ESE370: Circuit-Level Modeling, Design, and Optimization for Digital Systems

Lec 7: September 20, 2021

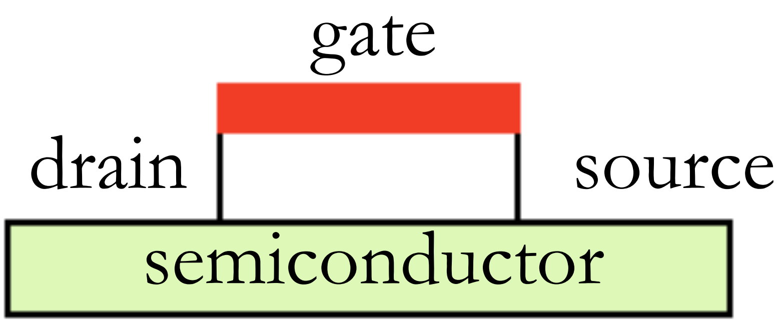
MOS Transistor Operating Regions

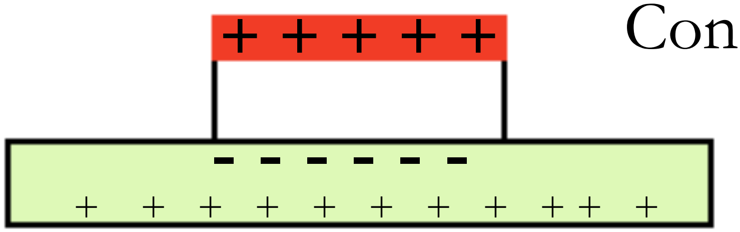
Part 1

Today

- MOS Transistor Topology
- Threshold
- Operating Regions
  - Resistive
  - Saturation
  - Subthreshold (next class)
  - Velocity Saturation (next class)

