ESE370: Circuit-Level Modeling, Design, and Optimization for Digital Systems

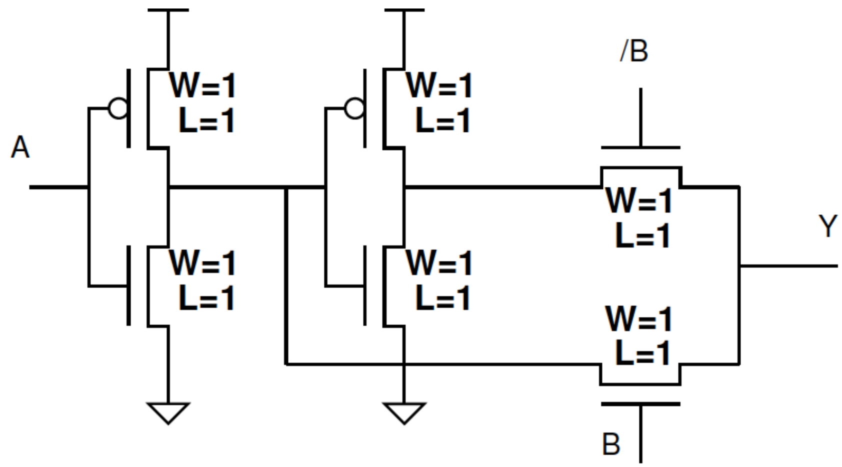
Lec 21: October 27, 2021

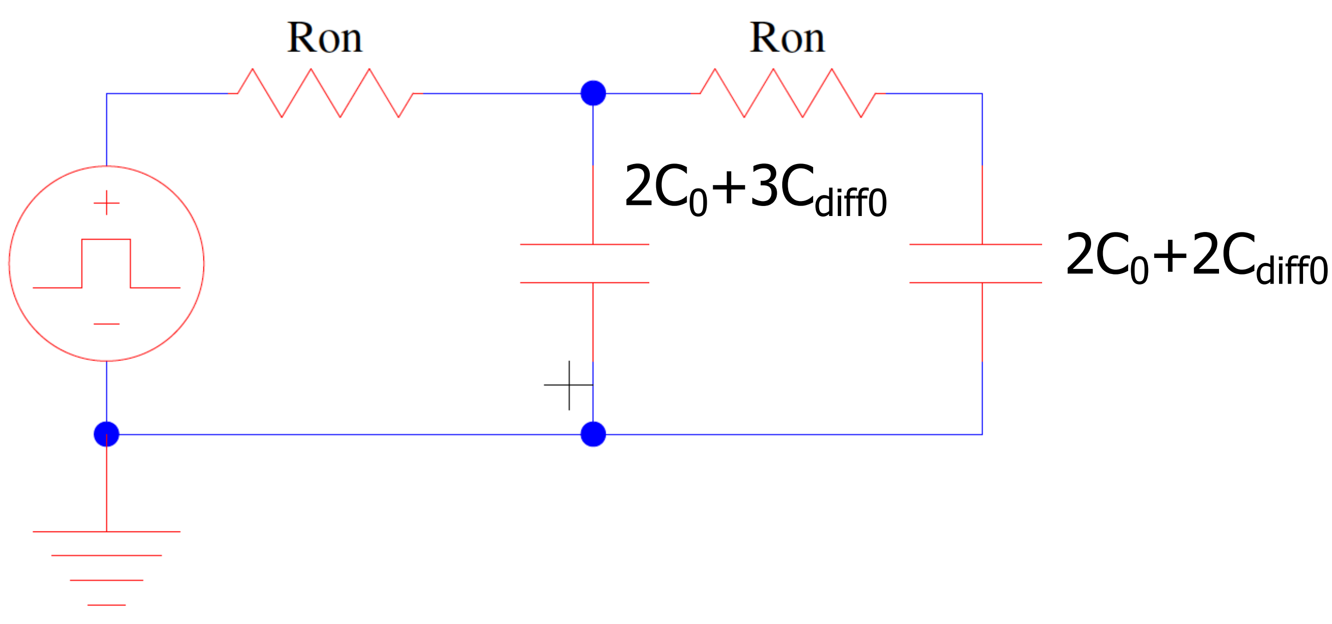
Distributed RC Wire and Elmore Delay

Today

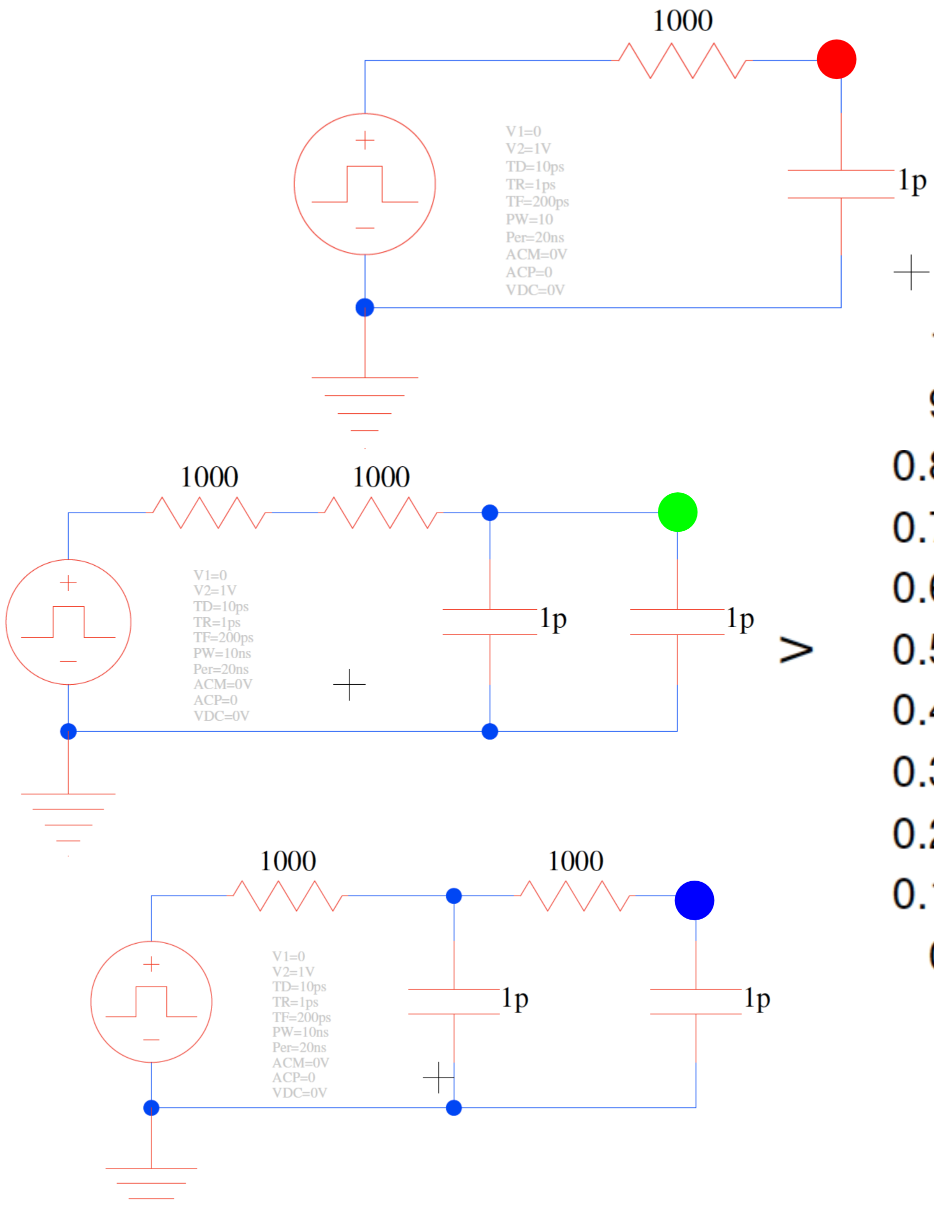
- Estimate delay in RC Network
  - Elmore delay calculation
- Apply to wire delay
- Apply to pass transistor circuits

Previously: Equivalent RC

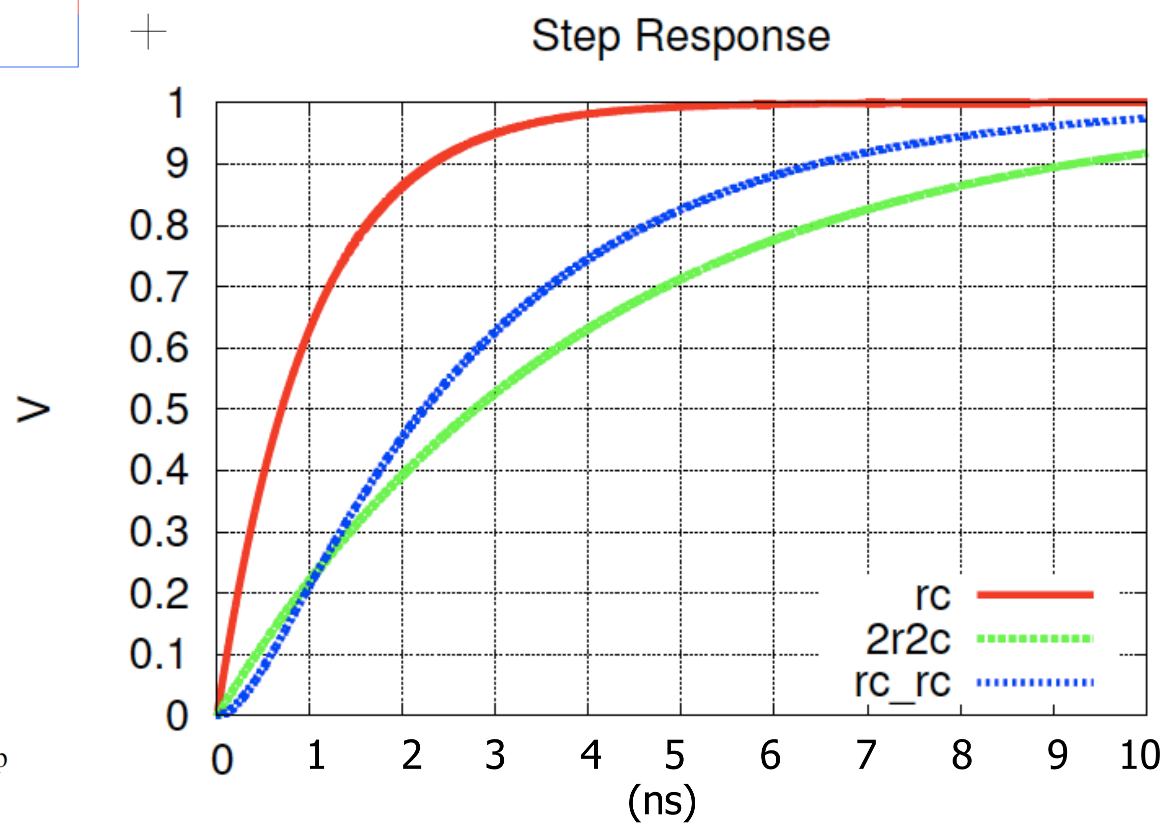




Previously: Distributed RC (preclass 1)



- What are the time constants?

