ESE370: Circuit-Level Modeling, Design, and Optimization for Digital Systems

Lec 3: September 10, 2021

Transistor Introduction (first order)

1

Today

- Basic Digital Gates
- Boolean Logic
  - Basic Algebra
    - DeMorgan's Law
  - Minimum Sum of Products/K-maps
- Cascading Gates
- Transistor first order model
  - For performance estimates (i.e propagation delay!)
  - There are always Rs and Cs!

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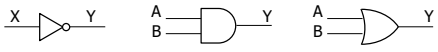
Basic Digital Gates

NOT $Y = \overline{X}$ | AND $Y = A \cdot B$ | OR $Y = A + B$



X	Y
0	1
1	0

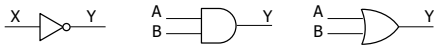
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

3

Basic Digital Gates

NOT $Y = \overline{X}$ | AND $Y = A \cdot B$ | OR $Y = A + B$

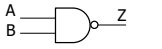


X	Y
0	1
1	0

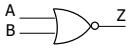
A	B	Y	Z
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

A	B	Y	Z
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

NAND $Z = \overline{A \cdot B}$



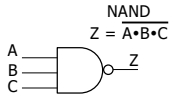
NOR $Z = \overline{A + B}$



4

Basic Digital Gates

NAND $Z = \overline{A \cdot B \cdot C}$



A	B	C	Z
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

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Boolean Algebra

TABLE 2-3
Basic Identities of Boolean Algebra

1.	$X + 0 = X$	2.	$X \cdot 1 = X$	
3.	$X + 1 = 1$	4.	$X \cdot 0 = 0$	
5.	$X + X = X$	6.	$X \cdot X = X$	
7.	$X + \overline{X} = 1$	8.	$X \cdot \overline{X} = 0$	
9.	$\overline{\overline{X}} = X$			
10.	$X + Y = Y + X$	11.	$XY = YX$	Commutative
12.	$X + (Y + Z) = (X + Y) + Z$	13.	$X(YZ) = (XY)Z$	Associative
14.	$X(Y + Z) = XY + XZ$	15.	$X + YZ = (X + Y)(X + Z)$	Distributive
16.	$\overline{X + Y} = \overline{X} \cdot \overline{Y}$	17.	$\overline{X \cdot Y} = \overline{X} + \overline{Y}$	DeMorgan's

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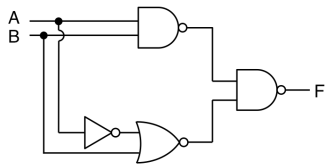
Combination



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Boolean Expressions

- Sum-of-products form (SOP)
  - Eg. ABC+DEF+GHI
- Product-of-sums form (POS)
  - Eg. (A+B+C)(D+E+F)(G+H+I)
- Convert between the two with Boolean algebra
  - DeMorgan's Law

$$A + B = \overline{\overline{A} \cdot \overline{B}}$$

$$A \cdot B = \overline{\overline{A} + \overline{B}}$$

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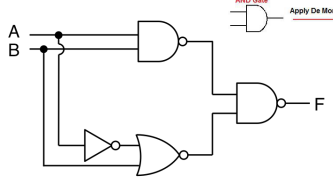
Combination



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Canonical Form

- Every minterm in your expression has every variable

Row Number	A	B	C	f
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

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Canonical Form

- Every minterm in your expression has every variable

Row Number	A	B	C	f
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

$$f(A,B,C) = ABC + A\overline{B}C + \overline{A}BC$$

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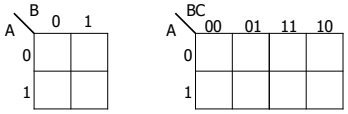
What is a K(arnaugh)-map?

- A grid of squares (representing truth table)
- Each square represents one minterm
- The minterms are ordered according to Gray code
  - Only one variable changes between adjacent squares
- Squares on edges are considered adjacent to squares on opposite edges
  - I.e Table wraps around
- K-maps are clumsy with more than 4 variables



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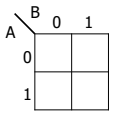
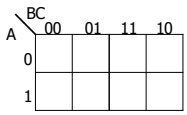
K-map Examples (Preclass 1)

- 2-variable



Eg: Z = A'B' + AB' + A'B

- 3-variable



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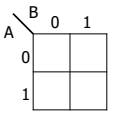
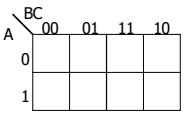
K-map Examples (Preclass 2)

