ESE370: Circuit-Level Modeling, Design, and Optimization for Digital Systems

Lec 32: December 1, 2021

Transmission Line

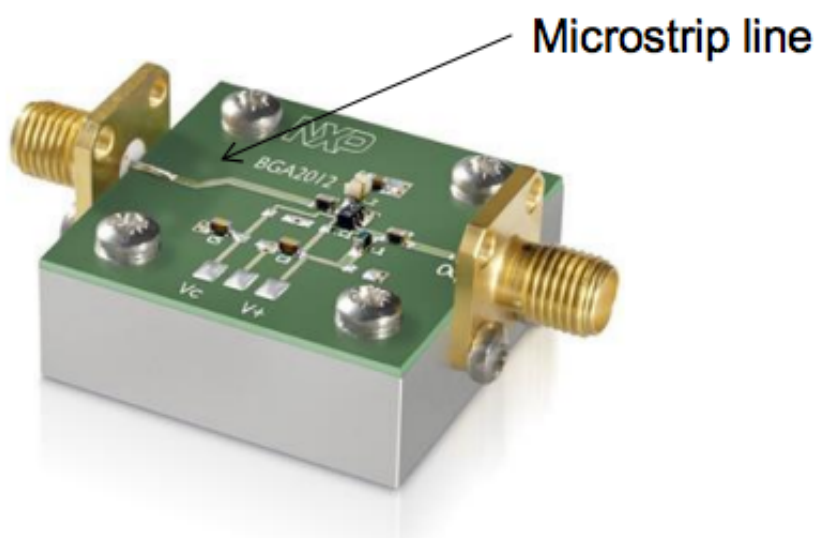
Next few Lectures

- Saw in action in lab
- Where transmission lines arise?
- General wire formulation
- Lossless Transmission Line
- End of Transmission Line?
- Termination
- Discuss Lossy
- Implications

Where Transmission Lines Arise

Transmission Lines

- Cable: coaxial
- PCB
  - Strip line
  - Microstrip line
- Twisted Pair (Cat5)



Transmission Lines

- How did the traces behave in lab?
- How does this differ from
  - Ideal equipotential?
  - RC-wire on chip?

Transmission Lines

- This is what long wires/cables look like
  - Aren't an ideal equipotential
  - Signals take time to propagate
  - Maintain shape of input signal
    - Within limits
  - Shape and topology of wiring effects how signals propagate

Transmission Lines

- Need theory/model to support design
  - Reason about behavior
  - Understand what can cause noise
  - Engineer high performance/speed communication

Wire Formulation

From the Way Way Back

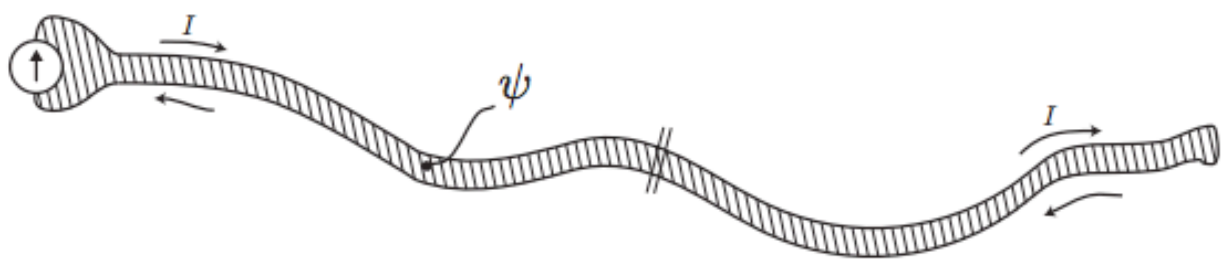
- Problem: A long cable – the trans-atlantic telephone cable – is laid out connecting NY to London. We would like analyze the electrical properties of this cable.
- For simplicity, assume the cable has a uniform cross-sectional configuration (shown as two wires here)

Trans-Atlantic Cable

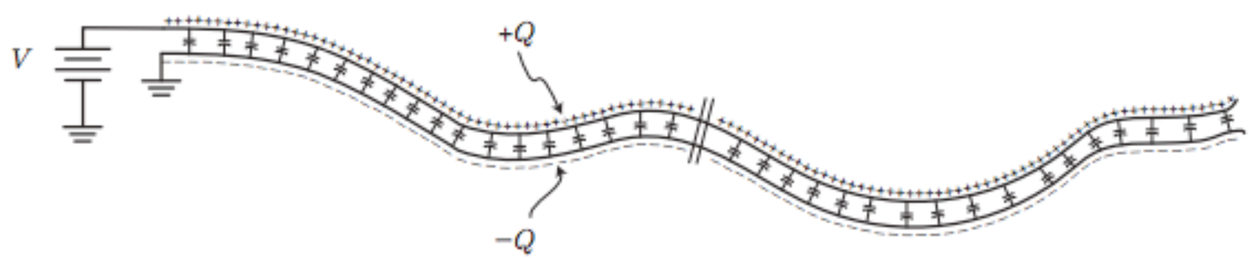
- Can we do it with circuit theory?
- Fundamental problem with circuit theory is that it assumes that the speed of light is infinite. So all signals are in phase: $V(z) = V(z + \ell)$
- Consequently, all variations in space are ignored: $\partial/\partial z \rightarrow 0$
- This allows the lumped circuit approximation

Lumped Circuit Properties of Cable

- Shorted Line: The long loop has inductance since the magnetic flux ψ is not negligible (long cable) (ψ = LI)

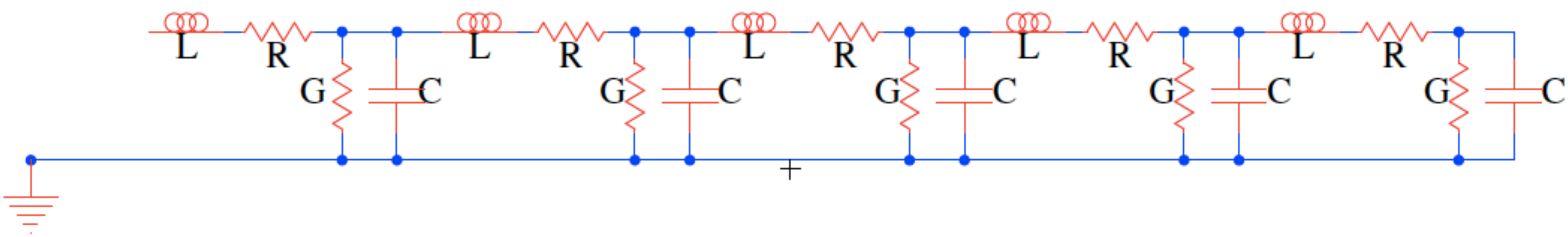


- Open Line: The cable also has substantial capacitance (Q = CV)



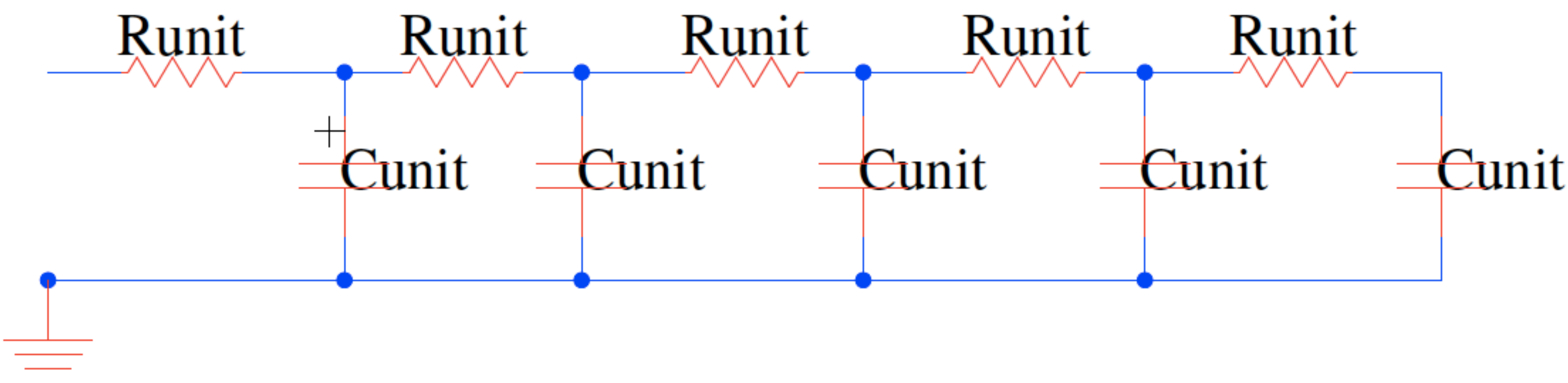
Wires

- In general, our "wires" have distributed R, L, C components



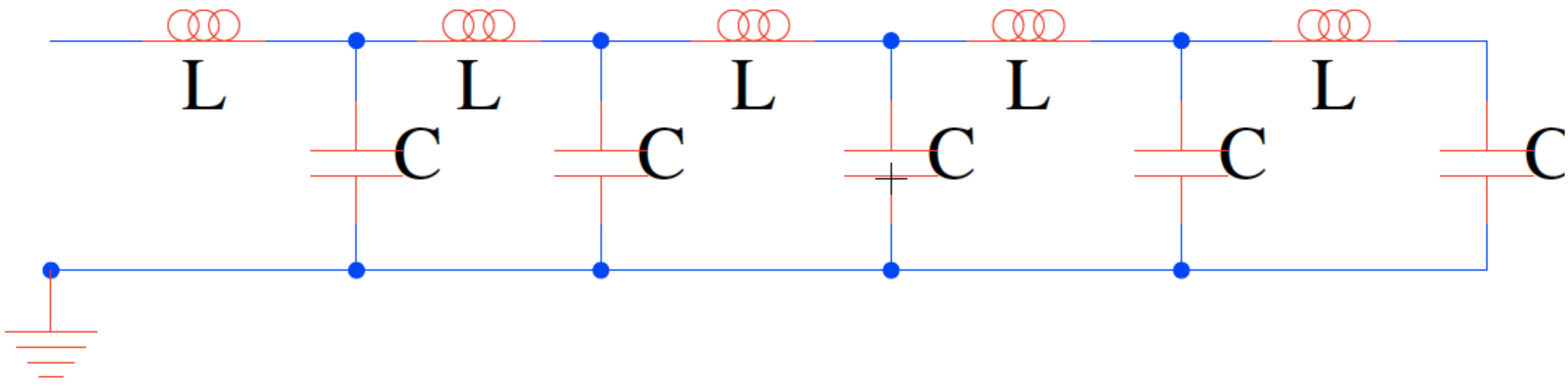
RC Wire (Preclass 1)

- When R dominates L
  - We have the distributed RC Wires
  - Typical of on-chip wires in ICs
  - What is RC response to step?



