ESE370: Circuit-Level Modeling, Design, and Optimization for Digital Systems

Lec 30: November 19, 2021

Inductive Noise

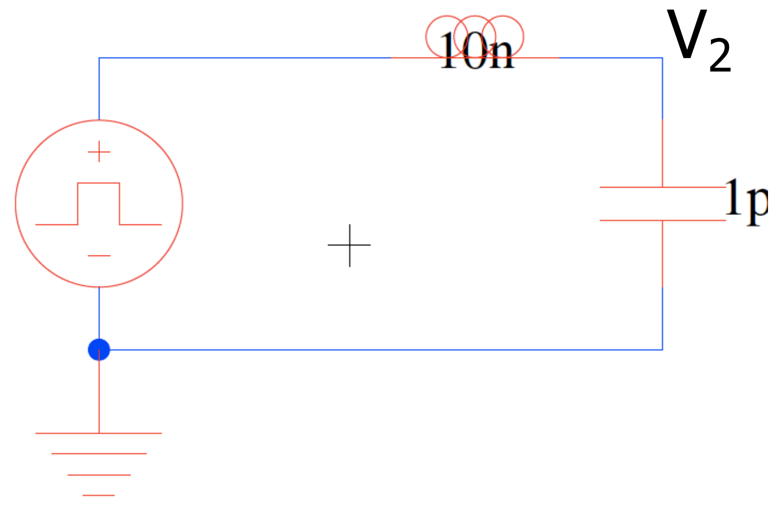
Today

- Inductive Responses
- Calculating L
- Where inductances show up
- Impact of inductance on digital circuits
- How to address inductive noise

LC Response (preclass 1)

- What happens here?

$$L\frac{dI_L}{dt} = V_L \qquad I_C = C\frac{dV_C}{dt}$$



LC Response

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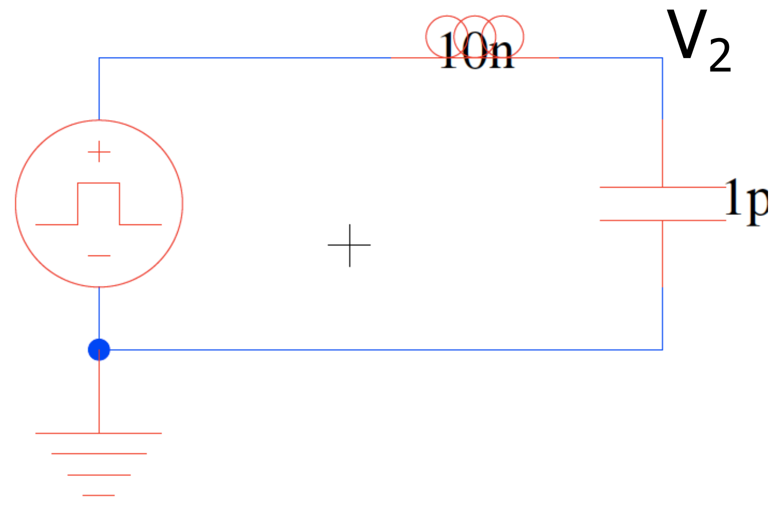


$$L\frac{dI}{dt} + V_2 = V_S$$

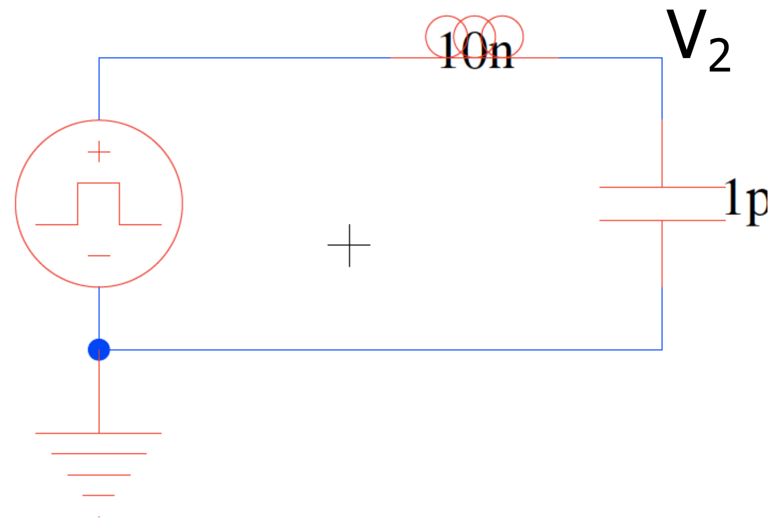
$$I = C\frac{dV_2}{dt} \Rightarrow \frac{dI}{dt} = C\frac{d^2V_2}{dt^2}$$

$$LC\frac{d^2V_2}{dt^2} + V_2 = V_S$$

LC Response

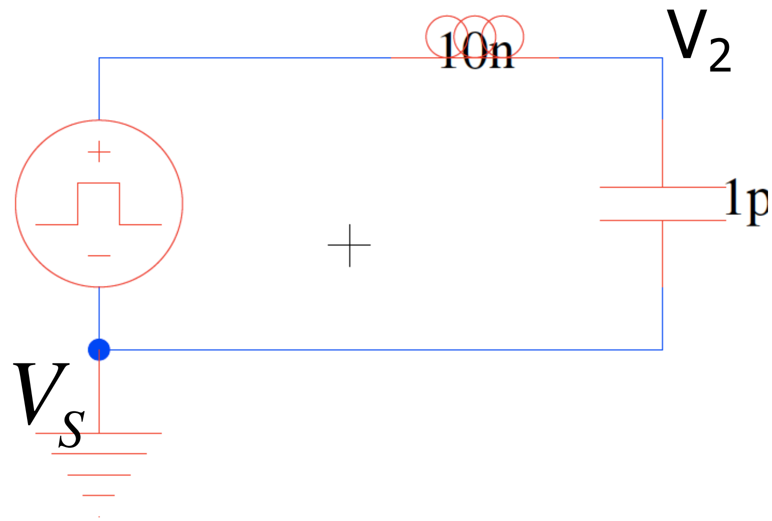
- What happens here?

$$LC\frac{d^2V_2}{dt^2}+V_2=V_S$$



LC Response

- What happens here?



$$LC\frac{d^2V_2}{dt^2} + V_2 = V_S$$

$$LC(-\omega^2 Be^{j\omega t}) + A + Be^{j\omega t} = V_S$$

$$A = V_S$$

$$-LC\omega^2 B + B = 0 \Rightarrow$$

$$\omega = \sqrt{\frac{1}{LC}}$$

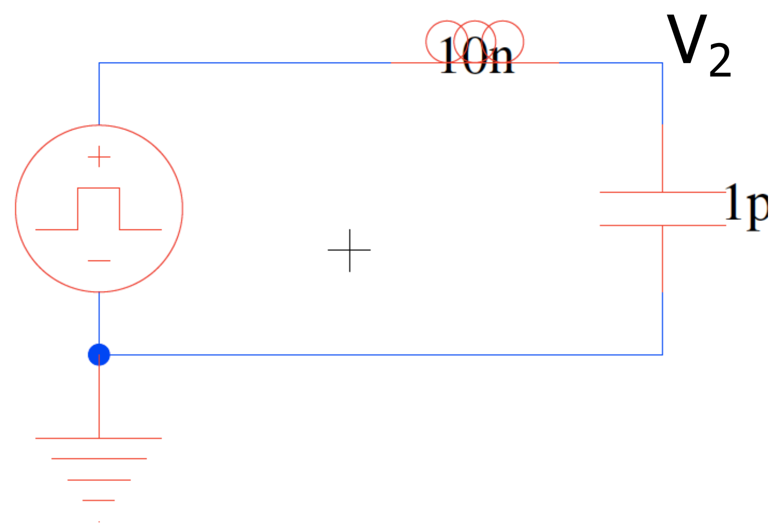
$$V_2 = A + Be^{j\omega t}$$

$$\frac{d^2V_2}{dt^2} = -\omega^2 Be^{j\omega t}$$

LC Response

- What happens here?