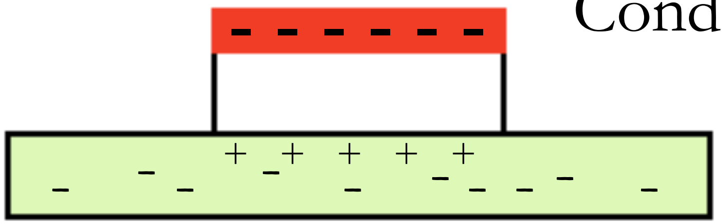
Last Time – MOS model





V_{gs} positive
Conducts

NMOS

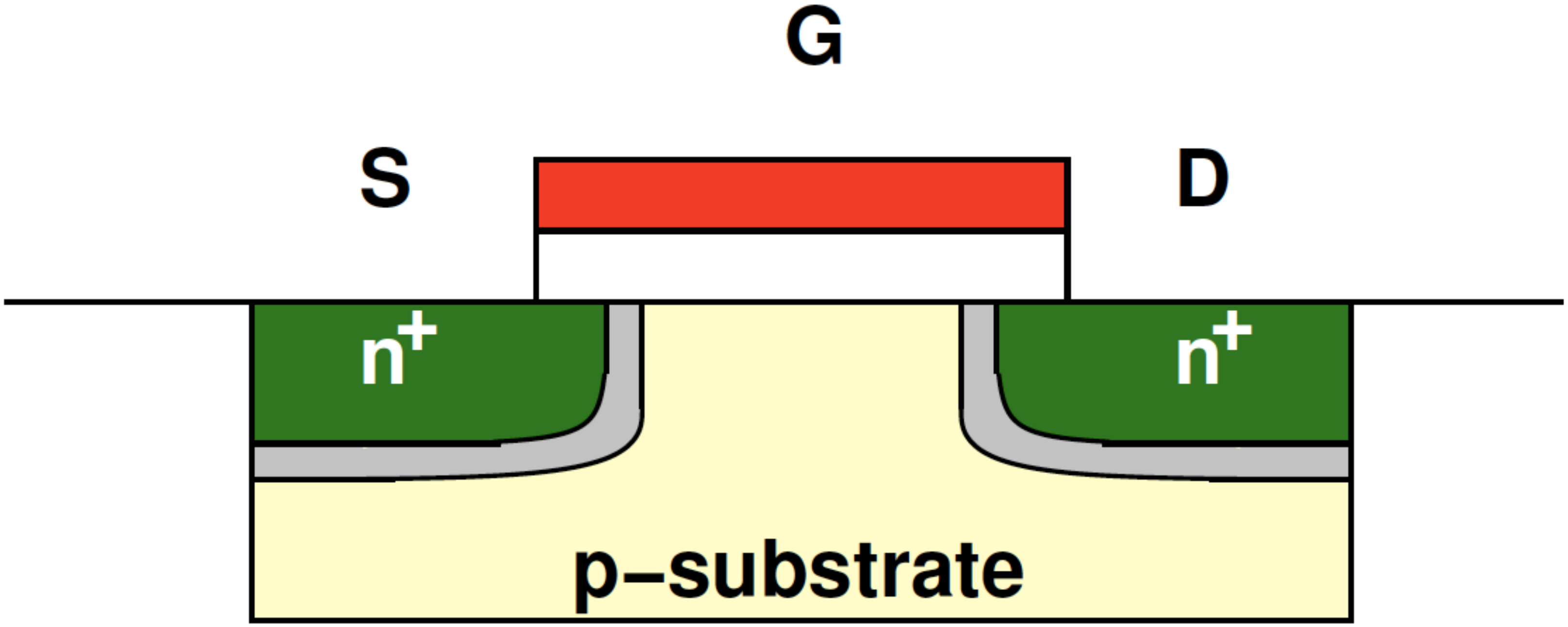


V_{gs} negative
Conducts

PMOS

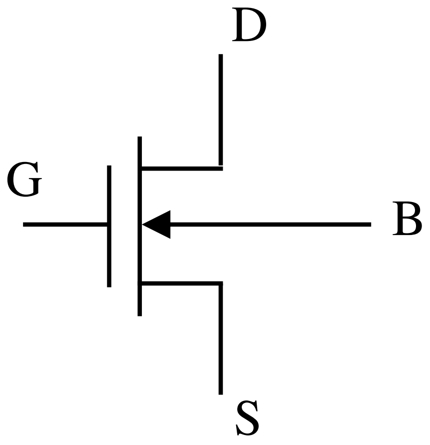
Refinement

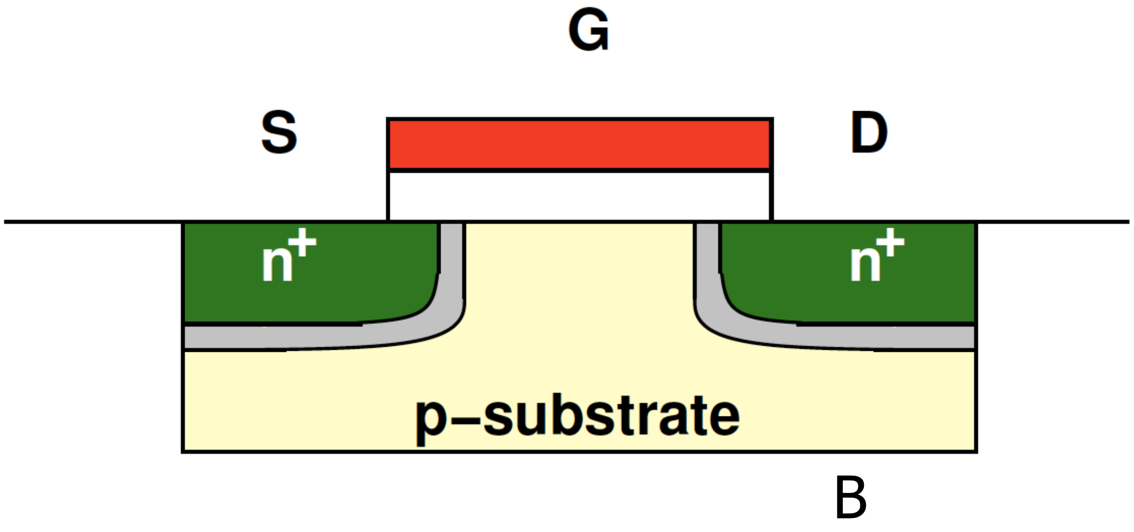
- Depletion region around D/S→ excess carriers depleted



Bulk/Body Contact

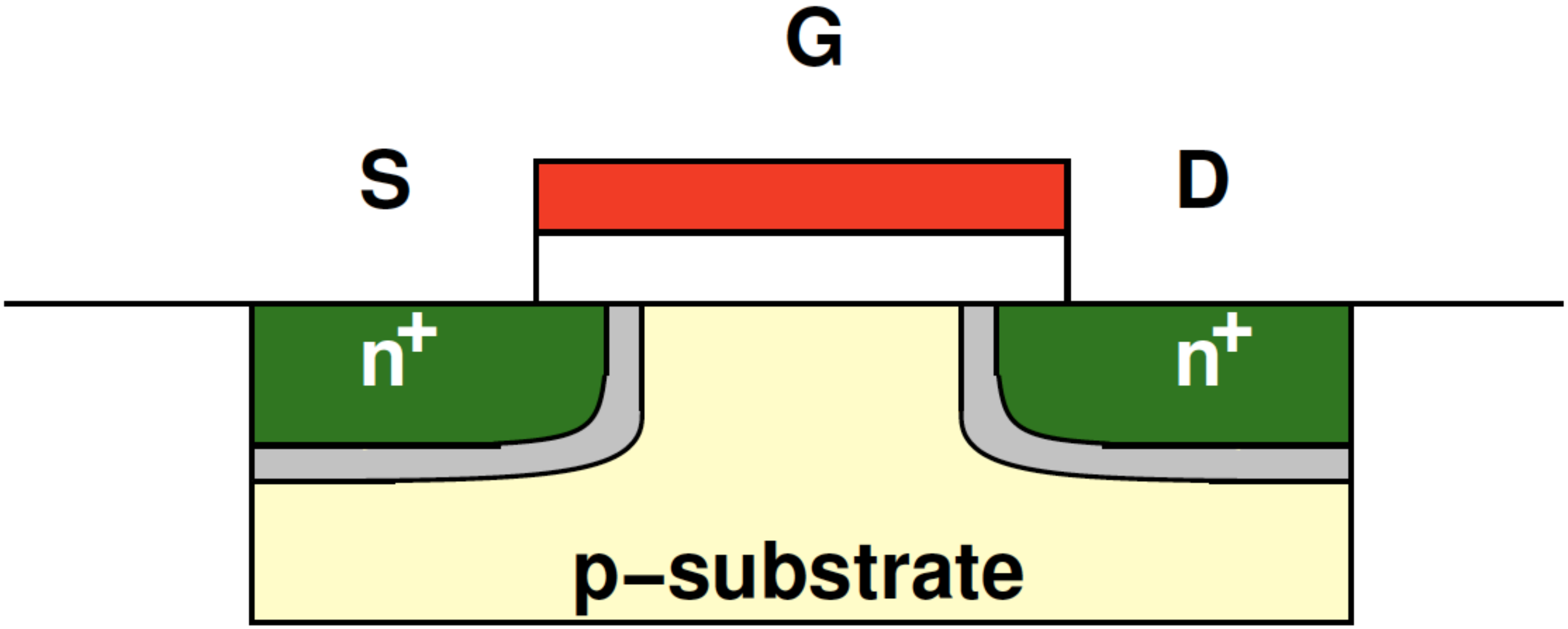
- MOS actually has four contacts
- Also effects fields
- Ideally substrate and source connected
  - Settle for substrate being <= source
  - Gnd for nmos (V_{dd} for pmos)