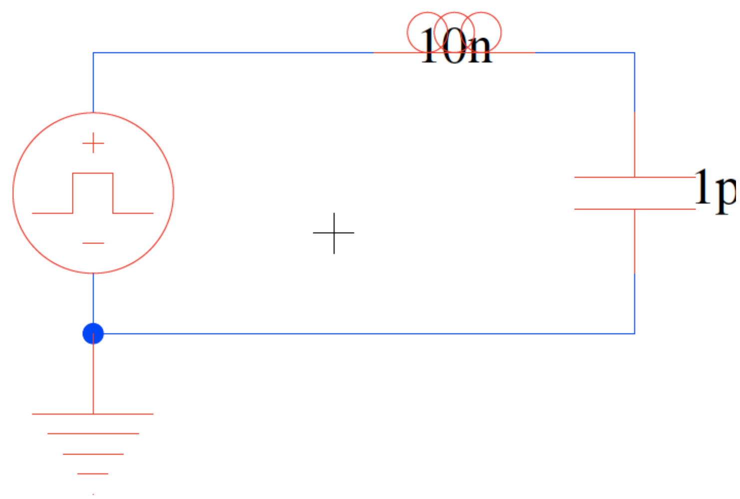
Transmission Line

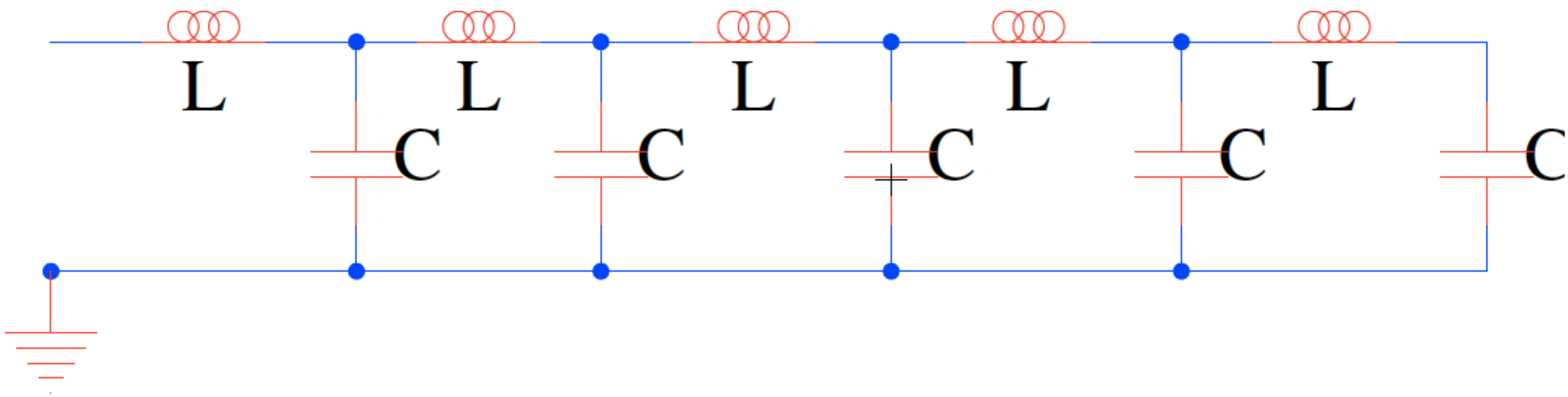
- When resistance is negligible
  - Have LC wire = Lossless Transmission Line
    - No energy dissipation (loss) through R's
  - More typical of printed circuit board (PCB) wires and bond wires



Build Intuition from LC (Preclass 2)

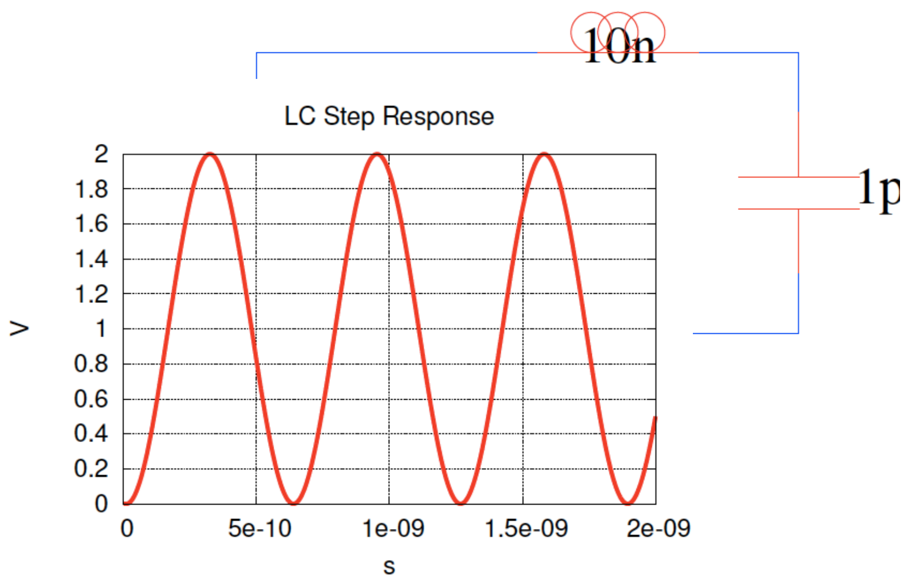
- What did one LC do?
- What will chain do?

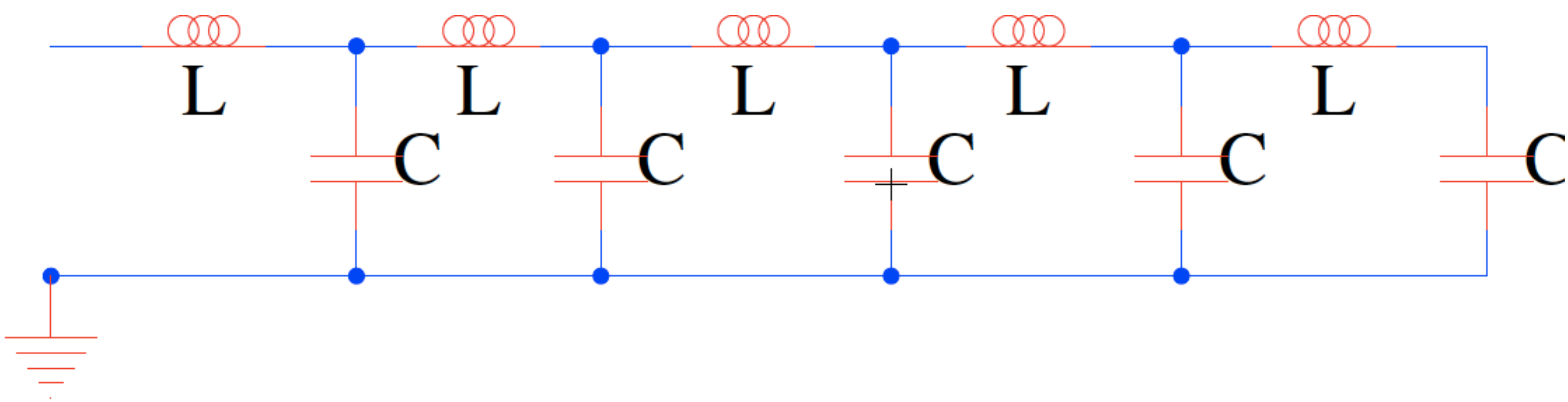




Build Intuition from LC (Preclass 2)

- What did one LC do?
- What will chain do?

