ESE370: Circuit-Level Modeling, Design, and Optimization for Digital Systems

Lec 35: December 8, 2021

Repeaters in Wiring

Previously

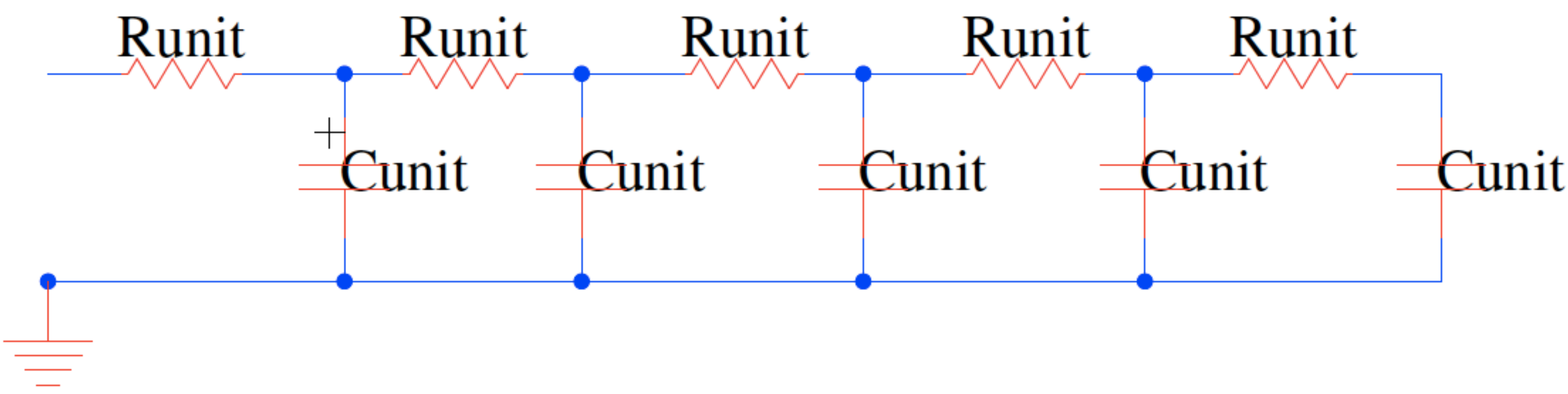
- Transmission line (LC wire) wire delay scales linearly with length
- (Unbuffered) RC wire delay scales quadratically with length

Reminder: Wire Delay

- Wire N units long:
  - $=R_{unit}*C_{unit}*N^2/2$
- With
  - $R_{unit}=1k\Omega$
  - $C_{unit}=1pF$