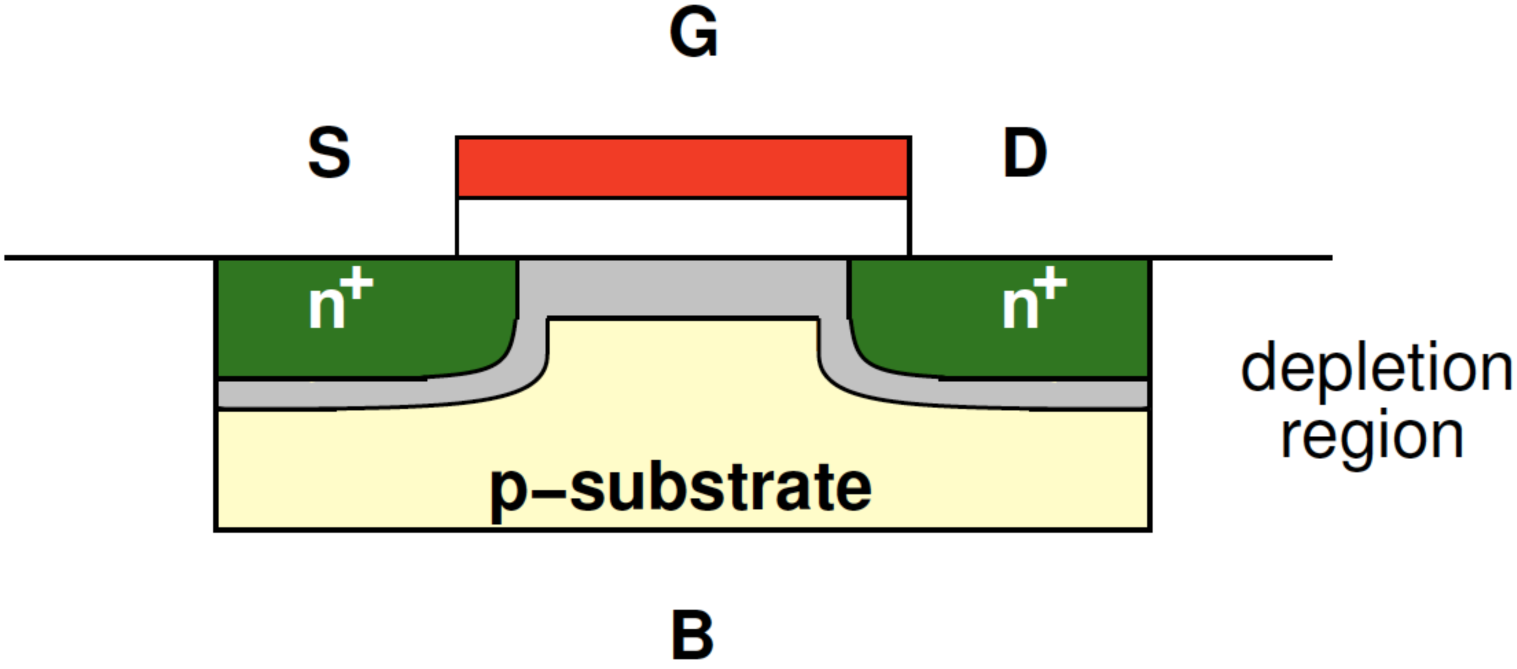
No Field

- $V_{GS}=0$, $V_{DS}=0$

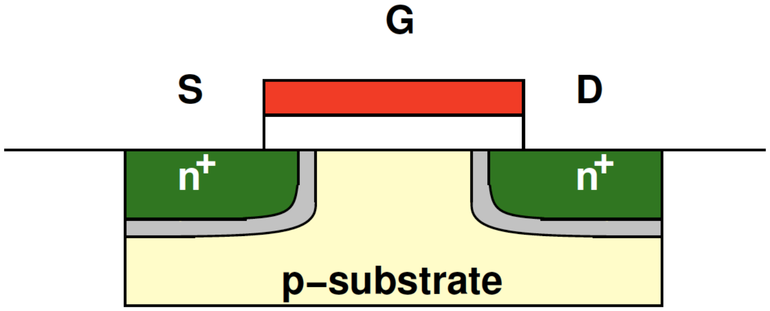


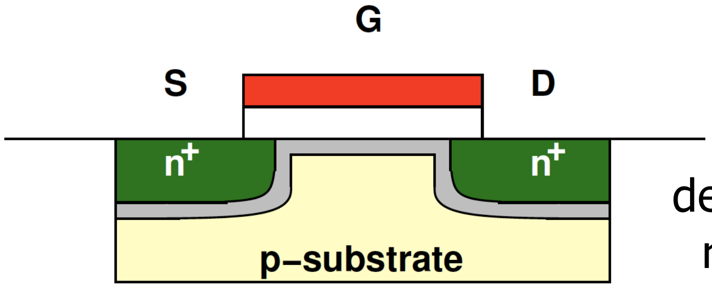
Apply $V_{GS}>0$

- Deplete excess positive charge under oxide
- Left with negative charge
  - Repel holes