$$\omega = \sqrt{\frac{1}{LC}}$$

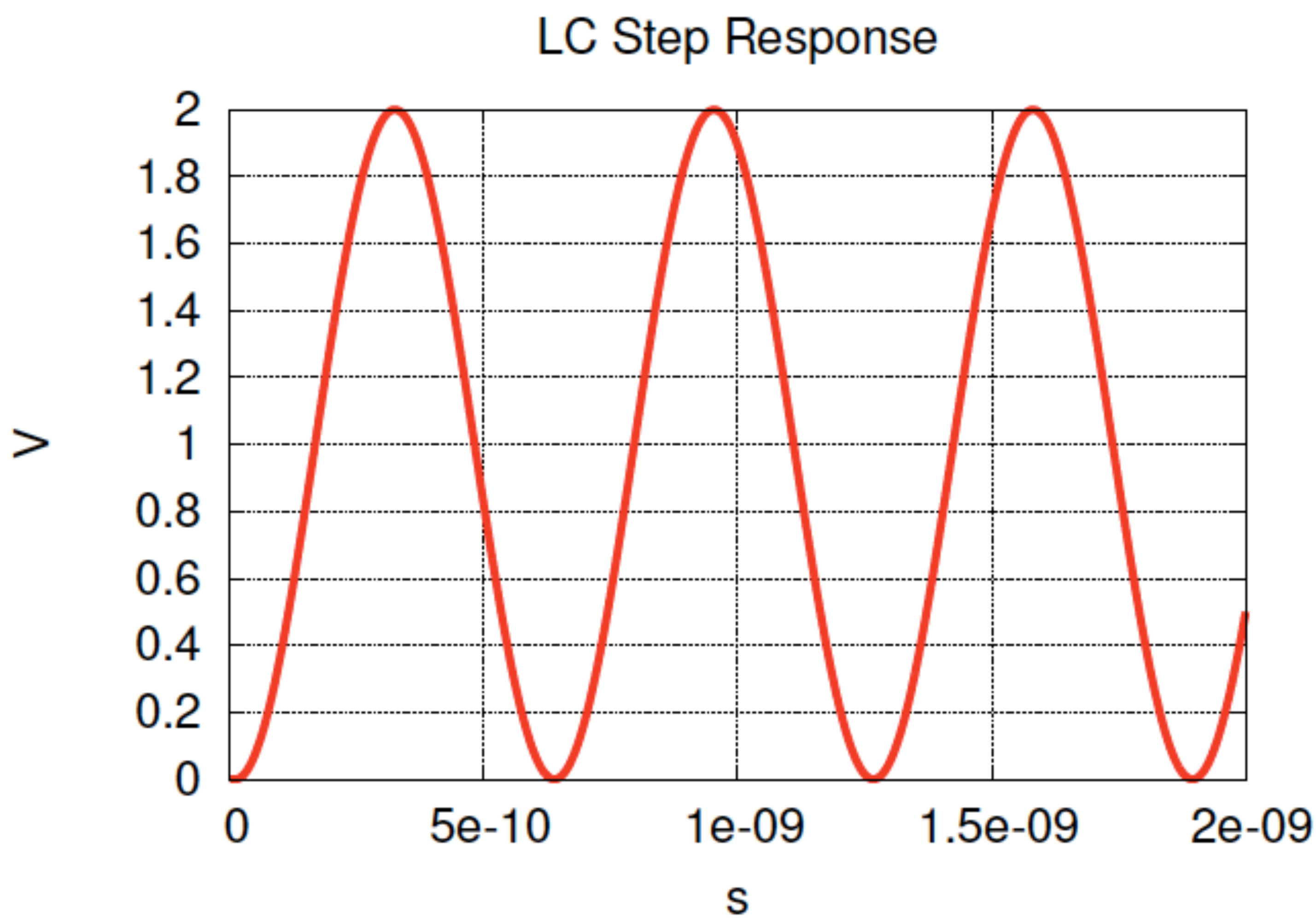


$$V_2 = A + Be^{j\omega t}$$

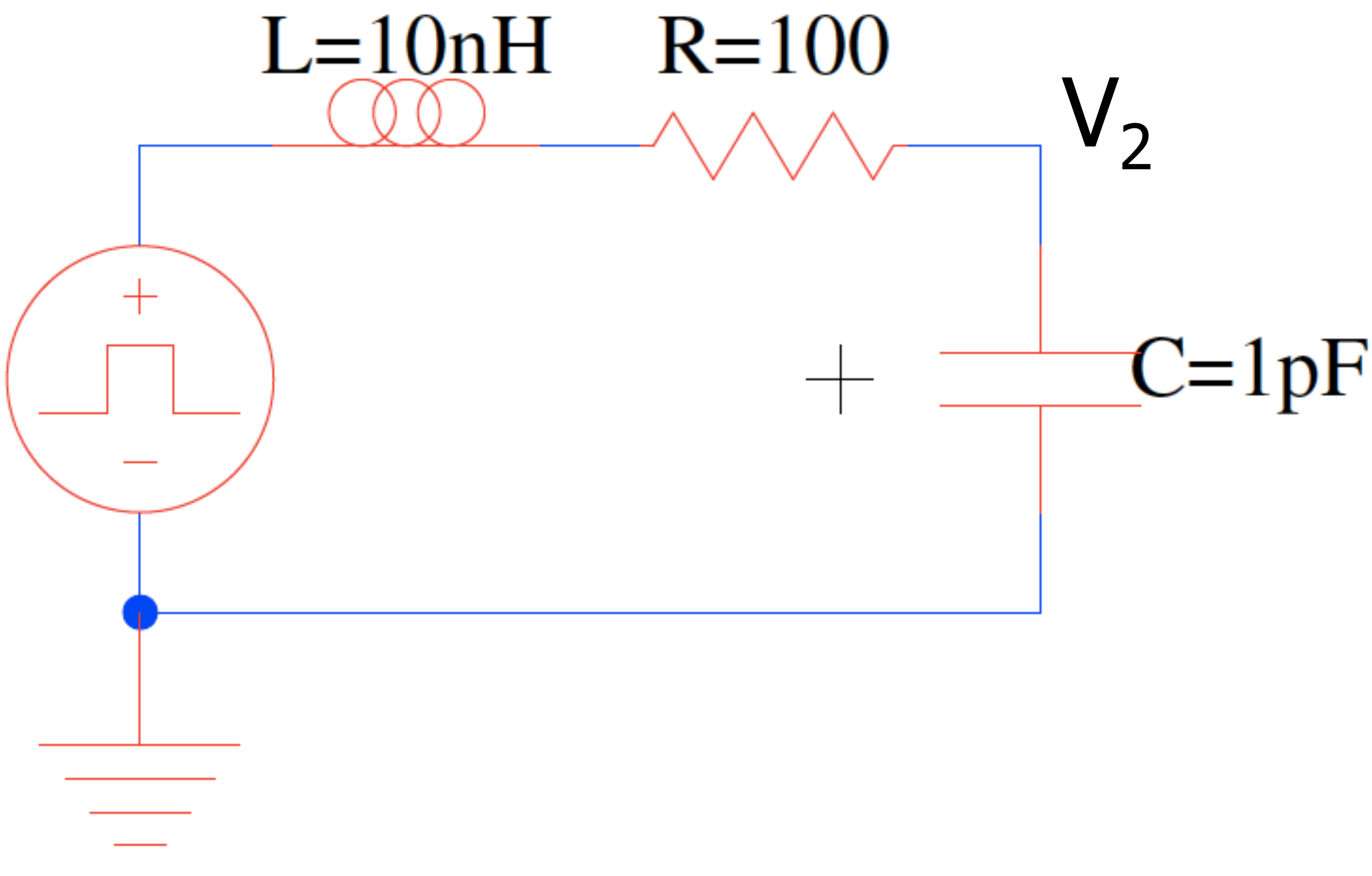
$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$V_2 = V + Be^{j\left(\sqrt{\frac{1}{LC}}\right)t}$$

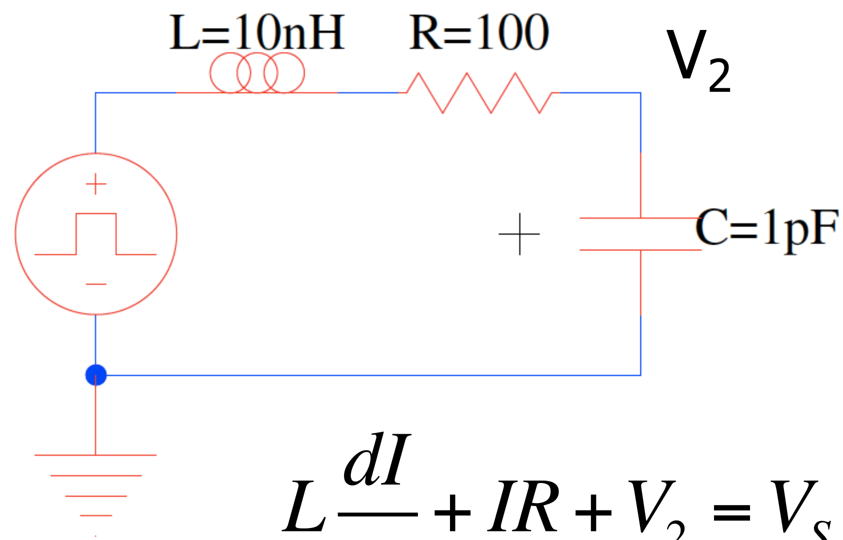
LC Response


LC Step Response
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
V
0
5e-10
1e-09
1.5e-09
2e-09
s

Response? (preclass 1)



RLC Response



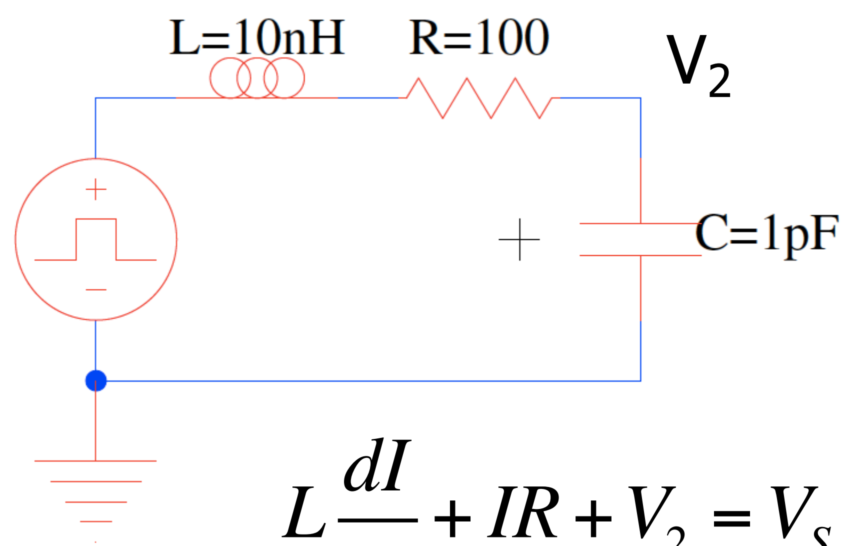
$$L\frac{dI}{dt} + IR + V_2 = V_S$$

RLC Response

$$V_2 = A + Be^{j\omega t}$$

$$\frac{dV_2}{dt} = j\omega Be^{j\omega t}$$

$$\frac{d^2V_2}{dt^2} = -\omega^2 Be^{j\omega t}$$



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RLC Response

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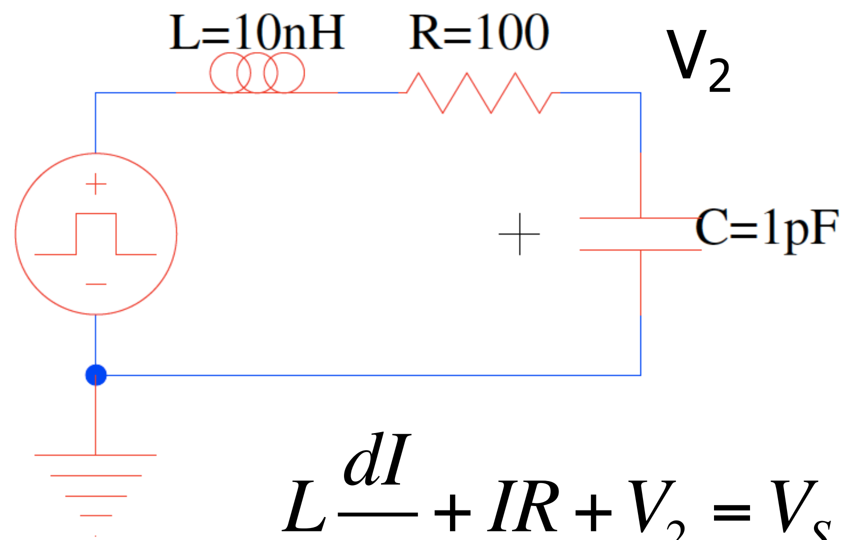


$$L\frac{dI}{dt} + IR + V_2 = V_S$$

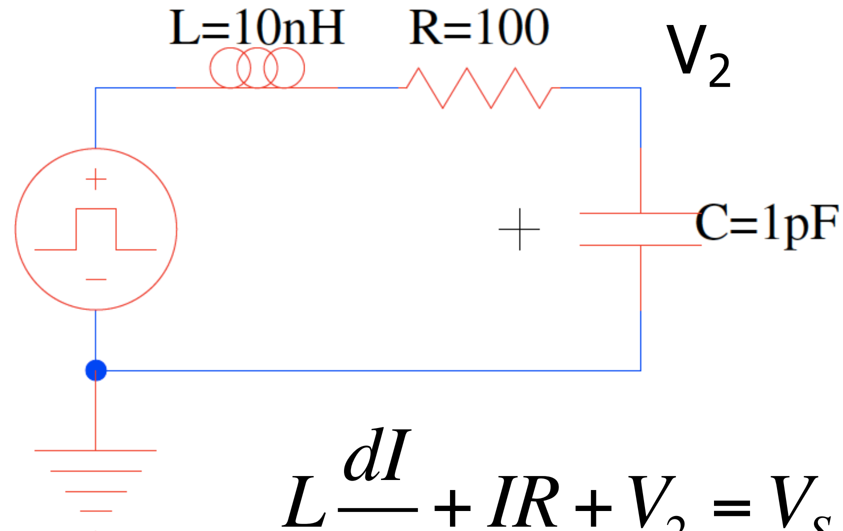
$$LC\left(\frac{d^2V_2}{dt^2}\right) + RC\left(\frac{dV_2}{dt}\right) + V_2 = V_S$$

RLC Response

$$V_2 = A + Be^{j\omega t}$$

$$\frac{dV_2}{dt} = j\omega Be^{j\omega t}$$

$$\frac{d^2V_2}{dt^2} = -\omega^2 Be^{j\omega t}$$



$$L\frac{dI}{dt} + IR + V_2 = V_S$$

$$LC\left(\frac{d^2V_2}{dt^2}\right) + RC\left(\frac{dV_2}{dt}\right) + V_2 = V_S$$

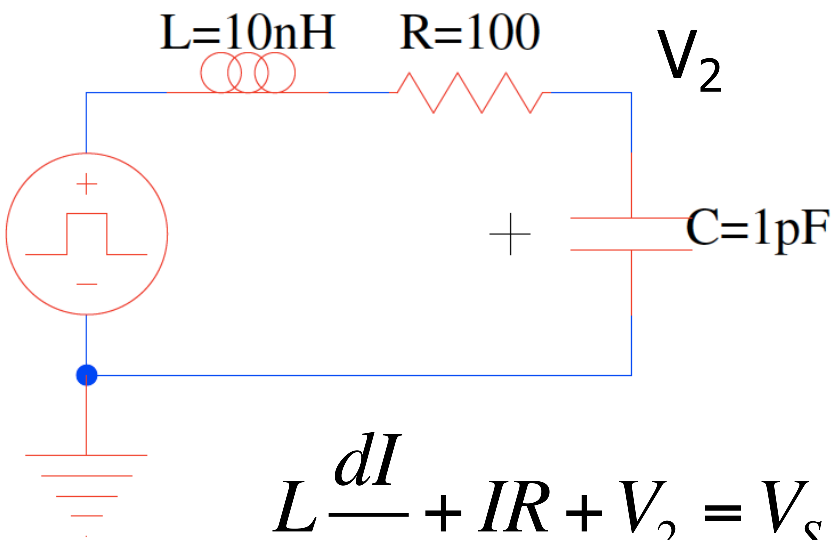
$$LC\left(-\omega^2 Be^{j\omega t}\right) + RC\left(j\omega Be^{j\omega t}\right) + A + Be^{j\omega t} = V_S$$

RLC Response

$$V_2 = A + Be^{j\omega t}$$

$$\frac{dV_2}{dt} = j\omega Be^{j\omega t}$$

$$\frac{d^2V_2}{dt^2} = -\omega^2 Be^{j\omega t}$$



$$L\frac{dI}{dt} + IR + V_2 = V_S$$

$$LC\left(-\omega^2 Be^{j\omega t}\right) + RC\left(j\omega Be^{j\omega t}\right) + A + Be^{j\omega t} = V_S$$