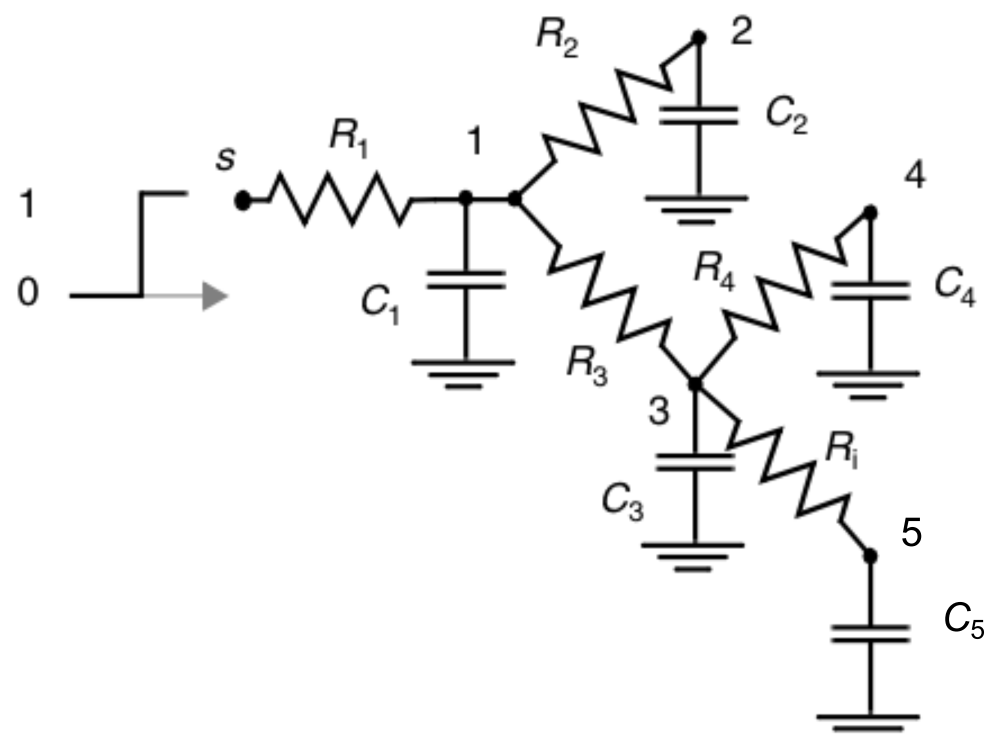


Elmore Delay: Distributed RC network

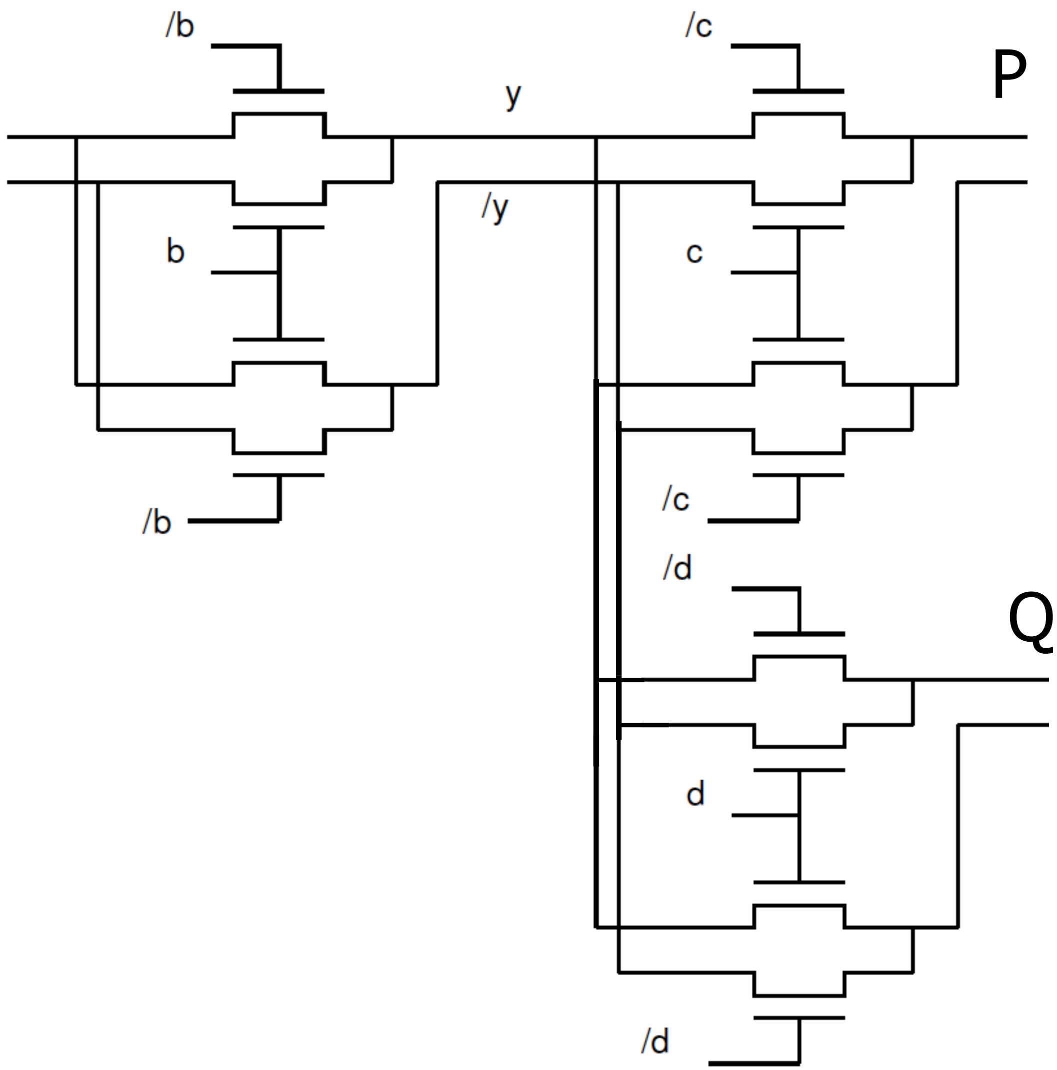
- The delay from source *s* to node *5*
  - N = number of nodes in circuit

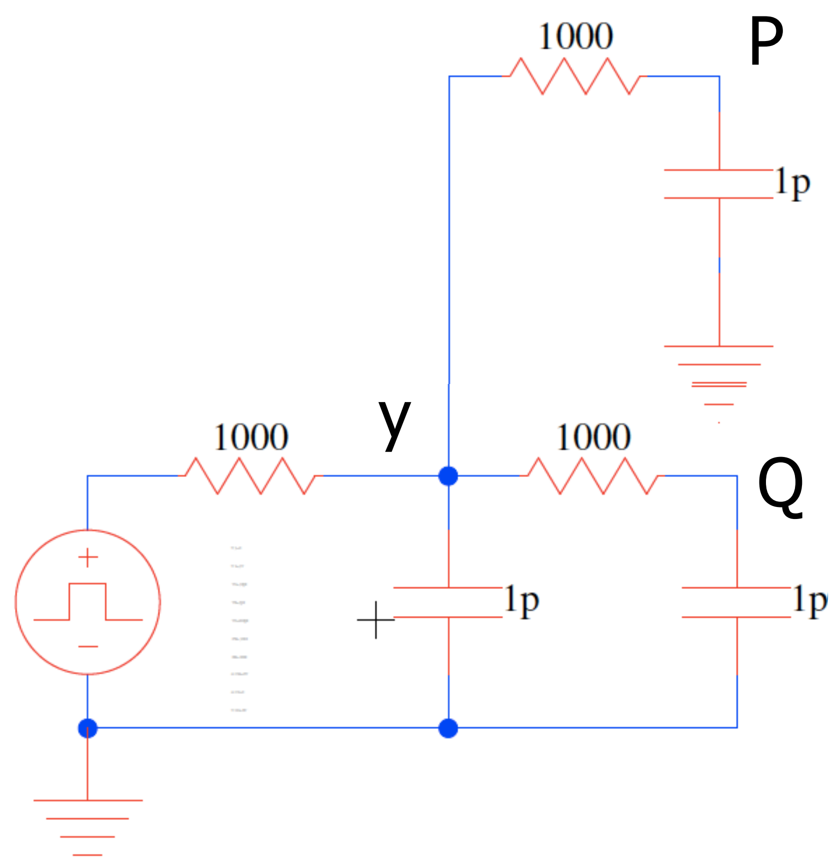
$$R_{5k} = \sum R_j \Rightarrow (R_j \in [path(s \to 5) \cap path(s \to k)])$$

$$\tau_{D5} = \sum_{k=1}^{N} C_k R_{5k}$$



What is response?