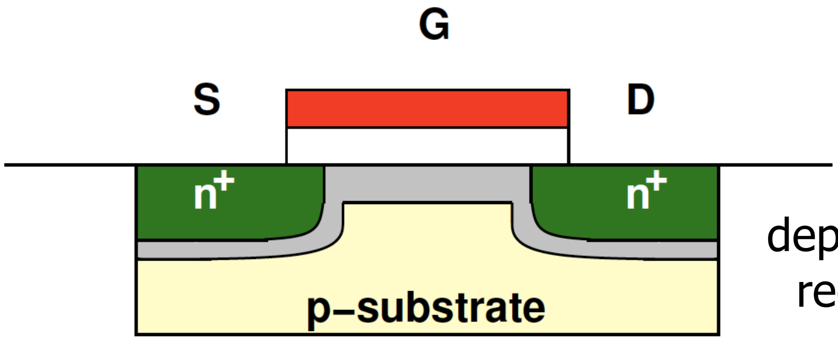


Channel Evolution -- Increasing Vgs





depletion
region



depletion
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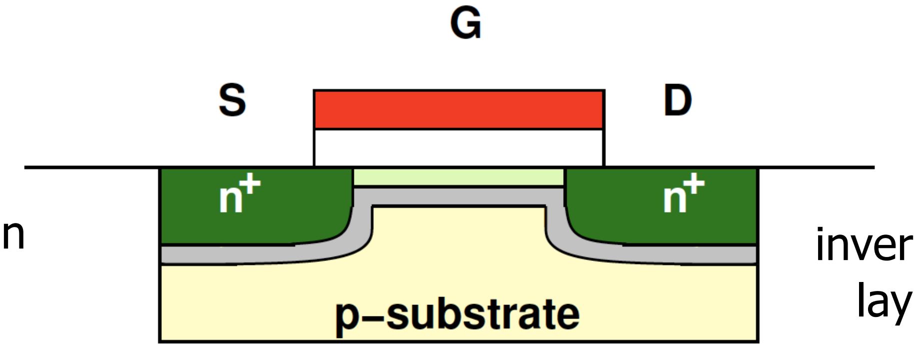
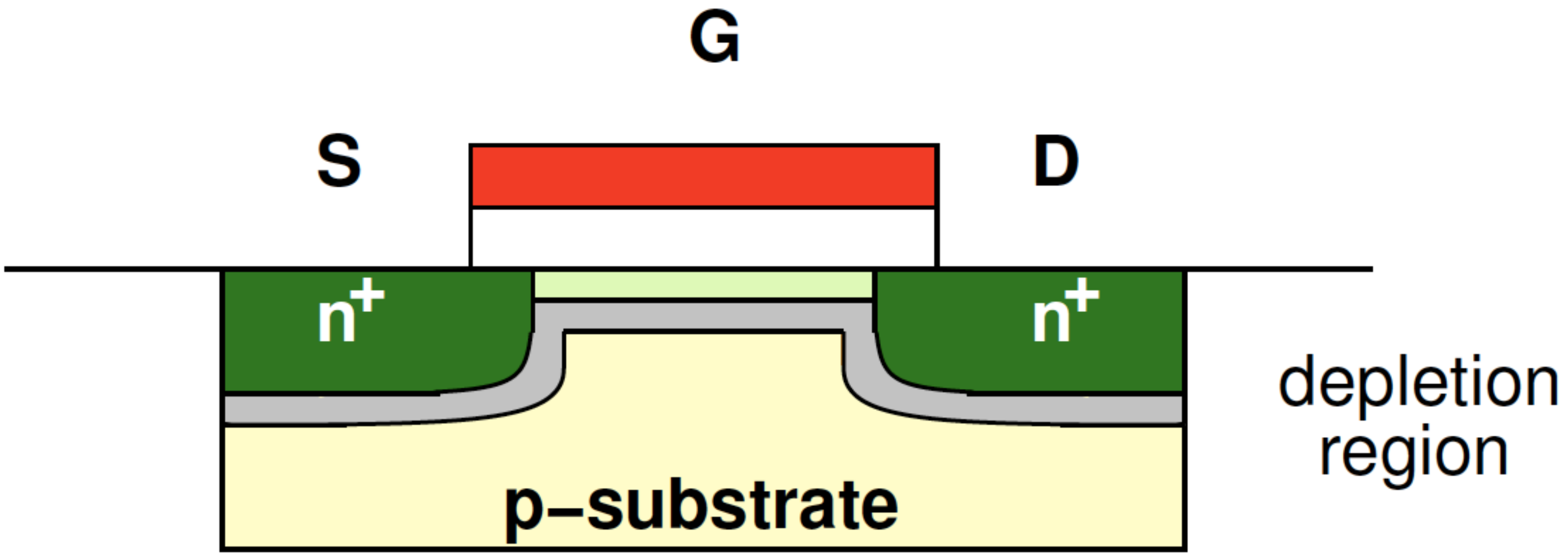


