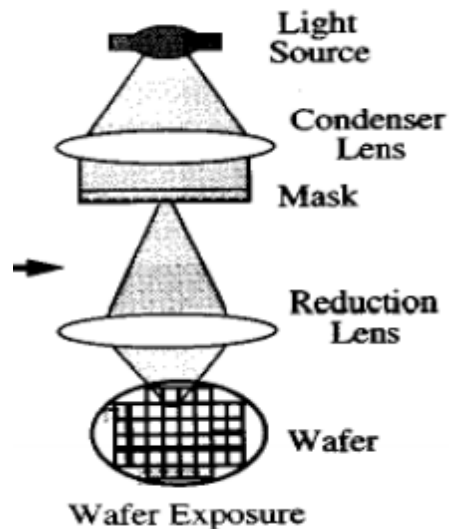
- Parameters change consistently across wafer or chip based on location

- Sources

- Chemical-Mechanical Polishing (CMP)

- Dishing

- Lens distortion



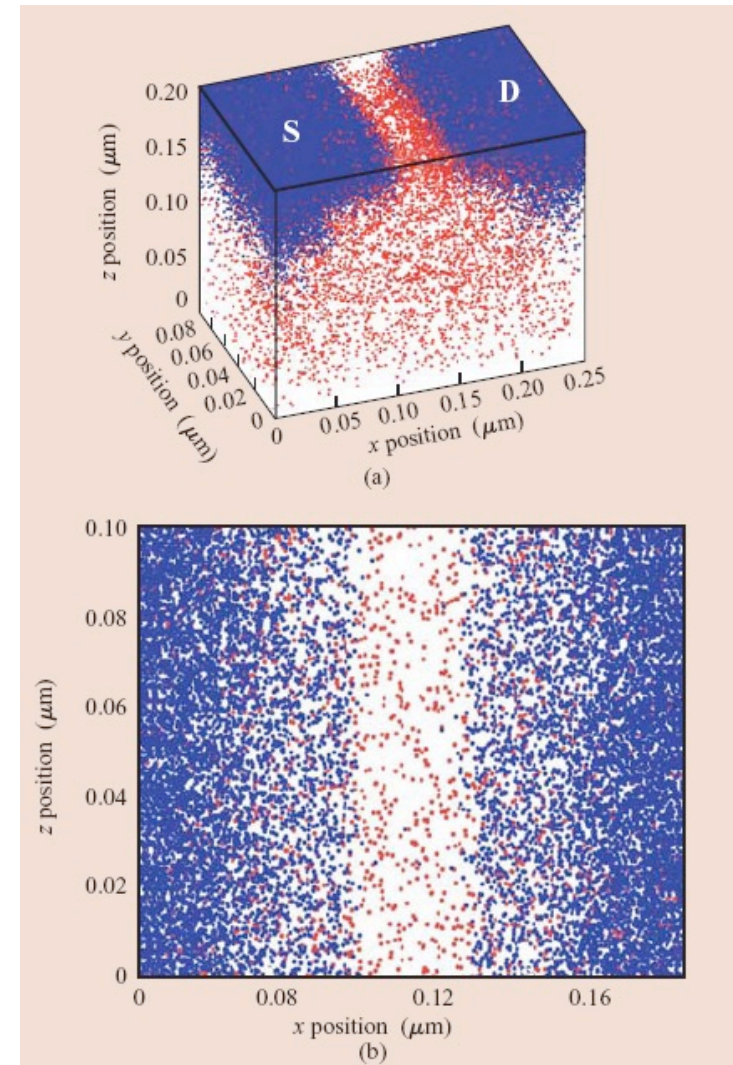
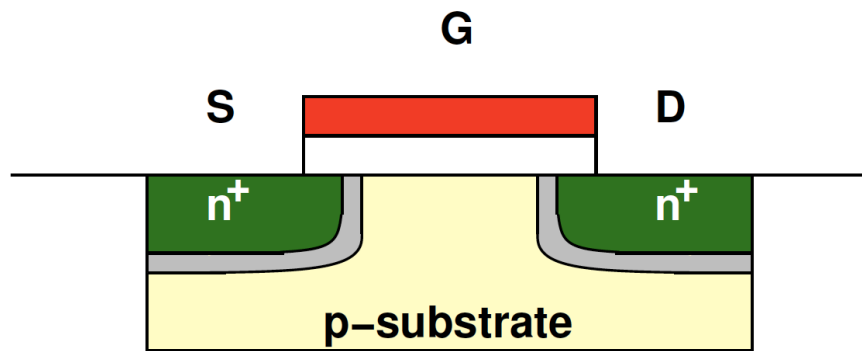


# Random Transistor-to-Transistor

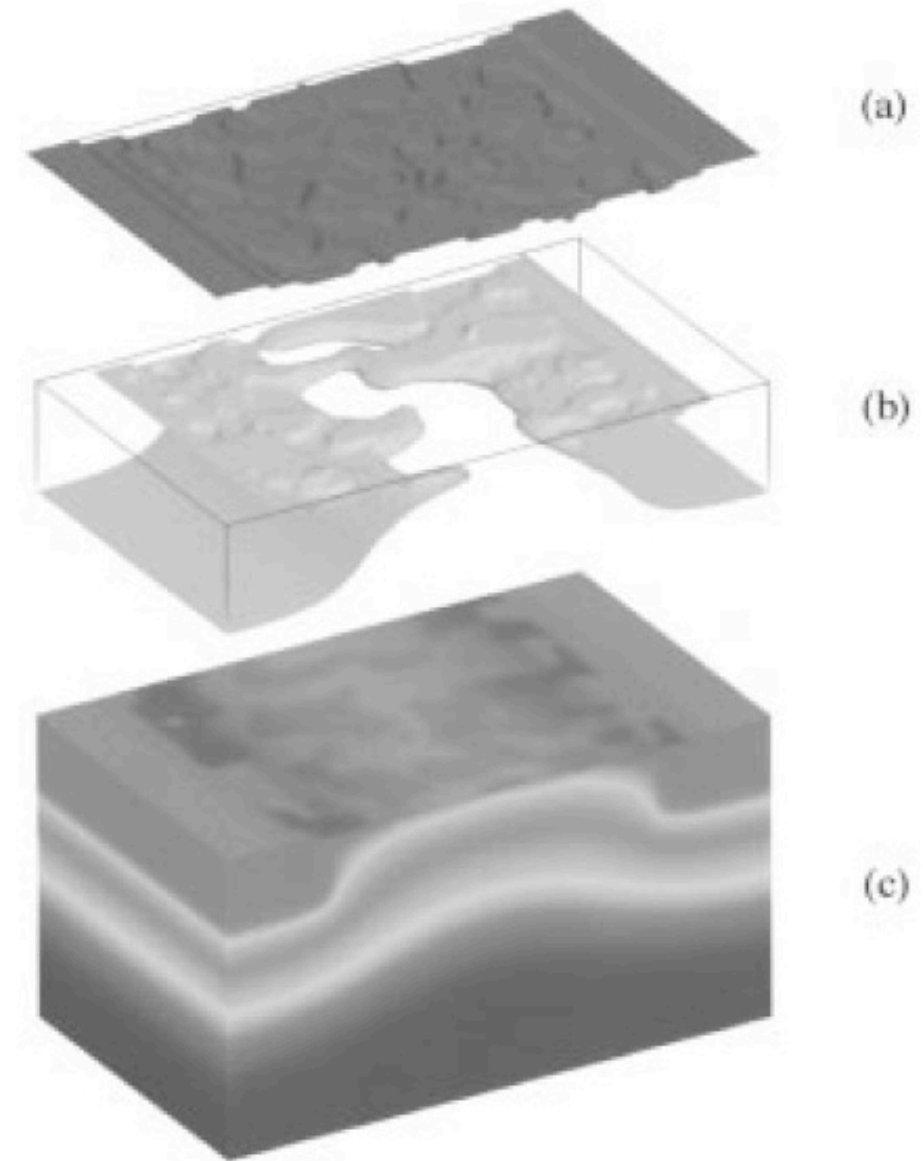
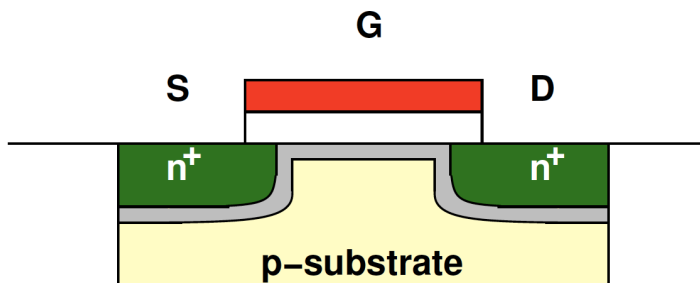
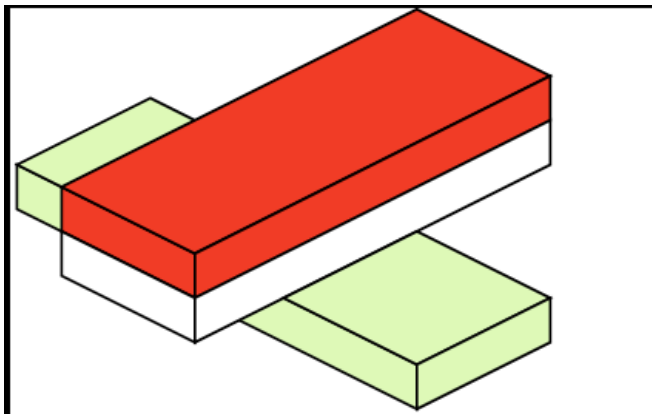
---

- ❑ Random dopant fluctuation
  - ❑ Local oxide variation
  - ❑ Line edge roughness
  - ❑ Etch and growth rates
- 
- ❑ Transistors differ from each other in random ways