- 2-variable



Eg: Z = A'B' + AB' + A'B

- 3-variable



Eg: Z = A'B'C' + A'B + ABC' + AC

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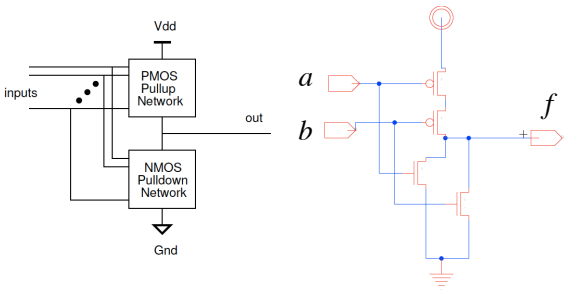
Static CMOS Gate Structure



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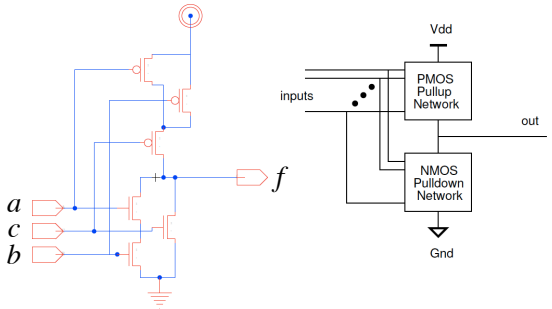
Gate Design Example

- Design gate to perform: $f = (\bar{a} + \bar{b}) \cdot \bar{c}$



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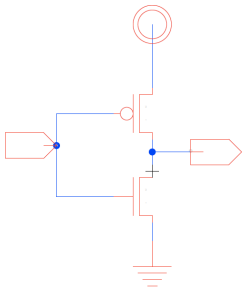
Inverting Stage

- Each stage of Static CMOS gate is inherently inverting



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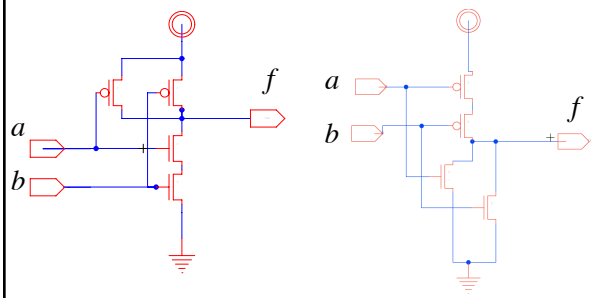
NAND/NOR Fundamental Gates



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How implement OR?

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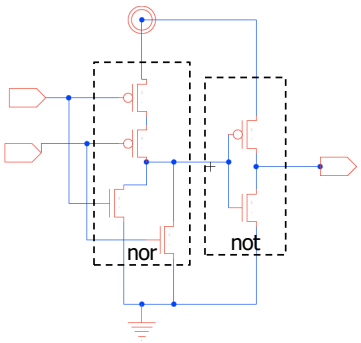
How implement OR?



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Cascading Stages

- Can always cascade "stages" to build more complex gates
- Could simply build nor2 or nand2 at circuit level and assemble arbitrary logic by combining – universality
  - but may not be smallest/fastest/least power

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Implement: $f = a \cdot \bar{b}$

- Pullup?
- Pulldown?

Hint: use cascading stages

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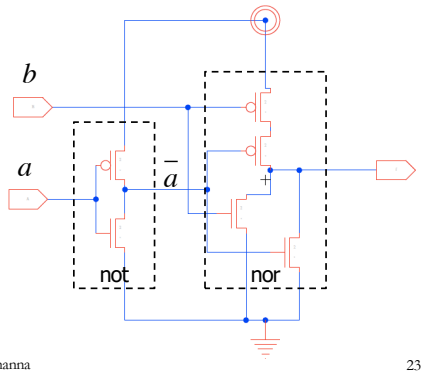
Implement: $f = a \cdot \bar{b}$

- Pullup?
- Pulldown?



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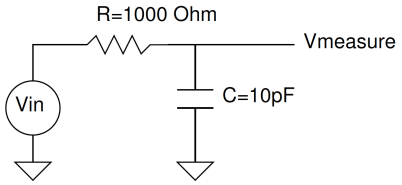
Final Voltage? (Preclass 4)

Assume V_{in} is 0 for $t < 0$ and steps to 1V at $t = 0$.

What value does $V_{measure}$ take on as $t \to \infty$?



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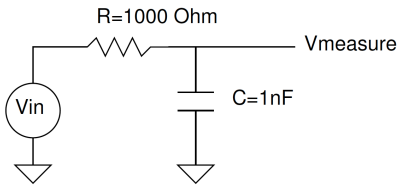
Final Voltage?