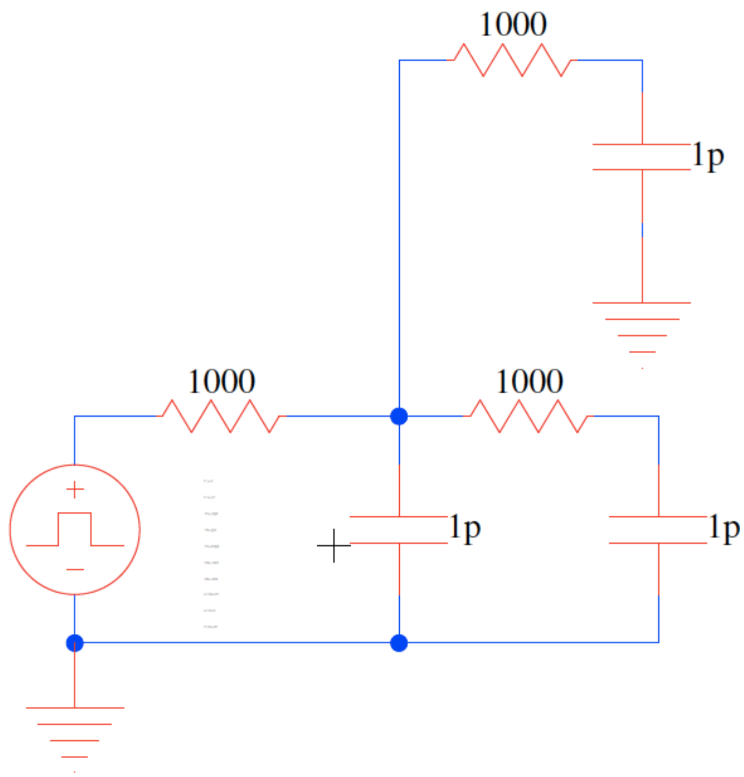


Elmore Delay: Practice (preclass 1)

- The delay from source *s* to node *i*
  - N = number of nodes in circuit

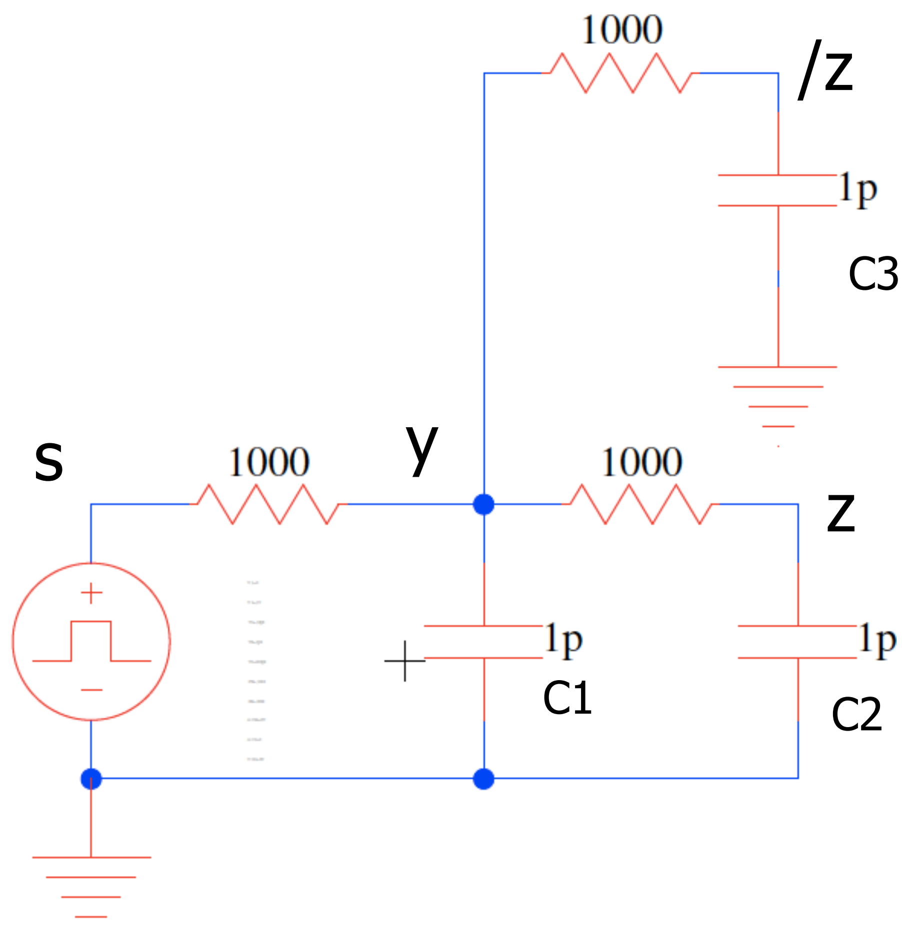
$$R_{ik} = \sum R_j \Rightarrow (R_j \in [path(s \rightarrow i) \cap path(s \rightarrow k)])$$

$$\tau_{Di} = \sum_{k=1}^{N} C_k R_{ik}$$

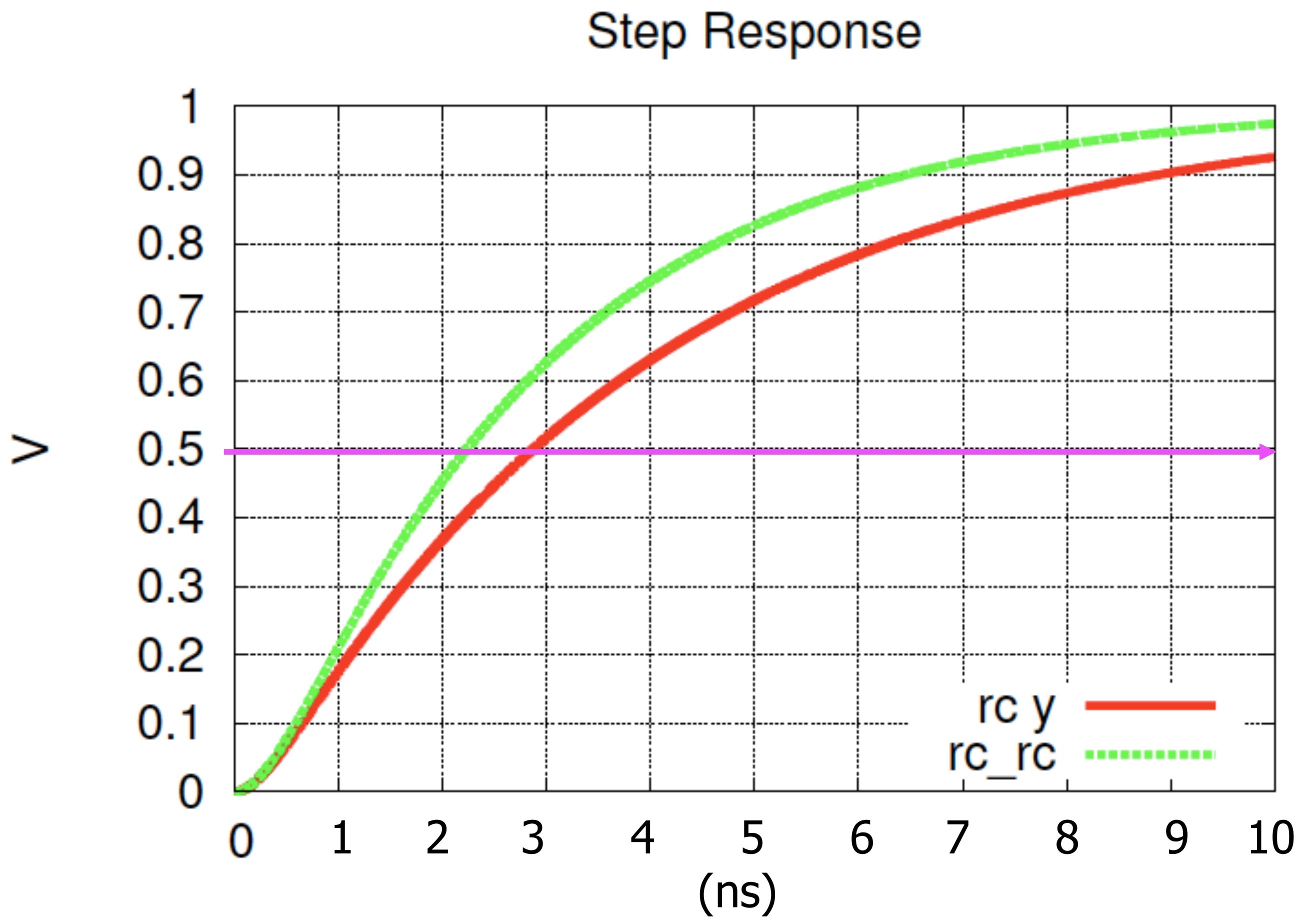


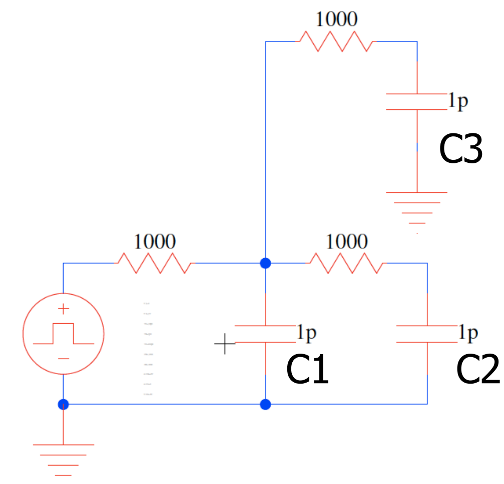
Apply (preclass 1)

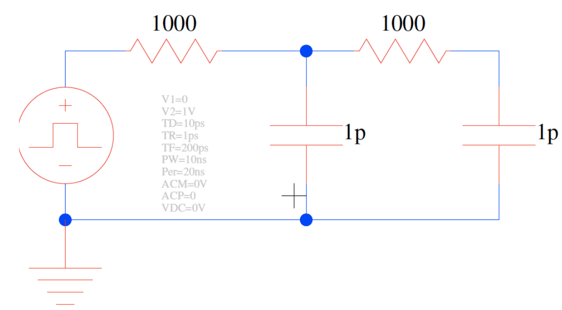
- What is Elmore delay?
  - S → Z



SPICE Response



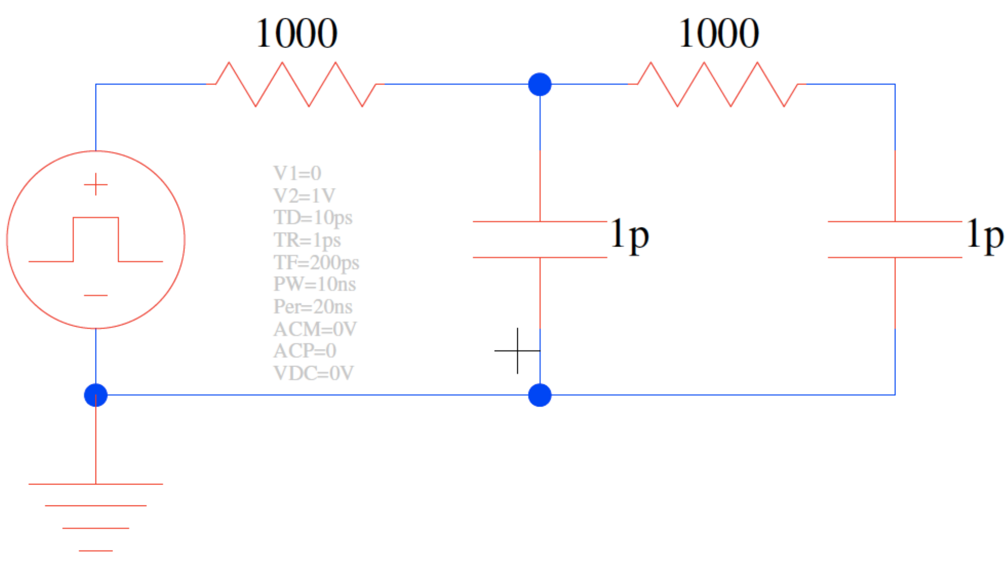




Elmore Delay: Special Ladder Case

- For each resistor C_k in path
  - Compute R_{kk} = sum of all Rs upstream of C_k

$$\tau_{DN} = \sum_{k=1}^{N} C_k \sum_{j=1}^{k} R_j = \sum_{k=1}^{N} C_k R_{kk}$$