# Statistical Dopant Placement



# Oxide Thickness and Interface roughness

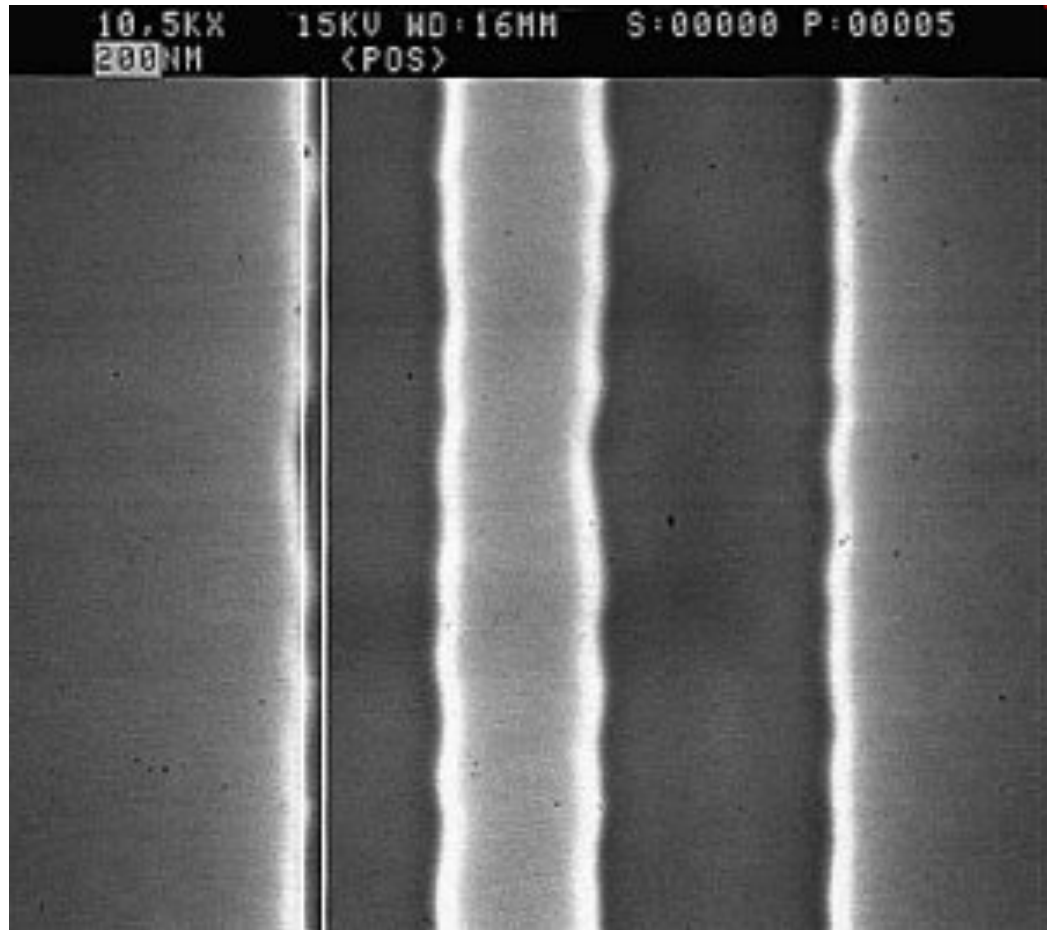
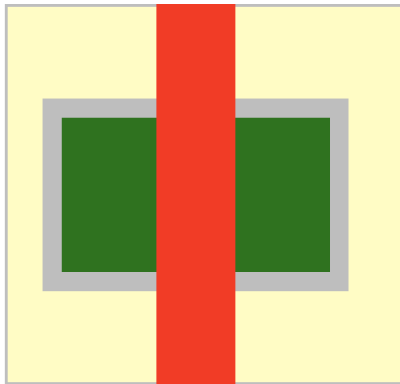


[Asenov et al. TRED 2002]

Fig. 1. (a) Typical profile of the random Si/SiO<sub>2</sub> interface in a 30 × 30 nm<sup>2</sup> MOSFET, followed by (b) an equiconcentration contour obtained from DG simulations, and (c) the potential distribution.



# Line Edge Roughness



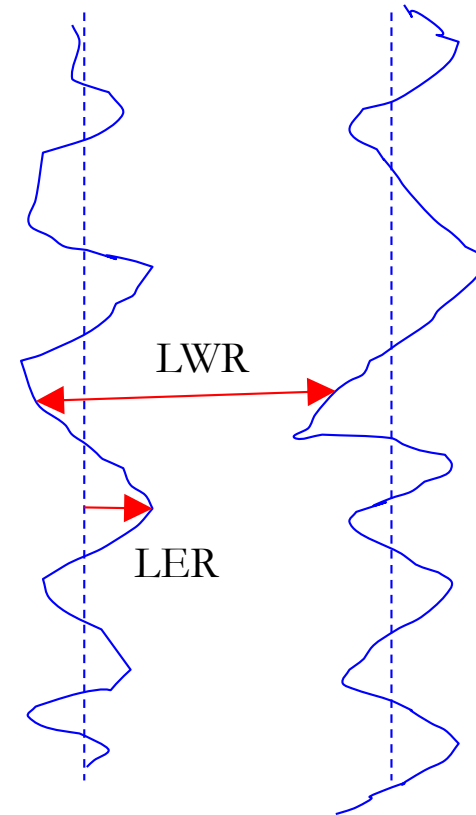
From:

[http://www.microtechweb.com/2d/lw\\_pict.htm](http://www.microtechweb.com/2d/lw_pict.htm)



# Line Edge & Line-Width Roughness

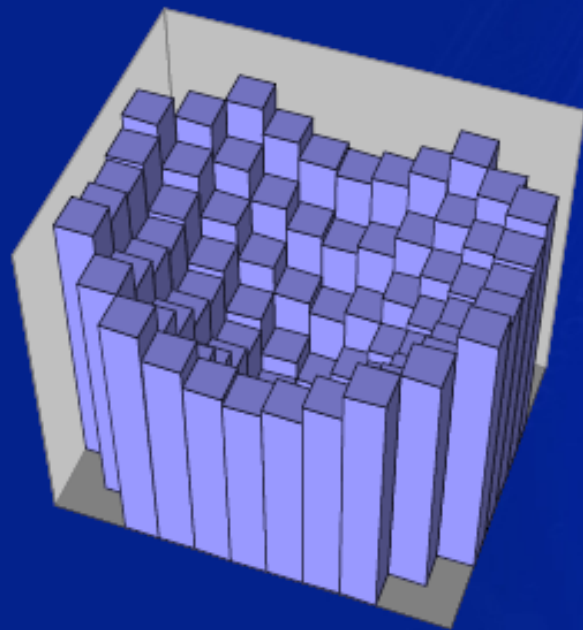
- ❑ LER = lines of device features are not straight
- ❑ LWR = distance between lines is not uniform
- ❑ LER & LWR arise from the lithography and etching processes
- ❑ They are most pronounced in poly-gate patterning



Source: Kuhn et al.

# Scale of Variations

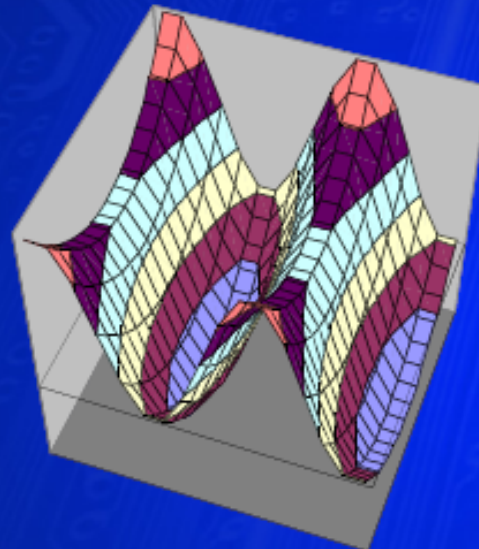
**Die-to-Die (D2D)  
Variations**



**Wafer Scale**

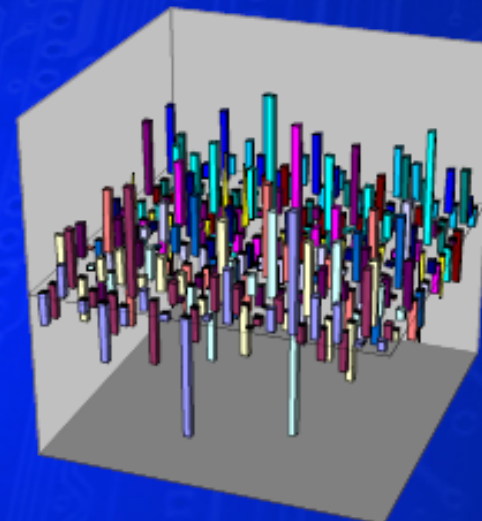
**Within-Die (WID)  
Variations**

**Systematic**



**Die Scale**

**(Uncorrelated)  
Random**



**Feature Scale**

Source: Noel Menezes, Intel ISPD2007

# Impact on $I_d$ (preclass 1)

- Changes parameters
  - $W, L, t_{OX}, V_{th}$ , etc.
- Change transistor behavior
  - $W$  increase?
  - $L$  increase?
  - $t_{OX}$  increase?
  - $V_{th}$  increase?

$$I_{DS} \approx v_{sat} C_{OX} W \left( V_{GS} - V_T - \frac{V_{DSAT}}{2} \right)$$

$$I_{DS} = \mu_n C_{OX} \left( \frac{W}{L} \right) \left[ (V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