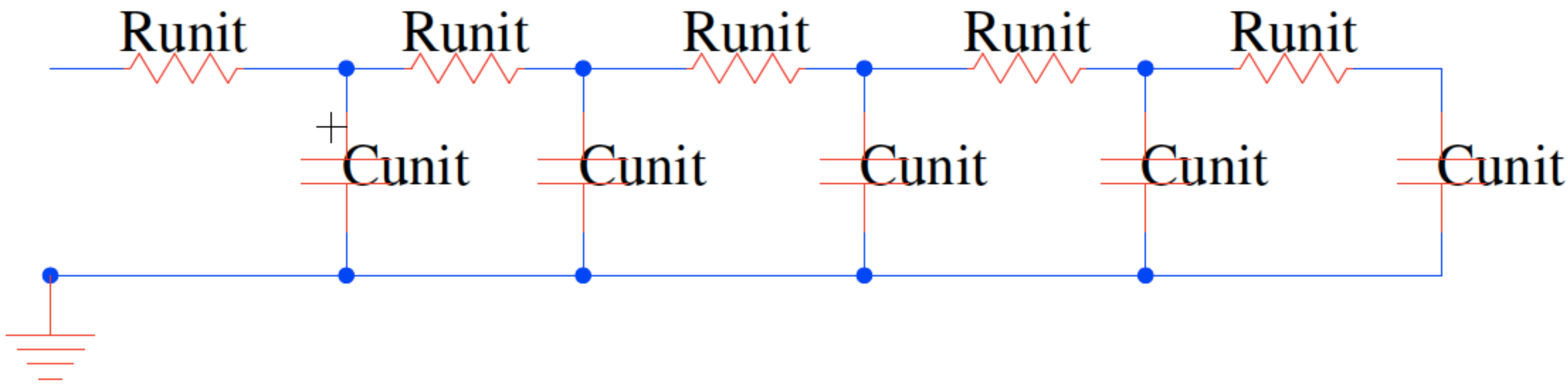
$R_{wire} = N \times R_{unit}$

$C_{wire} = N \times C_{unit}$



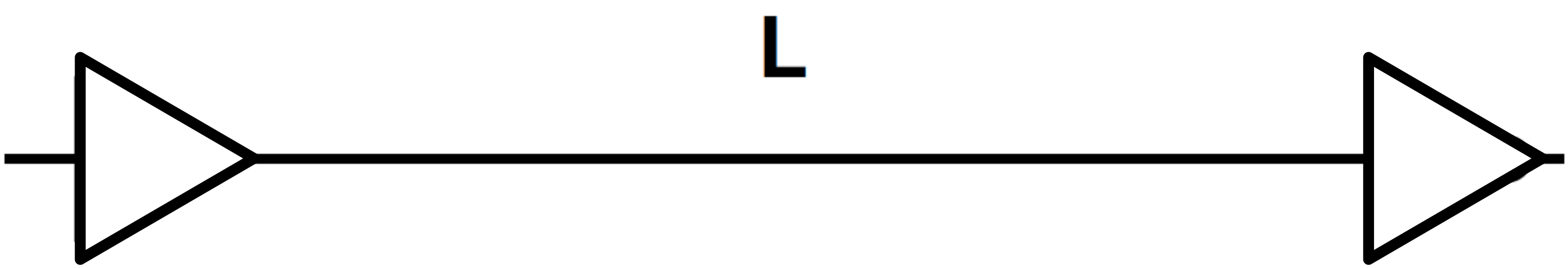
Today: Back to RC Wire

- RC (on-chip) Interconnect Buffering

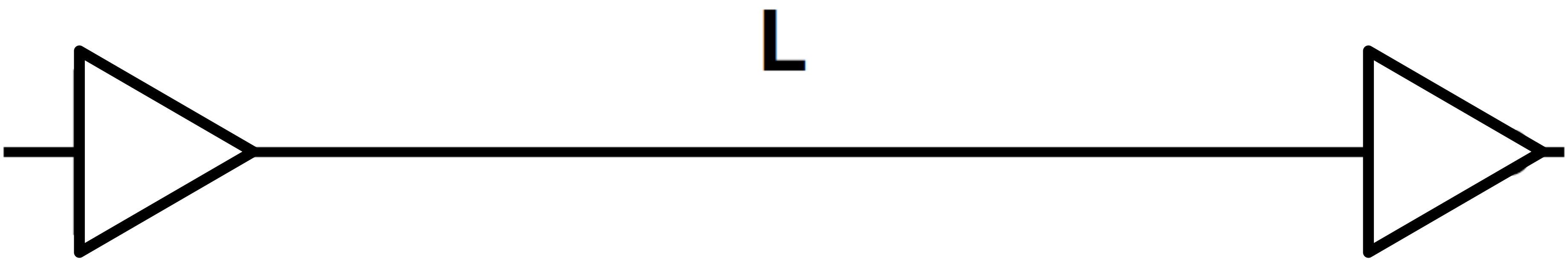


Delay of Wire (preclass 1)

- Long Wire: 1mm
- R_u = 60K Ω per 1mm of wire
- C_u = 0.16 pF per 1mm of wire
- Driven by buffer (R_0=25KΩ, C_0=0.01fF)
  - R_{buf} = 25K Ω
  - C_{self} = 0.02 fF
- Loaded by identical buffer



Formulate Delay (preclass 1)



Delay of buffer driving wire?

Calculate Delay

- $C_{load} = 2\ C_0 = .02fF$
- $R_{buf} = 25K\ \Omega$
- $C_{self} = 0.02fF$
- $C_{wire} = L*C_u = .16pF$
- $R_{wire} = L*R_u = 60K\ \Omega$

$$R_{buf} \times \left(C_{self} + C_{wire} + C_{load}\right) + 0.5 R_{wire} \times C_{wire} + R_{wire} \times C_{load}$$