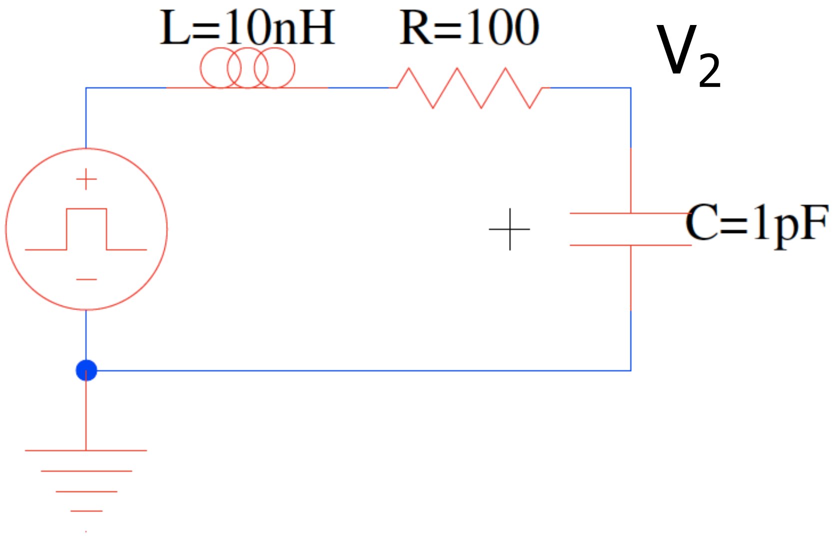
$$A = V_S$$

$$LC\left(-\omega^2\right) + RC\left(j\omega\right) + 1 = 0$$

RLC Response

$$LC\left(-\omega^2\right)+RC\left(j\omega\right)+1=0$$

$$\omega^2 - j\omega\frac{R}{L}-\frac{1}{LC}=0$$



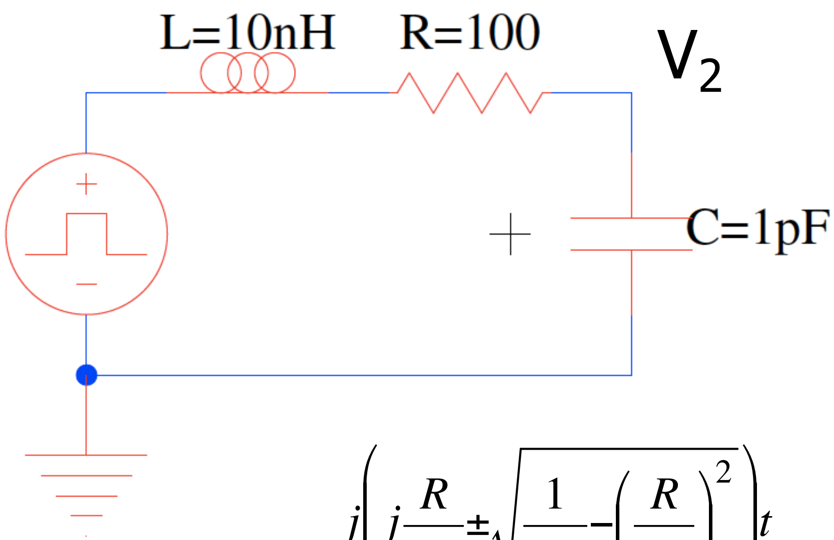
$$\omega=\frac{j\frac{R}{L}\pm\sqrt{\left(-j\frac{R}{L}\right)^2+\frac{4}{LC}}}{2}$$

$$\omega=j\frac{R}{2L}\pm\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}$$

RLC Response

$$LC\left(-\omega^2\right)+RC\left(j\omega\right)+1=0$$

$$\omega^2 - j\omega\frac{R}{L}-\frac{1}{LC}=0$$



$$\omega=\frac{j\frac{R}{L}\pm\sqrt{\left(-j\frac{R}{L}\right)^2+\frac{4}{LC}}}{2}$$

$$\omega=j\frac{R}{2L}\pm\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}$$

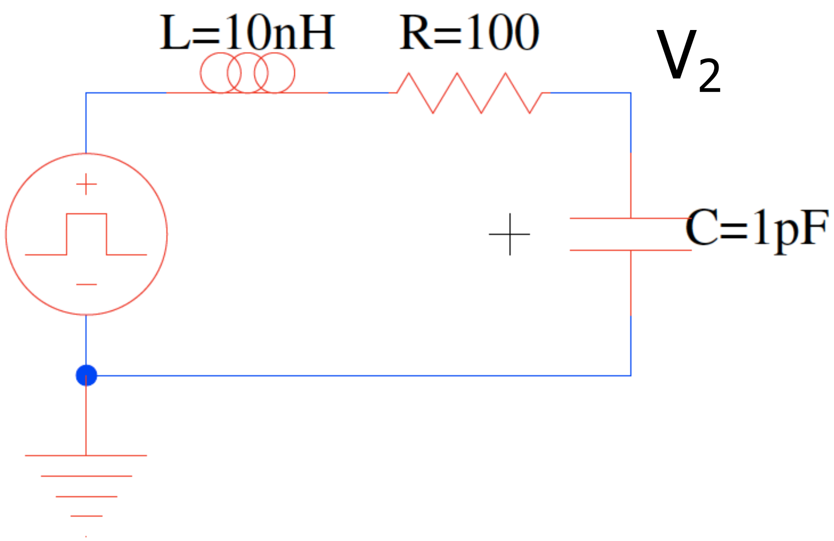
$$V_2=V_S+Be^{j\left(j\frac{R}{2L}\pm\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}\right)t}$$

$$V_2=V_S+Be^{\left(-\frac{R}{2L}\pm j\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}\right)t}$$

$$V_2=V_S+Be^{\left(-\frac{R}{2L}\right)t}e^{\left(j\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}\right)t}$$

RLC Response

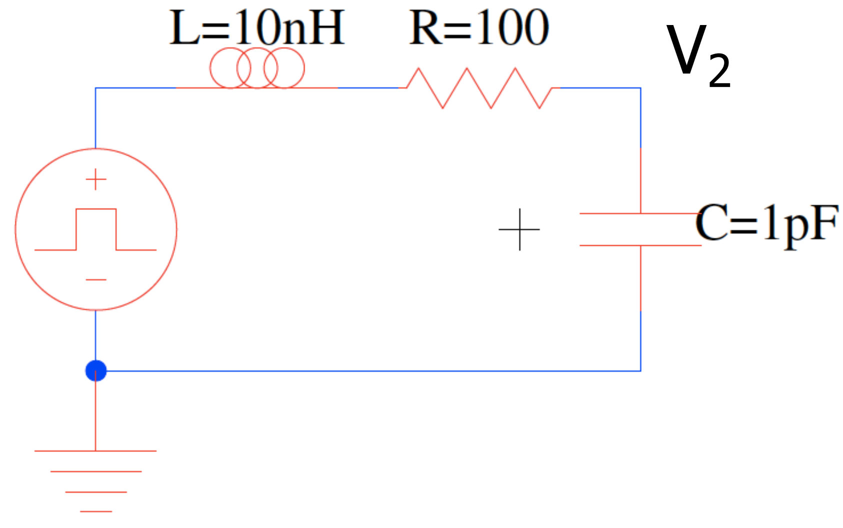
$$V_2 = V_S + Be^{\left(-\frac{R}{2L}\right)t} e^{\left(j\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}\right)t}$$



- For what R does this circuit oscillate?

RLC Response

$$V_2 = V_S + Be^{\left(-\frac{R}{2L}\right)t} e^{\left(j\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}\right)t}$$



- For what R does this circuit oscillate?

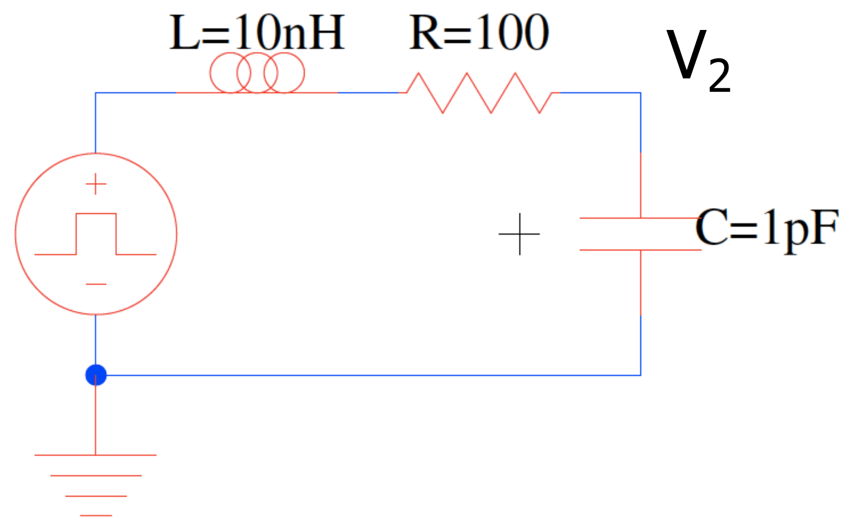
$$\frac{1}{LC}-\left(\frac{R}{2L}\right)^2 > 0$$

$$\frac{1}{LC} > \left(\frac{R}{2L}\right)^2 \Rightarrow \sqrt{\frac{4L}{C}} > R$$

RLC Response

Decay | Oscillation

$$V_2 = V_S + Be^{\left(-\frac{R}{2L}\right)t} e^{\left(j\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}\right)t}$$



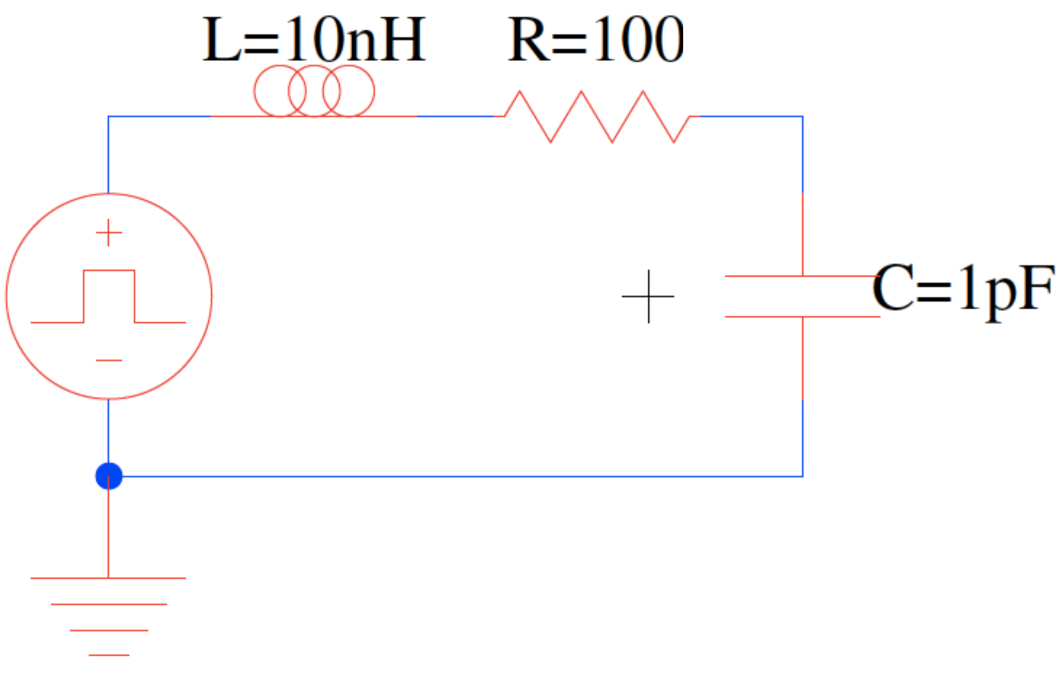
- For what R does this circuit oscillate?

$$\frac{1}{LC} - \left(\frac{R}{2L}\right)^2 > 0$$

$$\frac{1}{LC} > \left(\frac{R}{2L}\right)^2 \Rightarrow \sqrt{\frac{4L}{C}} > R$$

When Oscillate

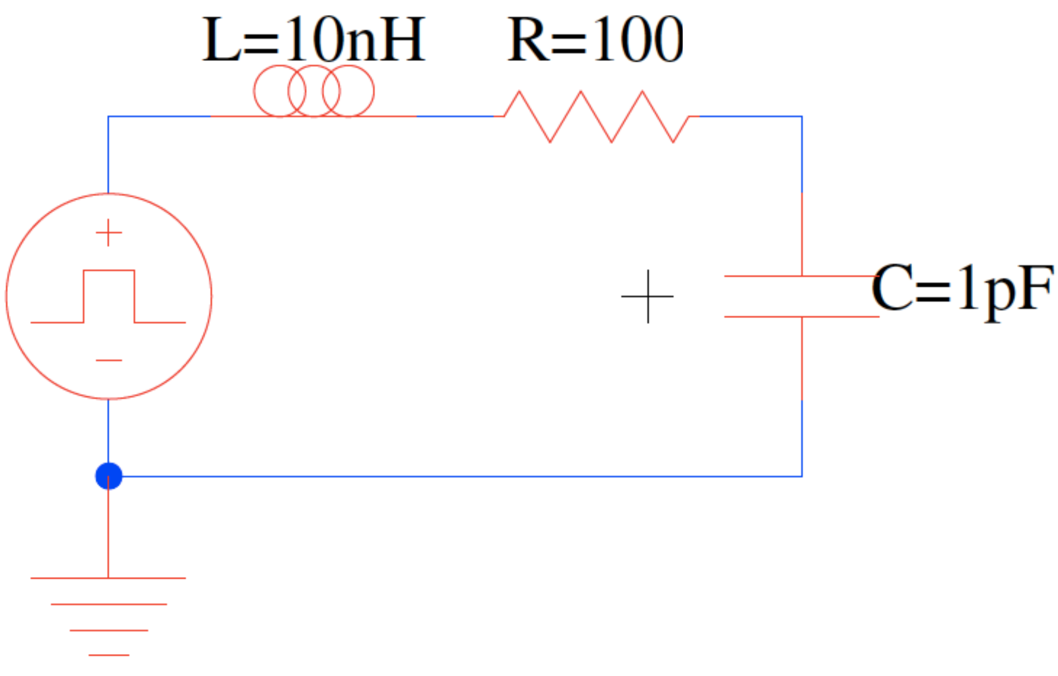
- For what R does this particular circuit oscillate?