inversion
layer

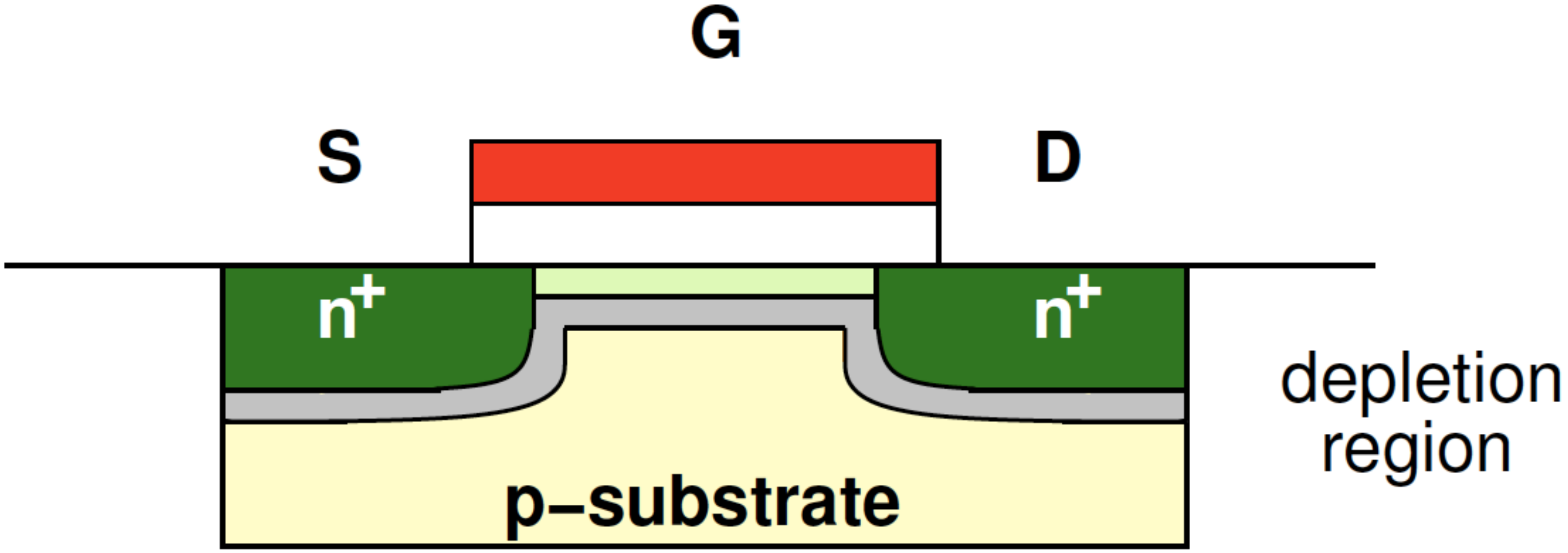
Inversion

- Surface builds electrons
  - Inverts to n-type
  - Draws electrons from n$^+$ source terminal



Threshold

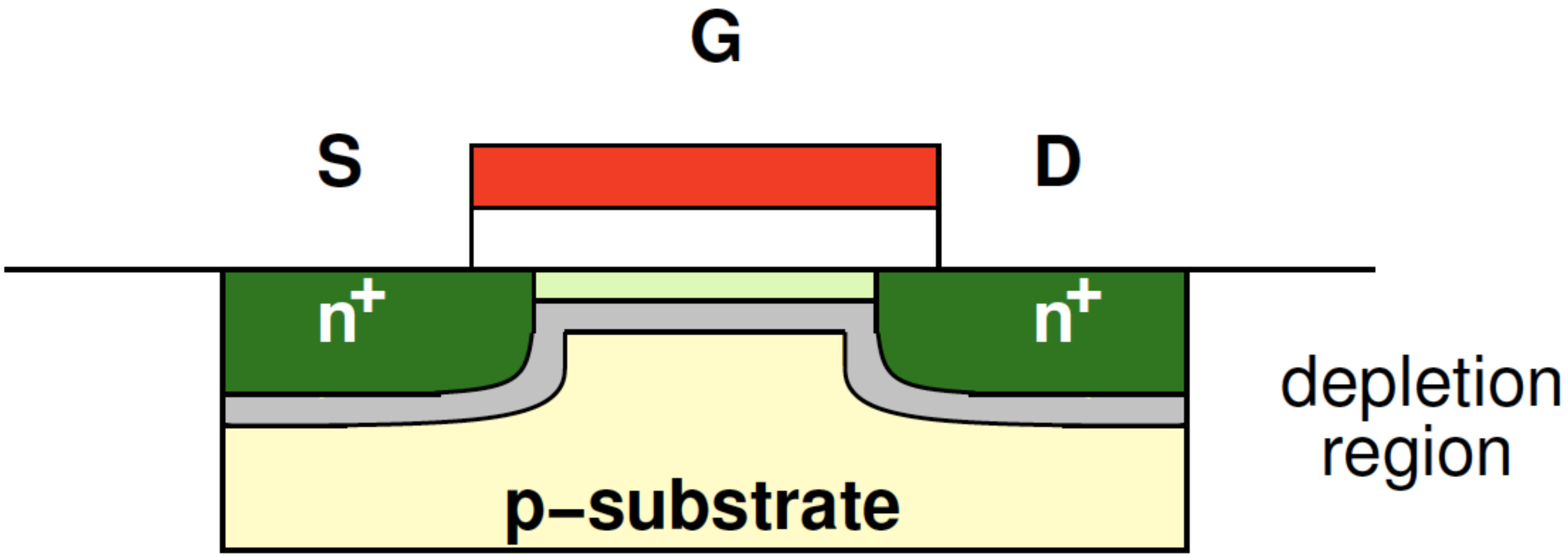
- Voltage where strong inversion occurs → threshold voltage
  - $V_{th} \sim= 2\phi_F$