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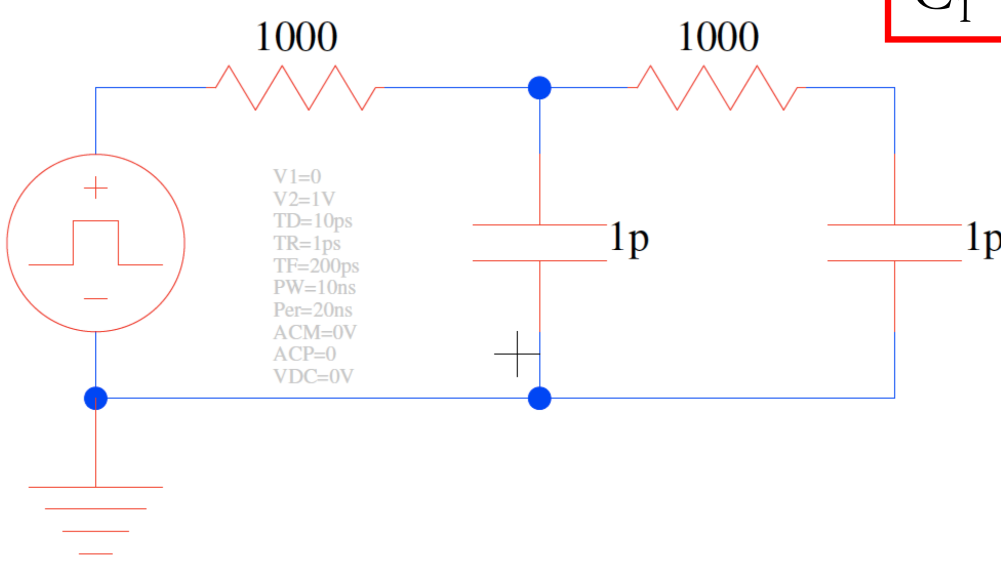
$C_1*(R_1)+C_2*(R_1+R_2)$



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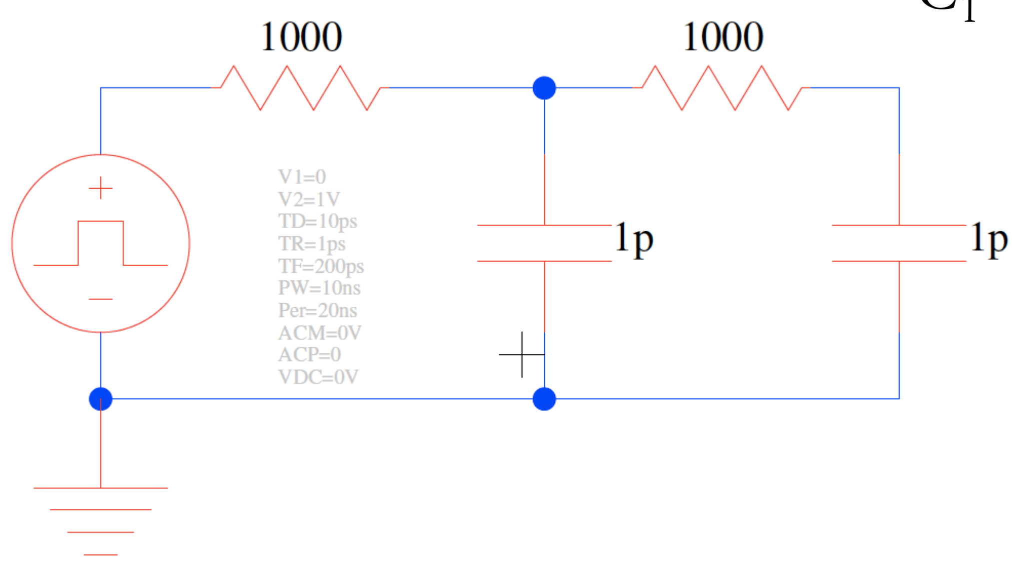


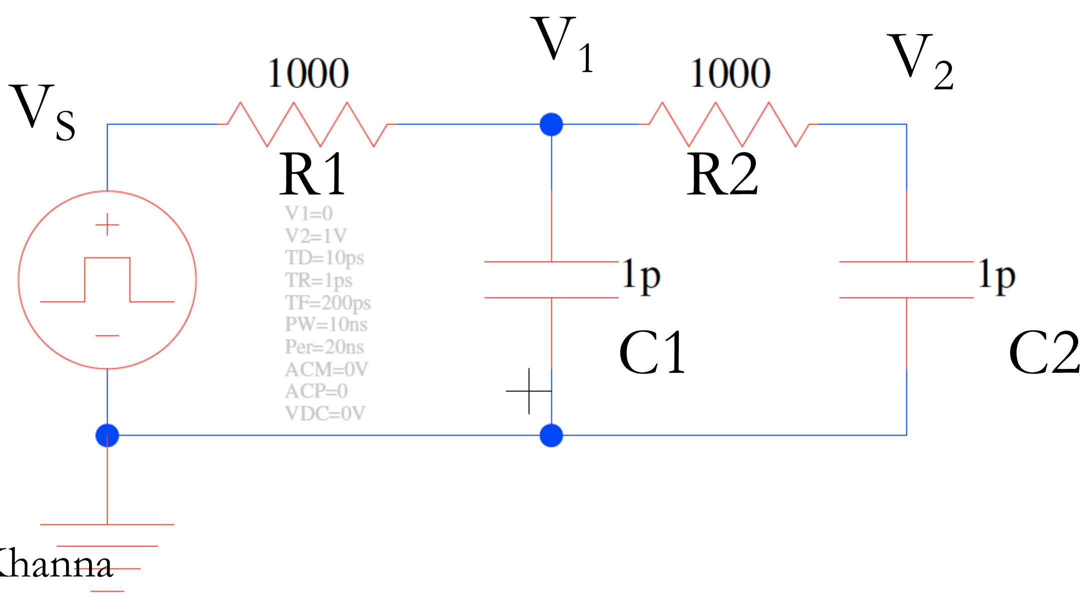
=3RC
=3ns

Compare KCL: Setup

- Equations from KCL?

$$@V_1: \quad \frac{V_S - V_1}{R_1} = \frac{V_1 - V_2}{R_2} + C_1 \frac{dV_1}{dt}$$

$$@V_2: \quad \frac{V_1 - V_2}{R_2} = C_2 \frac{dV_2}{dt}$$



Compare KCL: Math

@V_1:

$$\frac{V_S - V_1}{R_1} = \frac{V_1 - V_2}{R_2} + C_1 \frac{dV_1}{dt}$$

$$\frac{V_S}{R_1} = \frac{V_1}{R_1} + \frac{V_1}{R_2} - \frac{V_2}{R_2} + C_1 \frac{dV_1}{dt}$$

$$V_S = V_1\left(1 + \frac{R_1}{R_2}\right) - \frac{R_1 V_2}{R_2} + R_1 C_1 \frac{dV_1}{dt}$$

@V_2:

$$\frac{V_1 - V_2}{R_2} = C_2 \frac{dV_2}{dt}$$

$$\frac{V_1}{R_2} = \frac{V_2}{R_2} + C_2 \frac{dV_2}{dt}$$

$$V_1 = V_2 + R_2 C_2 \frac{dV_2}{dt}$$

$$\frac{dV_1}{dt} = \frac{dV_2}{dt} + R_2 C_2 \frac{d^2 V_2}{dt^2}$$

Compare KCL: Math

$$V_S = V_1\left(1+\frac{R_1}{R_2}\right) - \frac{R_1}{R_2}V_2 + R_1C_1\frac{dV_1}{dt}$$

$$V_S = \left(V_2 + R_2C_2\frac{dV_2}{dt}\right)\left(1+\frac{R_1}{R_2}\right) - \frac{R_1}{R_2}V_2 + R_1C_1\left(\frac{dV_2}{dt} + R_2C_2\frac{d^2V_2}{dt^2}\right)$$

$$V_S = V_2 + \left(R_2C_2\left(1+\frac{R_1}{R_2}\right) + R_1C_1\right)\frac{dV_2}{dt} + R_1C_1R_2C_2\frac{d^2V_2}{dt^2}$$

$$V_S = V_2 + (R_2C_2 + R_1C_2 + R_1C_1)\frac{dV_2}{dt} + R_1C_1R_2C_2\frac{d^2V_2}{dt^2}$$

Compare KCL: Math

$$R_1 = R_2 = R$$

$$C_1 = C_2 = C$$

$$V_S = V_2 + (R_2C_2 + R_1C_2 + R_1C_1)\frac{dV_2}{dt} + R_1C_1R_2C_2\frac{d^2V_2}{dt^2}$$

$$V_S = V_2 + 3RC\frac{dV_2}{dt} + R^2C^2\frac{d^2V_2}{dt^2}$$

Compare KCL: Math

$$R_1 = R_2 = R$$

$$C_1 = C_2 = C$$

$$V_S = V_2 + (R_2C_2 + R_1C_2 + R_1C_1)\frac{dV_2}{dt} + R_1C_1R_2C_2\frac{d^2V_2}{dt^2}$$

$$V_S = V_2 + 3RC\frac{dV_2}{dt} + R^2C^2\frac{d^2V_2}{dt^2}$$

$$V_2 = A(1 + e^{-\alpha t}) \rightarrow$$

$$V_S = A(1 + e^{-\alpha t}) - 3RC \cdot \alpha A e^{-\alpha t} + R^2C^2 \cdot \alpha^2 A e^{-\alpha t}$$

Compare KCL: Math

$$V_2 = A(1 + e^{-\alpha t}) \rightarrow$$

$$V_S = A(1 + e^{-\alpha t}) - 3RC \cdot \alpha A e^{-\alpha t} + R^2 C^2 \cdot \alpha^2 A e^{-\alpha t}$$

$$t = \infty \rightarrow V_2 = V_S = A$$

Compare KCL: Math

$$V_2 = A(1+e^{-\alpha t}) \rightarrow$$

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$$t = \infty \rightarrow V_2 = V_S = A$$