When Oscillate

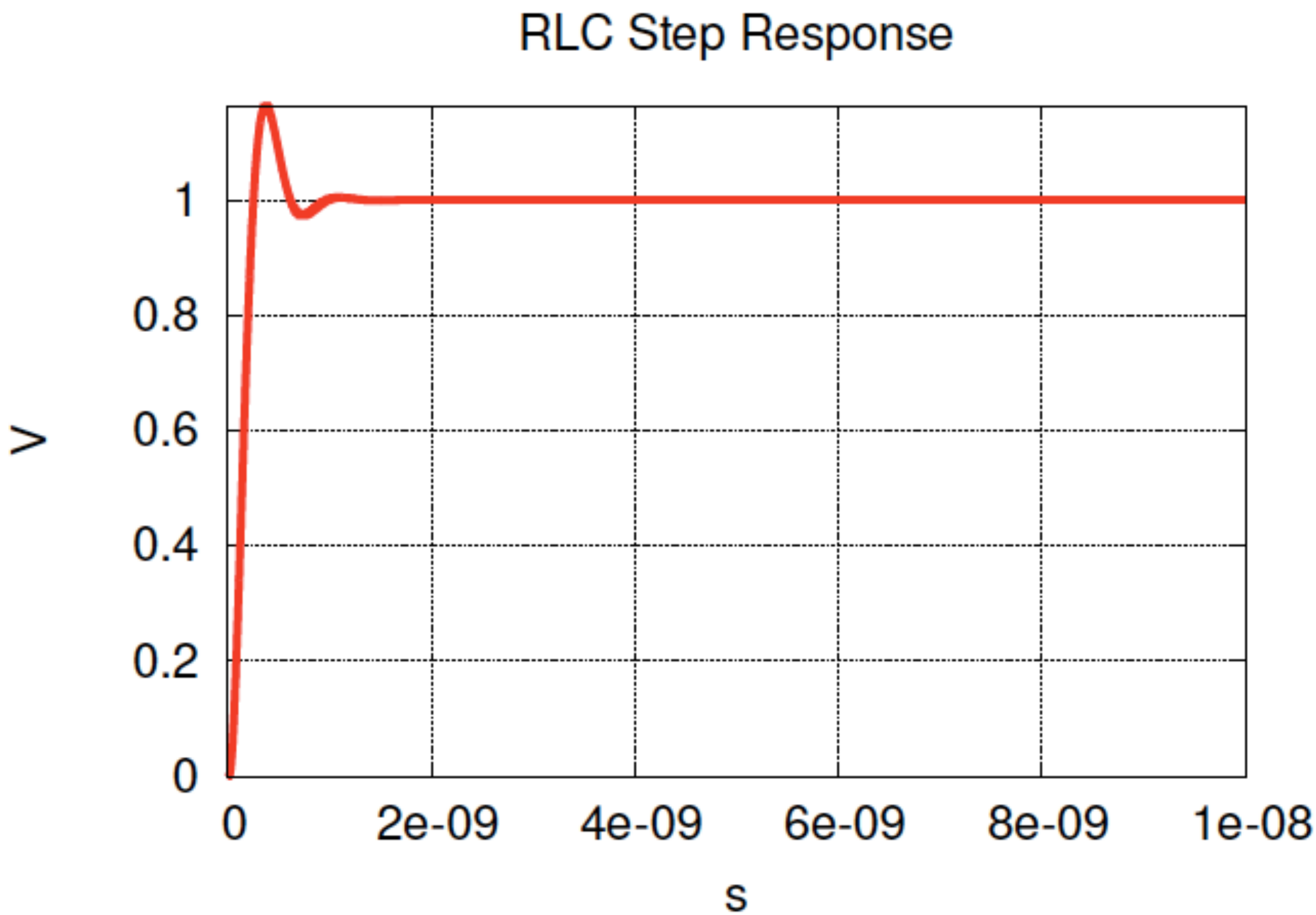
- For what R does this particular circuit oscillate?

$$R < \sqrt{\frac{4L}{C}}$$

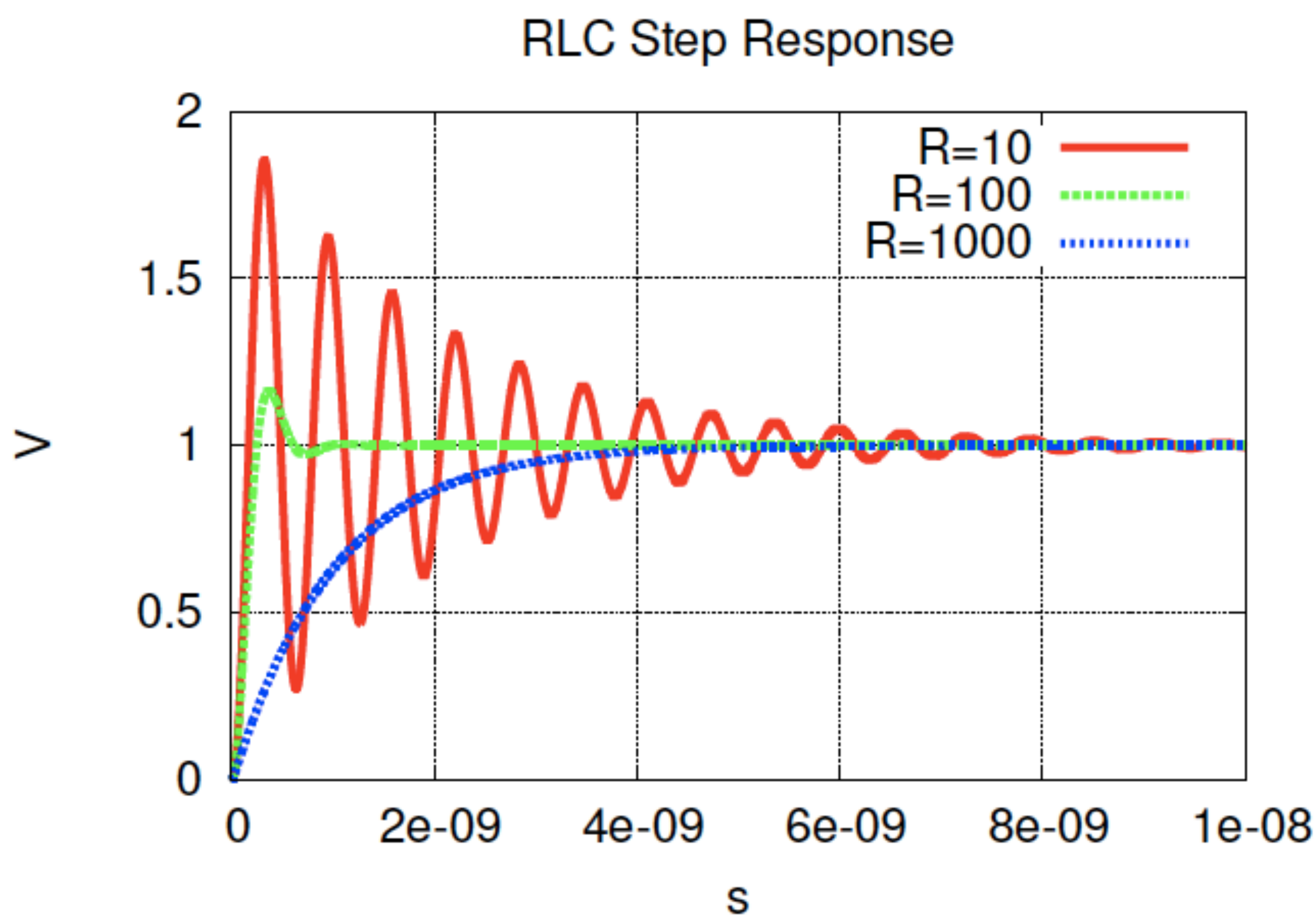


$$\sqrt{\frac{4L}{C}} = 200$$

RLC Response (R=100)