## Example: $V_{th}$

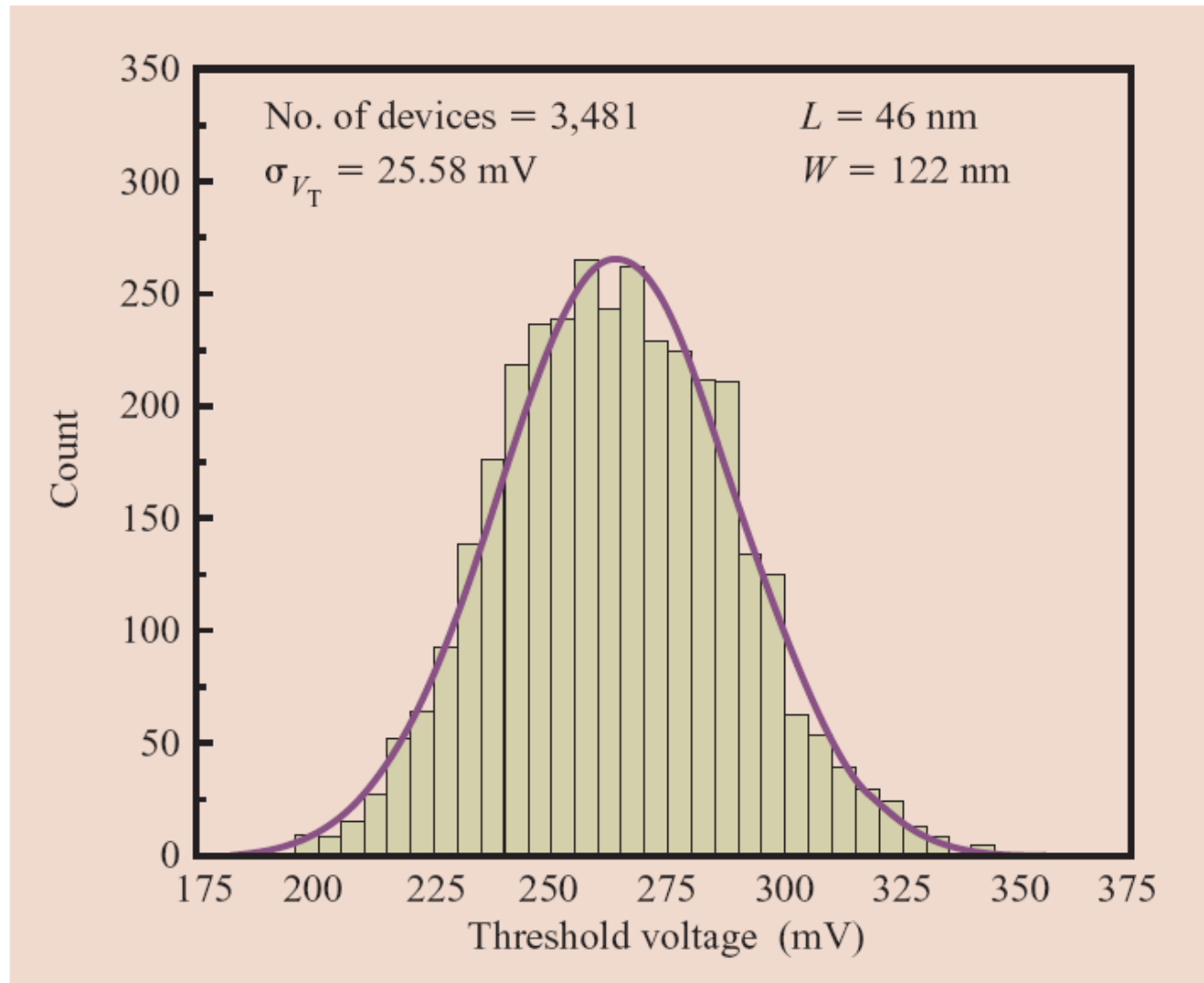
- Many physical effects impact  $V_{th}$ 
  - Doping, dimensions, roughness
- Behavior highly dependent on  $V_{th}$

$$I_{DS} \approx v_{sat} C_{OX} W \left( V_{GS} - V_T - \frac{V_{DSAT}}{2} \right)$$

$$I_{DS} = I'_S \left( \frac{W}{L} \right) e^{\left( \frac{V_{GS} - V_T}{nkT/q} \right)}$$



# $V_{th}$ Variability @ 65nm



[Bernstein et al, IBM JRD 2006]



# Impact of $V_{th}$ Variation?

---

- Higher  $V_{th}$ ?
  - Not drive as strongly
  - $I_{d,vsat} \propto (V_{gs} - V_{th})$
  - Performance?



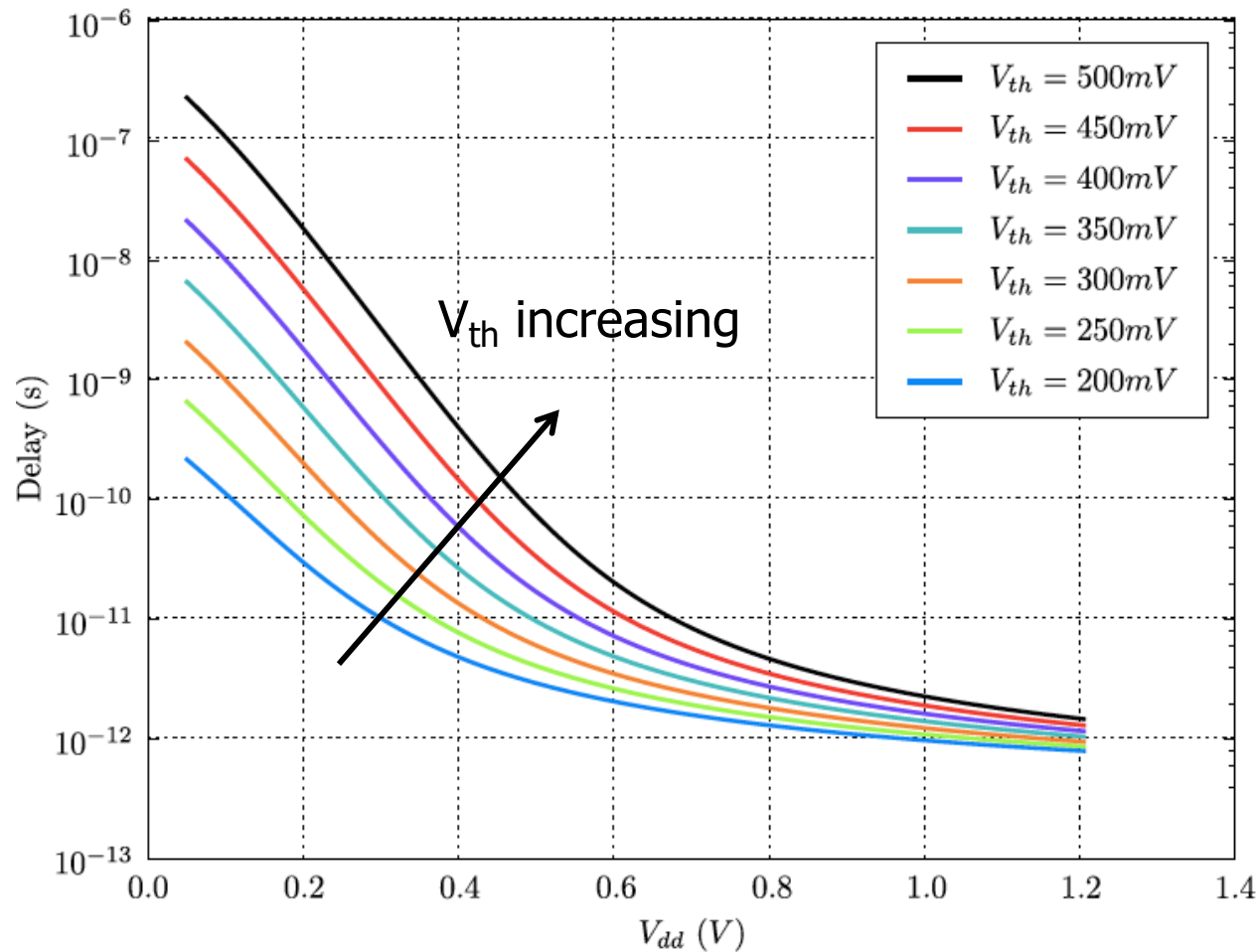
# Impact Performance

---

□ Higher  $V_{th}$   $\rightarrow$  lower  $I_{ds}$   $\rightarrow$  Delay ( $R_{on} * C_{load}$ )?

# Impact Performance

Higher  $V_{th}$   $\rightarrow$  lower  $I_{ds}$   $\rightarrow$  Delay ( $R_{on} * C_{load}$ )?

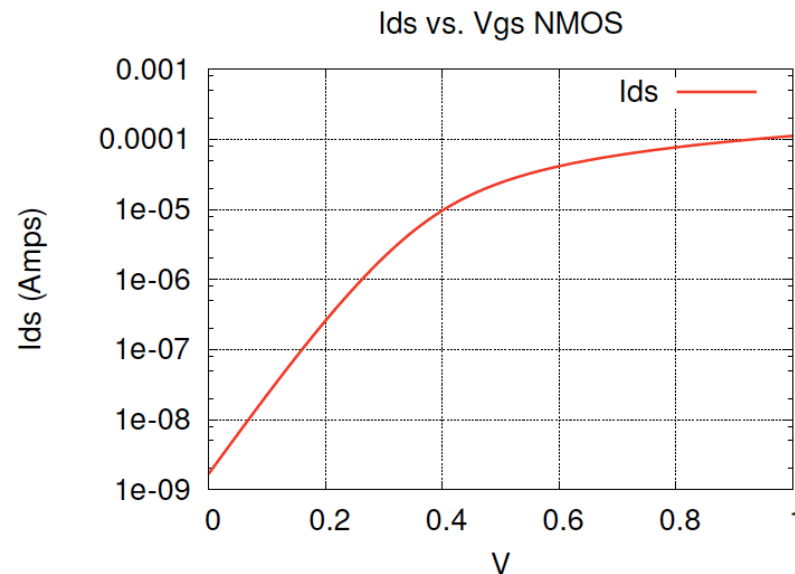




# Impact of $V_{th}$ Variation?

## □ Lower $V_{th}$ ?

- Not turn off as well → leaks more



$$I_{DS} = I'_S \left( \frac{W}{L} \right) e^{\left( \frac{V_{GS} - V_T}{nkT/q} \right)}$$