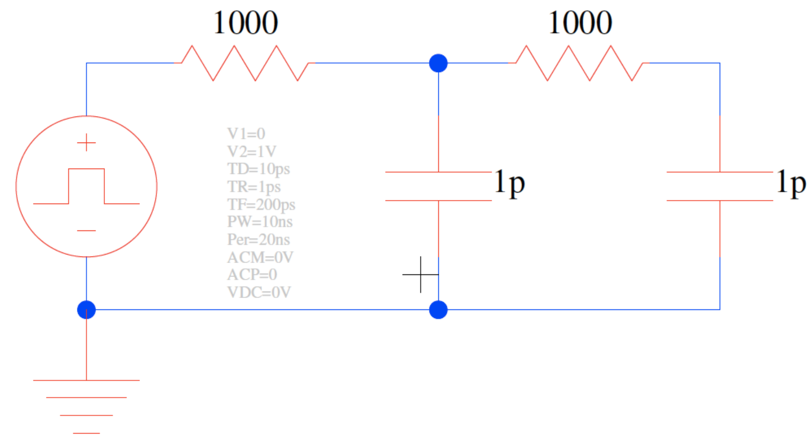
$$V_S = V_S(1+e^{-\alpha t}) - 3RC \cdot \alpha V_S e^{-\alpha t} + R^2C^2 \cdot \alpha^2 V_S e^{-\alpha t}$$

$$0 = e^{-\alpha t} - 3RC \cdot \alpha e^{-\alpha t} + R^2C^2 \cdot \alpha^2 e^{-\alpha t}$$

$$0 = e^{-\alpha t}(1 - 3RC\alpha + R^2C^2 \cdot \alpha^2)$$

$$0 = 1 - 3RC\alpha + R^2C^2 \cdot \alpha^2$$

Compare KCL: Math



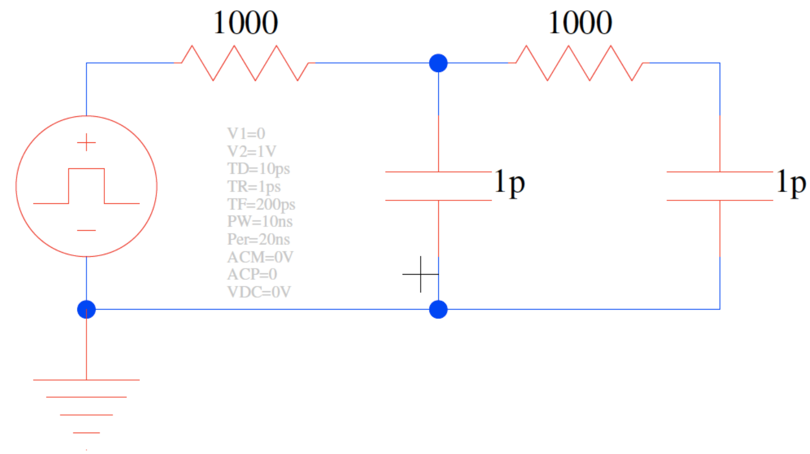
$$V_2 = V_S(1 + e^{-\alpha t})$$

$$0 = 1 - 3RC\alpha + R^2C^2 \cdot \alpha^2$$

$$\alpha = \frac{3RC \pm \sqrt{9(RC)^2 - 4(RC)^2}}{2(RC)^2} = \frac{3RC \pm \sqrt{5}RC}{2(RC)^2} = \frac{3 \pm \sqrt{5}}{2RC}$$

$$\rightarrow \tau = \frac{2}{3 \pm \sqrt{5}} RC \approx 2.6RC$$

Compare KCL: Math



$$V_2 = V_S(1 + e^{-\alpha t})$$

$$0 = 1 - 3RC\alpha + R^2C^2 \cdot \alpha^2$$

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$$\rightarrow \tau = \frac{2}{3 \pm \sqrt{5}} RC \approx 2.6RC$$

=3RC
=3ns

Elmore Delay: Distributed RC network

- The delay from source to node *i*
  - N = number of nodes in circuit

$$R_{ik} = \sum R_j \Rightarrow (R_j \in [path(s \rightarrow i) \cap path(s \rightarrow k)])$$

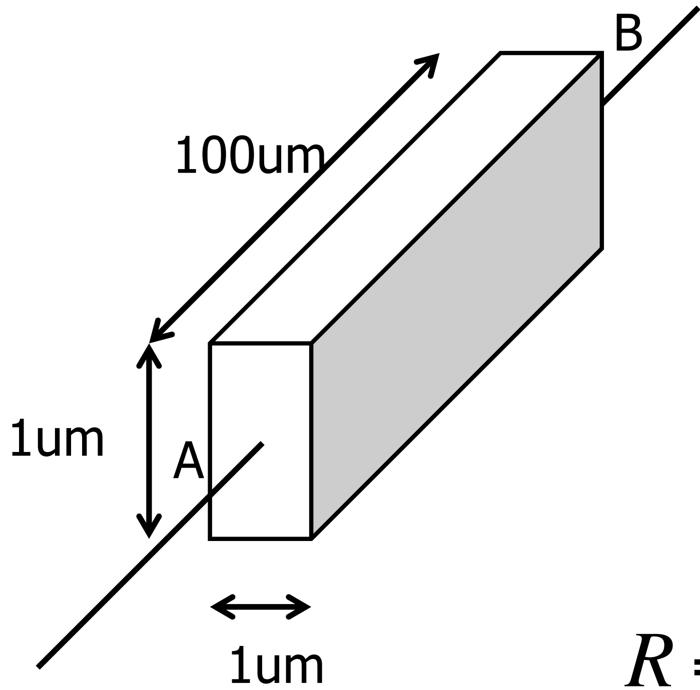
$$\tau_{Di} = \sum_{k=1}^{N} C_k R_{ik}$$

- Special ladder case

$$\tau_{DN} = \sum_{k=1}^{N} C_k \sum_{j=1}^{k} R_j = \sum_{k=1}^{N} C_k R_{kk}$$

Wire Delay

Wire Resistance

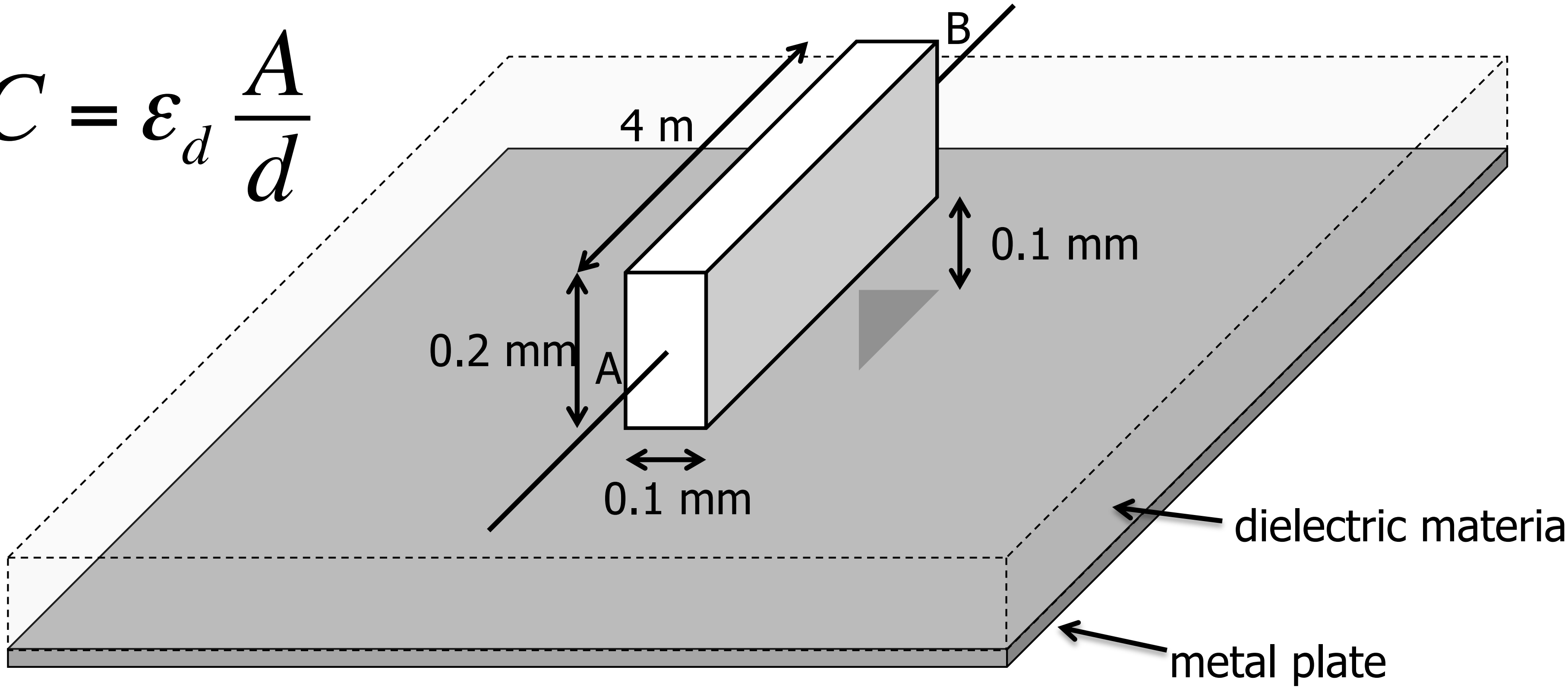


$$R = \frac{\rho L}{A}$$

$$R = \frac{10^{-7}\,\Omega \cdot m \cdot 100\,\mu m}{1\,\mu m \cdot 1\,\mu m} = 10\,\Omega$$