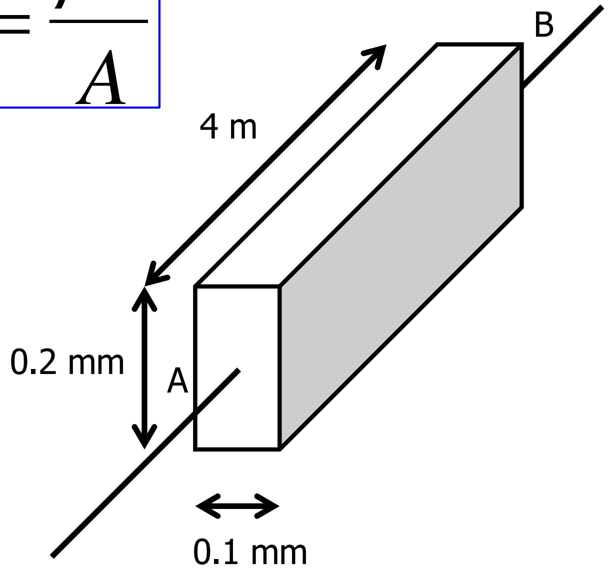
RLC Response



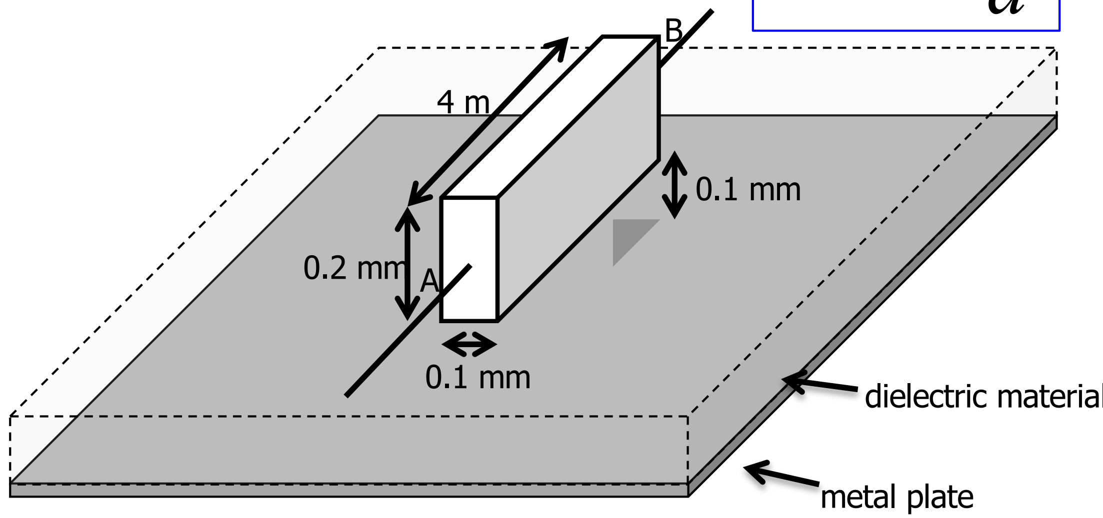
Inductance of Wire

Isolated Wire RC

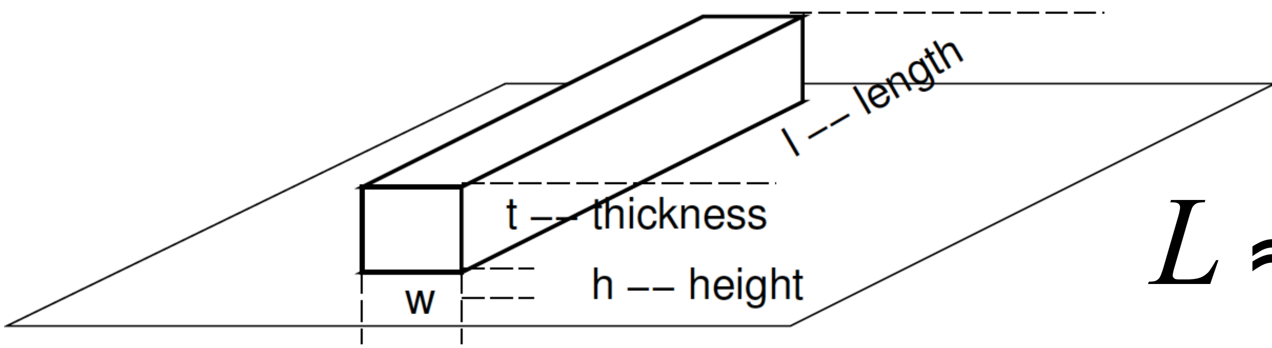
$$R = \frac{\rho L}{A}$$



$$C = \varepsilon_d \frac{A}{d}$$

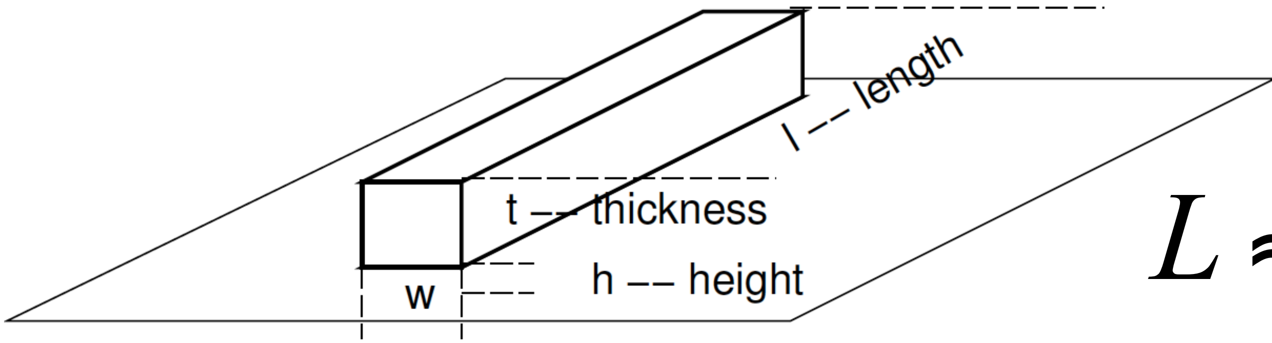


Inductance: Wire over Ground Plane



$$L \approx l\left(\frac{\mu_0 \mu_r h}{w}\right)$$

Inductance: Wire over Ground Plane



$$L \approx l\left(\frac{\mu_0 \mu_r h}{w}\right)$$

$$C = \varepsilon_r \varepsilon_0 \frac{A}{d} = \varepsilon_r \varepsilon_0 \frac{wl}{h} = l\left(\varepsilon_r \varepsilon_0 \frac{w}{h}\right)$$

Inductance: Wire over Ground Plane

$$C' = \varepsilon_r \varepsilon_0 \frac{w}{h} \qquad L' \approx \left(\frac{\mu_0 \mu_r h}{w} \right)$$

$$C' L' = \varepsilon \mu$$

C’ and *L’* per unit length

$$L' = \frac{\varepsilon \mu}{C'}$$

On Chip Inductance

- C_{wire} = 0.16 pF (for the 1mm)
- $C_{wire} =$ 0.16nF/m

- Permeability $\mu_0 \approx \mu_{Si02} = 12.6 \cdot 10^{-7}$ H/m
- Permitivity $\varepsilon_{ox} = 3.5 \cdot 10^{-11}$ F/m

$$L' = \frac{\varepsilon\mu}{C'}$$

On Chip Inductance

- C_{wire} = 0.16 pF (for the 1mm)
- C_{wire} = 0.16nF/m

- Permeability $\mu_0 \approx \mu_{Si02} = 12.6 \cdot 10^{-7}$ H/m
- Permitivity $\varepsilon_{ox} = 3.5 \cdot 10^{-11}$ F/m
- 276 pH (for 1 mm)

$$L' = \frac{\varepsilon\mu}{C'}$$

PCB Trace Inductance (preclass 2)

- Inductance per cm of PCB trace with h=3mil, w=5mil (1mil = .001 inch)
  - $\mu_0 = 1.26 \times 10^{-6}$
  - $\mu_r = 1$

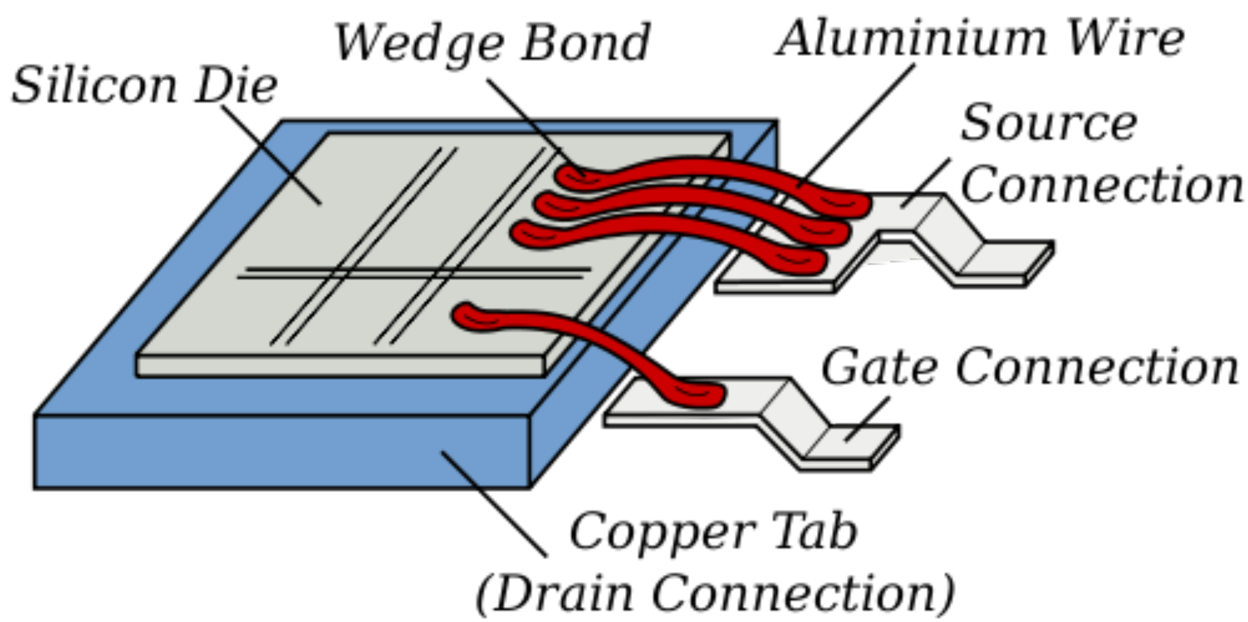
$$L \approx l\left(\frac{\mu_0 \mu_r h}{w}\right)$$

Comparisons

- 5mil trace on PCB
  - 7.56nH/cm
- Breadboard wires (0.6mm diameter)
  - About 7nH/cm
- On chip wire
  - 0.28nH/mm = 2.8nH/cm