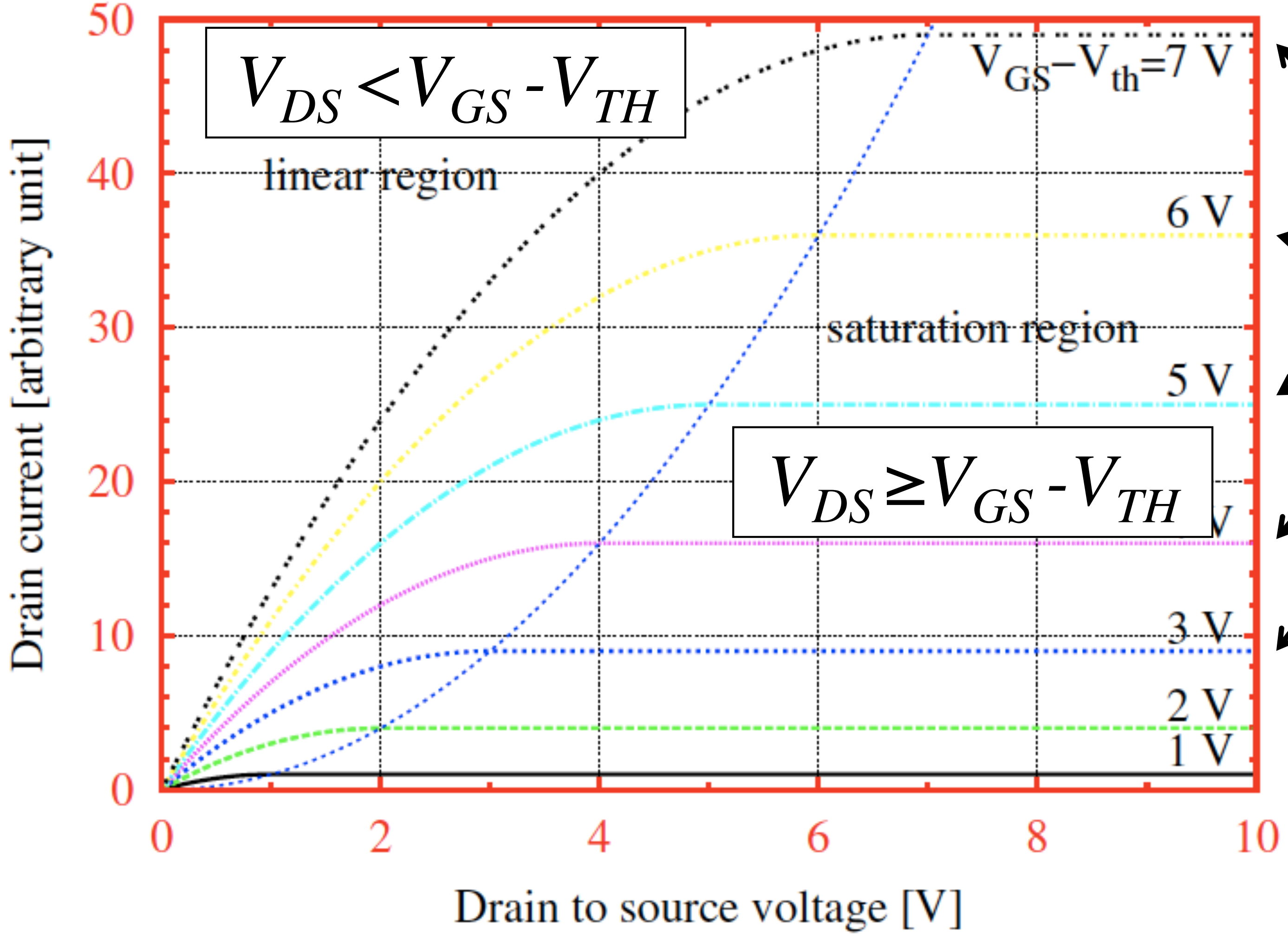
Threshold

- Voltage where strong inversion occurs → threshold voltage
  - $V_{th} \sim= 2\phi_F$
  - Engineer by controlling doping (N_A)

$$\phi_F = \frac{kT}{q}\ln\left(\frac{N_A}{n_i}\right)$$



MOSFET – IV Characteristics



$V_{DS} < V_{GS} - V_{TH}$

$V_{DS} \geq V_{GS} - V_{TH}$

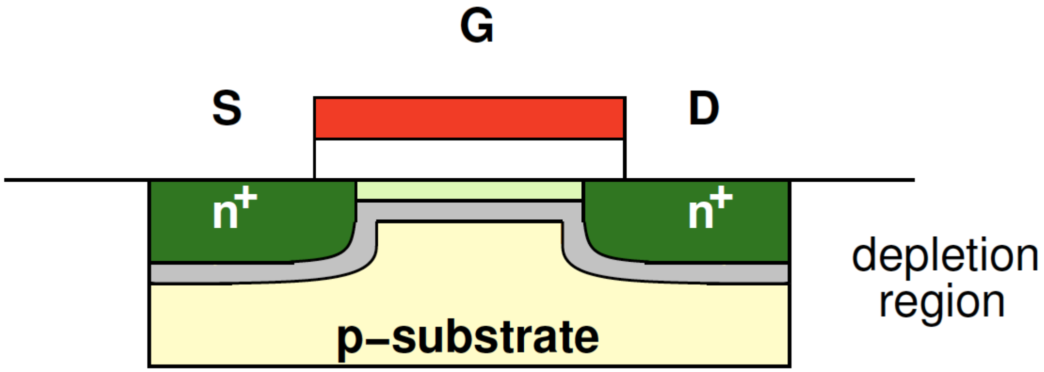
I_{DS}

V_{DS}

$V_{GS} - V_{th}$

Linear Region

- $V_{GS} > V_{th}$ and V_{DS} small



$$C_{OX} = \frac{\varepsilon_{OX}}{t_{OX}}$$

$$I_{DS} = \mu_n C_{OX} \left(\frac{W}{L}\right) \left[\left(V_{GS} - V_{th}\right) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

Linear Region

- $V_{GS} > V_{th}$ and V_{DS} small
- V_{GS} fixed → looks like resistor
  - Current linear in V_{DS}

$$I_{DS} = \mu_n C_{OX} \left(\frac{W}{L}\right) \left[\left(V_{GS} - V_{th}\right) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