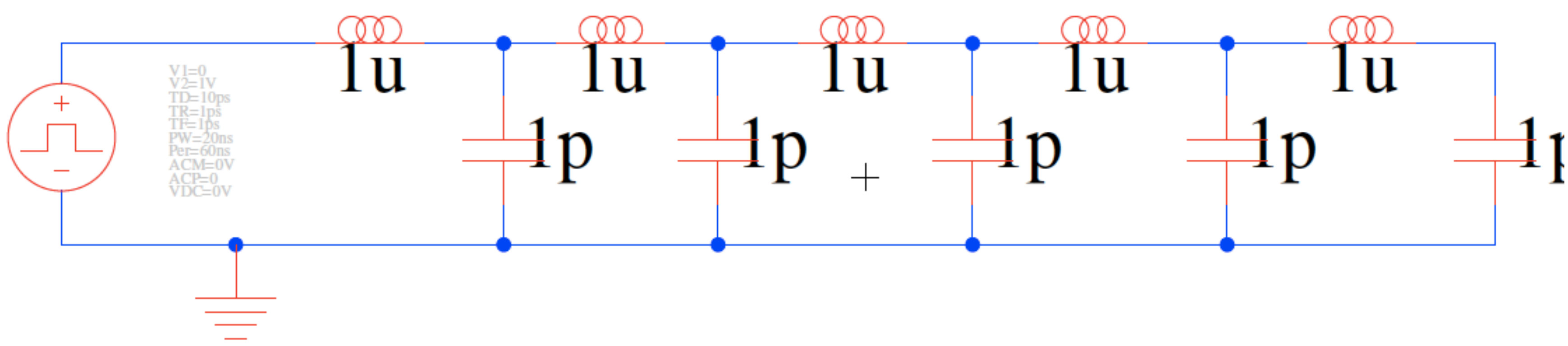




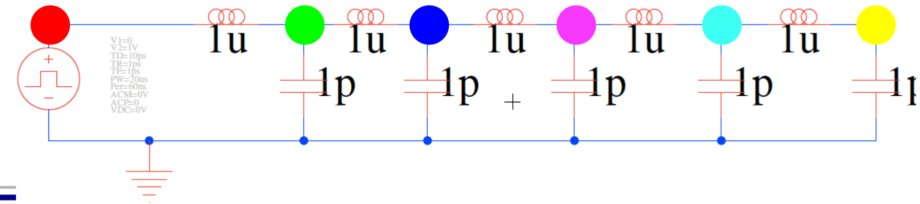
Intuitive: Lossless

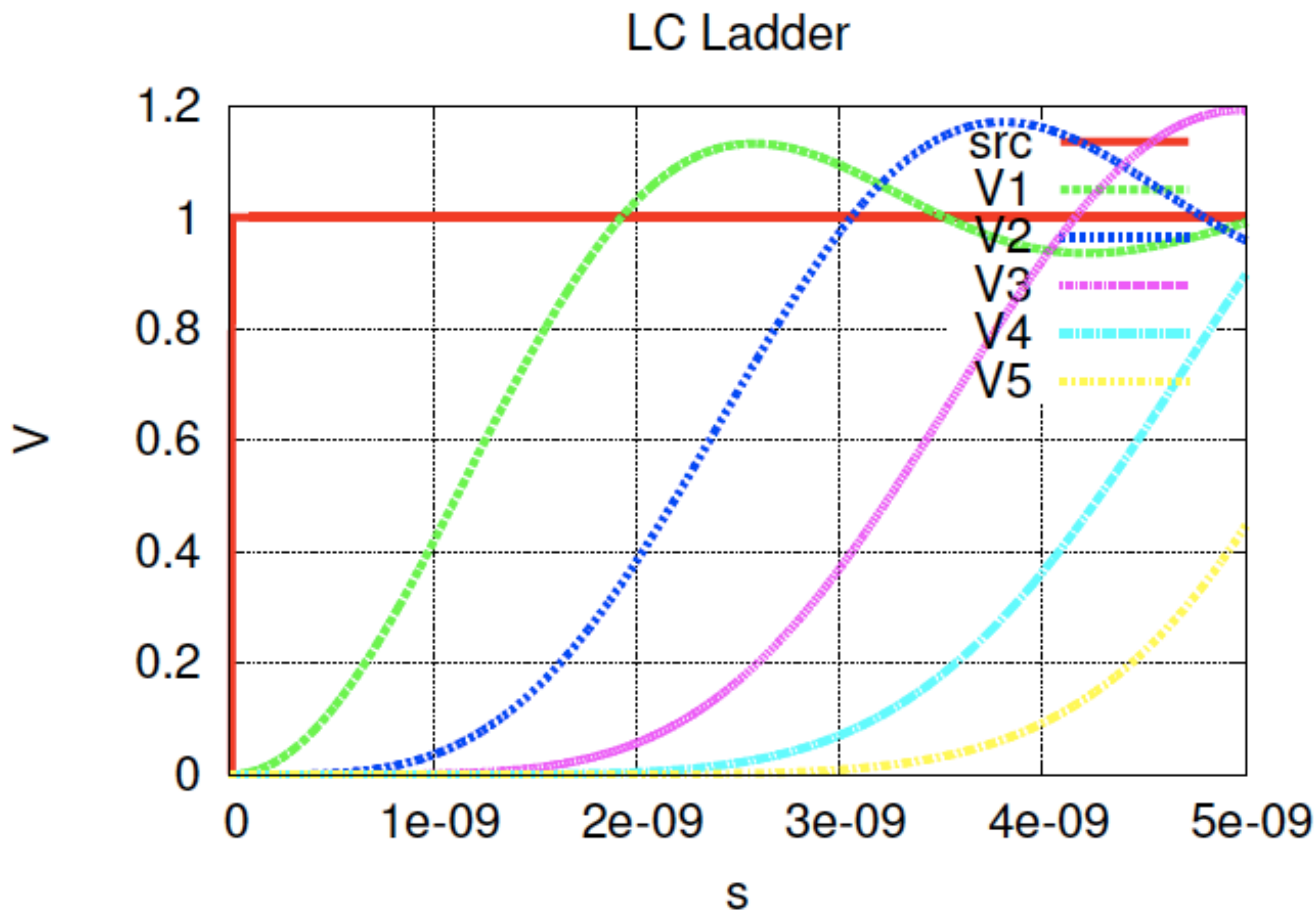
- Pulses travel as waves without distortion
  - (up to a characteristic frequency)