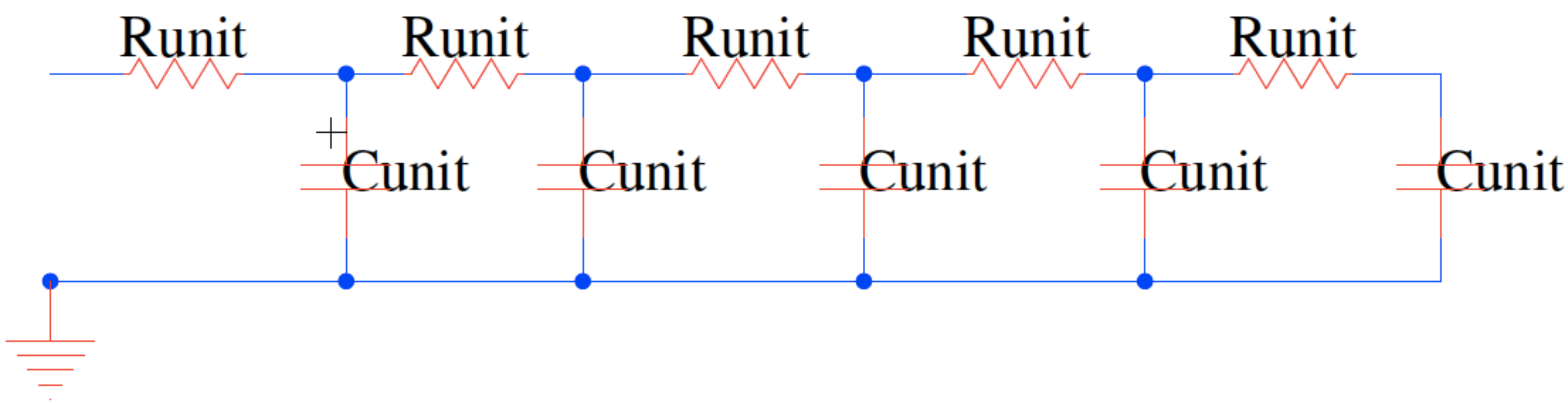
Wire Capacitance

$$C = \varepsilon_d \frac{A}{d}$$



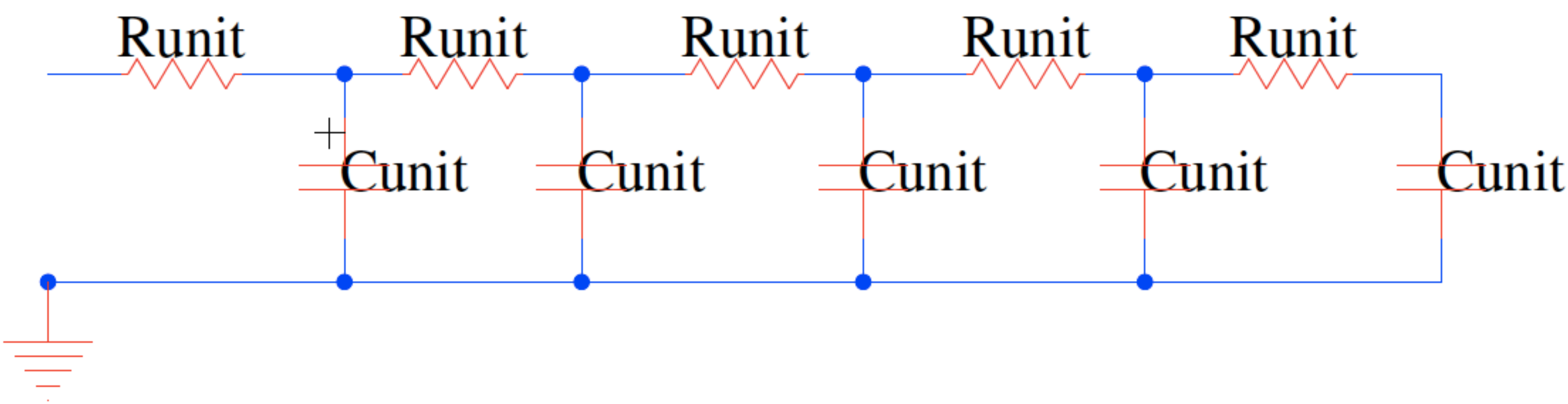
Wire as Distributed RC Ladder

- Measure wire length in units
  - Say λ
  - Each λ unit has C_{unit}, R_{unit}
    - Unit capacitance and resistance of wire of length λ



Wire Delay

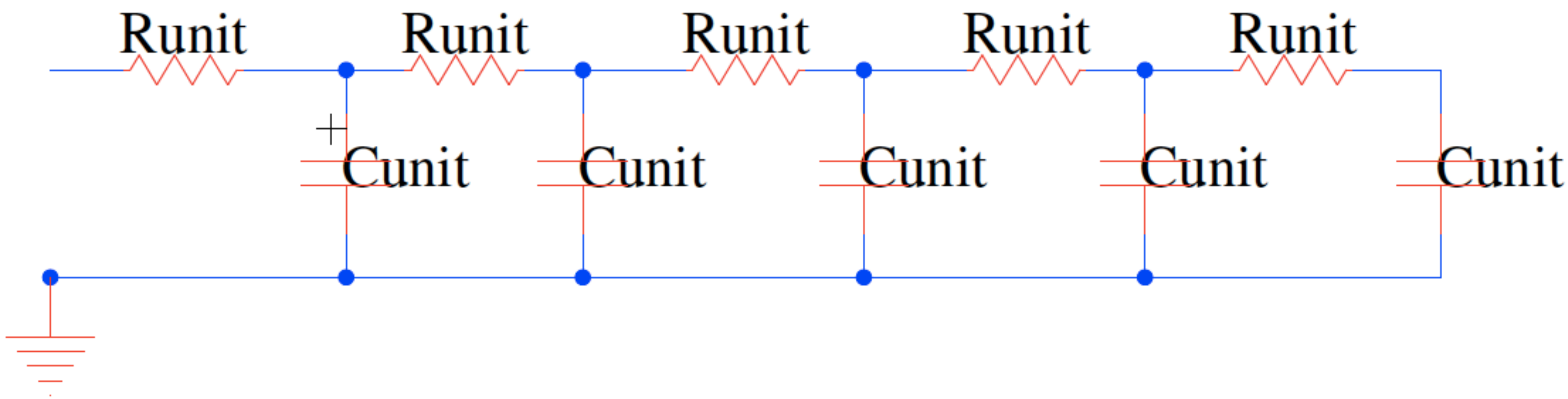
- Delay of Wire N units long:



Wire Delay

- Delay of Wire N units long:

  $R_{unit}C_{unit}$

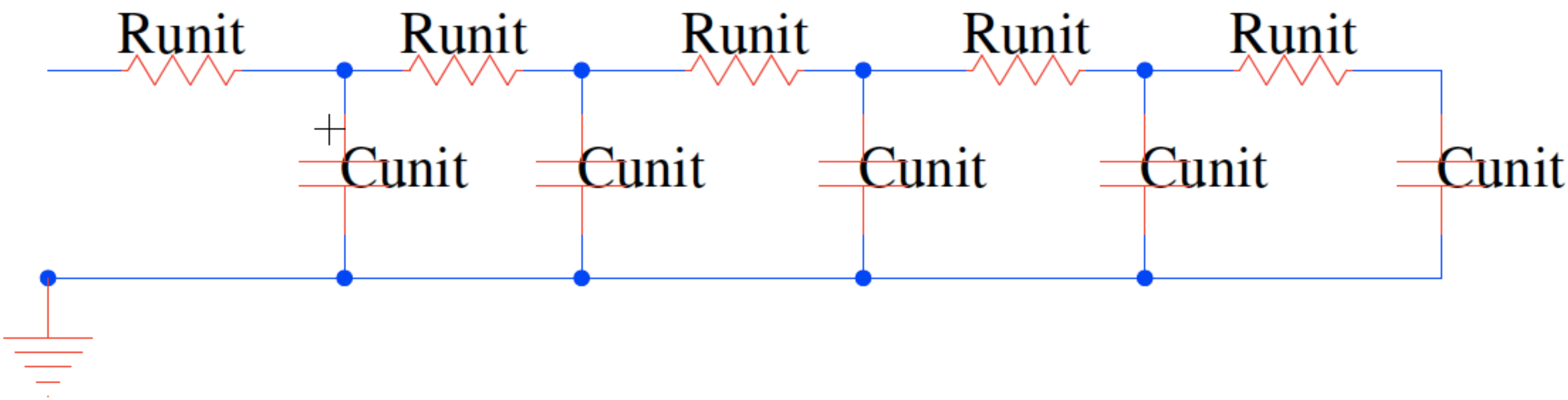


Wire Delay

- Delay of Wire N units long:

  $R_{unit}C_{unit}$

  $+ 2R_{unit}C_{unit}$



Wire Delay

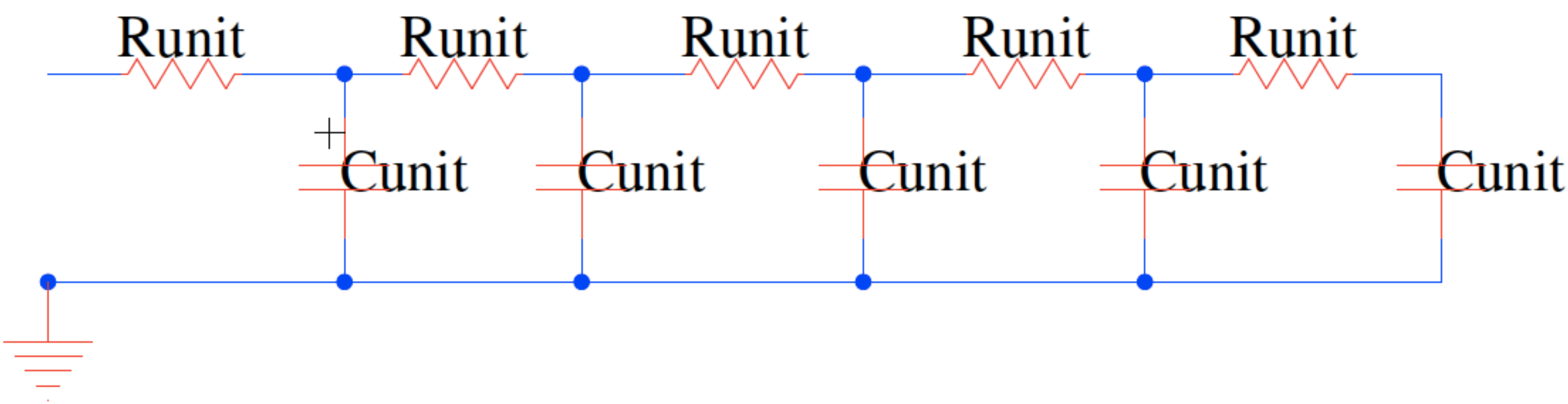
- Delay of Wire N units long:

  $R_{unit}C_{unit}$

  $+ 2R_{unit}C_{unit}$

  $+ 3R_{unit}*C_{unit} + \ldots +NR_{unit}*C_{unit}$

  $=(R_{unit}*C_{unit})*(N+(N-1)+(N-2)+\ldots.1)$



Sum of integers (preclass 2)

- What's the sum of the integer 1 to N?

$$\sum_{k=0}^{N} k =$$

Sum of integers

- What's the sum of the integer 1 to N?

$$\sum_{k=0}^{N} k = \frac{N(N+1)}{2} \approx \frac{N^2}{2}$$

Wire Delay (preclass 3)

- Delay of Wire N units long:

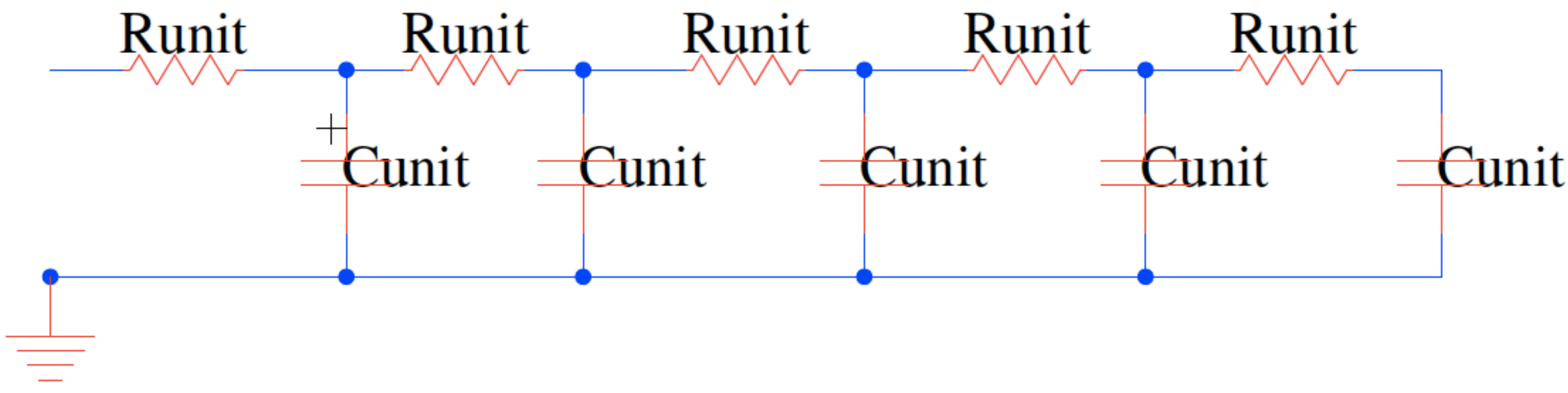
$R_{unit}*(N*C_{unit})$

$+ R_{unit}((N-1)*C_{unit}$

$+ R_{unit}*(N-2)*C_{unit} + \ldots +R_{unit}*C_{unit}$

$=(R_{unit}*C_{unit})*(N+N-1+N-2+\ldots.1)$

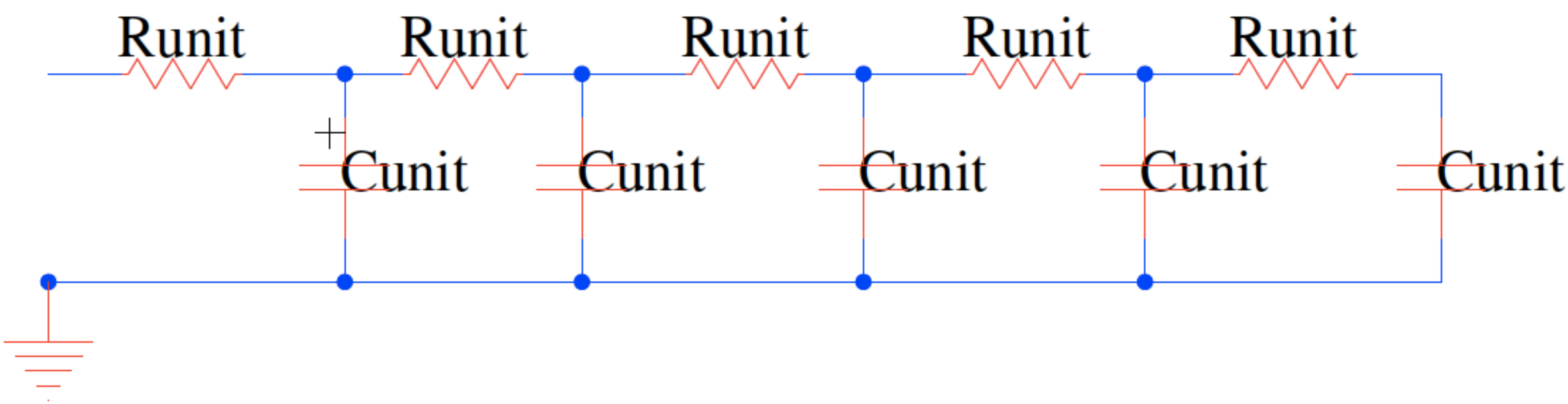
$\sim=R_{unit}C_{unit}*N^2/2$



Lumped RC Wire? (preclass 4)

- What would the delay be if we treated the wire as lumped R and C?

$$R_{wire} = N \times R_{unit} \qquad C_{wire} = N \times C_{unit}$$



Wire Delay

- $R_{wire} = N*R_{unit}$
- $C_{wire}=N*C_{unit}$
- Lumped RC wire delay = $R_{unit}*C_{unit}*N^2$
- Distributed RC Wire delay = $R_{unit}*C_{unit}*N^2/2$

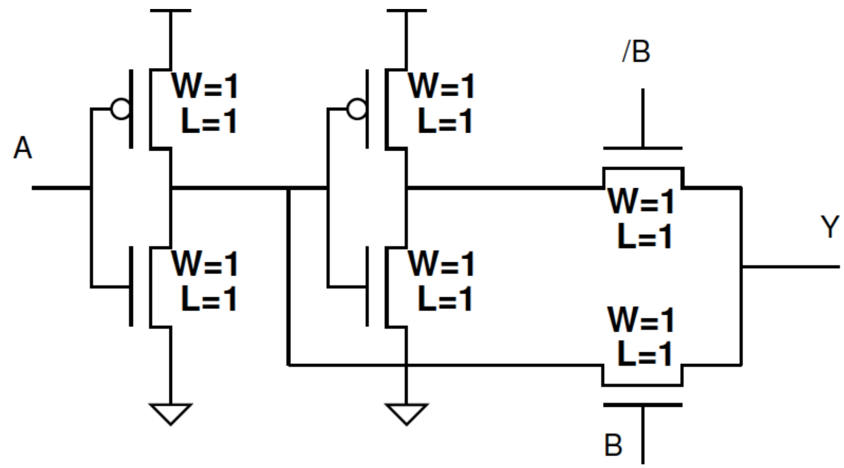
- Distributed has half the delay of lumped RC product
- Delay is quadratic in length of wire in both cases

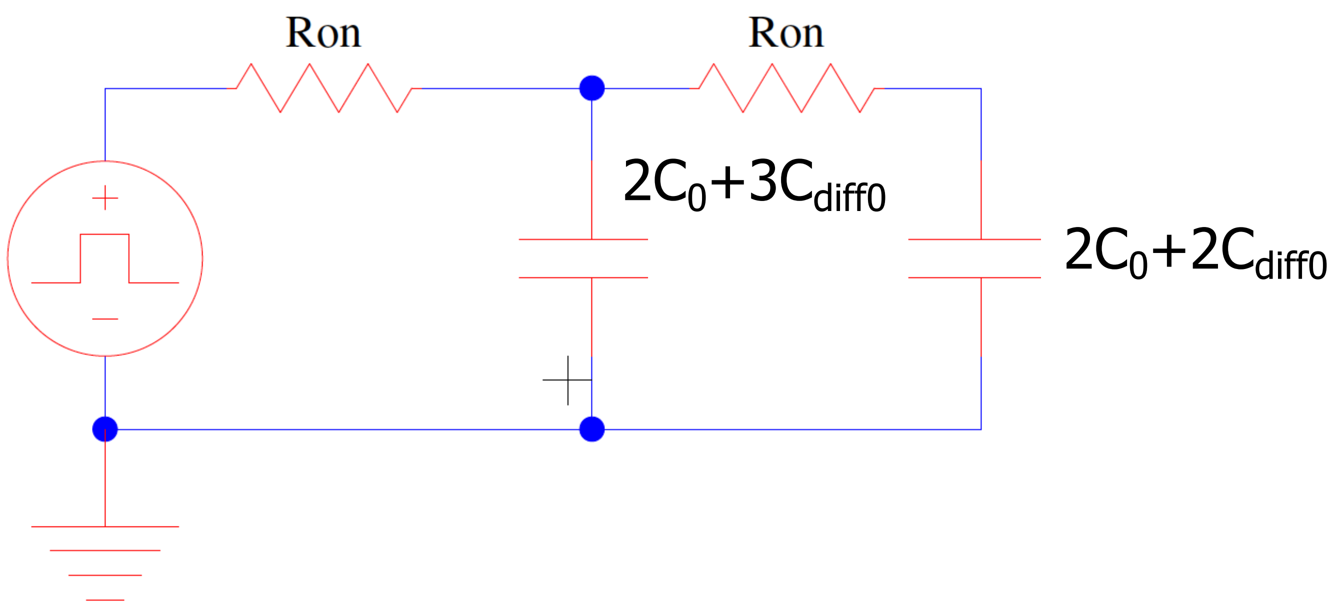
Apply to Gates

Pass Transistor and CMOS

Pass Transistor XOR

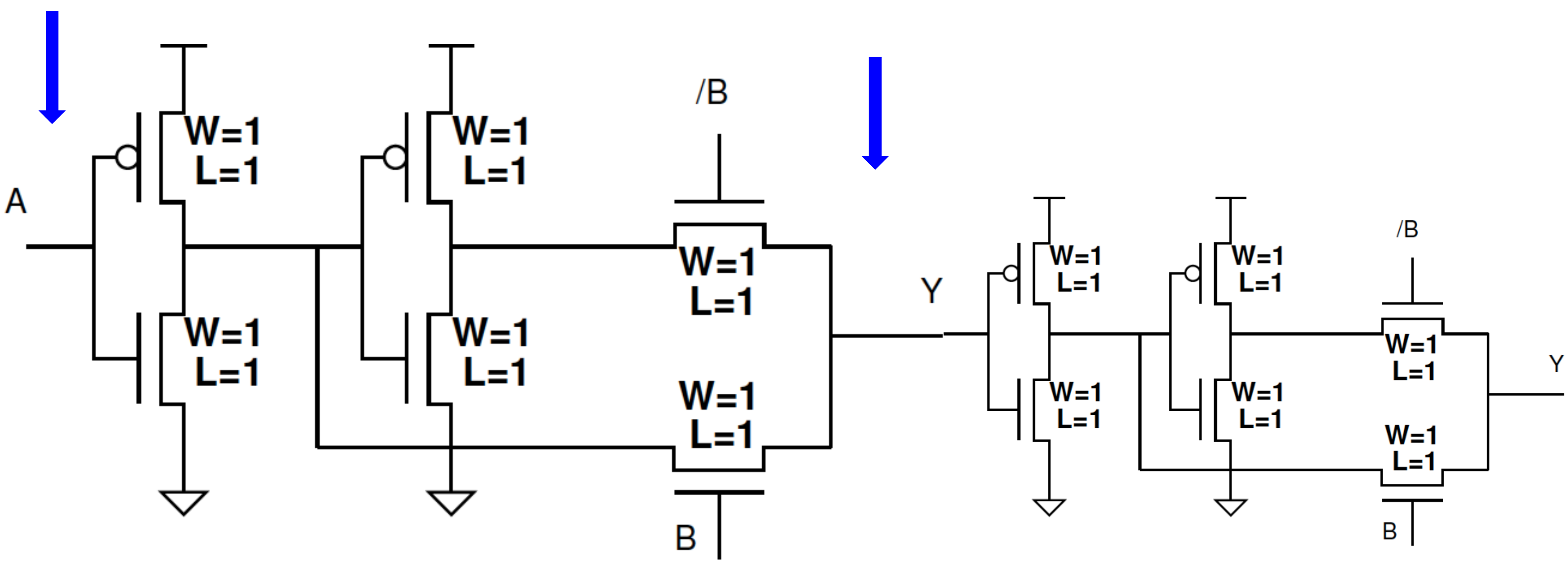
- Delay when A=B=1?