Linear Region

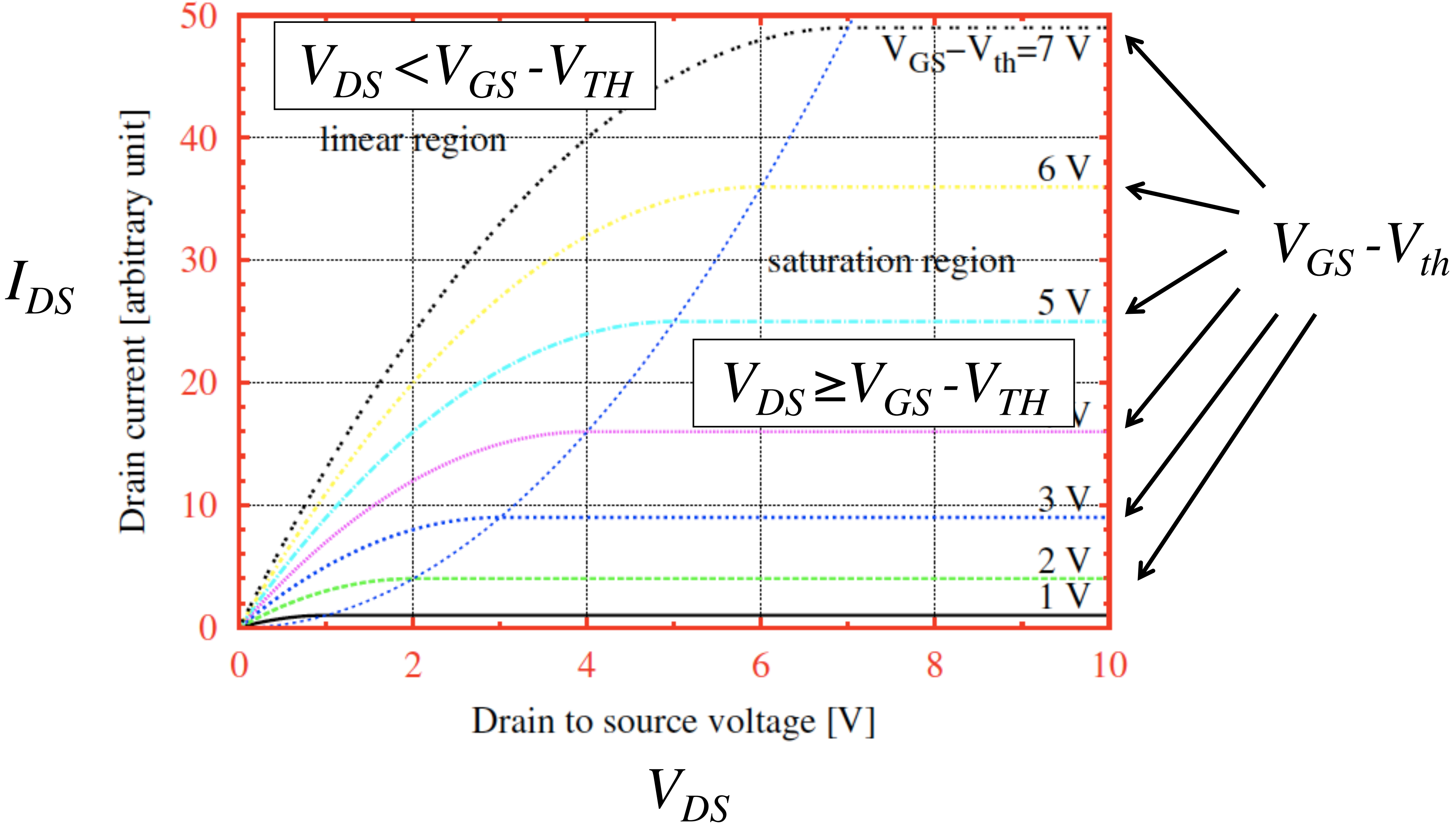
- $V_{GS} > V_{th}$ and V_{DS} small
- V_{GS} fixed → looks like resistor
  - Current linear in V_{DS}

$$I_{DS} = \mu_n C_{OX} \left(\frac{W}{L}\right)\left[(V_{GS} - V_{th})V_{DS} - \cancelto{\sim 0}{\frac{V_{DS}^2}{2}}\right]$$

$$I_{DS} \approx \mu_n C_{OX} \left(\frac{W}{L}\right)(V_{GS} - V_{th})V_{DS}$$

$$I_{DS} \propto V_{DS}$$

MOSFET – IV Characteristics



$V_{DS} < V_{GS} - V_{TH}$

$V_{DS} \geq V_{GS} - V_{TH}$

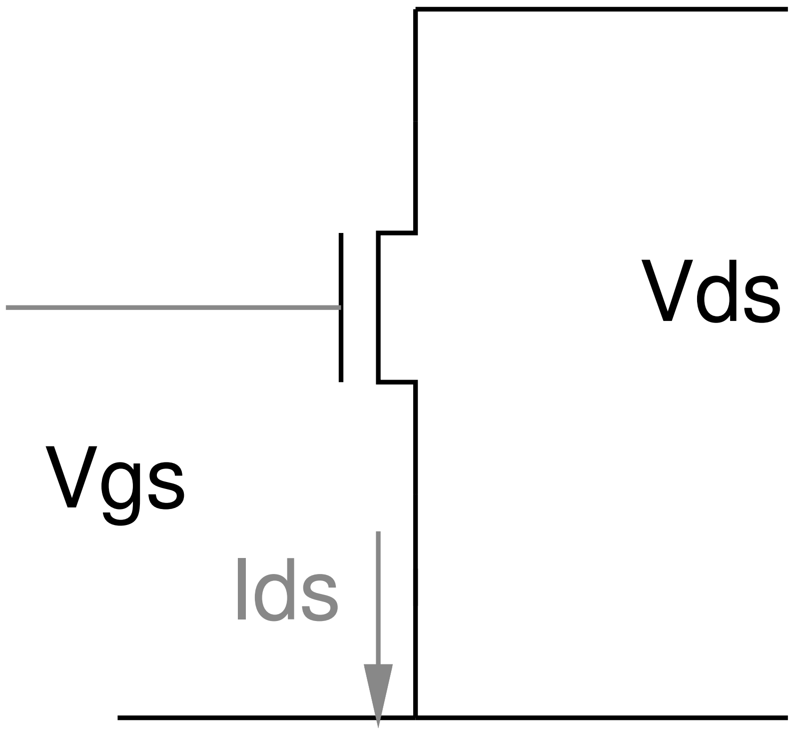
$V_{GS} - V_{th}$

I_{DS}

V_{DS}

Preclass 1

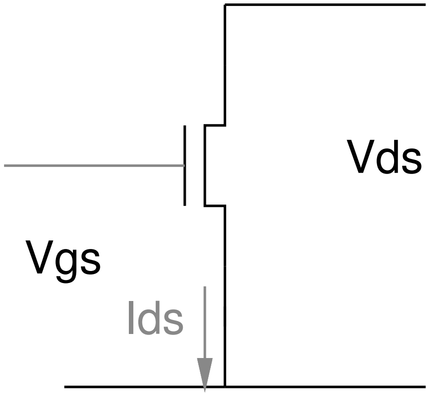
- Reference: I_{ds} for single transistor with Vgs and Vds bias