SPICE Simulation

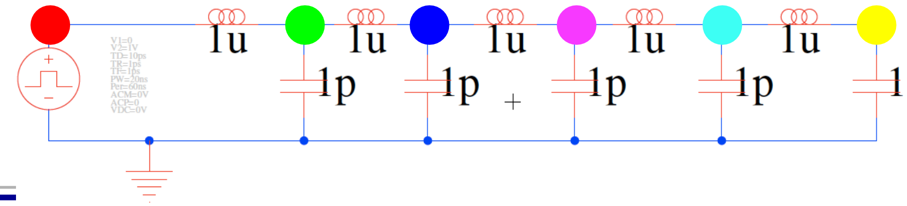


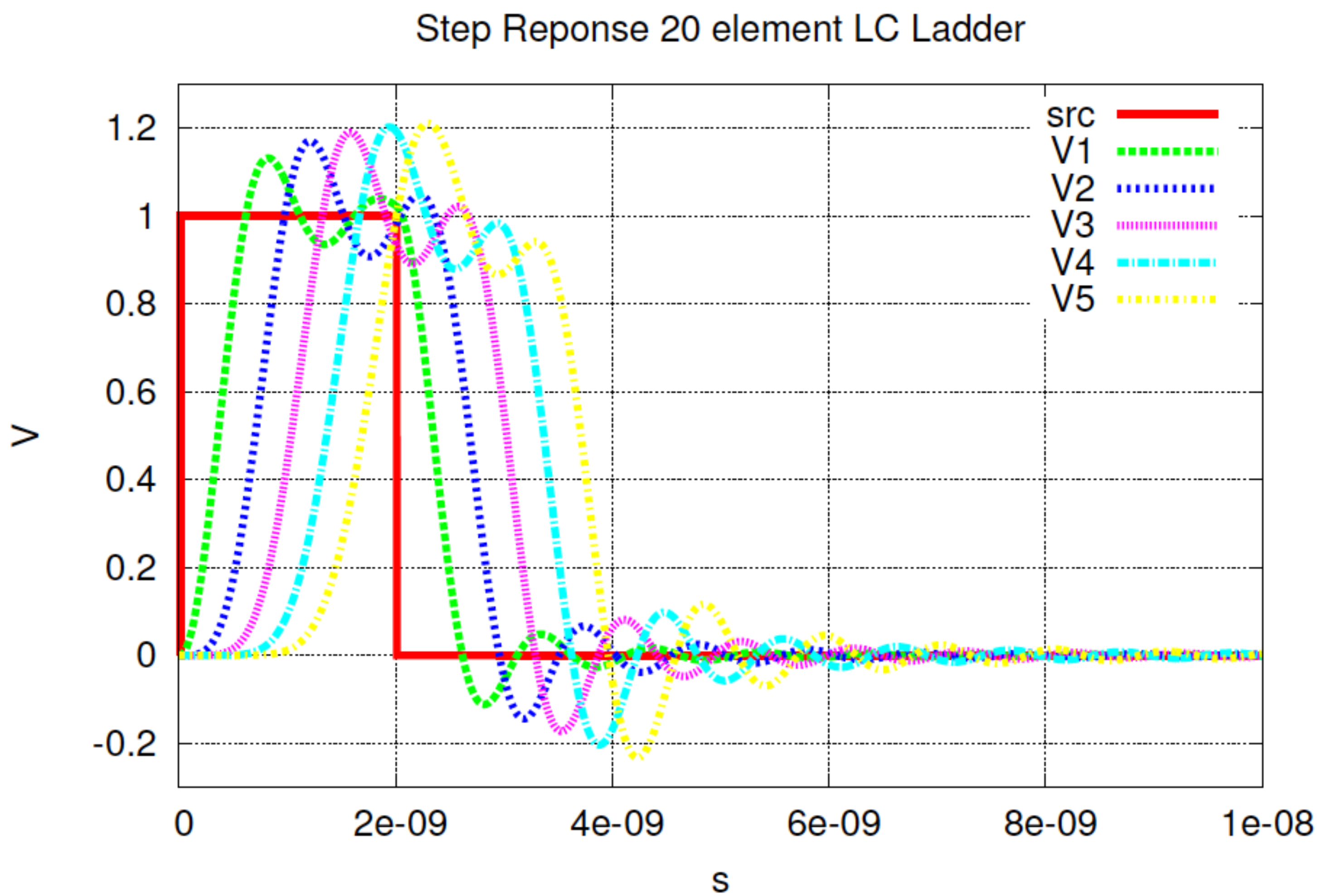
Step Response SPICE



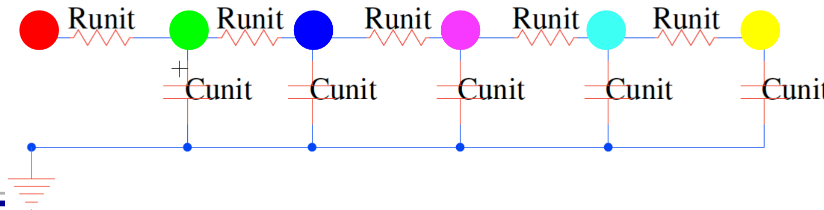


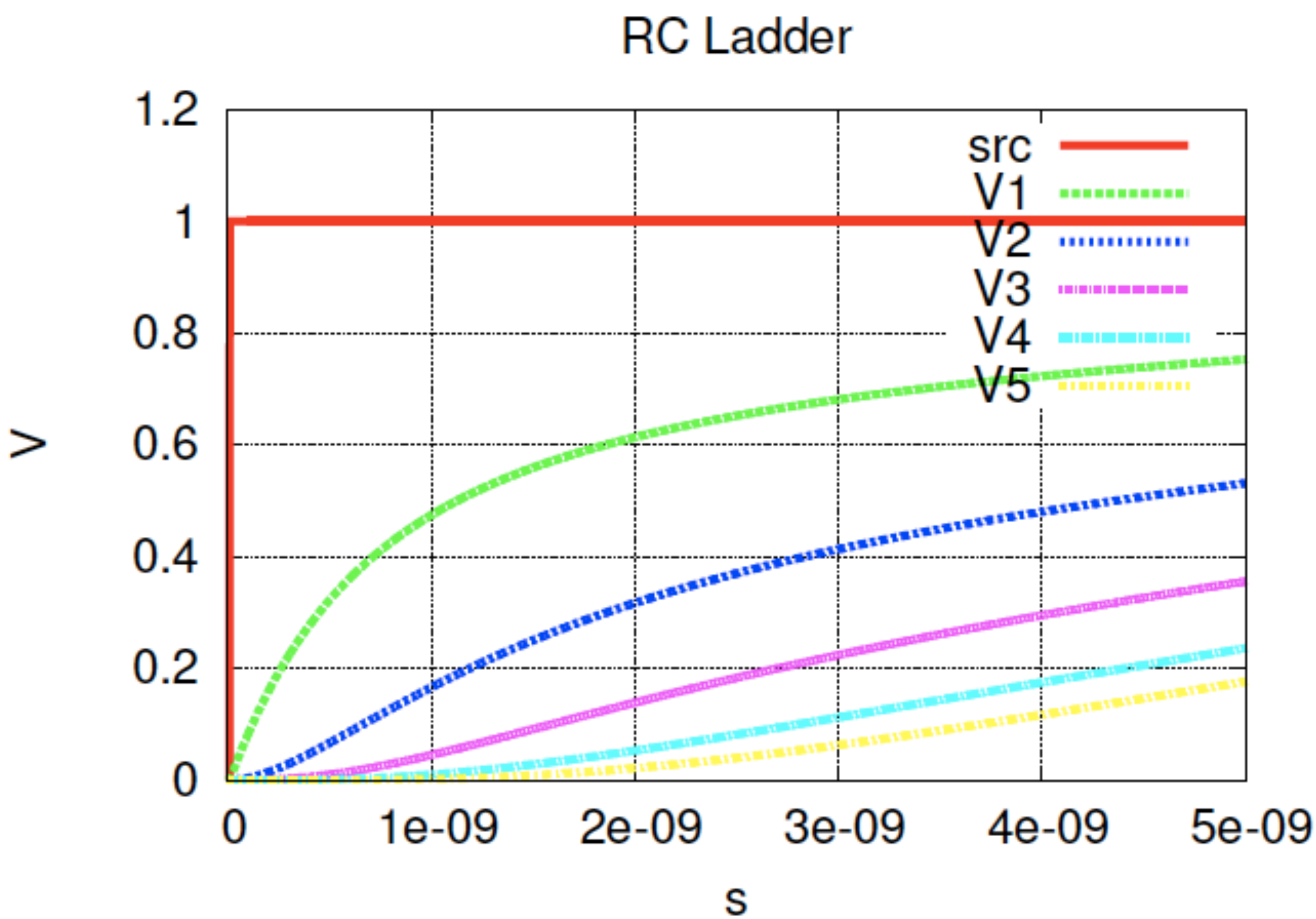
Pulse Response SPICE



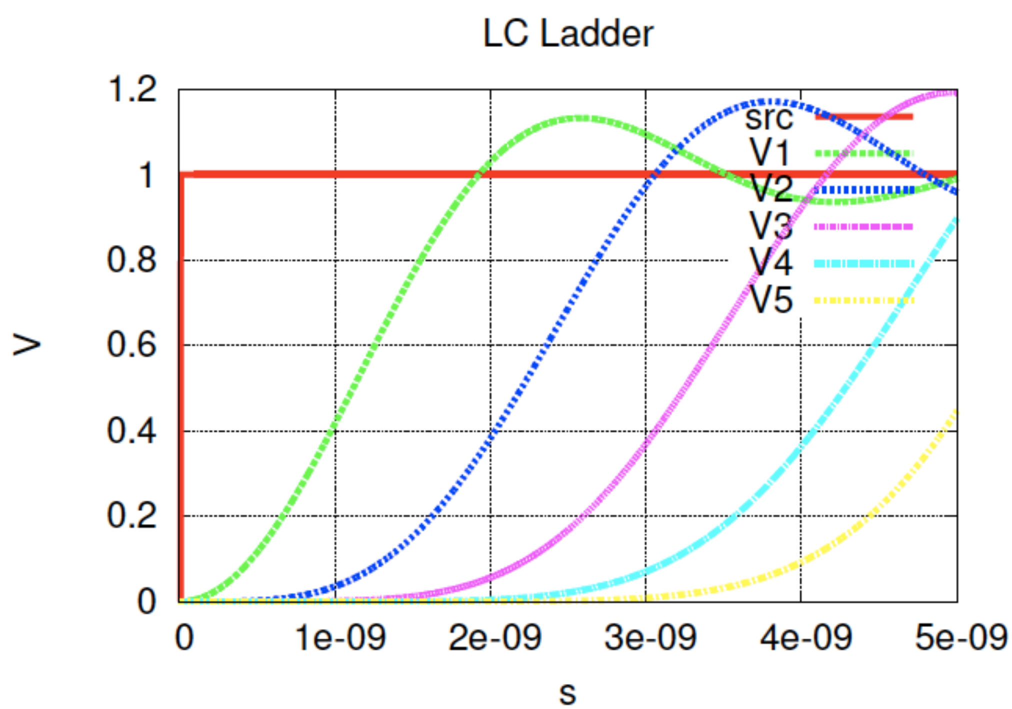


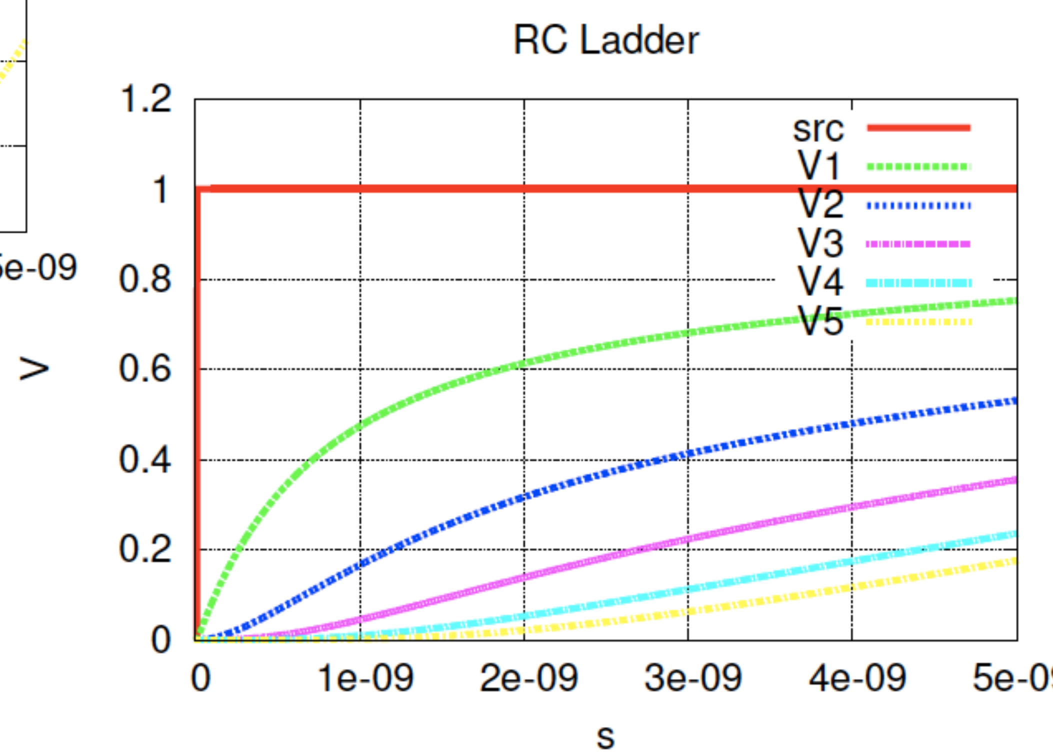
Contrast RC Wire





Contrast


LC Ladder
1.2
1
0.8
0.6
0.4
0.2
0
V
0
1e-09
2e-09
3e-09
4e-09
5e-09
s
src
V1
V2
V3
V4
V5

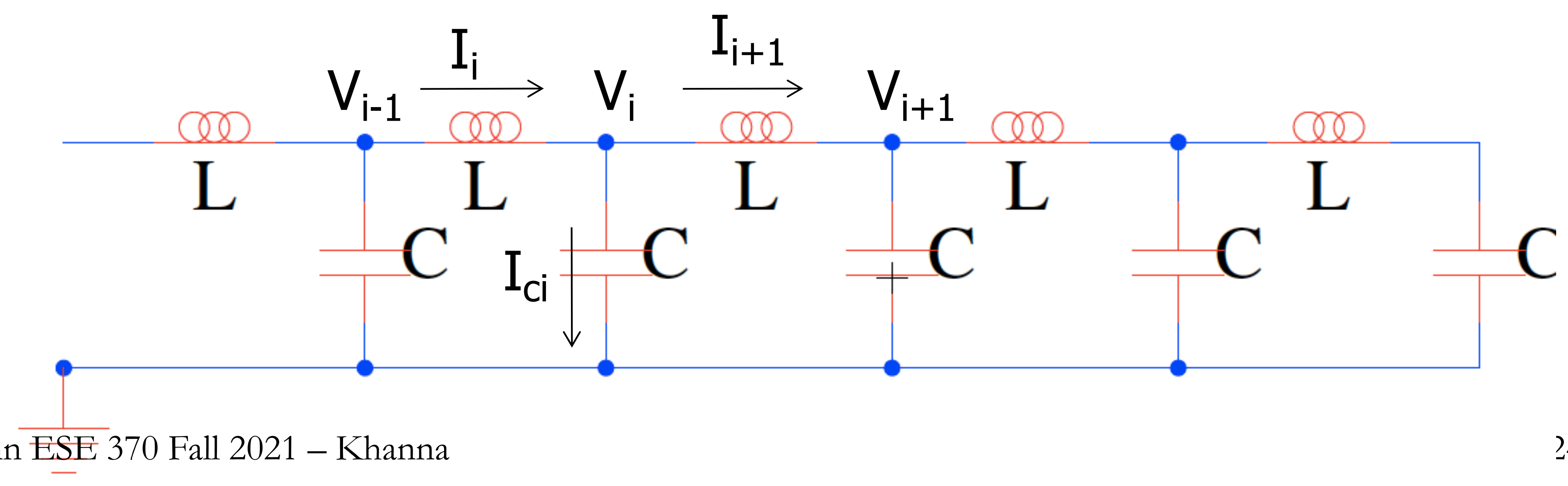

RC Ladder
1.2
1
0.8
0.6
0.4
0.2
0
V
0
1e-09
2e-09
3e-09
4e-09
s
src
V1
V2
V3
V4
V5

Model

- Now voltage is a function of time **and** position
  - Position along wire – distance from source
- Want to get V(x,t)
  - And I(x,t)

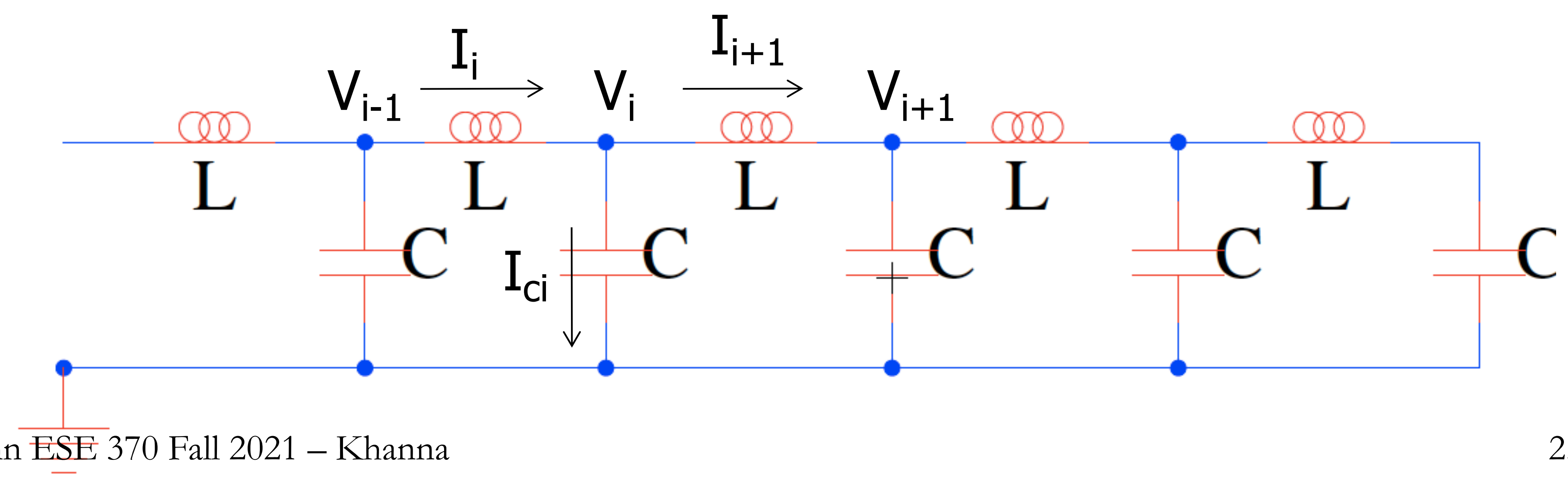
Setup Relations (Preclass 4)

- i is a node, x is the distance from source, Δx is distance between nodes
- Position along wire: $x = i \times \Delta x$
- So $V_i(t) = V(x = i\Delta x, t)$



Setup Relations (Preclass 4)