Calculate Delay

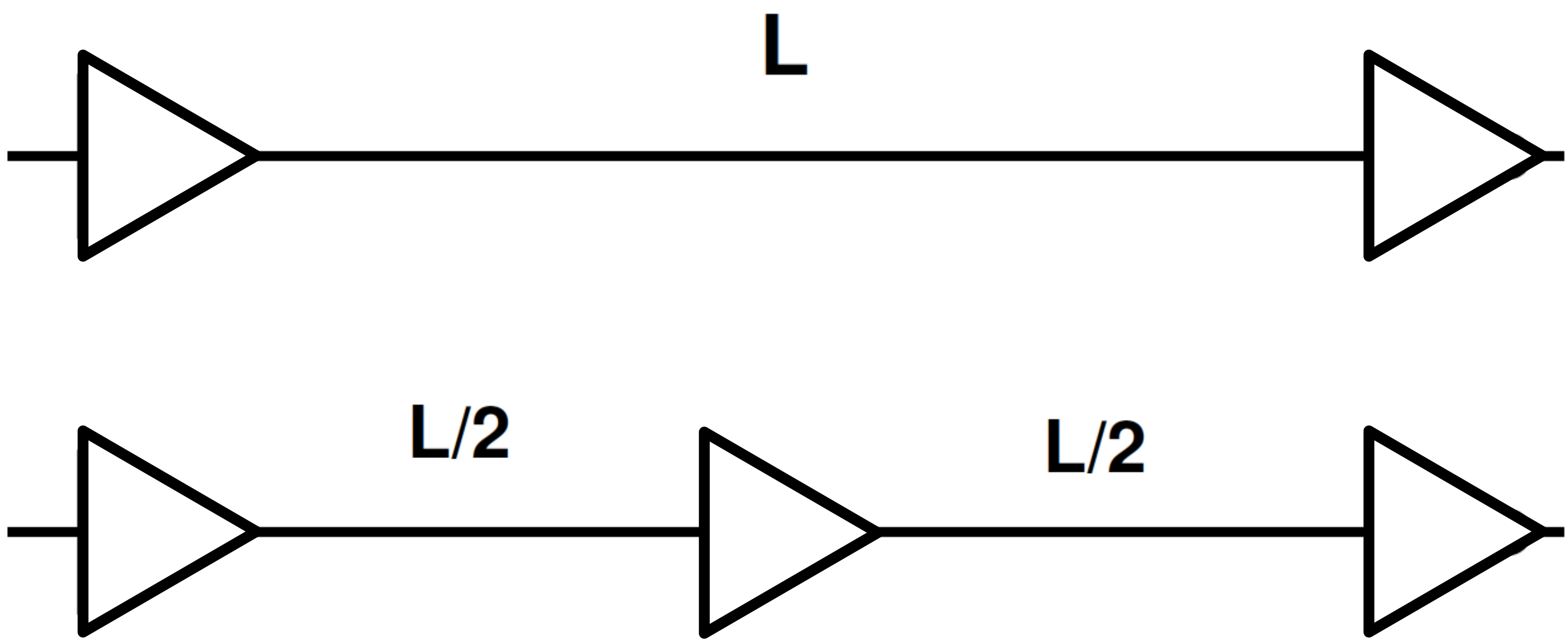
- $C_{load} = 2\ C_0 = .02fF$
- $R_{buf} = 25K\ \Omega$
- $C_{self} = 0.02fF$
- $C_{wire} = L*C_u = .16pF$
- $R_{wire} = L*R_u = 60K\ \Omega$

8.8ns

$$R_{buf} \times \left(C_{self} + C_{wire} + C_{load}\right) + 0.5R_{wire} \times C_{wire} + R_{wire} \times C_{load}$$