Assume V_{in} is 0 for $t < 0$ and steps to 1V at $t = 0$.

What value does $V_{measure}$ take on as $t \to \infty$?



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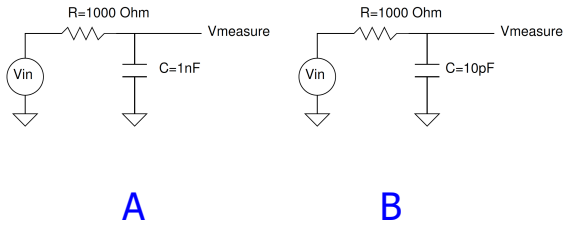
Final Voltage?

- Bonus question: Which one will settle faster?



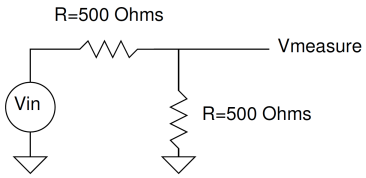
A B

26

Final Voltage?

Assume V_{in} is 0 for $t < 0$ and steps to 1V at $t = 0$.

What value does $V_{measure}$ take on as $t \to \infty$?



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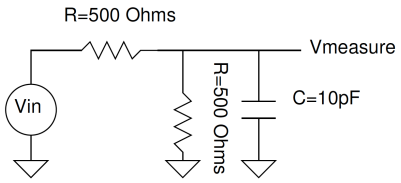
Final Voltage?

Assume V_{in} is 0 for $t < 0$ and steps to 1V at $t = 0$.

What value does $V_{measure}$ take on as $t \to \infty$?



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Conclude?

- DC/Steady-State
  - Ignore the capacitors
  - Look like "open circuit"

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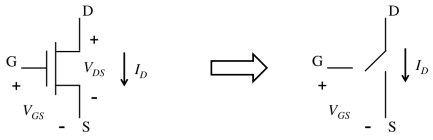
MOSFET – Zeroeth Order Model



- Ideal Switch
  - $V_{GS} > V_{th}$ → switch is closed, conducts
  - $V_{GS} < V_{th}$ → switch is open, does not conduct
- Gate draws no current from input
  - Loads input capacitively (gate capacitance)

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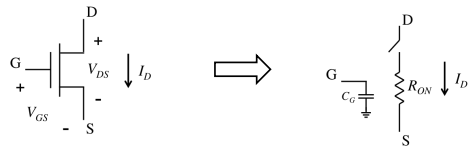
First Order Model



- Switch
  - Loads gate input capacitively
    - C_g
  - Has finite drive strength
    - R_{on}

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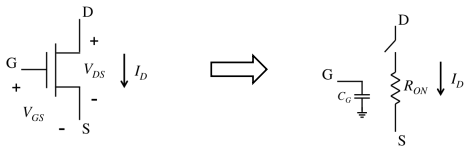
First Order Model



- Switch
  - Loads gate input capacitively
    - C_g
  - Has finite drive strength
    - R_{on}

What do drive and load mean?

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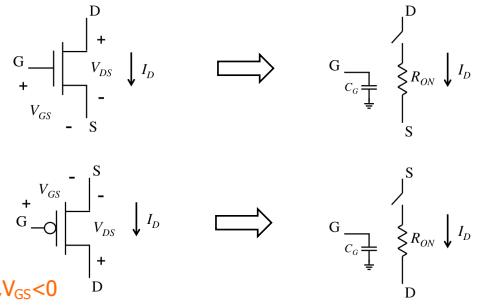
First Order Model - PMOS



$V_{DS}, V_{GS} < 0$

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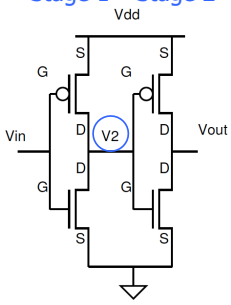
CMOS Buffer Gate

Stage 1 Stage 2



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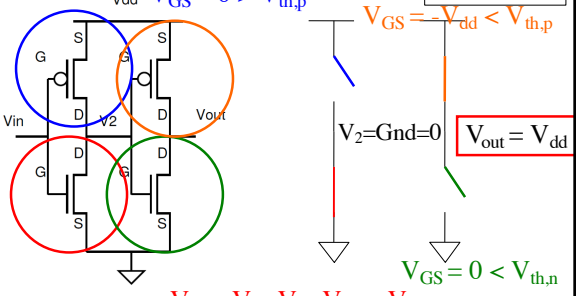
Reminder: Zero-Order Model?