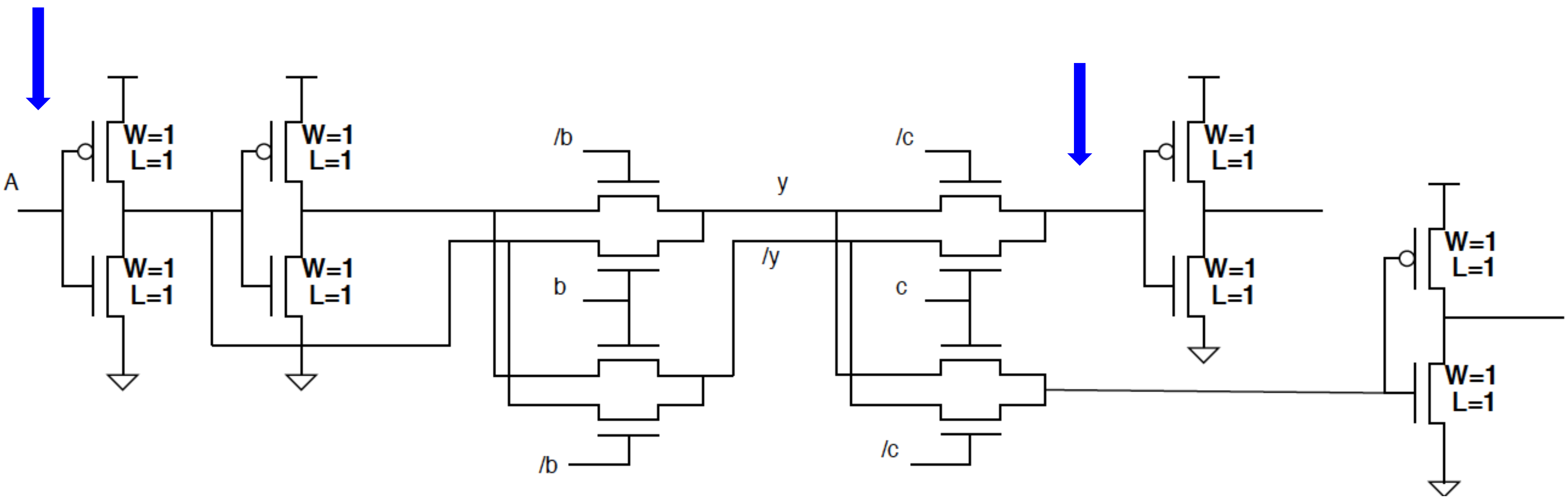
Pass Transistor XOR (preclass 5)

- Delay when A=1, B=0?
  - Start with equivalent RC circuit



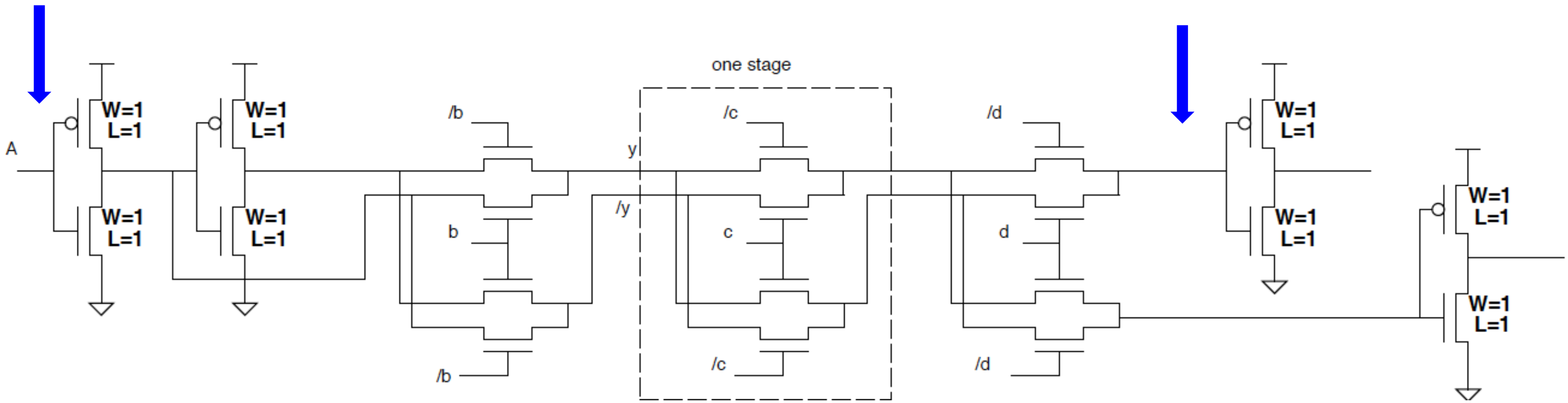
Unbuffered (preclass 6)

- Circuit → Delay?
  - 2 stages



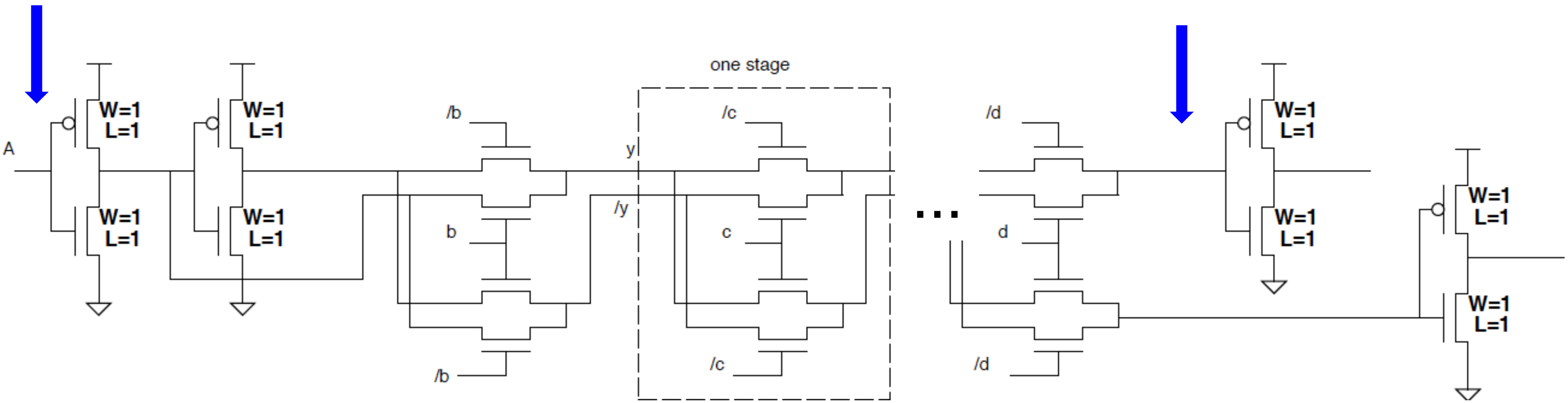
Unbuffered (preclass 7)

- Circuit → Delay?
  - 3 stages



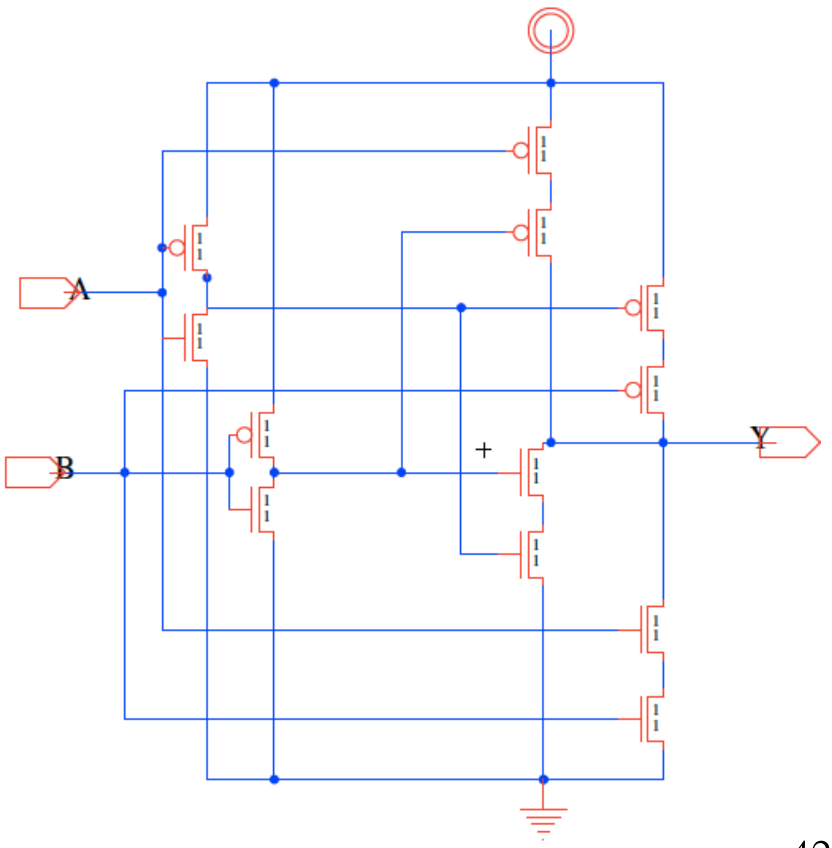
Unbuffered (preclass 8)

- Delay as a function of number of stages, k?



CMOS XOR (preclass 9)

- Delay with Cdiff>0?



Idea

- Elmore delay calculation allows us to estimate delay for distributed RC network
  - Necessary for pass transistors
- Wires are distributed RC
  - Half delay lumped calculation
  - Still quadratic in length

Admin

- Project 1
  - Final report due Friday 10/29 midnight