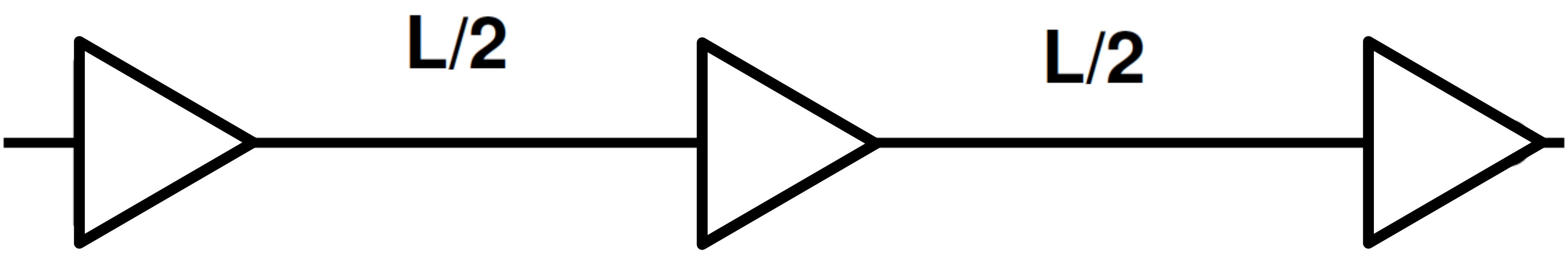
$$4ns + 4.8ns + 1.2ps$$

Buffering Wire



Buffering Wire: L/2

- $C_{load} = 2\ C_0 = .02fF$
- $R_{buf} = 25K\ \Omega$
- $C_{self} = .02fF$
- $C_{wire} = L/2 * C_u = .08pF$
- $R_{wire} = L/2 * R_u = 30K\ \Omega$

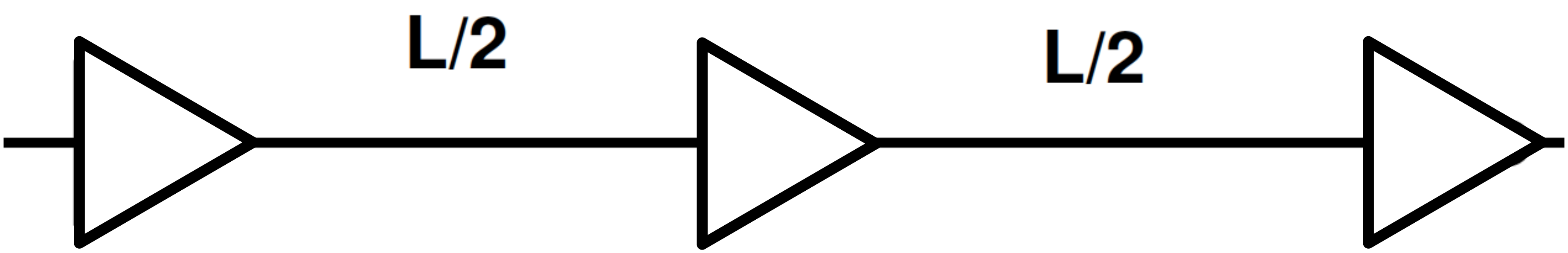


Buffering Wire: L/2

- $C_{load} = 2\ C_0 = .02fF$
- $R_{buf} = 25K\ \Omega$
- $C_{self} = .02fF$
- $C_{wire} = L/2 * C_u = .08pF$
- $R_{wire} = L/2 * R_u = 30K\ \Omega$

$$R_{buf} \times \left(C_{self} + C_{wire} + C_{load}\right) + 0.5 R_{wire} \times C_{wire} + R_{wire} \times C_{load}$$

$$2ns + 1.2ns + .6ps = 3.2ns$$



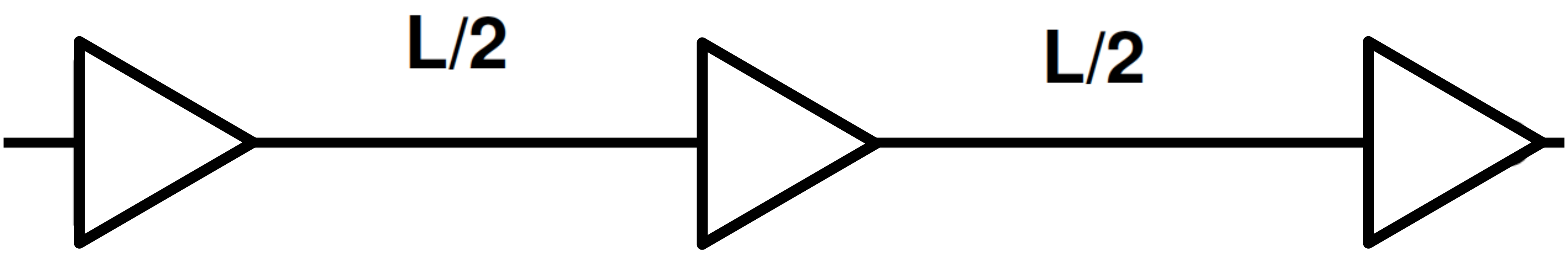
Buffering Wire: L/2

- $C_{load} = 2\ C_0 = .02fF$
- $R_{buf} = 25K\ \Omega$
- $C_{self} = .02fF$
- $C_{wire} = L/2 * C_u = .08pF$
- $R_{wire} = L/2 * R_u = 30K\ \Omega$