# Operation

---

Temperature  
Voltage



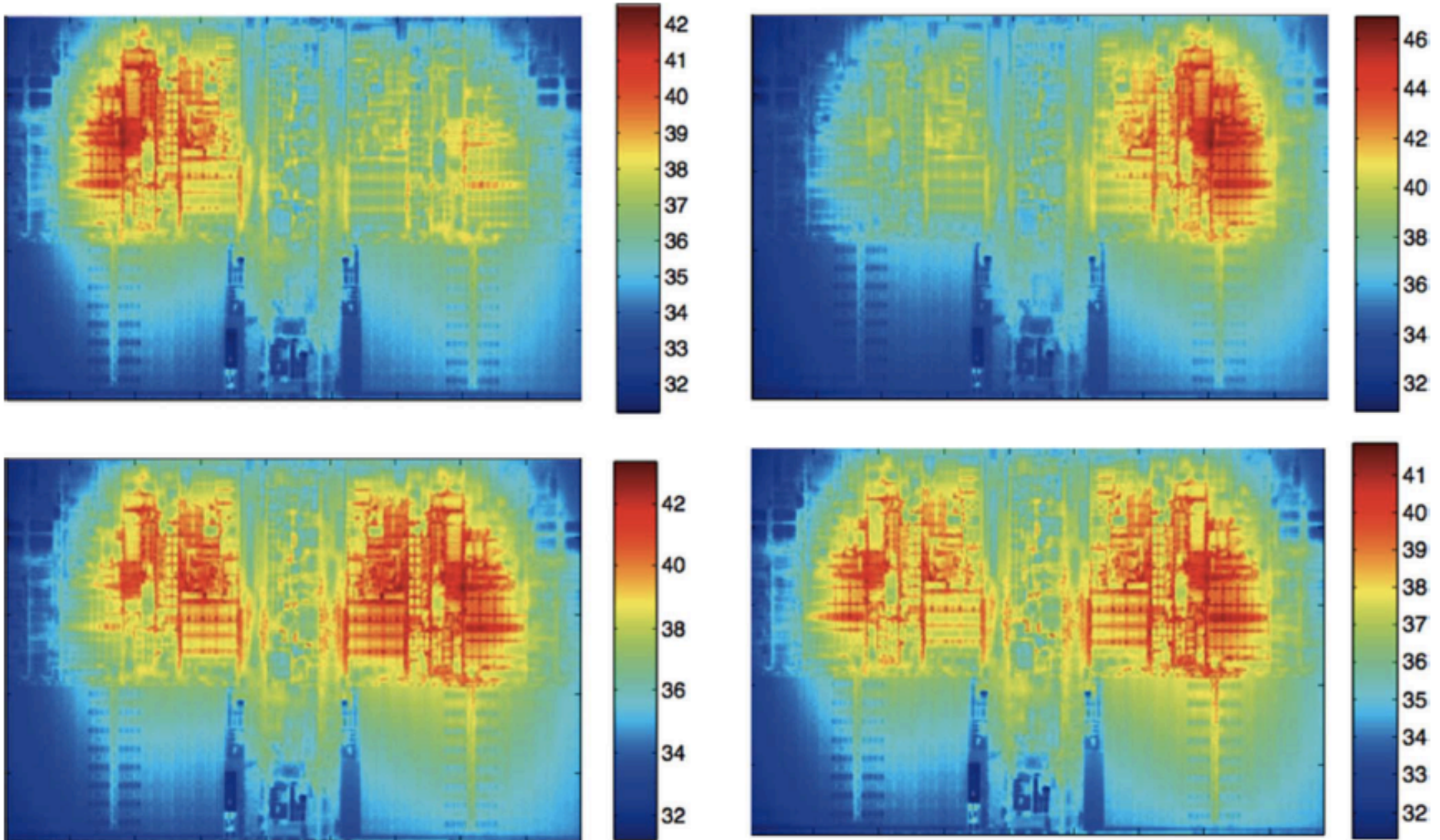
# Temperature Changes

---

- ❑ Different ambient environments
  - January in Maine
  - July in Philly
  - Air conditioned machine room
- ❑ Self heat from activity of chip
- ❑ Quality of heat sink (attachment thereof)
  - E.g cooling fan



# Thermal Profile for Processor





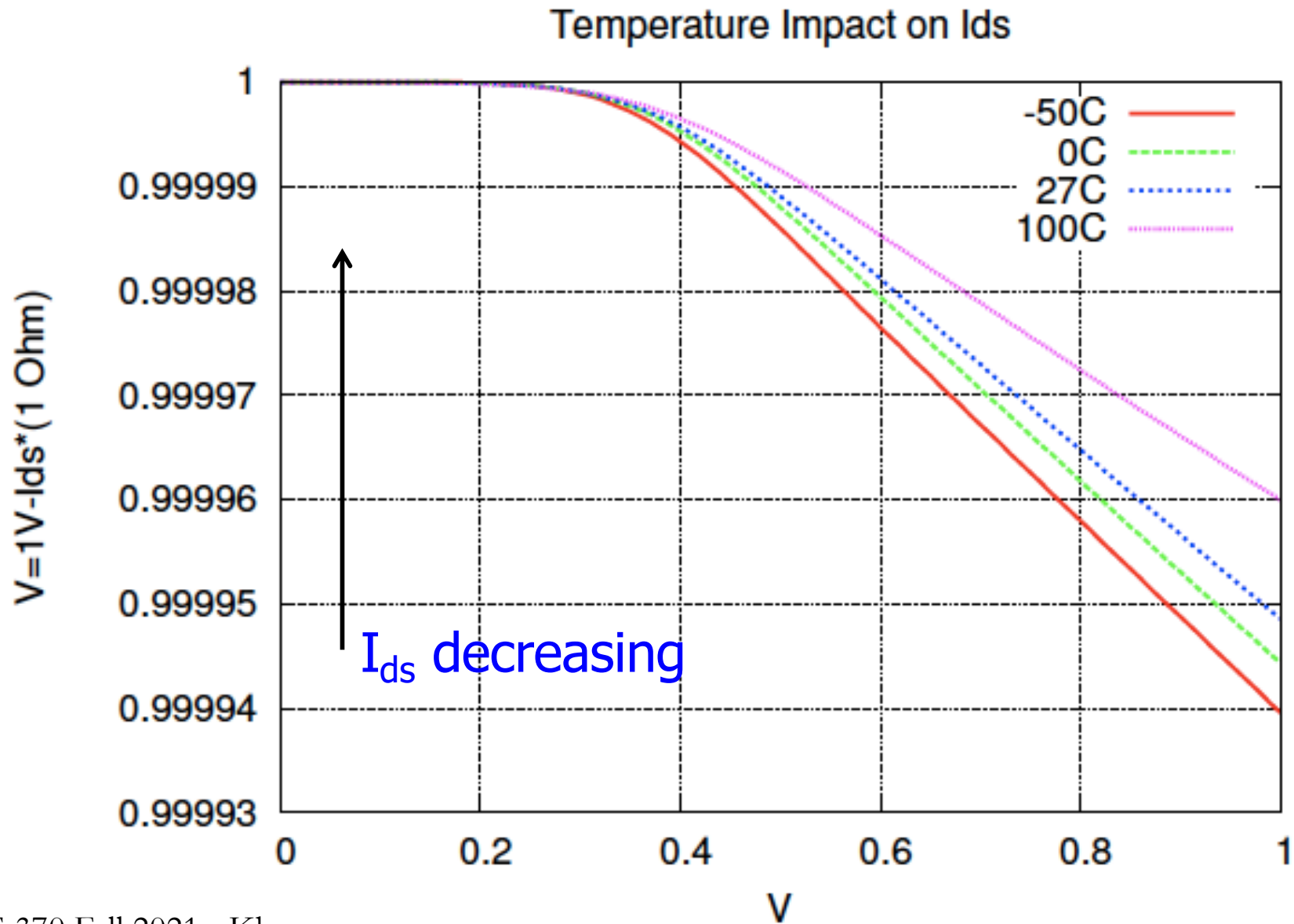
# How does temperature impact on-current?

---

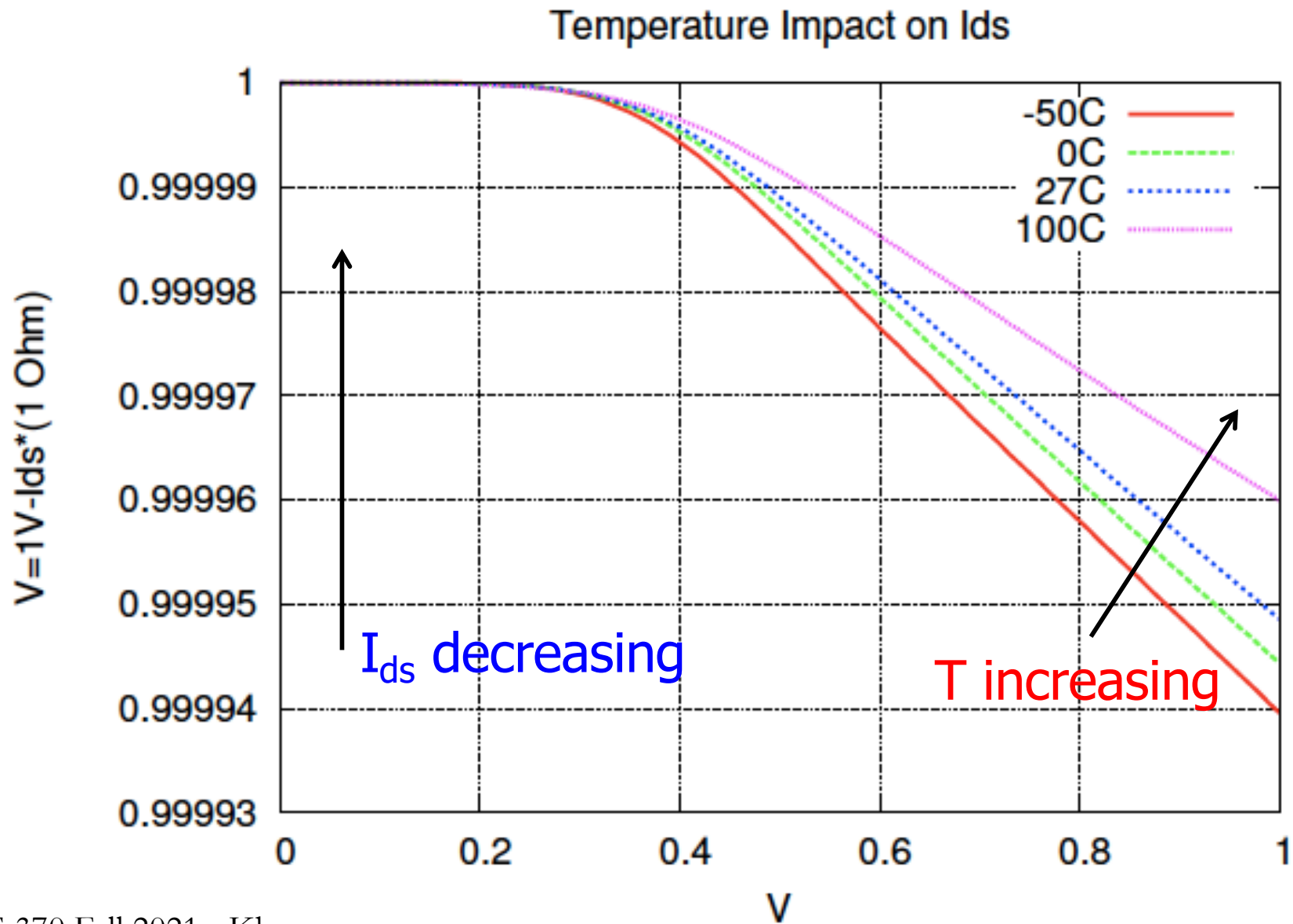
- High temperature
  - More free thermal energy
    - Easier to conduct
    - Lowers  $V_{th}$
  - Increase rate of collision
    - Lower saturation velocity
    - Lower saturation voltage
    - Lower peak  $I_{ds}$  → slows down
- One reason don't want chips to run hot