What happens when $V_{in} = V_{dd} > V_{thn}$?

$V_{th,p} = -V_{th,n}$

$V_{GS} = V_G - V_S$



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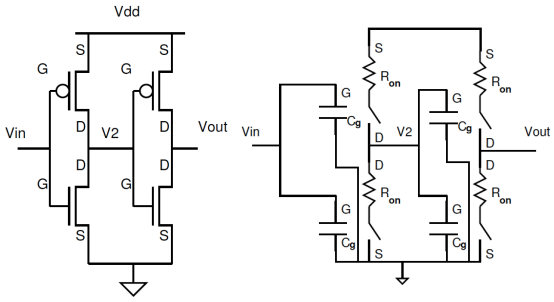
CMOS Buffer Gate - First Order



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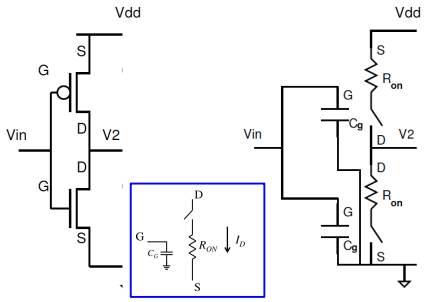
CMOS Buffer Gate - First Order



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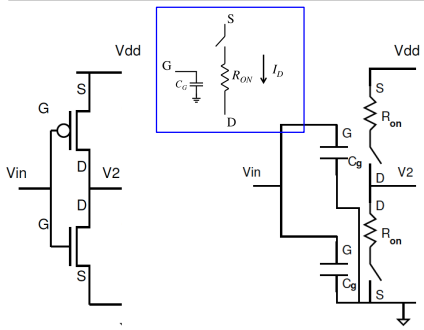
CMOS Buffer Gate - First Order



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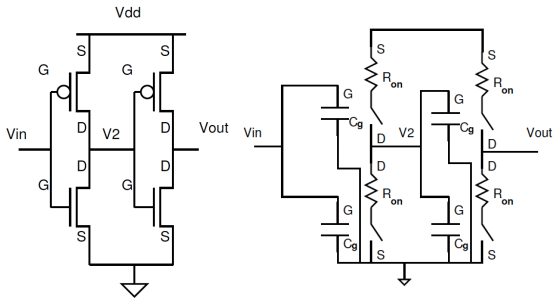
CMOS Buffer Gate - First Order



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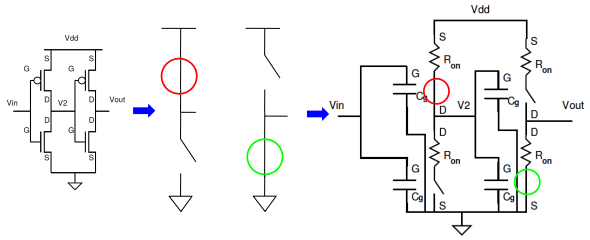
Zero-Order Model to Set Switches



- What is V_{IN} for this switch pattern?

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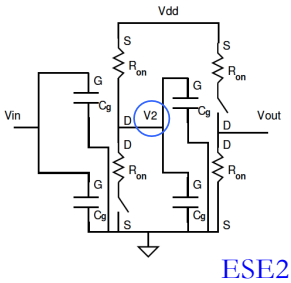
CMOS Buffer Gate - First Order



ESE215 problem

- Leaves an RC Circuit we can analyze

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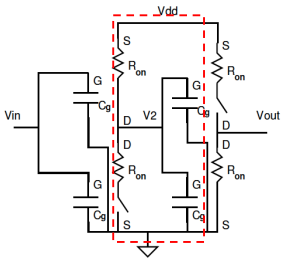
CMOS Buffer Gate - First Order



- Look at intermediary node V_2
  - Connected to output of stage 1 and input of stage 2

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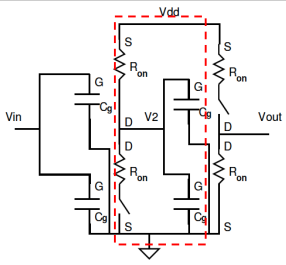
CMOS Buffer Gate - First Order



- What is equivalent circuit for the gate output of stage 1 driving V_2? What is load on the output of stage 1?