6.4ns

$$R_{buf} \times \left(C_{self} + C_{wire} + C_{load}\right) + 0.5 R_{wire} \times C_{wire} + R_{wire} \times C_{load}$$

$$2ns + 1.2ns + .6ps = 3.2ns$$



Buffering Wire: L/N (preclass 2)

Wire of Length	Delay (ns)	Number in 1mm (N)	Total Delay for 1mm (ns)
1 mm	8.8ns	1	8.8ns
0.5mm	3.2ns	2	6.4ns
0.1mm		10	
0.01 mm		100	
0.001 mm		1000	

$$R_{buf} \times \left(C_{self} + C_{wire} + C_{load}\right) + 0.5 R_{wire} \times C_{wire} + R_{wire} \times C_{load}$$

Buffering Wire: L/N (preclass 2)

Wire of Length	Delay (ns)	Number in 1mm (N)	Total Delay for 1mm (ns)
1 mm	8.8ns	1	8.8ns
0.5mm	3.2ns	2	6.4ns
0.1mm	0.45ns	10	4.5ns
0.01 mm	.041ns	100	4.1ns
0.001 mm	.005ns	1000	5ns

$$R_{buf} \times \left(C_{self} + C_{wire} + C_{load}\right) + 0.5 R_{wire} \times C_{wire} + R_{wire} \times C_{load}$$

N Buffers (preclass 3)

$$R_{buf} \times \left(C_{self} + C_{wire} + C_{load}\right) + 0.5R_{wire} \times C_{wire} + R_{wire} \times C_{load}$$

- Delay Equation for N buffers?

N Buffers (preclass 3)

$$R_{buf} \times \left(C_{self} + C_{wire} + C_{load}\right) + 0.5R_{wire} \times C_{wire} + R_{wire} \times C_{load}$$

- Delay Equation for N buffers?

$$N\left(R_{buf}\left(C_{self} + \frac{C_{wire}}{N} + C_{load}\right) + 0.5\left(\frac{R_{wire}}{N} \cdot \frac{C_{wire}}{N}\right) + \frac{R_{wire}}{N} \cdot C_{load}\right)$$

N Buffers

- Delay Equation for N buffers?

$$N\left(R_{buf}\left(C_{self}+\frac{C_{wire}}{N}+C_{load}\right)+0.5\left(\frac{R_{wire}}{N}\cdot\frac{C_{wire}}{N}\right)+\frac{R_{wire}}{N}\cdot C_{load}\right)$$

$$N\cdot R_{buf}\left(C_{self}+\frac{C_{wire}}{N}+C_{load}\right)+N\cdot 0.5\left(\frac{R_{wire}}{N}\cdot\frac{C_{wire}}{N}\right)+N\cdot\frac{R_{wire}}{N}\cdot C_{load}$$

$$N\cdot R_{buf}\left(C_{self}+C_{load}\right)+R_{buf}\times C_{wire}+0.5\left(\frac{1}{N}\right)R_{wire}C_{wire}+R_{wire}C_{load}$$

Minimize Delay (preclass 4)

- Minimize delay?

Minimize Delay (preclass 4)

- Minimize delay
- Derivative with respect to N

$$N \cdot R_{buf}\left(C_{self}+C_{load}\right)+R_{buf}\times C_{wire}+0.5\left(\frac{1}{N}\right)R_{wire}C_{wire}+R_{wire}C_{load}$$

$$0=R_{buf}\left(C_{self}+C_{load}\right)-0.5\left(\frac{1}{N^2}\right)R_{wire}C_{wire}$$

Solve for N (preclass 4)

$$0 = R_{buf}\left(C_{self} + C_{load}\right) - 0.5\left(\frac{1}{N^2}\right)R_{wire}C_{wire}$$

$$R_{buf}\left(C_{self} + C_{load}\right) = 0.5\left(\frac{1}{N^2}\right)R_{wire}C_{wire}$$

$$N^2 = \frac{0.5R_{wire}C_{wire}}{R_{buf}\left(C_{self} + C_{load}\right)}$$

$$N = \sqrt{\frac{0.5R_{wire}C_{wire}}{R_{buf}\left(C_{self} + C_{load}\right)}}$$

Minimize Delay (preclass 4)