# Temperature and $I_{ds}$



# Temperature and $I_{ds}$



# How does temp impact leakage current?

---

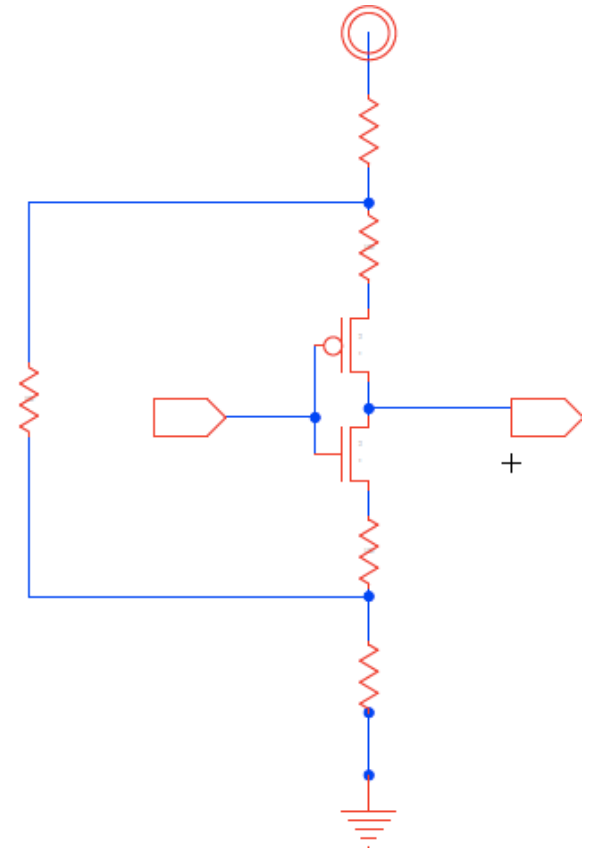
- High temperature lowers  $V_{th}$

$$I_{DS} = I'_S \left( \frac{W}{L} \right) e^{\left( \frac{V_{GS} - V_T}{nkT/q} \right)}$$



# Voltage

- ❑ Power supply isn't perfect
- ❑ Differs from design to design
  - Board to board?
  - How precise is regulator?
- ❑ IR-drop in distribution
- ❑ Bounce with current spikes



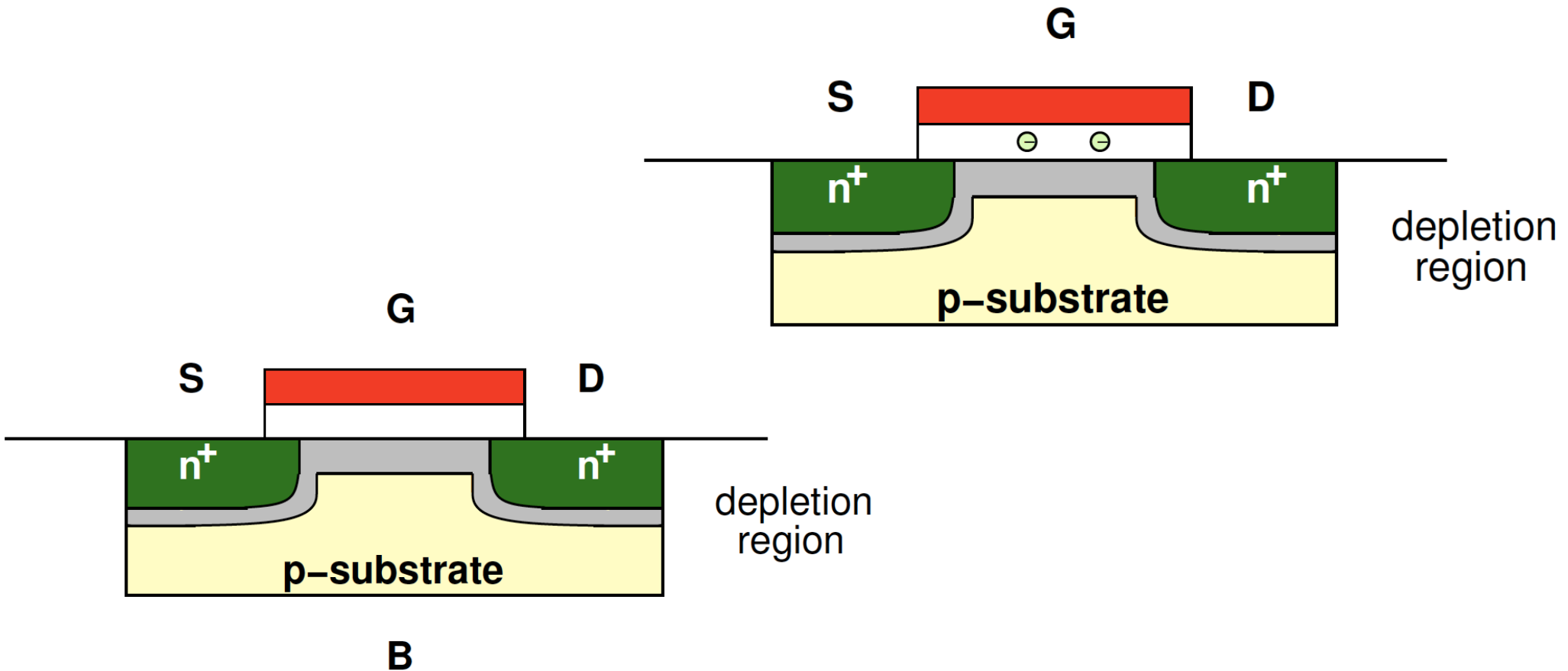
# Aging

---

## Hot Carrier Injection Negative Bias Temperature Instability (NBTI)

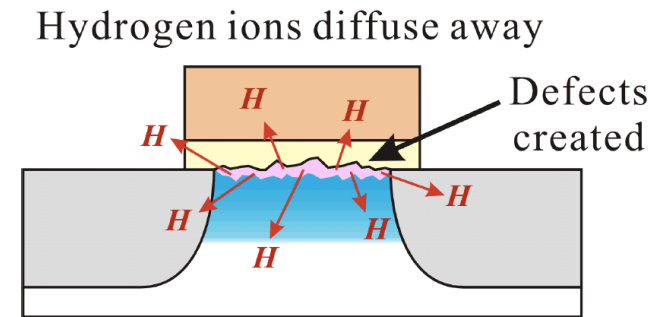
# Hot Carrier Injection

- Trap electrons in oxide
  - increases  $V_{th}$



# NBTI

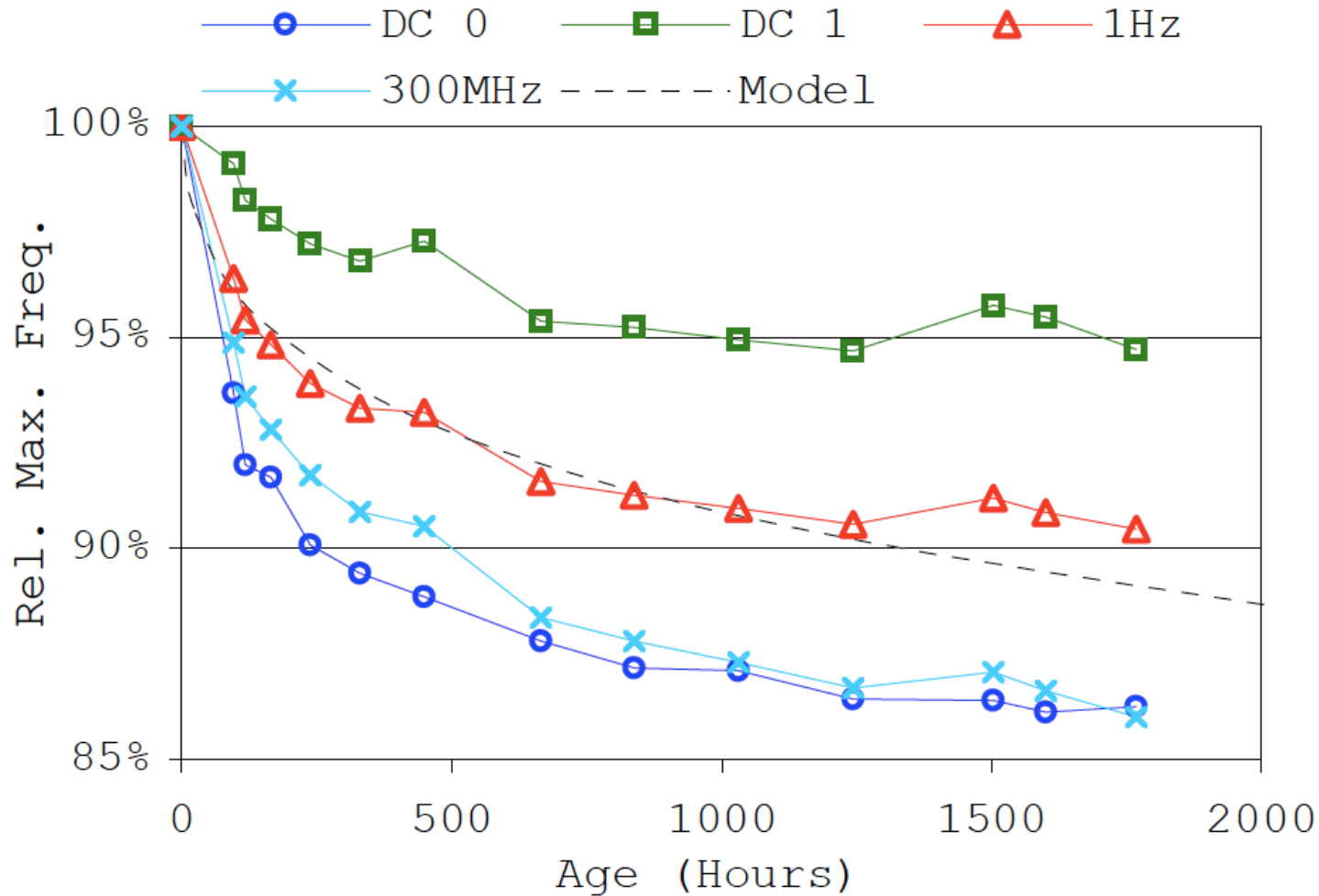
- ❑ Negative Bias Temperature Instability
  - Interface traps, Holes
- ❑ Long-term negative gate-source voltage
  - Affects PFET most
- ❑ Increase  $V_{th}$
- ❑ Temperature dependent