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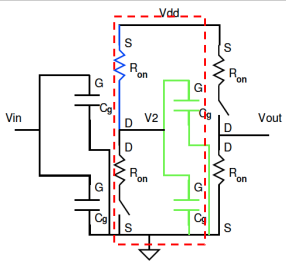
CMOS Buffer Gate - First Order



- Stage 1 equivalent circuit for the gate output
- Load on V_2
  - Capacitive, input of stage 2

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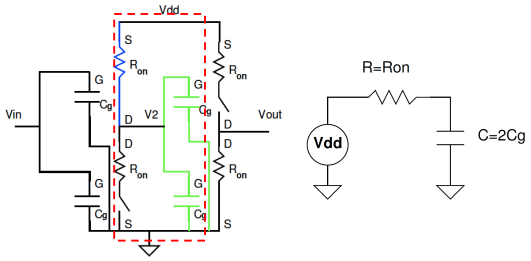
CMOS Buffer Gate - First Order



- Stage 1 equivalent circuit for the gate output
- Load on V_2
  - Capacitive, input of stage 2

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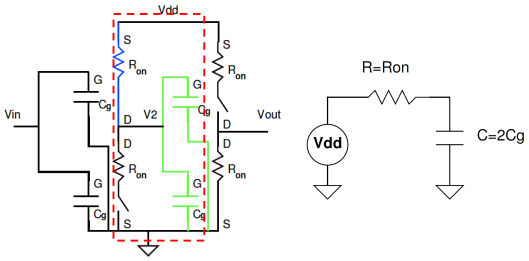
CMOS Buffer Gate - First Order



- What is time constant of V_2 when V_{in} switches from V_{DD} to 0?

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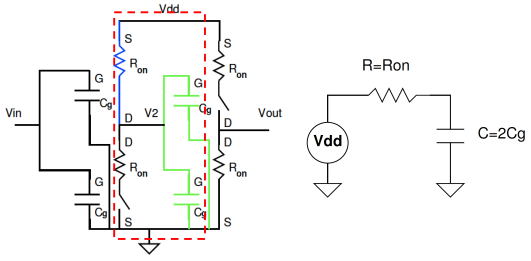
CMOS Buffer Gate - First Order



- What is time constant of V_2 when V_{in} switches from V_{DD} to 0?
  - $\tau = 2R_{on}C_g$

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First-Order Model

- Includes settling times/delay
- Voltage settling with capacitive loads
  - At least some basis for reasoning about delay

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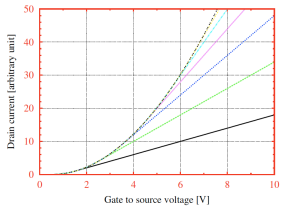
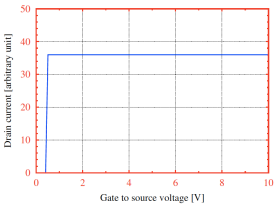
What is still missing?

IV curve



1st Order



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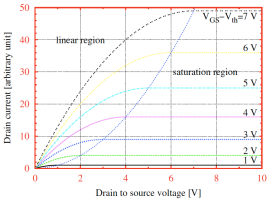
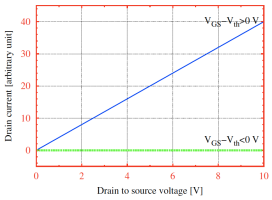
What is still missing?

IV curve



1st Order



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What is still missing?

- What happens at intermediate voltages?
  - When the input is not rail-to-rail (not just gnd or V_{dd} inputs)
- Details of dynamics, including…
  - Input transition is not an ideal step
  - Intermediate drive strengths change with V_{GS}
    - Drain resistance changes
- Sub-threshold operation
  - When $V_{gs}<V_{th}$

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Design: Engineering Control

- V_{th}
  - Process engineer
- Drive strength (R_{on})
  - Circuit engineer
  - Control with sizing transistors
- Supply voltages (V_{dd})
  - Range set by process engineer
  - Detail use by circuit engineer

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Big Ideas

- MOSFET Transistor as switch
  - With limited drive
- Purpose-driven simplified modeling
  - Aid reasoning, sanity check, simplify design
- Analysis methodology
  - Zero order to understand switch state (logic)
  - First-order to get equivalent RC circuit (delay)

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Admin

- Diagnostic grades recorded
  - Solutions posted online (pdf in Canvas)
  - Make sure you understand
  - Diagnose what you need to review and study
    - Karnaugh maps (minimum sum of products), RC capacitive settling (time constant), and wire resistive and capacitive properties
- HW 1 due Tuesday 9/14 next week
  - Because you need Monday lecture for noise margins
    - Can look at last year's lecture slides if you want to look ahead
  - HW due following Monday 9/20

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