Equalizes delays from buffer and wire

$$N = \sqrt{\frac{0.5 R_{wire} C_{wire}}{R_{buf}\left(C_{self} + C_{load}\right)}}$$

$$\boxed{N \cdot R_{buf}\left(C_{self} + C_{load}\right)} + R_{buf} C_{wire} + \boxed{0.5\left(\frac{1}{N}\right) R_{wire} C_{wire}} + R_{wire} C_{load}$$

Delay with Optimal N (preclass 4)

$$N = \sqrt{\frac{0.5 R_{wire} C_{wire}}{R_{buf}\left(C_{self} + C_{load}\right)}}$$

$$N \cdot R_{buf}\left(C_{self} + C_{load}\right) + R_{buf} C_{wire} + 0.5\left(\frac{1}{N}\right) R_{wire} C_{wire} + R_{wire} C_{load}$$

Delay with Optimal N

$$N = \sqrt{\frac{0.5 R_{wire} C_{wire}}{R_{buf}\left(C_{self} + C_{load}\right)}}$$

$$N \cdot R_{buf}\left(C_{self} + C_{load}\right) + R_{buf} C_{wire} + 0.5\left(\frac{1}{N}\right) R_{wire} C_{wire} + R_{wire} C_{load}$$

$$\sqrt{\frac{0.5 R_{wire} C_{wire}}{R_{buf}\left(C_{self} + C_{load}\right)}} \cdot R_{buf}\left(C_{self} + C_{load}\right) + R_{buf} C_{wire} + 0.5\left(\sqrt{\frac{R_{buf}\left(C_{self} + C_{load}\right)}{0.5 R_{wire} C_{wire}}}\right) R_{wire} C_{wire} + R_{wire} C_{load}$$

Delay with Optimal N

$$N = \sqrt{\frac{0.5R_{wire}C_{wire}}{R_{buf}\left(C_{self}+C_{load}\right)}}$$

$$N \cdot R_{buf}\left(C_{self}+C_{load}\right) + R_{buf}C_{wire} + 0.5\left(\frac{1}{N}\right)R_{wire}C_{wire} + R_{wire}C_{load}$$

$$\sqrt{\frac{0.5R_{wire}C_{wire}}{R_{buf}\left(C_{self}+C_{load}\right)}} \cdot R_{buf}\left(C_{self}+C_{load}\right) + R_{buf}C_{wire} + 0.5\left(\sqrt{\frac{R_{buf}\left(C_{self}+C_{load}\right)}{0.5R_{wire}C_{wire}}}\right)R_{wire}C_{wire} + R_{wire}C_{load}$$

$$\sqrt{0.5R_{wire}C_{wire}\left(R_{buf}\left(C_{self}+C_{load}\right)\right)} + R_{buf}C_{wire} + \sqrt{0.5R_{wire}C_{wire}\left(R_{buf}\left(C_{self}+C_{load}\right)\right)} + R_{wire}C_{load}$$

Delay with Optimal N

$$N = \sqrt{\frac{0.5R_{wire}C_{wire}}{R_{buf}\left(C_{self}+C_{load}\right)}}$$

$$N \cdot R_{buf}\left(C_{self}+C_{load}\right)+R_{buf}C_{wire}+0.5\left(\frac{1}{N}\right)R_{wire}C_{wire}+R_{wire}C_{load}$$

$$\sqrt{\frac{0.5R_{wire}C_{wire}}{R_{buf}\left(C_{self}+C_{load}\right)}} \cdot R_{buf}\left(C_{self}+C_{load}\right)+R_{buf}C_{wire}+0.5\left(\sqrt{\frac{R_{buf}\left(C_{self}+C_{load}\right)}{0.5R_{wire}C_{wire}}}\right)R_{wire}C_{wire}+R_{wire}C_{load}$$

$$\sqrt{0.5R_{wire}C_{wire}\left(R_{buf}\left(C_{self}+C_{load}\right)\right)}+R_{buf}C_{wire}+\sqrt{0.5R_{wire}C_{wire}\left(R_{buf}\left(C_{self}+C_{load}\right)\right)}+R_{wire}C_{load}$$

$$2\sqrt{0.5R_{wire}C_{wire}\left(R_{buf}\left(C_{self}+C_{load}\right)\right)}+R_{buf}C_{wire}+R_{wire}C_{load}$$

Calculate: Delay at Optimum Stages (preclass 4)