- a) KCL @ V_i: I_i-I_{i+1}=
- b) I_{ci}=
- d) V_i-V_{i-1} =



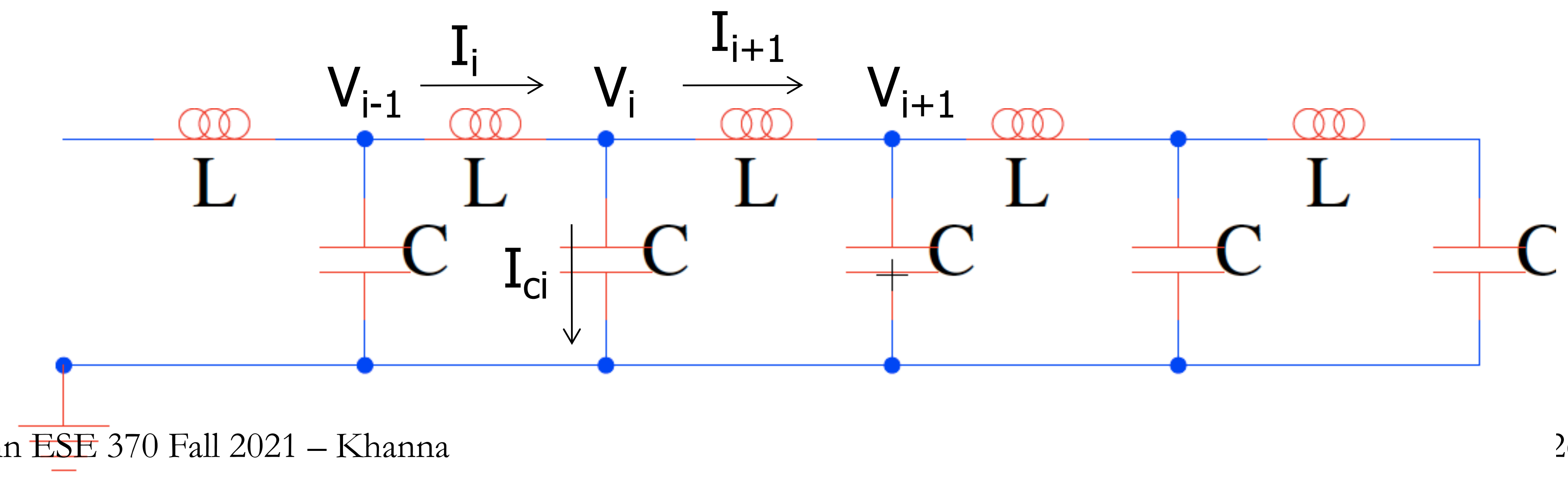
Setup Relations (Preclass 4)

- a) KCL @ V_i: I_i-I_{i+1}=
- b) I_{ci}=
- d) V_i-V_{i-1} =

$$I_i - I_{i+1} = C\frac{dV_i}{dt}$$

$$V_i - V_{i-1} = -L\frac{dI_i}{dt}$$

- *i* is a node, but has spatial dimension along the Tline
  - No longer agnostic of wire length
- V_i changes at different positions



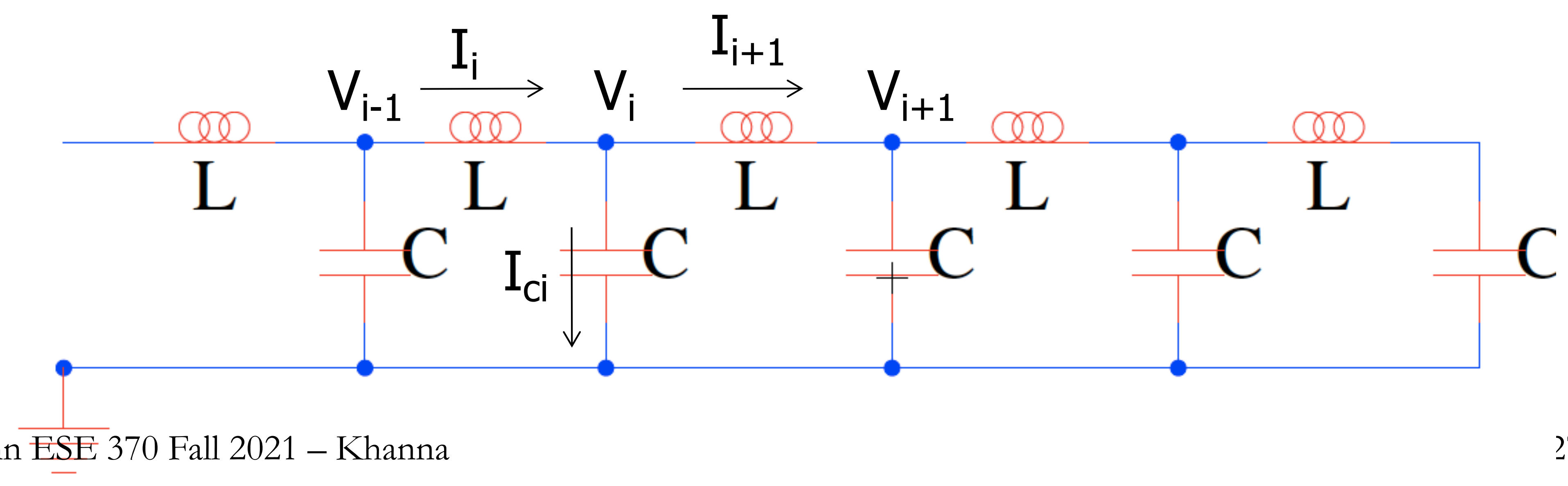
Setup Relations (Preclass 4)

- a) KCL @ V_i: I_i-I_{i+1}=
- b) I_{ci}=
- d) V_i-V_{i-1} =

$$I_i - I_{i+1} = C\frac{dV_i}{dt} \Rightarrow -\frac{dI_i}{dx} = C\frac{dV_i}{dt}$$

$$V_i - V_{i-1} = -L\frac{dI_i}{dt} \Rightarrow \frac{dV_i}{dx} = -L\frac{dI_i}{dt}$$

- *i* is a node, but has spatial dimension along the Tline
  - No longer agnostic of wire length
- V_i changes at different positions



Reduce to Single Equation

- Eliminate I? Differential equation in voltage only.

$$I_i - I_{i+1} = C\frac{dV_i}{dt}$$

- Take derivative with respect to time

$$\frac{dI_i}{dt} - \frac{dI_{i+1}}{dt} = C\frac{d^2V_i}{dt^2}$$

Reduce to Single Equation

$$\frac{dI_i}{dt} - \frac{dI_{i+1}}{dt} = C\frac{d^2V_i}{dt^2}$$

$$\frac{dV_i}{dx} = -L\frac{dI_i}{dt}$$

- Eliminate Is ?

Reduce to Single Equation

$$\frac{dI_i}{dt} - \frac{dI_{i+1}}{dt} = C\frac{d^2V_i}{dt^2}$$

$$\frac{dV_i}{dx} = -L\frac{dI_i}{dt}$$

$$\frac{dV_{i+1}}{dx} = -L\frac{dI_{i+1}}{dt}$$

- Eliminate Is ?

Reduce to Single Equation

$$\frac{dI_i}{dt} - \frac{dI_{i+1}}{dt} = C\frac{d^2V_i}{dt^2}$$

$$\frac{dV_i}{dx} = -L\frac{dI_i}{dt}$$

$$\frac{dV_{i+1}}{dx} = -L\frac{dI_{i+1}}{dt}$$

- Eliminate Is ?

$$\frac{dI_i}{dt} - \frac{dI_{i+1}}{dt} = C\frac{d^2V_i}{dt^2}$$

$$-\frac{1}{L}\frac{dV_i}{dx} - \left(-\frac{1}{L}\frac{dV_{i+1}}{dx}\right) = C\frac{d^2V_i}{dt^2}$$

$$\frac{dV_{i+1}}{dx} - \frac{dV_i}{dx} = LC\frac{d^2V_i}{dt^2}$$

$$\frac{d^2V_i}{dx^2} = LC\frac{d^2V_i}{dt^2}$$

Implication

- Wave equation:

$$\frac{\partial^2 V}{\partial x^2} = LC\frac{\partial^2 V}{\partial t^2}$$

- Solution:

$$V(x,t) = A + Be^{x-wt}$$

- What is w?

Implication

- Wave equation:

$$\frac{\partial^2 V}{\partial x^2} = LC\frac{\partial^2 V}{\partial t^2}$$

- Solution:

$$V(x,t) = A + Be^{x-wt}$$

- What is w?