Inductors

- Bond pads
- Chip leads
- Long wire runs
- Cables

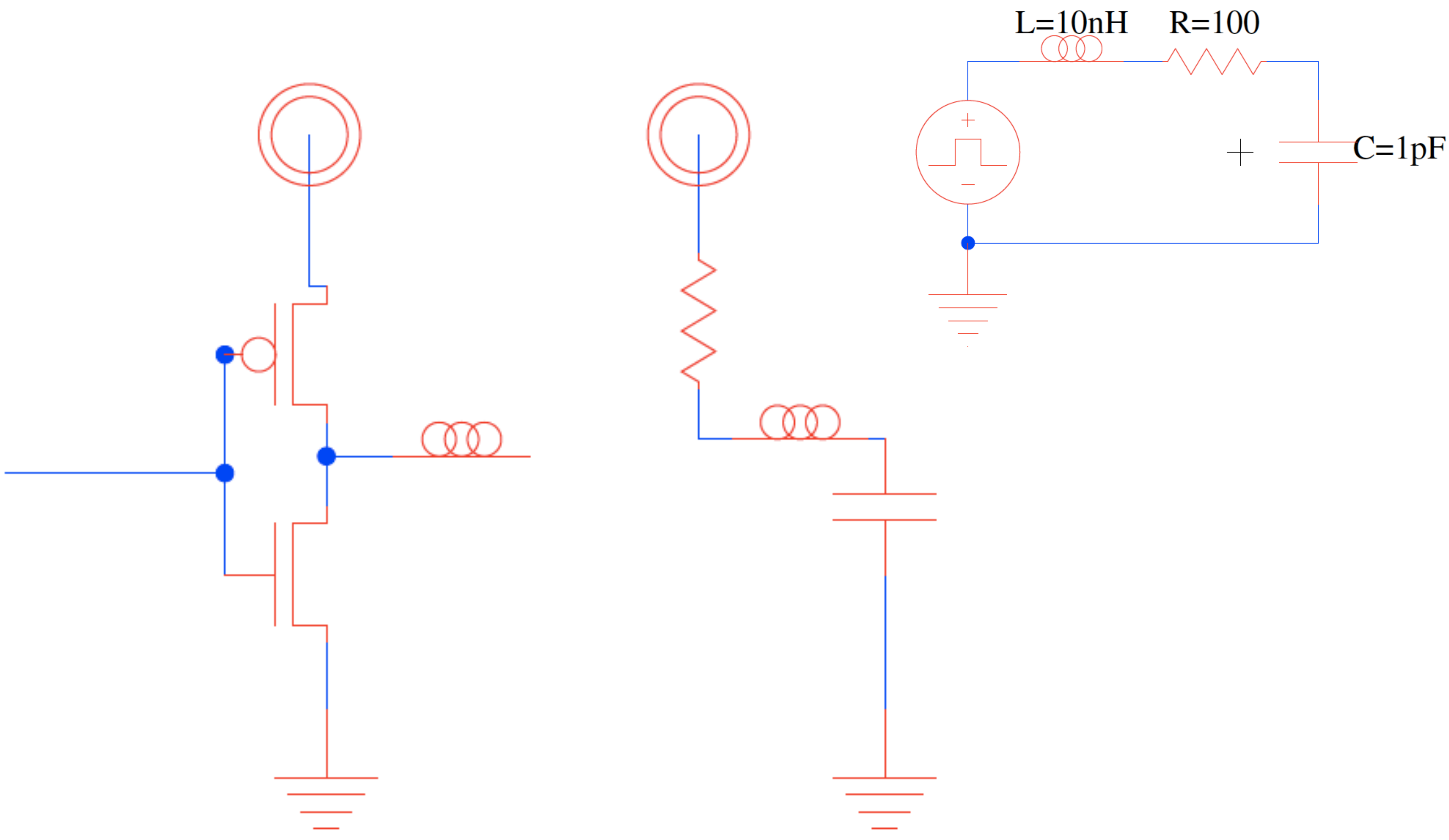


Src: http://en.wikipedia.org/wiki/File:Wirebonding2.svg

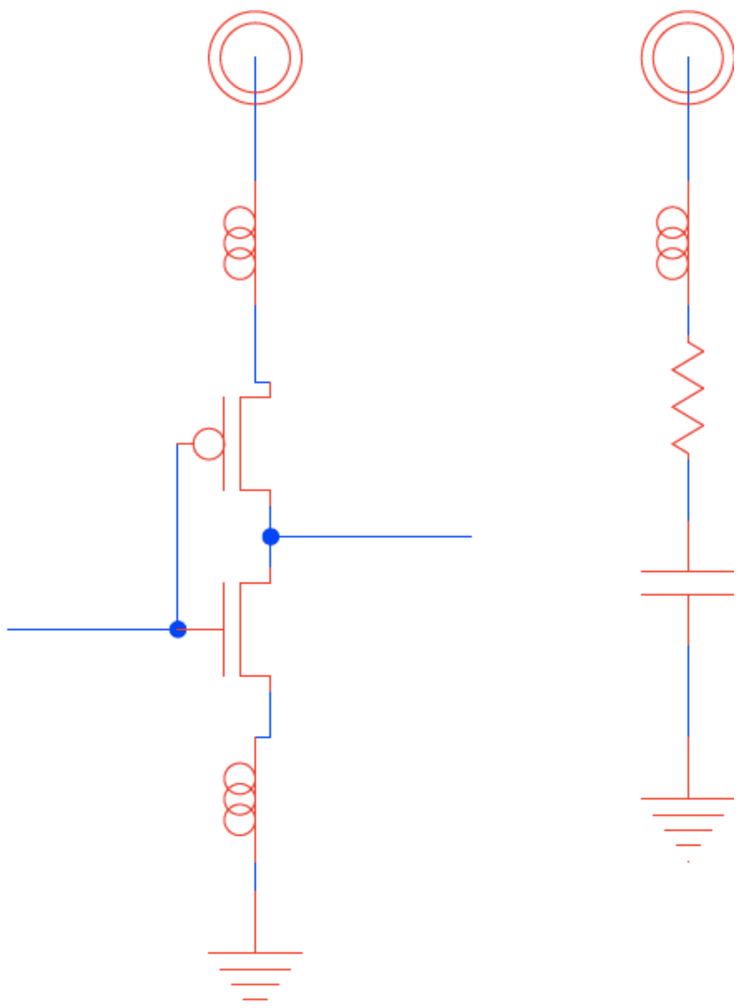
Where Inductance Arises

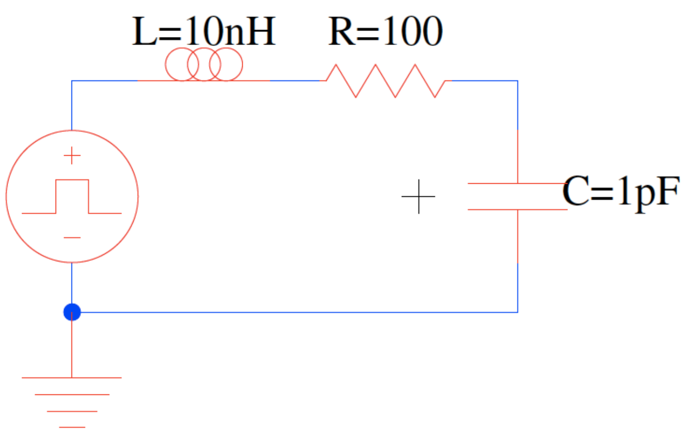
In our digital systems

Signal Path

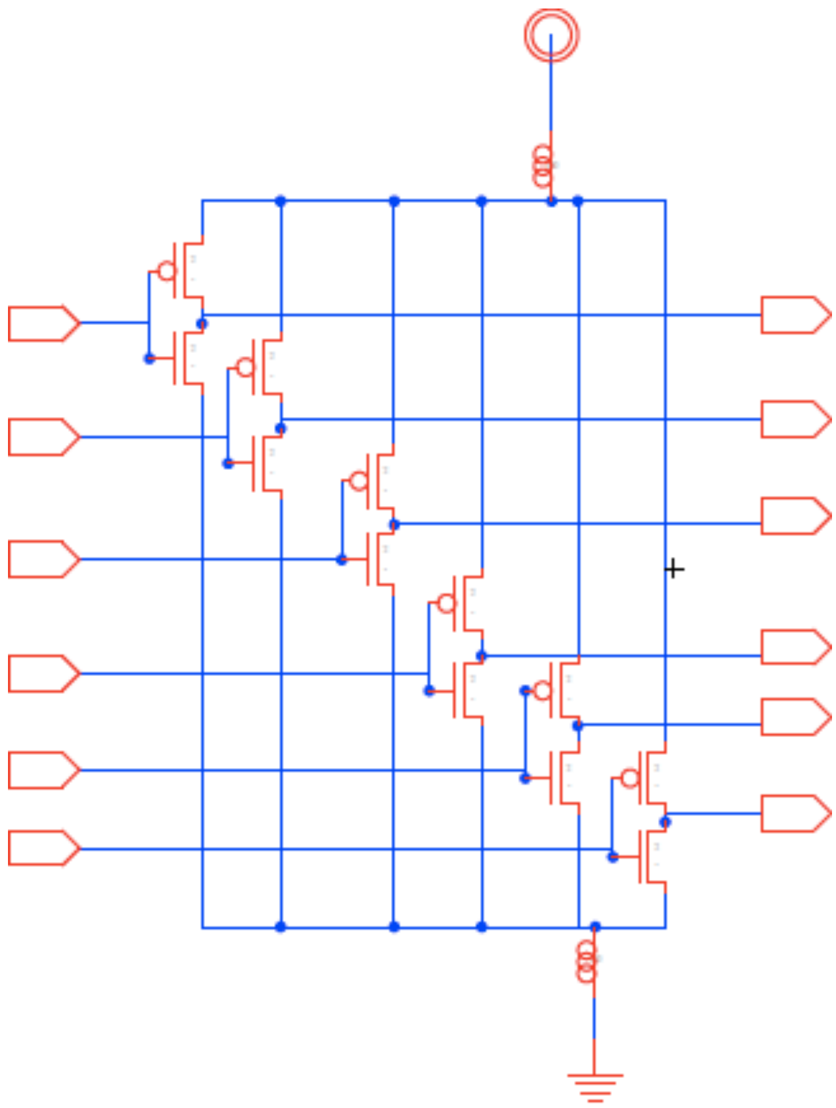


Power Ground





Shared Power/Ground Example: 74x04



Estimate

- R_{eq}, C_{eq} for gates in parallel
  - $R_0 = 25\text{K}\ \Omega$
  - $C_0 = 0.01\ \text{fF}$
    - say $10C_0=0.1\text{fF}$ for typical load
- 250 gates switching at clock

Decay Oscillation

$$V_2 = V_S + Be^{\left(-\frac{R}{2L}\right)t} e^{\left(j\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}\right)t}$$

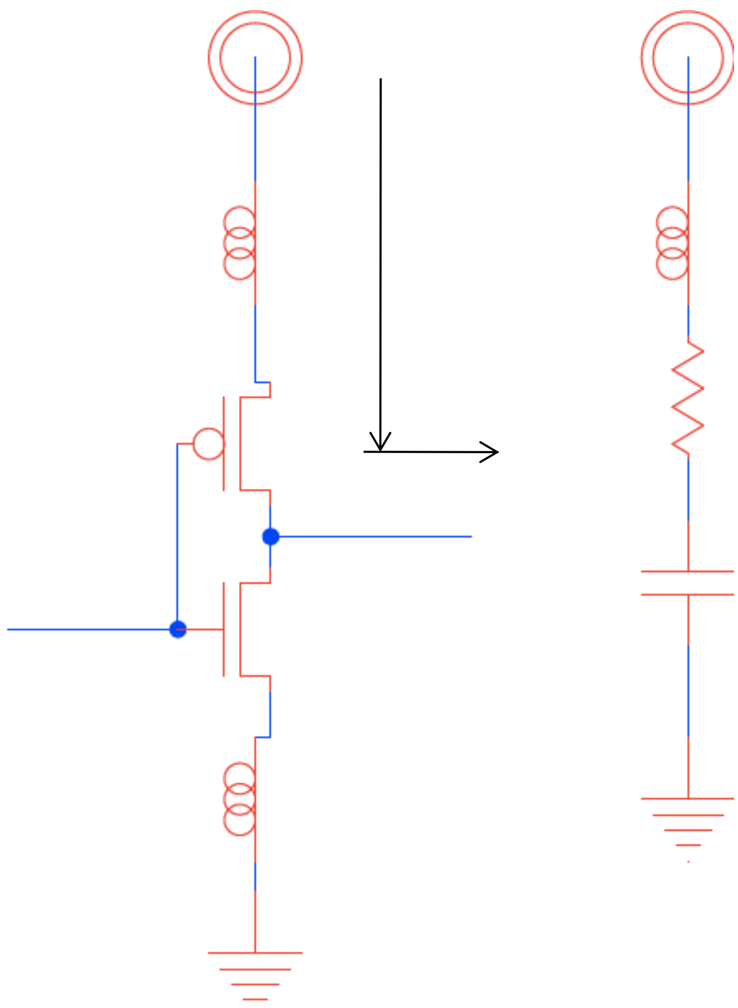
Estimate

- R_{eq}, C_{eq} for gates in parallel
  - $R_0 = 25K\ \Omega$
  - $C_0 = 0.01$ fF
    - say $10C_0$=0.1fF for typical load
- 250 gates switching at clock
- $R_{eq} = 100\Omega$ C_{eq}=25fF
- Assume L=1nH
- How long to settle? Oscillation freq?

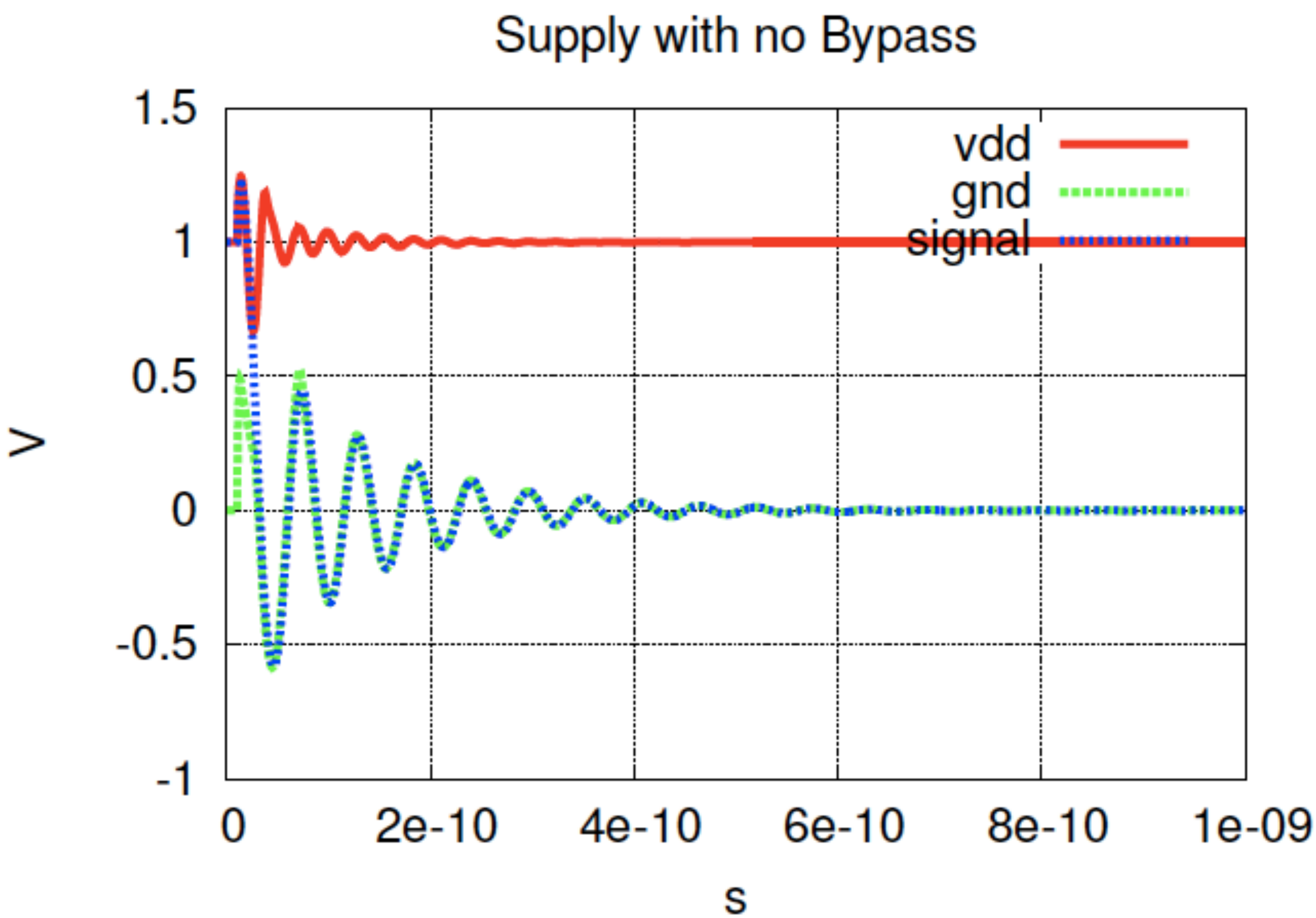
Decay | Oscillation

$$V_2 = V_S + Be^{\left(-\frac{R}{2L}\right)t} e^{\left(j\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}\right)t}$$