- R_u = 60K Ω per 1mm of wire
- C_u = 0.16 pF per 1mm of wire
- R_{buf}=25K Ω
- C_{self}=C_{load}=0.02fF

$$N = \sqrt{\frac{0.5R_{wire}C_{wire}}{R_{buf}\left(C_{self}+C_{load}\right)}}$$

$$2\sqrt{0.5R_{wire}C_{wire}\left(R_{buf}\left(C_{self}+C_{load}\right)\right)}+R_{buf}C_{wire}+R_{wire}C_{load}$$

Segment Length (preclass 4)

- $R_{wire} = L \times R_{unit}$
- $C_{wire} = L \times C_{unit}$

$$L^*_{seg} = \frac{L}{N}$$

$$N = \sqrt{0.5\left(\frac{R_{wire} \times C_{wire}}{R_{buf} \times \left(C_{self} + C_{load}\right)}\right)}$$

$$N = L\sqrt{0.5\left(\frac{R_u \times C_u}{R_{buf} \times \left(C_{self} + C_{load}\right)}\right)}$$

$$L^*_{seg} = \frac{L}{N} = \sqrt{2\left(\frac{R_{buf} \times \left(C_{self} + C_{load}\right)}{R_u \times C_u}\right)}$$

Optimal Segment Length

- Delay scales linearly with distance once optimally buffered

$$L_{seg}^{*} = \frac{L}{N} = \sqrt{2\left(\frac{R_{buf} \times \left(C_{self} + C_{load}\right)}{R_u \times C_u}\right)}$$

$$N = L\sqrt{0.5\left(\frac{R_u \times C_u}{R_{buf} \times \left(C_{self} + C_{load}\right)}\right)}$$

Buffer Size? (preclass 5)

- How big should buffer be?
  - $R_{buf} = R_0/W$
  - $C_{self} = 2\ W\ C_{dff} = 2\ W\ \gamma\ C_0$
  - $C_{load} = 2\ W\ C_0$

Buffer Size?

- How big should buffer be?
  - $R_{buf} = R_0/W$
  - $C_{self} = 2\ W\ C_{dff} = 2\ W\ \gamma\ C_0$
  - $C_{load} = 2\ W\ C_0$

$$2\sqrt{0.5R_{wire}C_{wire}\left(R_{buf}\left(C_{self}+C_{load}\right)\right)}+R_{buf}C_{wire}+R_{wire}C_{load}$$

$$2\sqrt{0.5R_{wire}C_{wire}\left(\frac{R_0}{W}\left(2WC_0\left(1+\gamma\right)\right)\right)}+\frac{R_0}{W}C_{wire}+R_{wire}\cdot 2WC_0$$

$$2\sqrt{0.5R_{wire}C_{wire}\left(2R_0C_0\left(1+\gamma\right)\right)}+\frac{R_0}{W}C_{wire}+R_{wire}\cdot 2WC_0$$

Optimal W (preclass 5)

- $R_{wire} = L \times R_{unit}$
- $C_{wire} = L \times C_{unit}$

$$2\sqrt{0.5R_{wire}C_{wire}\left(2R_0C_0\left(1+\gamma\right)\right)}+\frac{R_0}{W}C_{wire}+R_{wire}\cdot 2WC_0$$

$$0 = 2R_{wire}C_0 - R_0C_{wire}\frac{1}{W^2}$$

$$W=\sqrt{\frac{R_0C_{wire}}{2R_{wire}C_0}}=\sqrt{\frac{R_0C_{unit}}{2R_{unit}C_0}}$$

Implication W

- $R_{wire} = L \times R_{unit}$
- $C_{wire} = L \times C_{unit}$

$$2\sqrt{0.5R_{wire}C_{wire}\left(2R_0C_0\left(1+\gamma\right)\right)}+\frac{R_0}{W}C_{wire}+R_{wire}\cdot 2WC_0$$

$$0 = 2R_{wire}C_0 - R_0C_{wire}\frac{1}{W^2}$$

$$W = \sqrt{\frac{R_0C_{wire}}{2R_{wire}C_0}} = \sqrt{\frac{R_0C_{unit}}{2R_{unit}C_0}}$$

- ➔ W independent of wire length L
  - Depends on technology

Delay at Optimum W (preclass 5)

$$W = \sqrt{\frac{R_0 C_{wire}}{2R_{wire} C_0}}$$

$$2\sqrt{0.5R_{wire}C_{wire}\left(2R_0C_0\left(1+\gamma\right)\right)}+\frac{R_0}{W}C_{wire}+R_{wire}\cdot 2WC_0$$