$$Be^{x-wt} = LCw^2 Be^{x-wt}$$

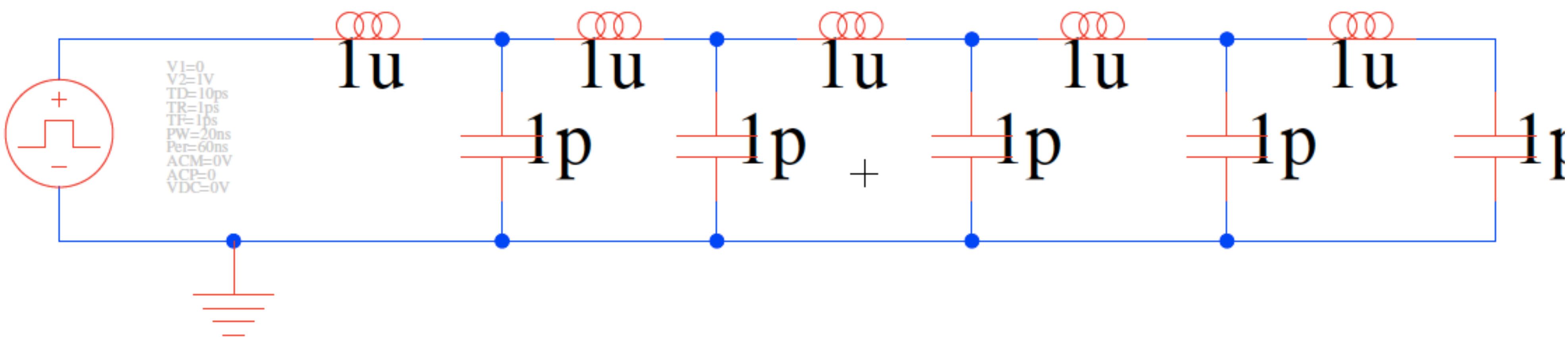
$$w = \sqrt{\frac{1}{LC}}$$

w is the rate of propagation

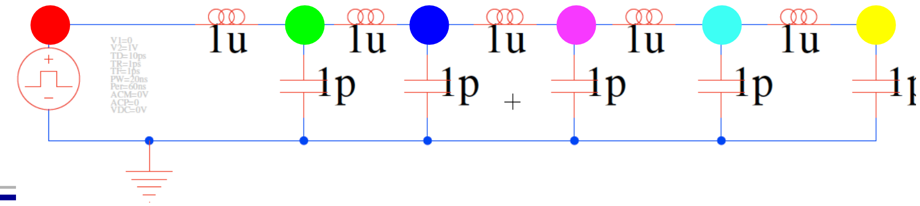
Propagation Rate in Example

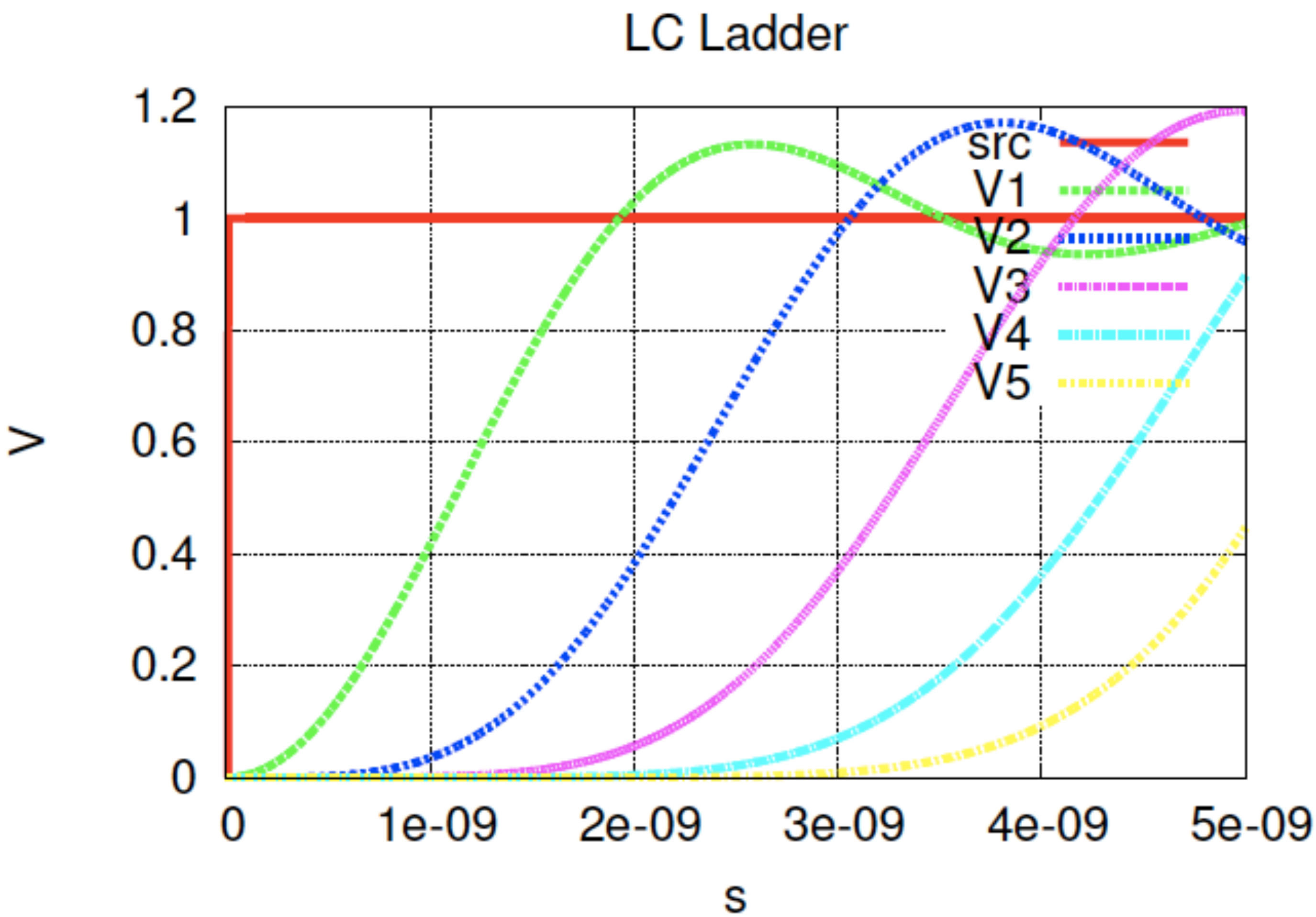
- L=1uH
- C=1pF
- What is w?

$$w = \frac{1}{\sqrt{LC}}$$

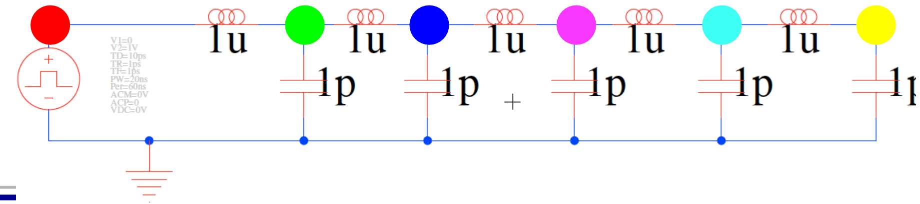


Signal Propagation

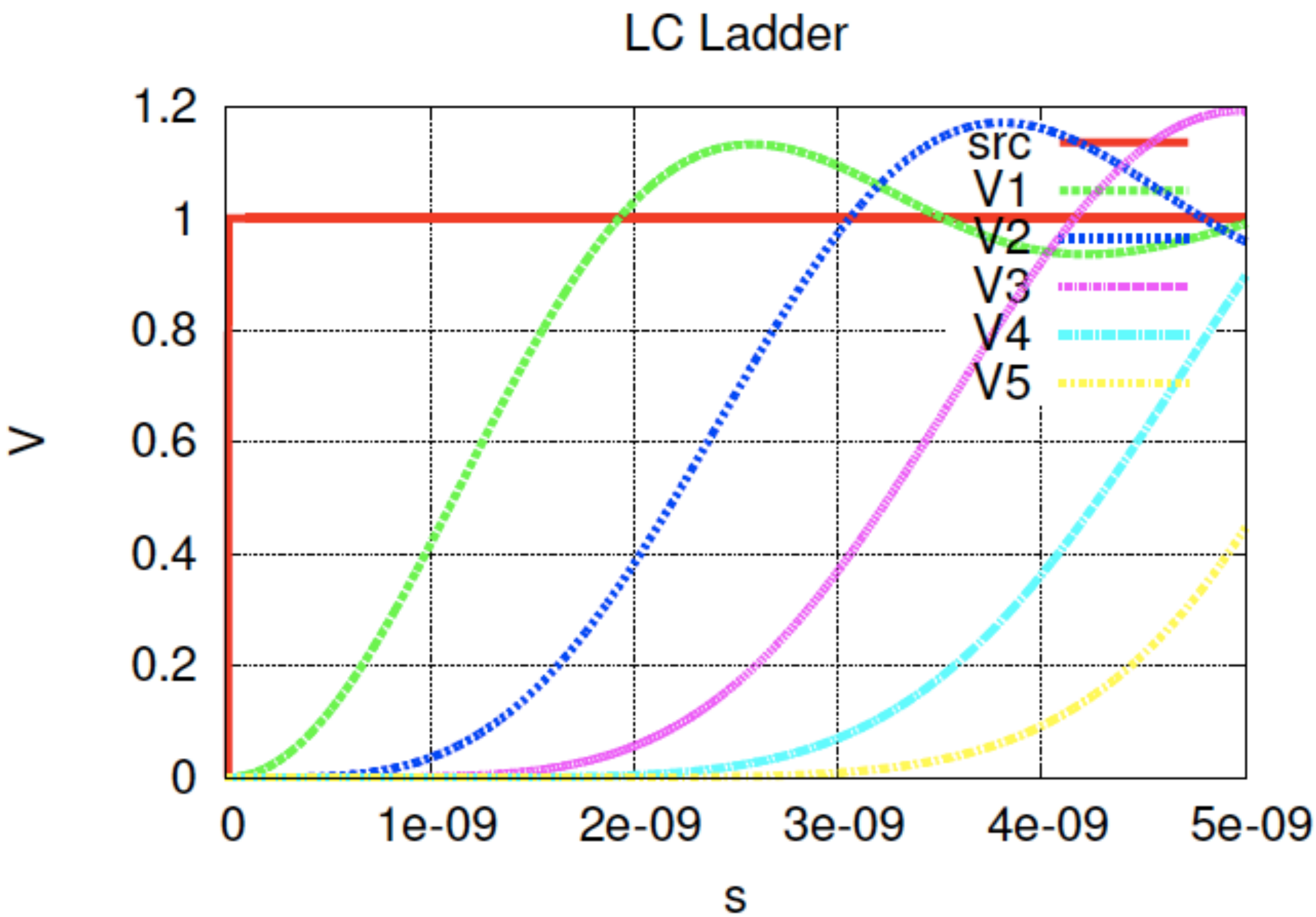




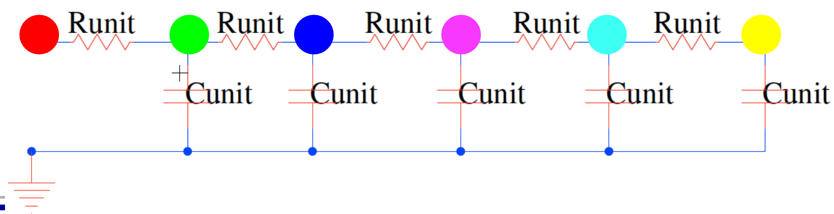
Signal Propagation



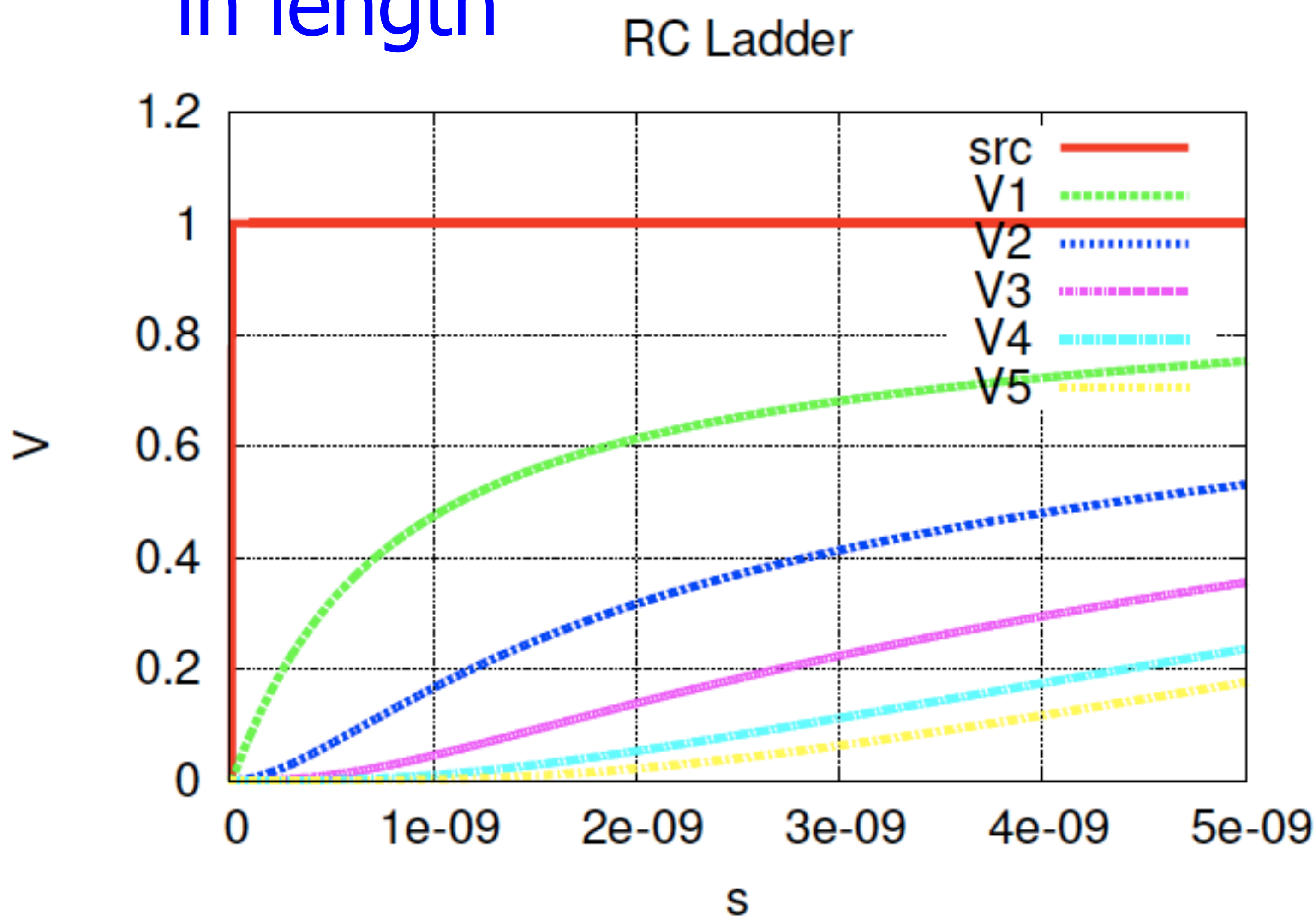
Delay linear in length



Contrast RC Wire



RC wire delay quadratic in length



Propagation

- Solution:

$$V(x,t) = A + Be^{x-wt}$$

- Rate of propagation, w:

$$w = \sqrt{\frac{1}{LC}}$$

- Previously:

$$CL = \varepsilon\mu$$

Propagation

- Solution:

$$V(x,t) = A + Be^{x-wt}$$

- Rate of propagation, w:

$$w = \sqrt{\frac{1}{LC}}$$

- Previously:

$$CL = \varepsilon\mu$$

$$w = \sqrt{\frac{1}{\varepsilon\mu}} = \sqrt{\frac{1}{\varepsilon_0\varepsilon_r\mu_0\mu_r}}$$

$$c^2 = \frac{1}{\varepsilon_0\mu_0} \Rightarrow$$

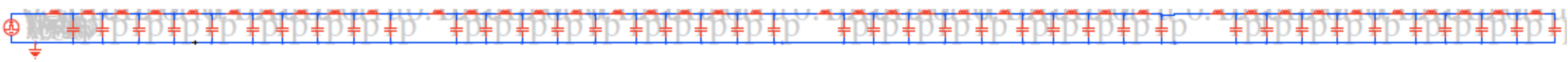
$$w = \frac{c}{\sqrt{\varepsilon_r\mu_r}}$$

Longer LC (open)

- 40 Stages
- L=100nH
- C=1pF

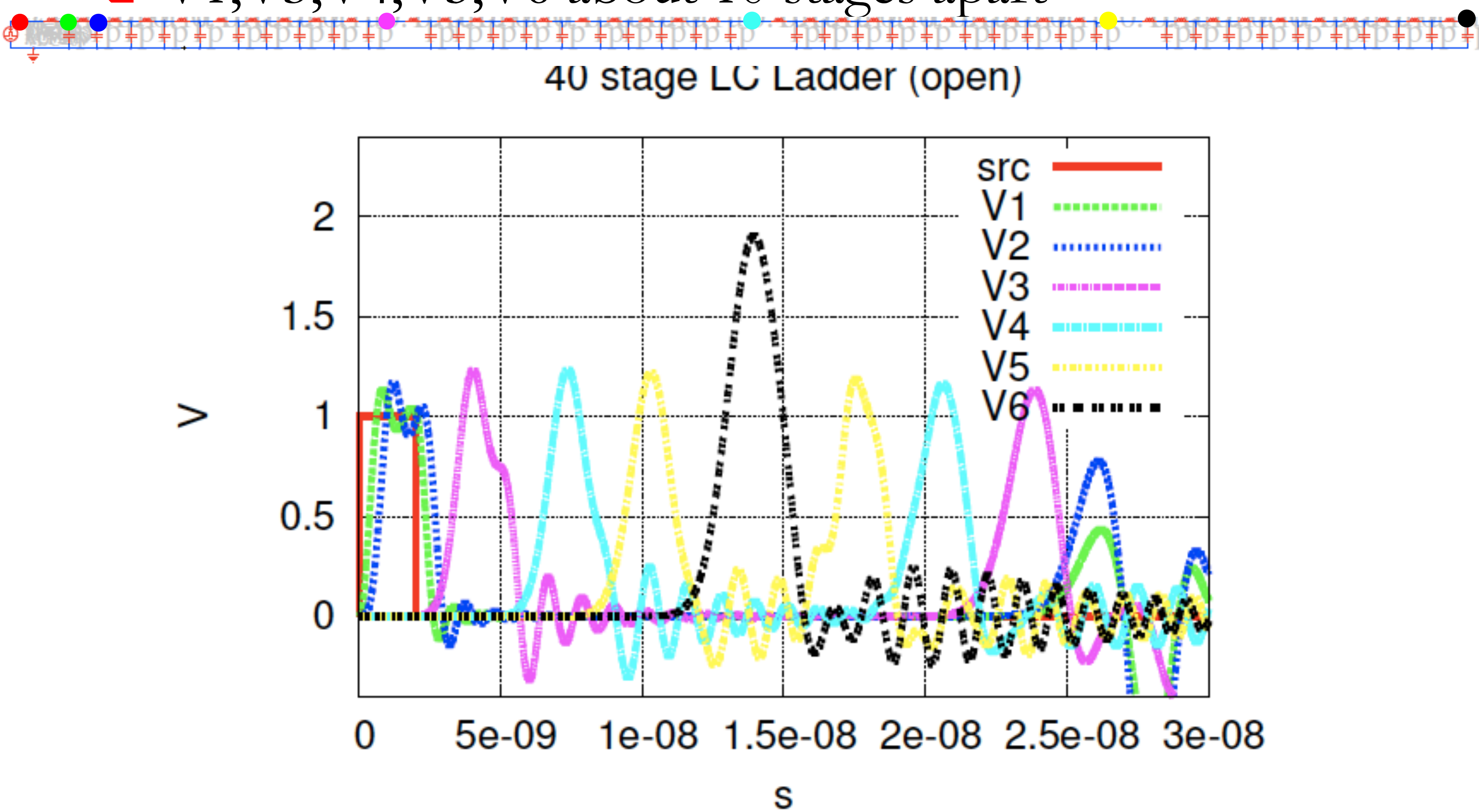
$$w = \frac{1}{\sqrt{LC}} = \frac{c_0}{\sqrt{\varepsilon_r \mu_r}}$$

Stage delay? How long to propagate?



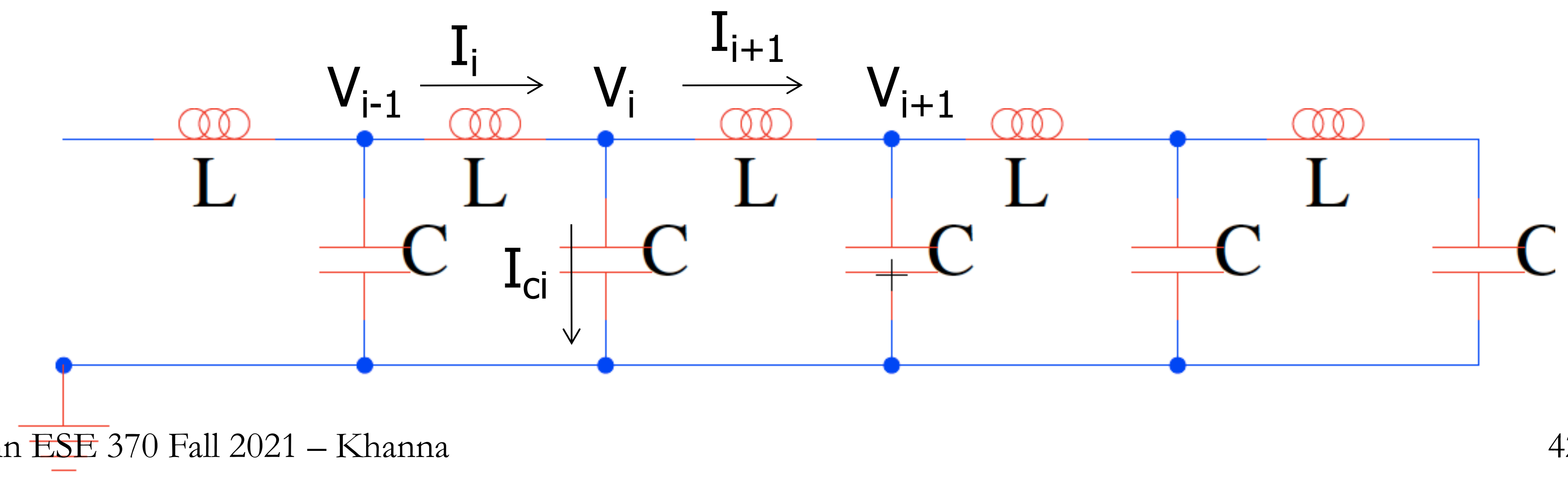
Reflection Teaser: Pulse Travel RC

- V1,V3,V4,V5,V6 about 10 stages apart



Wire "Resistance"

- What is the resistance at V_i ?
  - Q needed to charge C to V_i?
  - I_i given velocity w?
  - $R = V_i/I_i$?

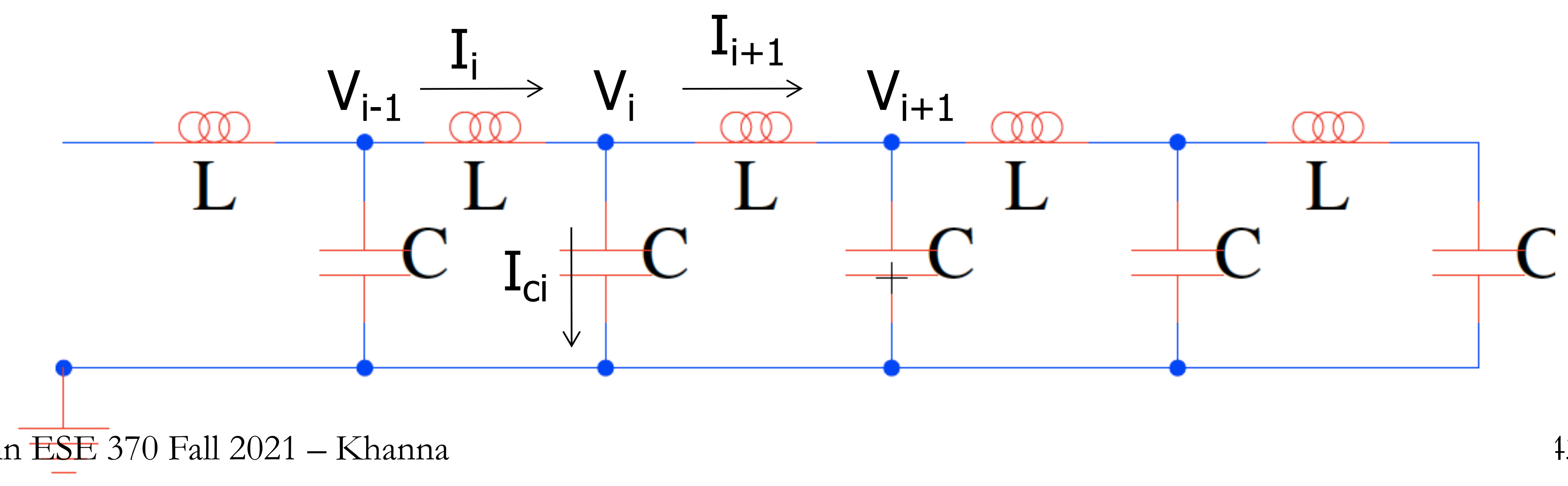


Wire "Resistance"