Power Ground

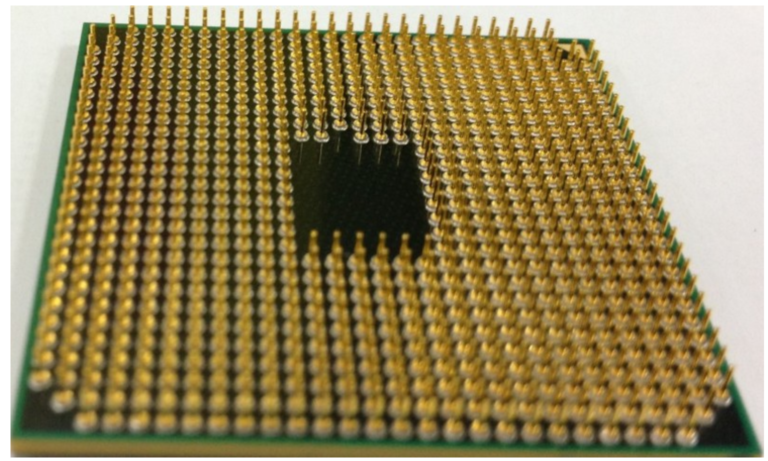


RLC Response



Today's chips: Multiple Power/Ground Pins

- Use many power/ground pins



- Divide switching gates by pins
  - To get effective load on each pin

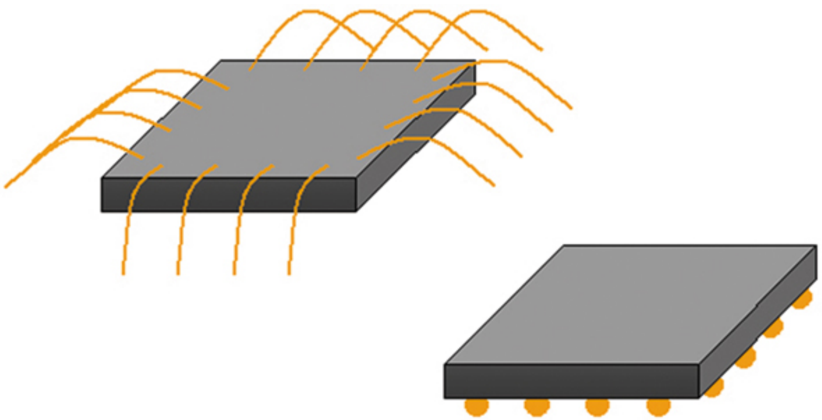
Improve Performance

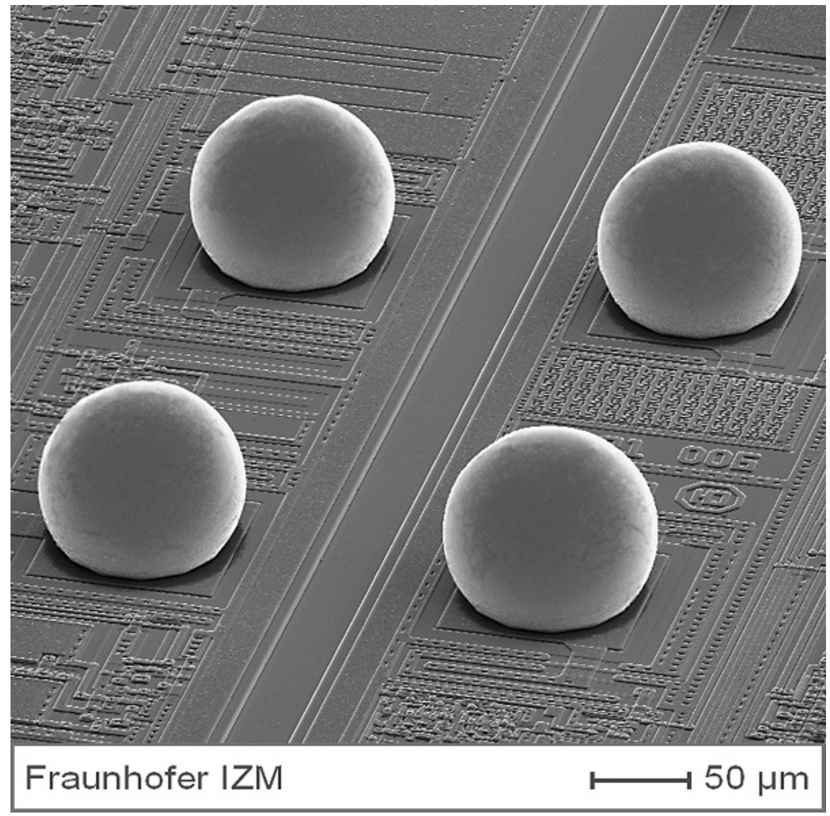
Minimize the L

- Make wires short
- Use power and ground planes
  - Think of power plane as a very wide wire

$$L \approx l\left(\frac{\mu_0 \mu_r h}{w}\right)$$

Flip Chip, Area IO

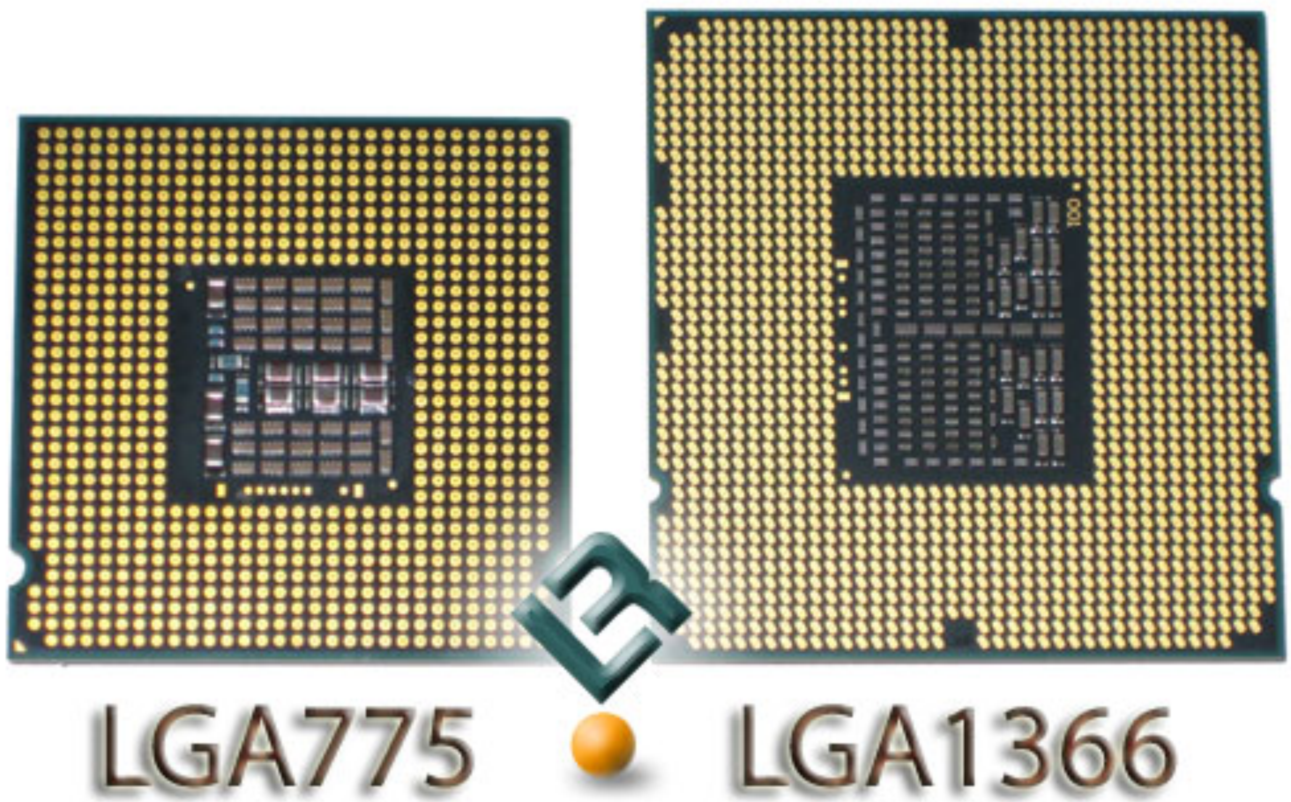




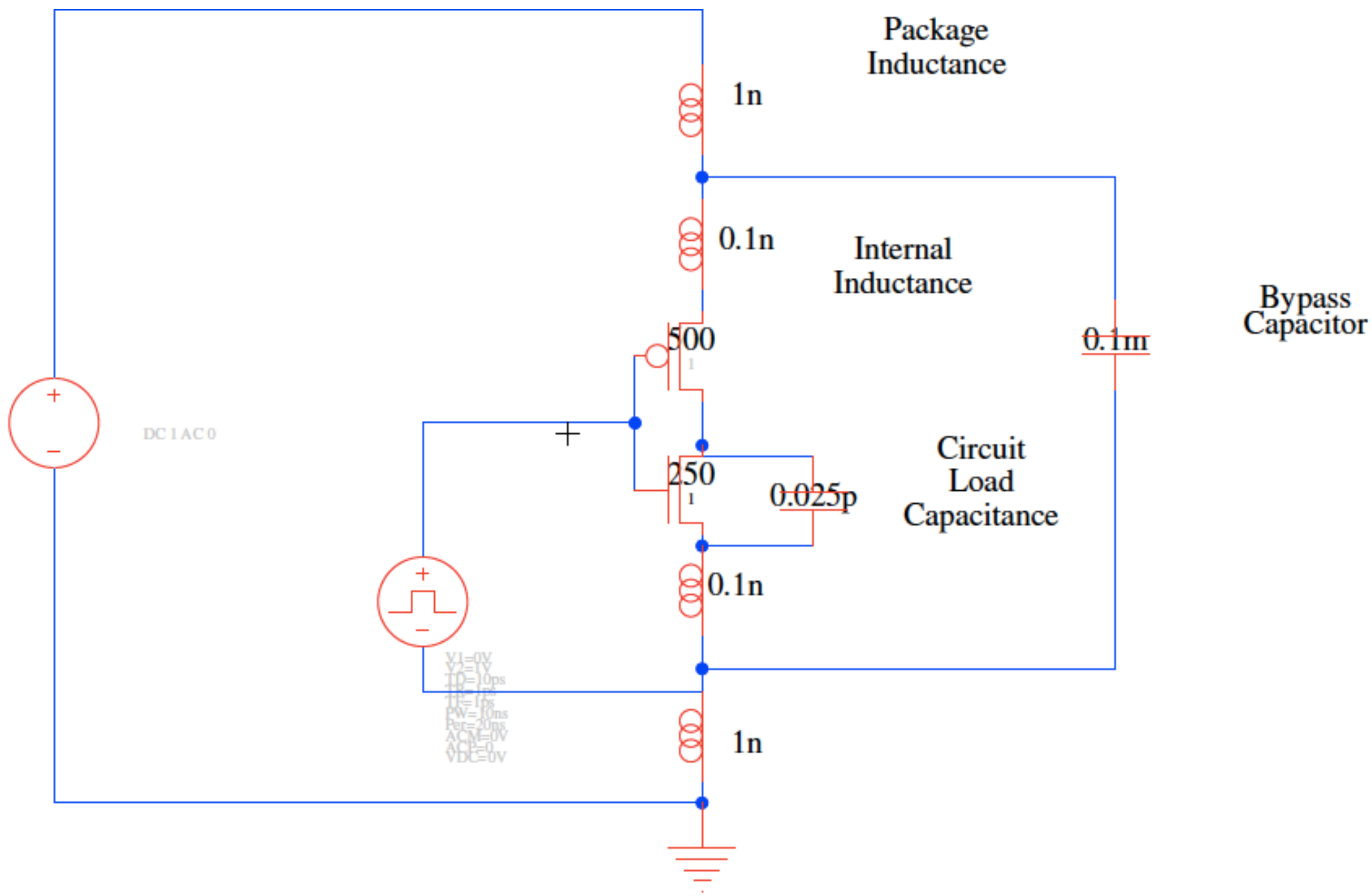
http://www.izm.fraunhofer.de/en/abteilungen/high_density_interconnectwaferlevelpackaging/arbeitsgebiete/arbeitsgebiet1.