Delay at Optimum W

$$W = \sqrt{\frac{R_0 C_{wire}}{2R_{wire} C_0}}$$

$$2\sqrt{0.5R_{wire}C_{wire}\left(2R_0C_0\left(1+\gamma\right)\right)} + \frac{R_0}{W}C_{wire} + R_{wire}\cdot 2WC_0$$

$$2\sqrt{0.5R_{wire}C_{wire}\left(2R_0C_0\left(1+\gamma\right)\right)} + \frac{R_0}{\sqrt{\frac{R_0 C_{wire}}{2R_{wire} C_0}}}C_{wire} + R_{wire}\cdot 2\sqrt{\frac{R_0 C_{wire}}{2R_{wire} C_0}}C_0$$

Delay at Optimum W

$$W=\sqrt{\frac{R_0C_{wire}}{2R_{wire}C_0}}$$

$$2\sqrt{0.5R_{wire}C_{wire}\left(2R_0C_0\left(1+\gamma\right)\right)}+\frac{R_0}{W}C_{wire}+R_{wire}\cdot 2WC_0$$

$$2\sqrt{0.5R_{wire}C_{wire}\left(2R_0C_0\left(1+\gamma\right)\right)}+\frac{R_0}{\sqrt{\frac{R_0C_{wire}}{2R_{wire}C_0}}}C_{wire}+R_{wire}\cdot 2\sqrt{\frac{R_0C_{wire}}{2R_{wire}C_0}}C_0$$

$$2\sqrt{0.5R_{wire}C_{wire}\left(2R_0C_0\left(1+\gamma\right)\right)}+\sqrt{R_0C_{wire}\cdot 2R_{wire}C_0}+\sqrt{R_0C_{wire}\cdot 2R_{wire}C_0}$$

Delay at Optimum W

$$W = \sqrt{\frac{R_0 C_{wire}}{2R_{wire}C_0}}$$

$$2\sqrt{0.5R_{wire}C_{wire}\left(2R_0C_0\left(1+\gamma\right)\right)} + \frac{R_0}{W}C_{wire} + R_{wire}\cdot 2WC_0$$

$$2\sqrt{0.5R_{wire}C_{wire}\left(2R_0C_0\left(1+\gamma\right)\right)} + \frac{R_0}{\sqrt{\frac{R_0C_{wire}}{2R_{wire}C_0}}}C_{wire} + R_{wire}\cdot 2\sqrt{\frac{R_0C_{wire}}{2R_{wire}C_0}}C_0$$

$$2\sqrt{0.5R_{wire}C_{wire}\left(2R_0C_0\left(1+\gamma\right)\right)} + \sqrt{R_0C_{wire}\cdot 2R_{wire}C_0} + \sqrt{R_0C_{wire}\cdot 2R_{wire}C_0}$$

$$2\sqrt{0.5R_{wire}C_{wire}\left(2R_0C_0\left(1+\gamma\right)\right)} + 2\sqrt{2R_0C_0C_{wire}R_{wire}}$$

Delay at Optimum W

- If γ=1

$$2\sqrt{R_{wire}C_{wire}\left(R_0C_0\left(1+1\right)\right)}+2\sqrt{2R_0C_0C_{wire}R_{wire}}$$

$$2\sqrt{2R_0C_0C_{wire}R_{wire}}+2\sqrt{2R_0C_0C_{wire}R_{wire}}$$

$$4\sqrt{2R_0C_0C_{wire}R_{wire}}$$

- Optimal design equalizes all delays!

Delay at Optimum W

- If γ=1

$$2\sqrt{R_{wire}C_{wire}\left(R_0C_0\left(1+1\right)\right)}+2\sqrt{2R_0C_0C_{wire}R_{wire}}$$
$$2\sqrt{2R_0C_0C_{wire}R_{wire}}+2\sqrt{2R_0C_0C_{wire}R_{wire}}$$
$$4\sqrt{2R_0C_0C_{wire}R_{wire}}$$

- Optimal design equalizes all delays!

8.8ns → 0.27ns

Ideas

- Wire delay linear once buffered optimally
- Optimal buffers equalizes delays
  - Buffer delay
  - Delay on wire between buffers
  - Delay of wire driving buffer

Admin

- Friday HW 7 due
- Final (F 12/17)
  - 12-2pm in Moore 212
  - Cumulative: Lec 1 – 35
    - Big Idea slides from each lecture
  - Finals 2010—2019 online
  - Friday lecture review
  - TA review session before exam
    - TBD, watch Piazza. Maybe a poll.