[Stott, FPGA2010]

$$\Delta V_t(t) \propto \exp(-\beta V_G) \exp\left(-\frac{E_a}{\kappa T}\right) t^n$$

# Measured Accelerated Aging (Cyclone III, 65nm FPGA)



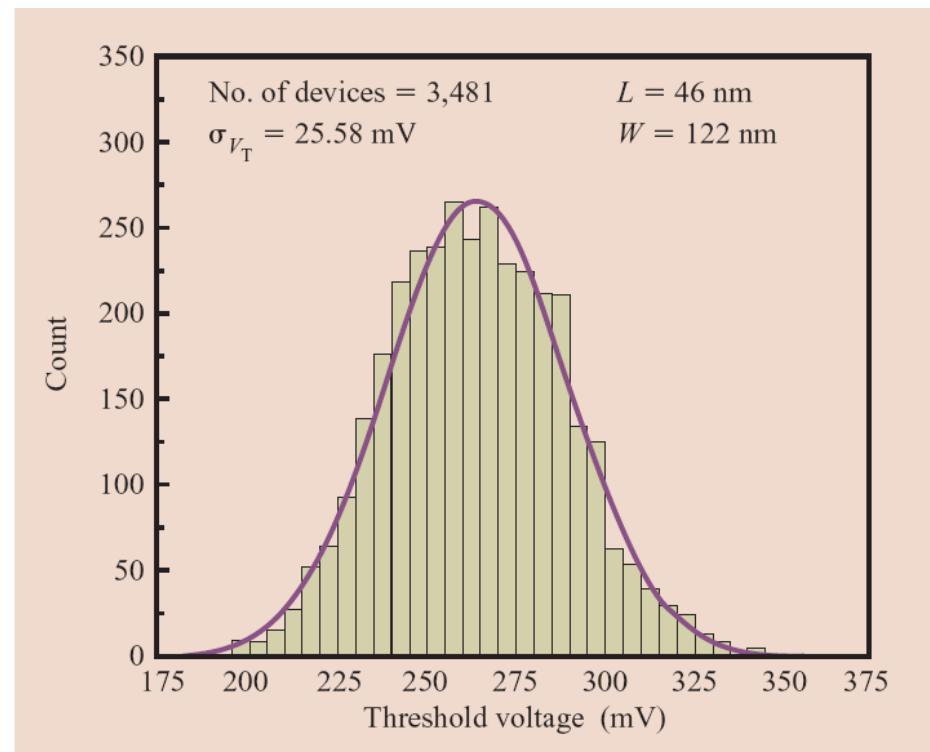
[Stott, FPGA2010]

# Coping with Variation

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# Variation

- See a range of parameters
  - L:  $L_{\min} - L_{\max}$
  - $V_{\text{th}}$ :  $V_{\text{th},\min} - V_{\text{th},\max}$



# Impact of $V_{th}$ Variation

- Higher  $V_{th}$ 
  - Not drive as strongly
  - $I_{d,vsat} \propto (V_{gs} - V_{TH})$

$$I_{DS} \approx v_{sat} C_{OX} W \left( V_{GS} - V_T - \frac{V_{DSAT}}{2} \right)$$

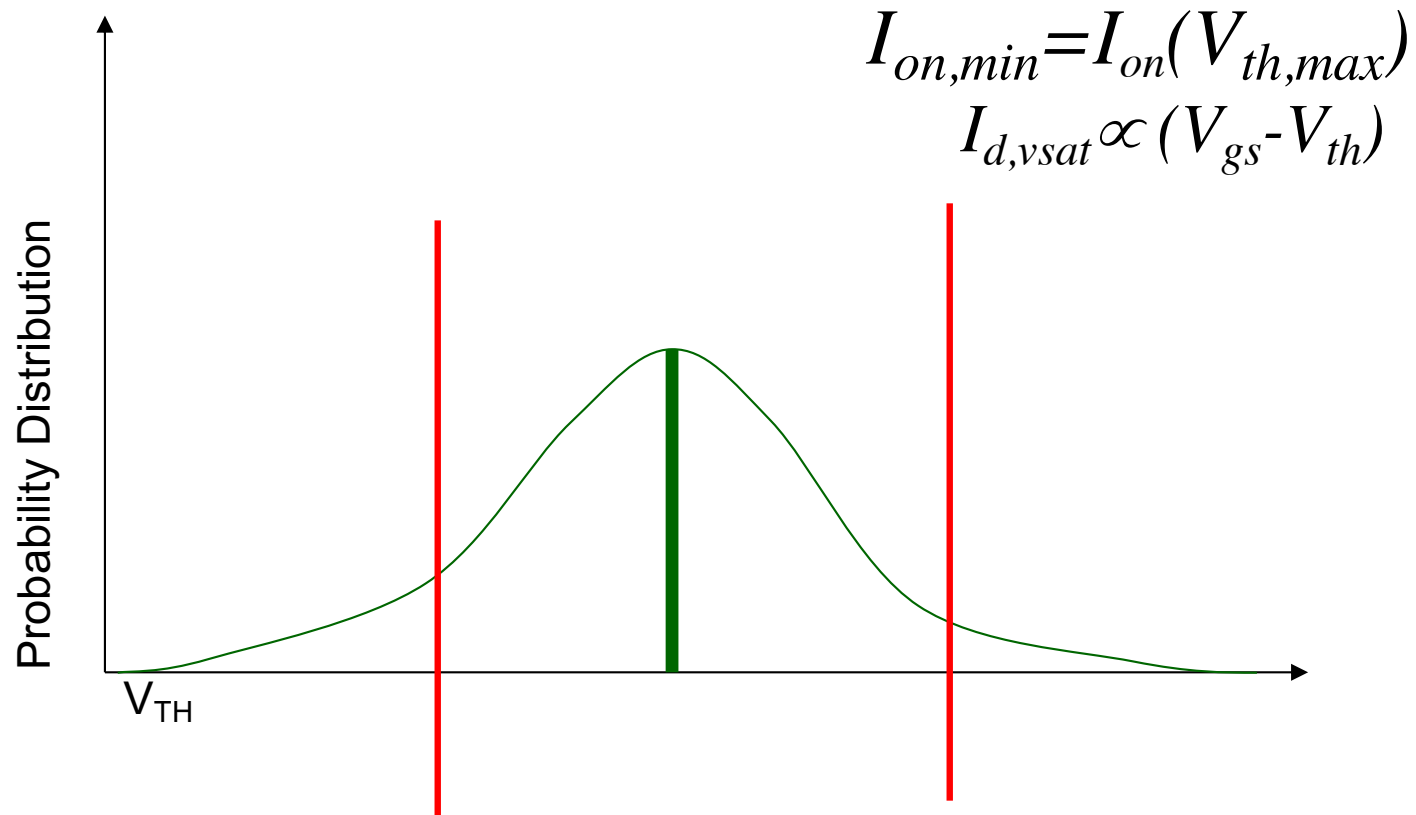
- Lower  $V_{th}$ 
  - Not turn off as well  $\rightarrow$  leaks more

$$I_{DS} = I'_S \left( \frac{W}{L} \right) e^{\left( \frac{V_{GS} - V_T}{nkT/q} \right)}$$