- $Q=CV_i$
- $I_i = dQ/dt$
- Moving at rate w
- $I_i=wCV_i$
- $R=V_i/I_i=1/(wC)$

$$w = \frac{1}{\sqrt{LC}}$$

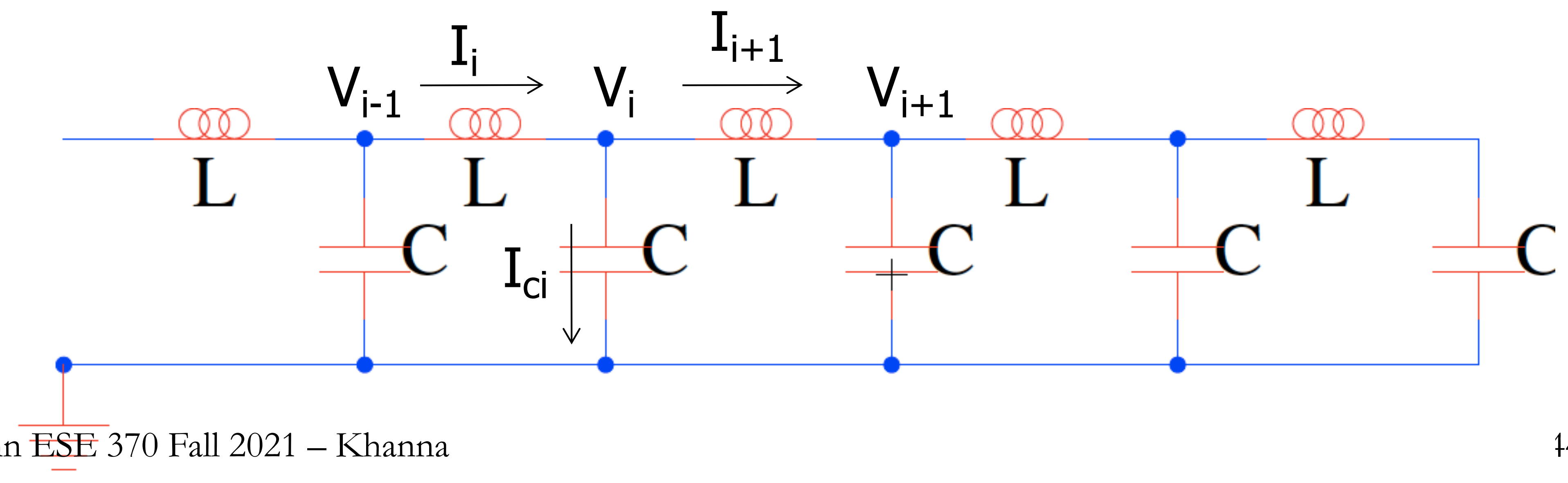
$$R = \frac{\sqrt{LC}}{C}$$



Characteristic Impedance

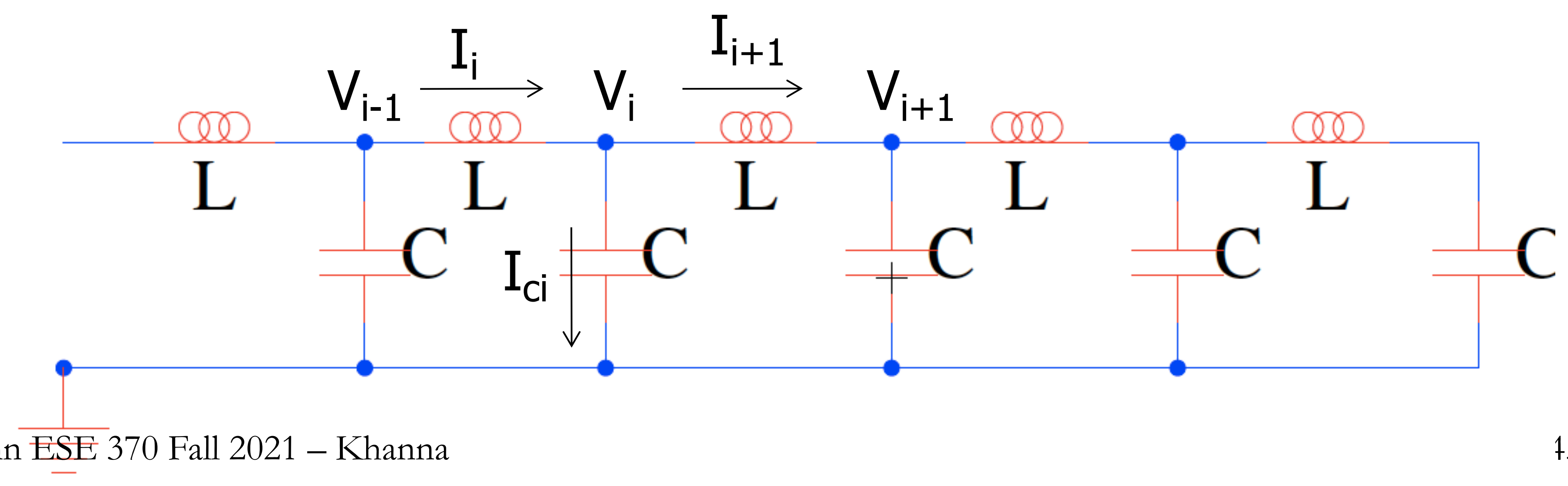
- $Z_0 = R$

$$R = \frac{\sqrt{LC}}{C} = \sqrt{\frac{L}{C}} = Z_0$$



Impedance

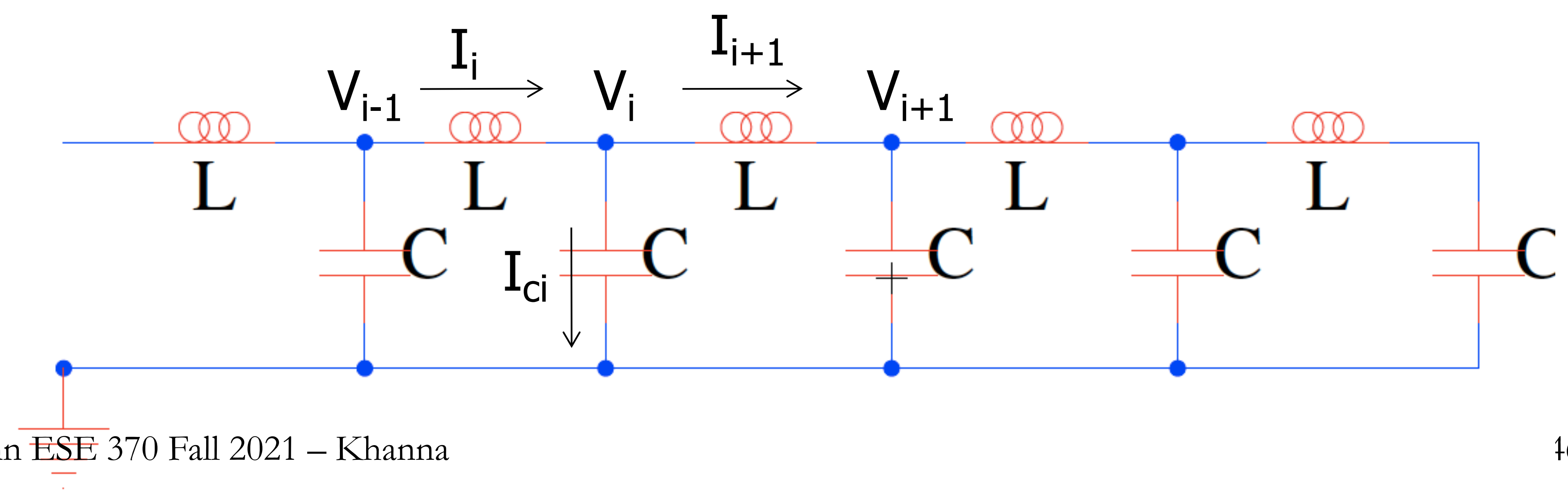
- Assuming infinitely long wire, how does the impedance look at V_i, V_{i+1}, V_{i+2}?



Impedance

- Assuming infinitely long wire, how does the impedance look at V_i, V_{i+1}, V_{i+2}?

$$Z_0 = \sqrt{\frac{L}{C}}$$



Impedance

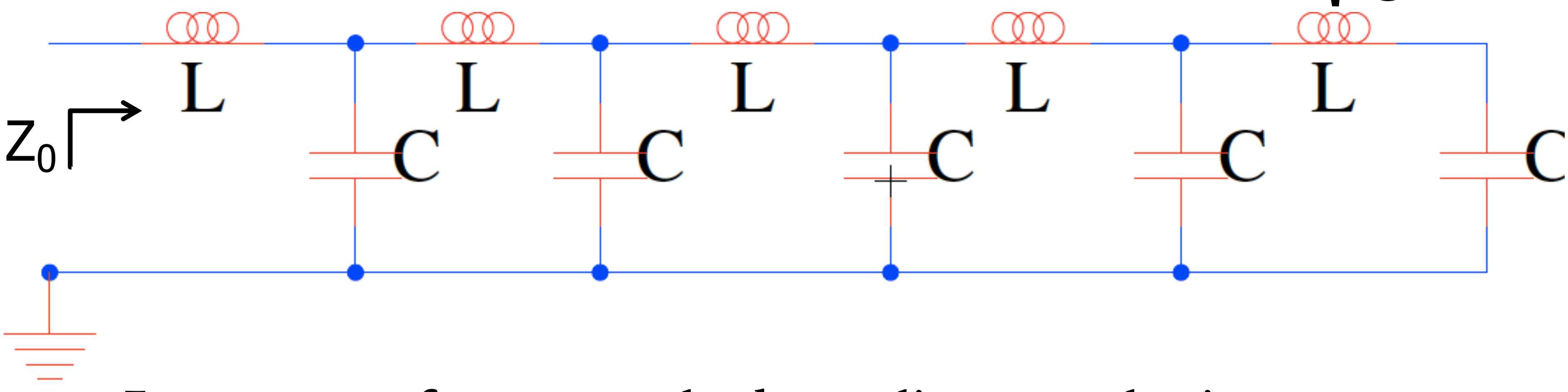
- Transmission lines have a characteristic impedance

$$Z_0 = \sqrt{\frac{L}{C}}$$

Infinite Lossless Transmission Line

- Transmission line looks like resistive load

$$Z_0 = \sqrt{\frac{L}{C}}$$



- Input waveform travels down line at velocity
  - Without distortion

$$w = \frac{1}{\sqrt{LC}}$$

Idea

- Signal propagates as wave down transmission line
  - Delay linear in wire length, if resistance negligible
  - Rate of propagation
  - Characteristic impedance

$$w = \frac{1}{\sqrt{LC}} = \frac{c}{\sqrt{\varepsilon_r \mu_r}}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

Admin

- Project 2 due Friday 12/3
- HW 7 out on Friday
  - Due 12/10