Add Good C's

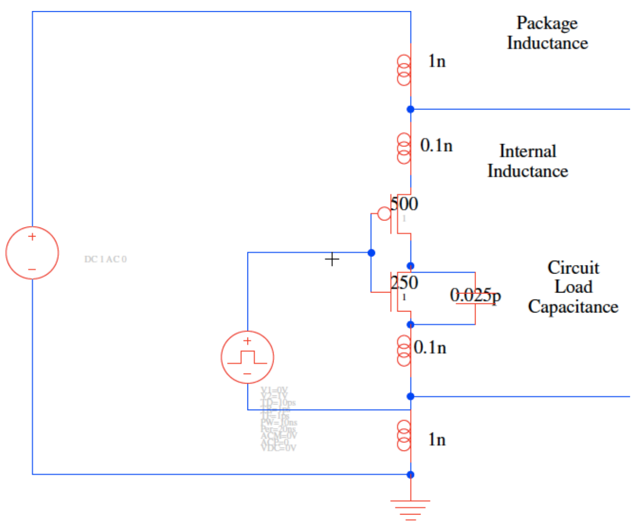
- Bypass Capacitors – inside the inductances
  - On board
  - On package
  - On chip



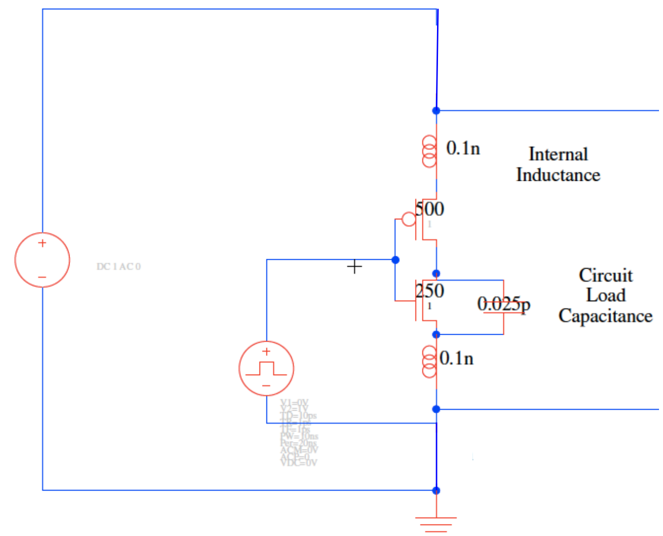
Bypass Capacitor Example



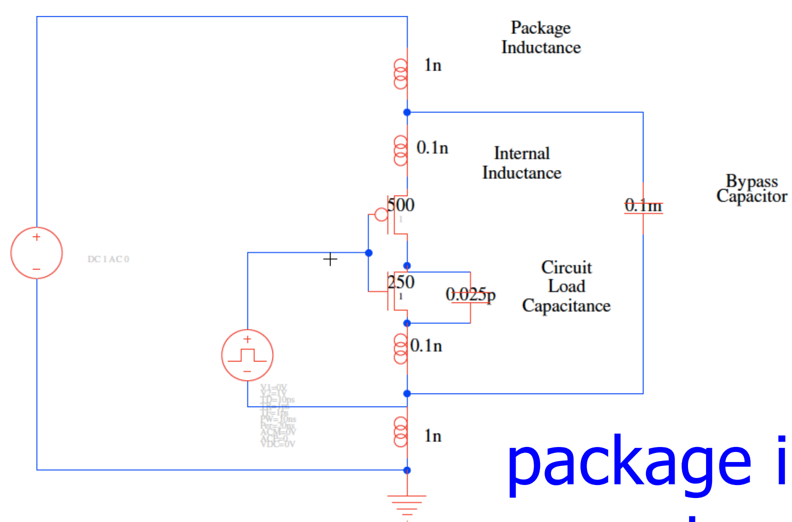
Bypass Capacitor Example



No bypass cap

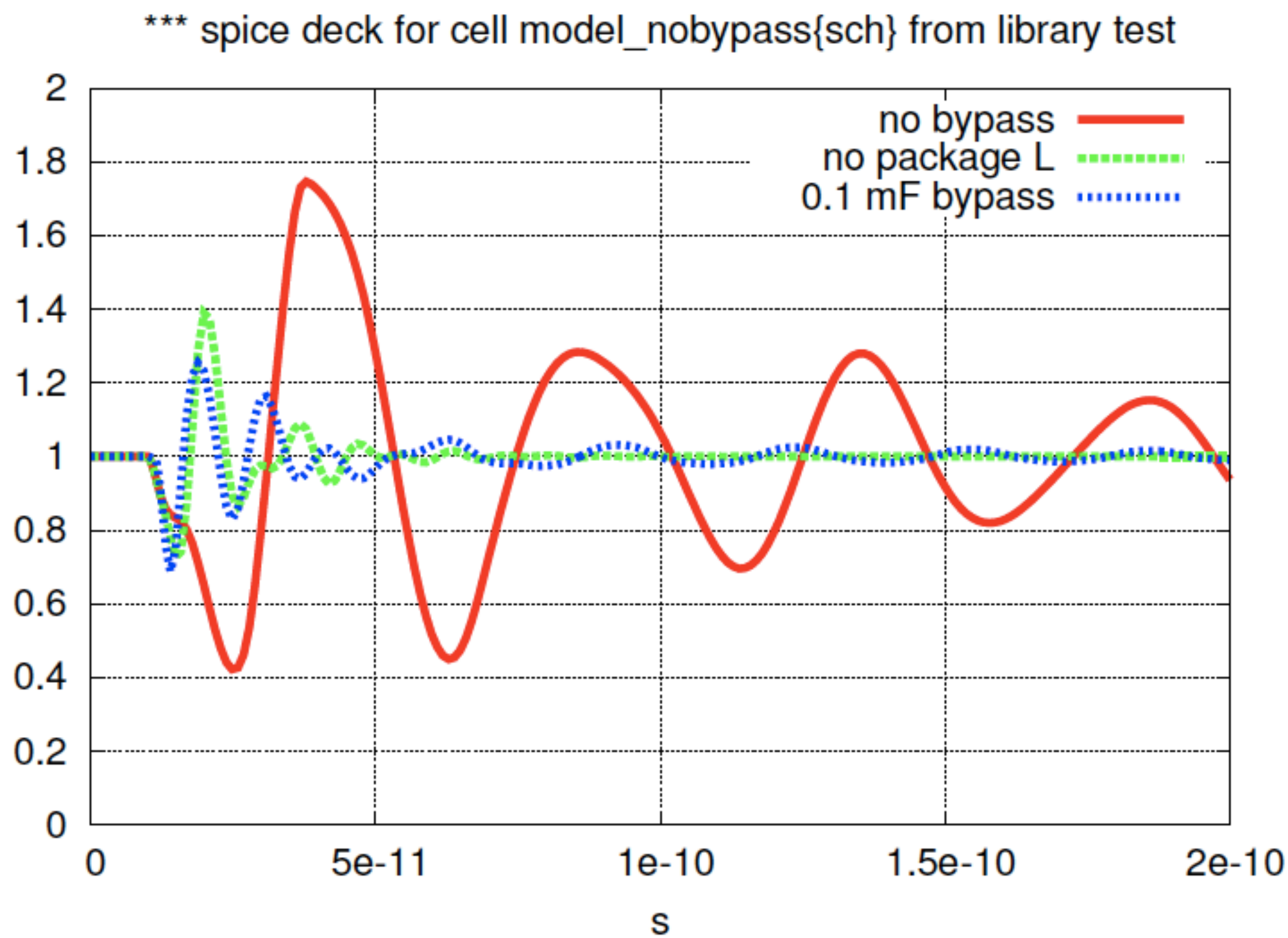


No package inductance

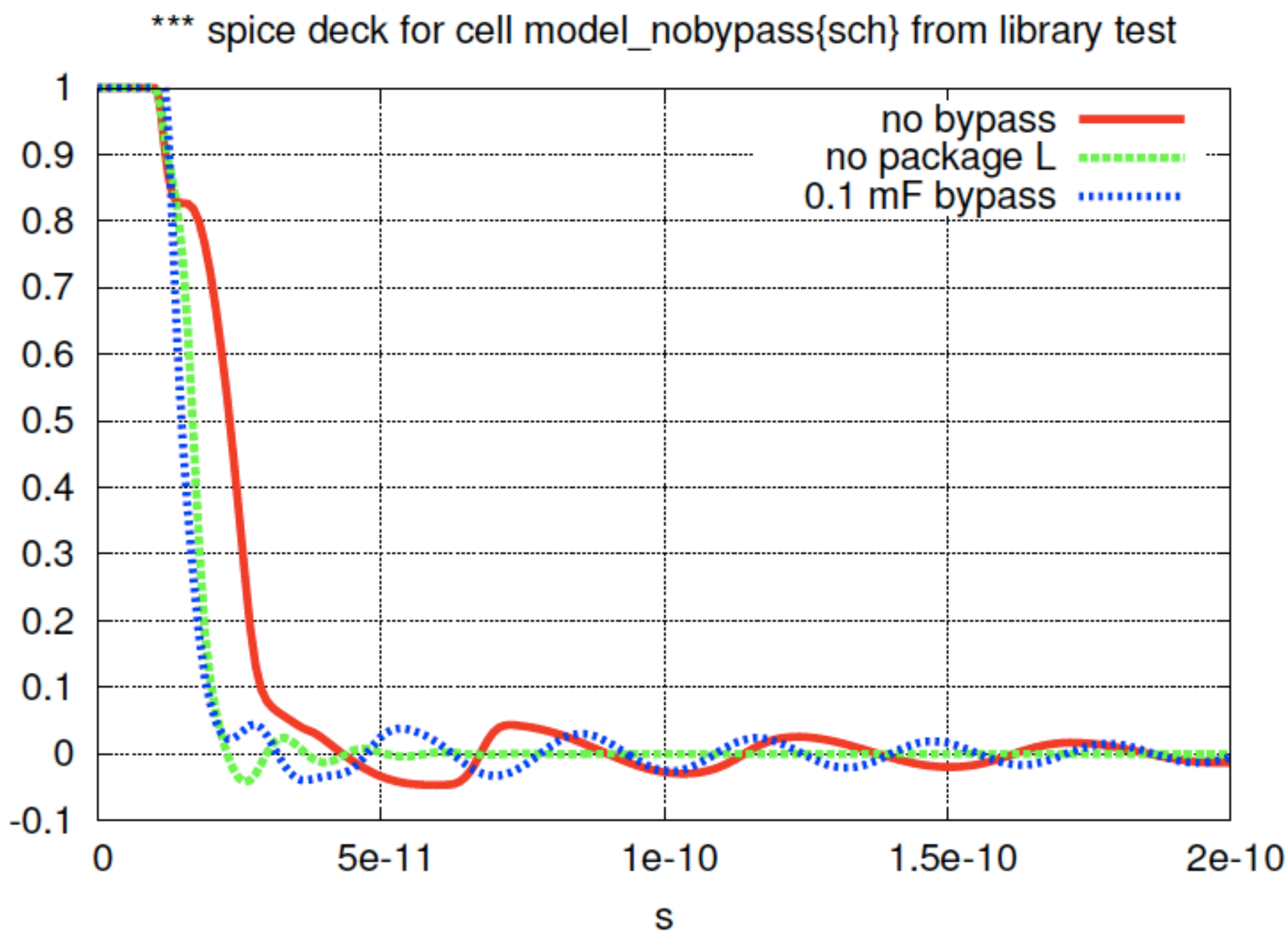


package inductance w/
bypass cap

Bypassed Supplies (@ transistor)

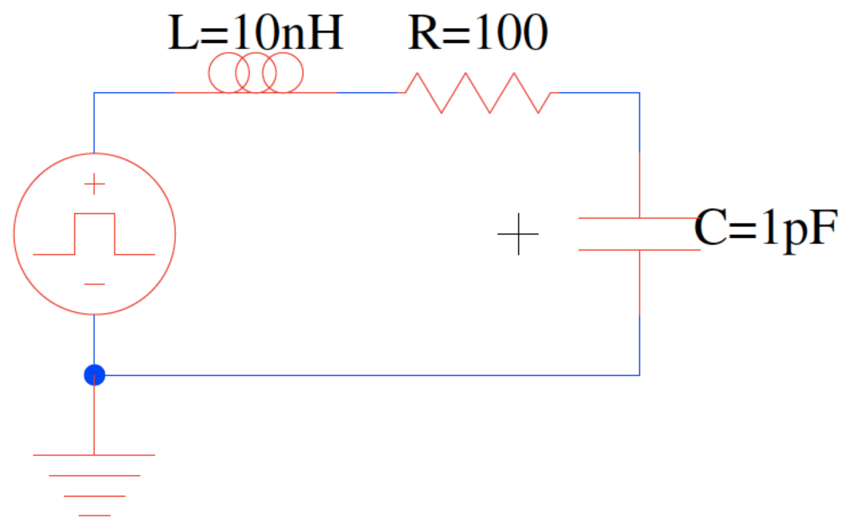


Bypassed Output



Idea

- Long wires are inductive
  - **Avoid** them
  - Especially on power supplies
- Bypass capacitors help



Decay | Oscillation

$$V_2 = V_S + Be^{\left(-\frac{R}{2L}\right)t} e^{\left(j\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}\right)t}$$

Admin

- Project 2
  - Milestone due Monday
    - Feedback by Wednesday
  - Final due Friday 12/3