# Variation

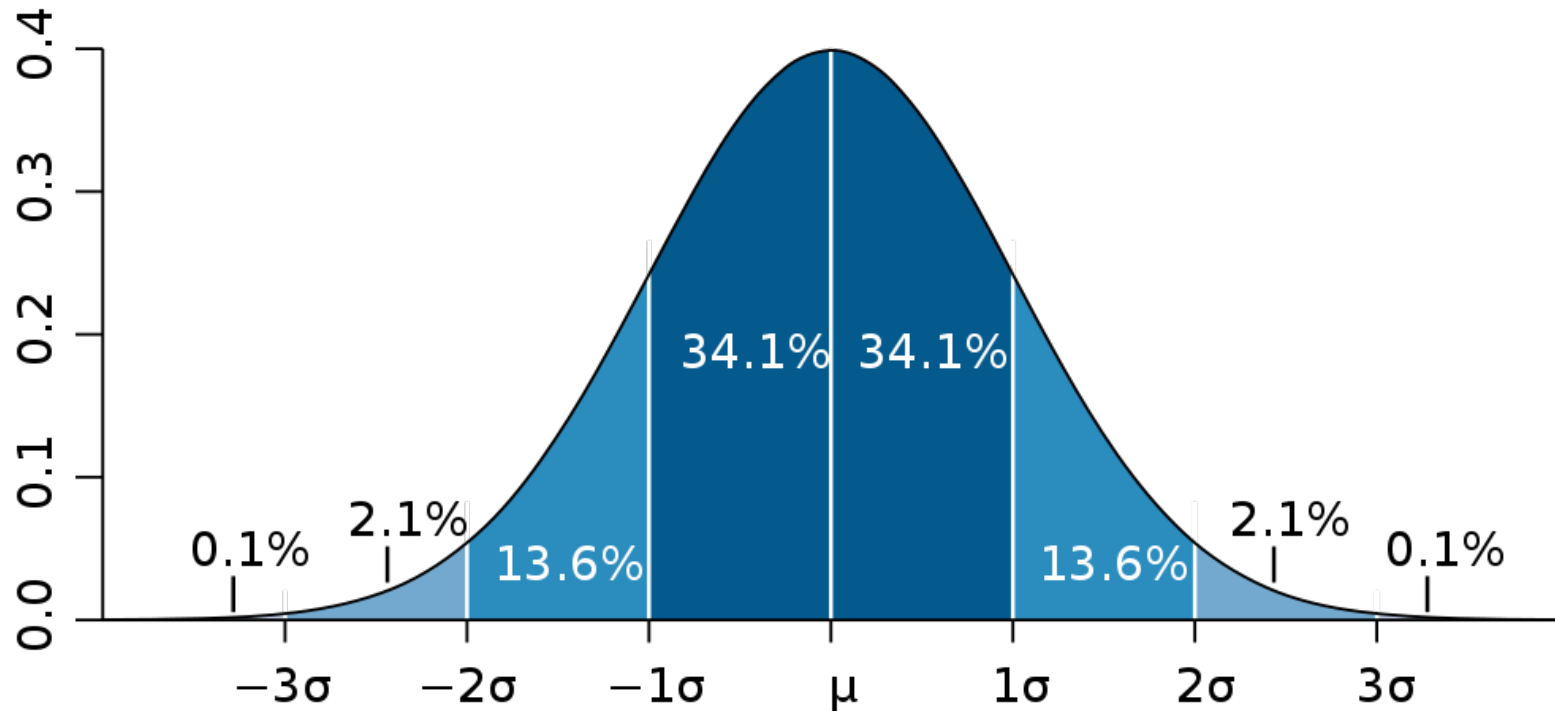
- Margin for expected variation
- Must assume  $V_{th}$  can be any value in range
  - Speed  $\rightarrow$  assume  $V_{th}$  slowest value





# Gaussian Distribution

---



From: [http://en.wikipedia.org/wiki/File:Standard\\_deviation\\_diagram.svg](http://en.wikipedia.org/wiki/File:Standard_deviation_diagram.svg)



# Impact (preclass 2&3)

---

## □ Given

- $V_{th,nom} = 250mV$
- Standard deviation:  $\sigma = 25mV$

## □ Probability of 100 transistor circuit having all transistors with threshold in range $200mV < V_{th} < 300mV$

- When each transistors has 96% prob of being in range?
- When each has 99.8% probability?



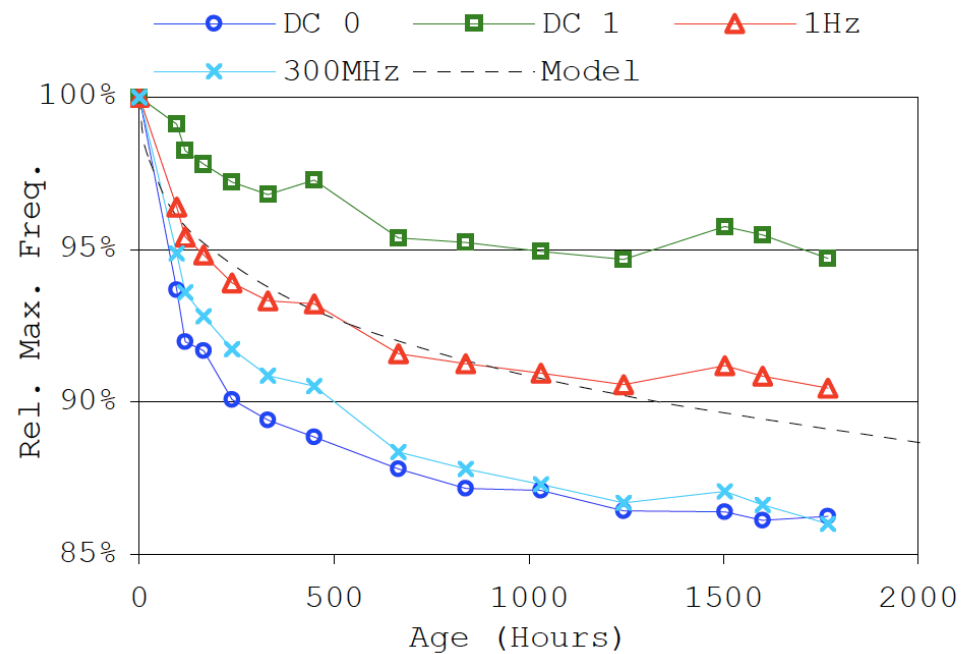
# Variation

---

- See a range of parameters
  - L:  $L_{\min} - L_{\max}$
  - $V_{th}$ :  $V_{th,\min} - V_{th,\max}$
  
- Validate design at extremes
  - Work for both  $V_{th,\min}$  and  $V_{th,\max}$  ?
  - Design for worst-case scenario

# Margining

- Also margin for
  - Temperature
  - Voltage
  - Aging: end-of-life



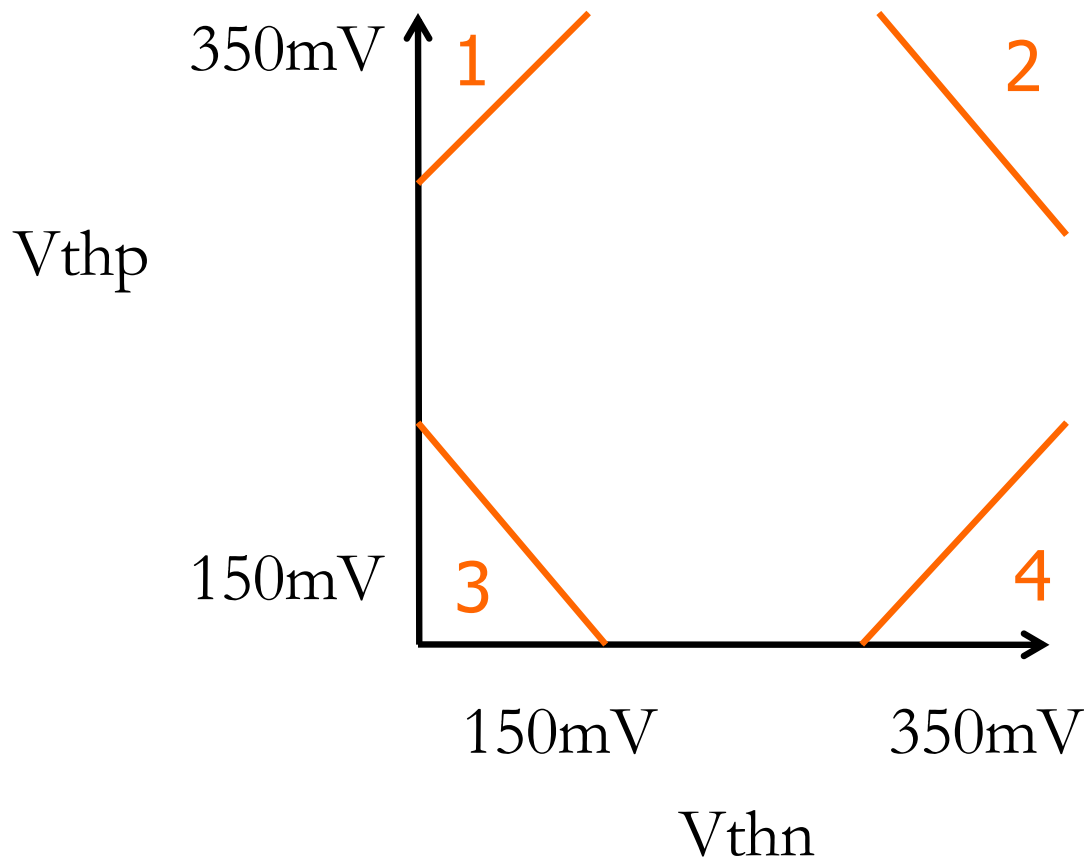


# Process Corners (preclass 4)

---

- ❑ Many effects independent
- ❑ Many parameters
- ❑ With  $N$  parameters,
  - Look only at extreme ends (low, high)
  - *How many cases?*
- ❑ Try to identify the {worst,best} set of parameters
  - Slow corner of design space, fast corner
- ❑ Use corners to bracket behavior

# Simple Corner Example



What happens  
at various  
corners?



# Process Corners

---

- ❑ Many effects independent
- ❑ Many parameters
- ❑ Try to identify the {worst,best} set of parameters
  - E.g. Lump together things that make slow
    - $V_{tn}$ ,  $V_{tp}$ , temperature, Voltage
    - Try to reduce number of unique corners
  - Slow corner of design space
- ❑ Use corners to bracket behavior





# Worst-case Corner Model

---

- ❑ corners for analog applications
  - For modeling worst-case speed
    - Slow NMOS and slow PMOS(SS) corner
  - For modeling worst-case power
    - Fast NMOS and fast PMOS(FF) corner
- ❑ corners for digital applications
  - For modeling worst-case 1
    - Fast NMOS and slow PMOS(FS) corner
  - For modeling worst-case 0
    - Slow NMOS and fast PMOS(SF) corner



# Worst-case Corner Model

---

## □ Advantages

- Worst case corner models give designers the capability to simulate the pass/fail results of a typical design and are usually pessimistic.

## □ Disadvantages

- The fixed-corner method is too wide
- Some valid designs can not be accepted in worst-case corner model
- The correlations between the device parameters are ignored



# Statistical Corner Model

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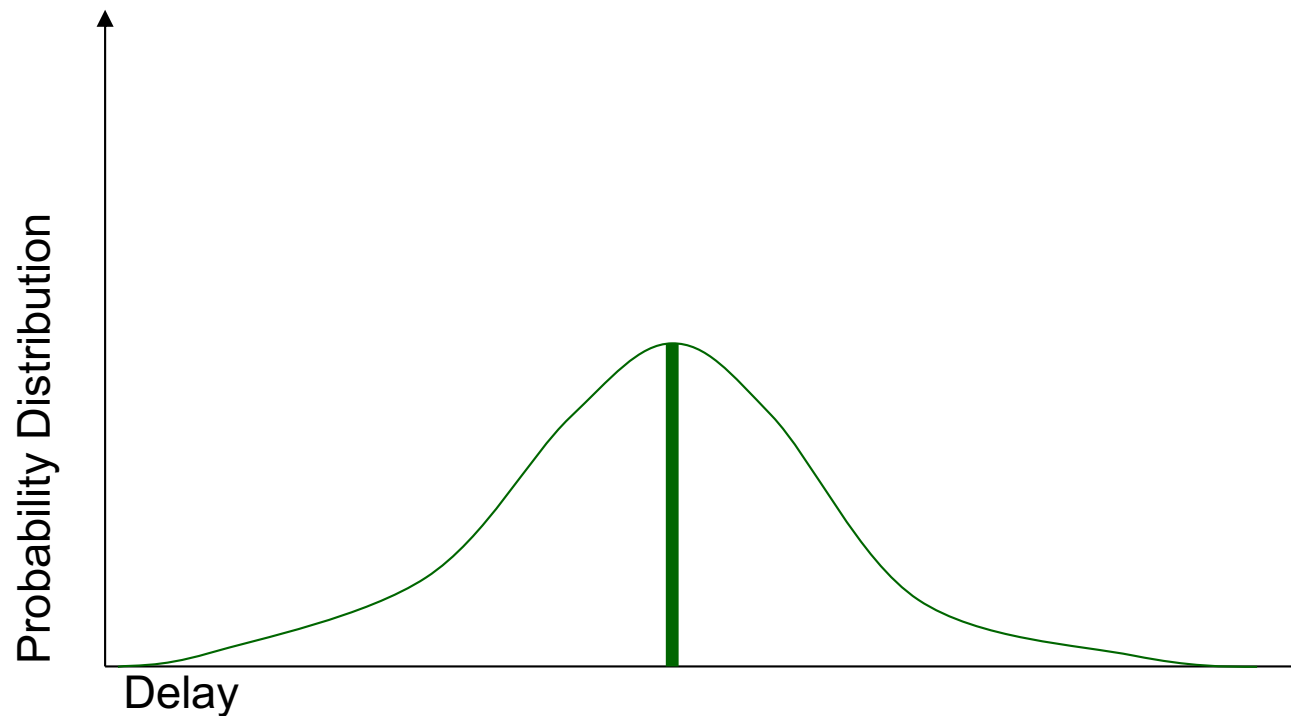
- ❑ For more realistic modeling for process variability than worst-case corner model.
  - Using data from different dies, wafers, and wafer lots collected over a long enough period of time to represents realistic process variability of the target technology
  
- ❑ The difference between statistical corner model and worst-case corner-model
  - Statistical corner model use the realistic PDF of the corresponding model parameter of its typical model
    - PDF is obtained from the distribution of a large set of production data
  - Statistical models can pass a valid design, which were rejected in worst-corner model



# Range of Behavior

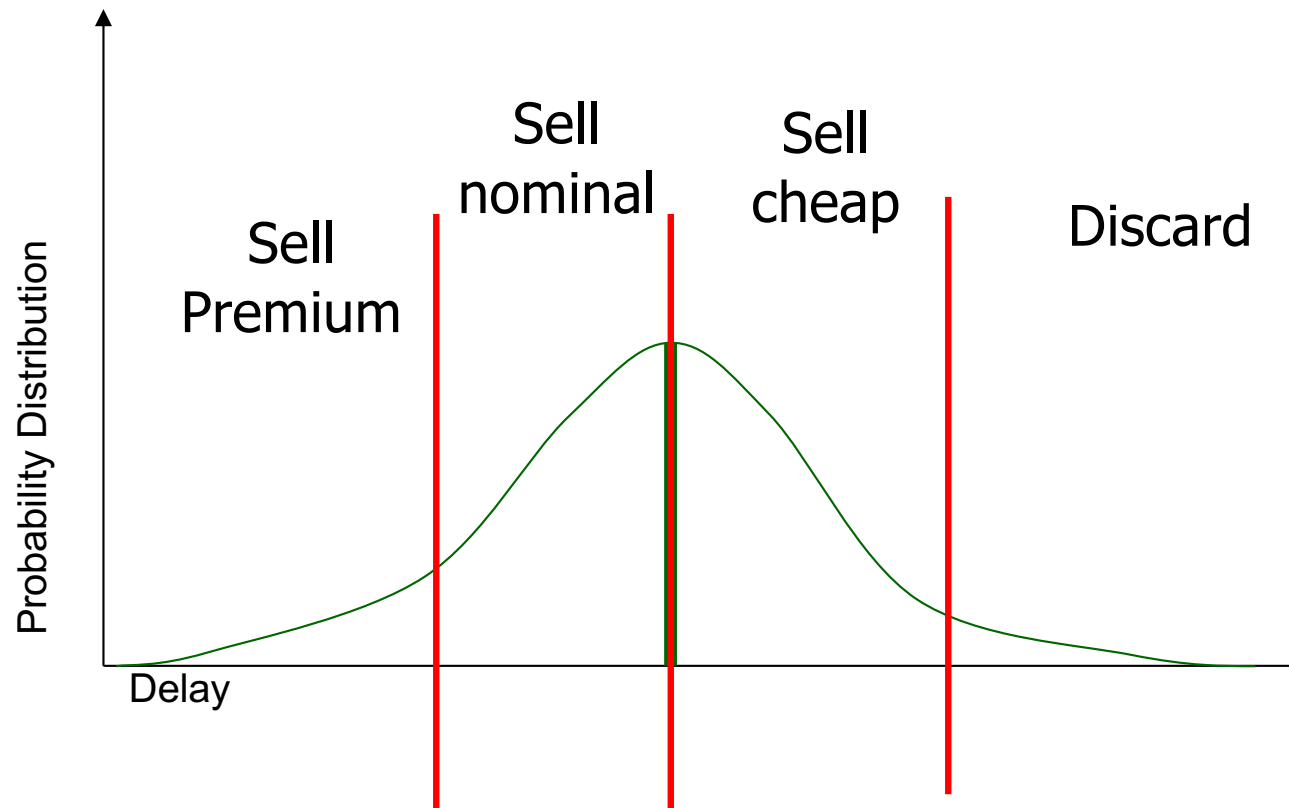
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- ❑ Still get range of performances
- ❑ *Any way to exploit the fact some are faster?*



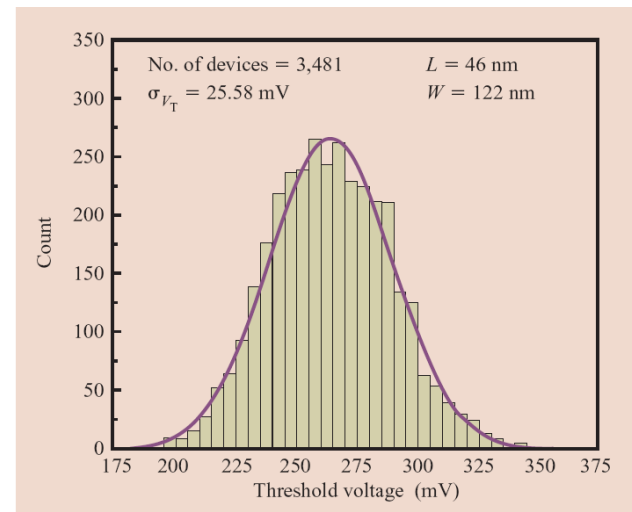
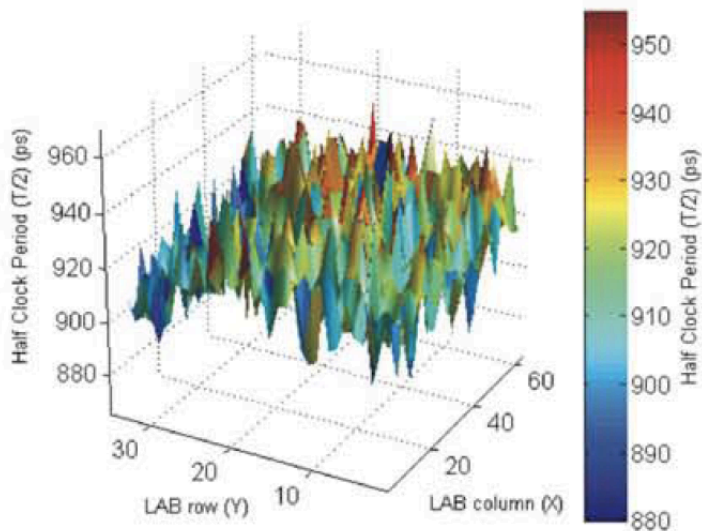


# Speed Binning



# Big Idea

- Parameters Approximate
- Differ
  - Chip-to-chip, transistor-to-transistor, over time
- Robust design accommodates
  - Tolerance and Margins
  - Doesn't depend on precise behavior





# Admin

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- ❑ HW 3 due tonight @midnight
- ❑ Midterm 1 Friday 10/1 (next week)
  - 7-9pm in Towne 309
  - No Lecture, virtual office hours
  - Virtual review session on Wednesday, will be recorded
    - See Piazza for updates
- ❑ HW 4 posted Friday 10/1 after midterm
  - Due following **Friday** 10/8 @midnight