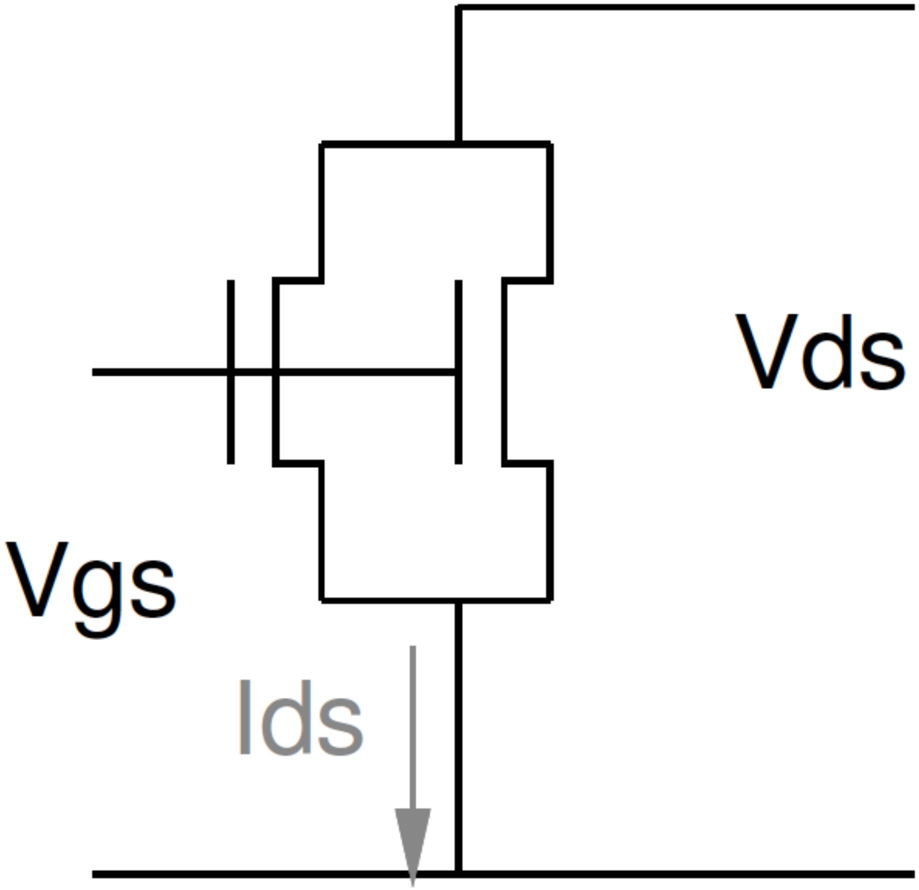


Preclass 1

- I_{ds} for identical transistors in parallel?

Reference:

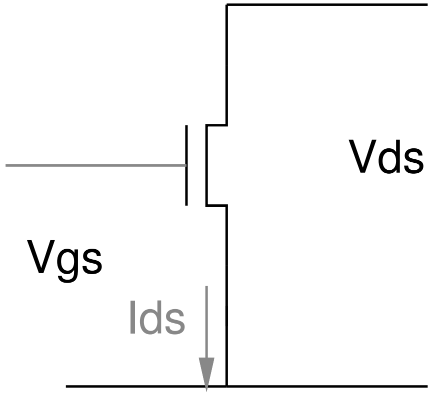


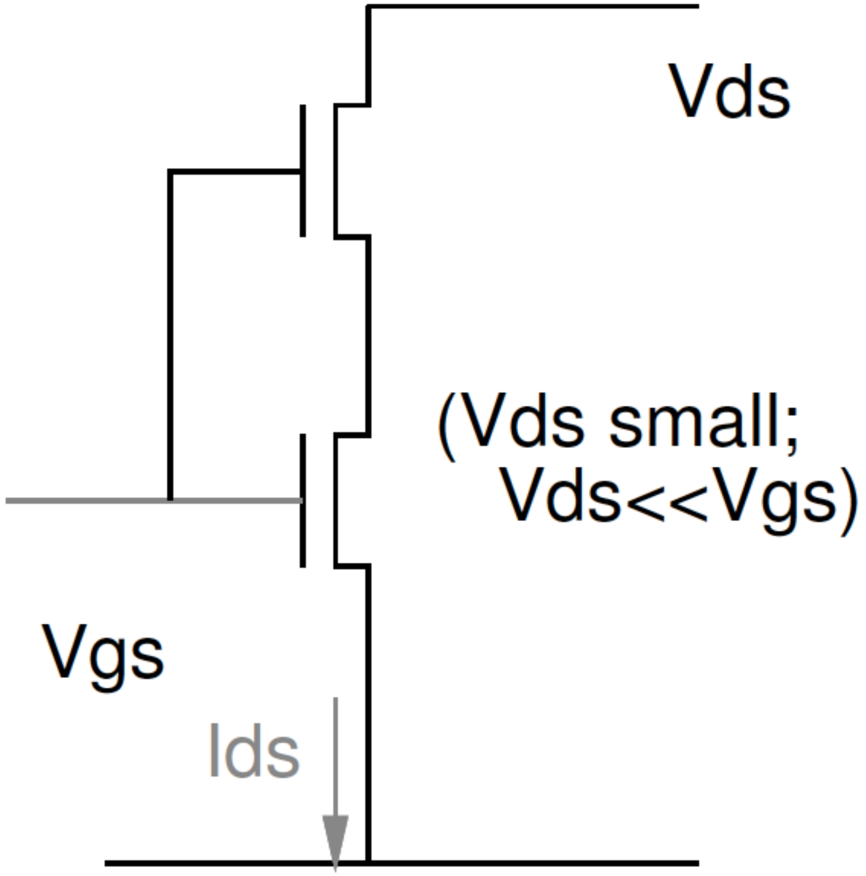


Preclass 1

- I_{ds} for identical transistors in series?
  - (Vds small)

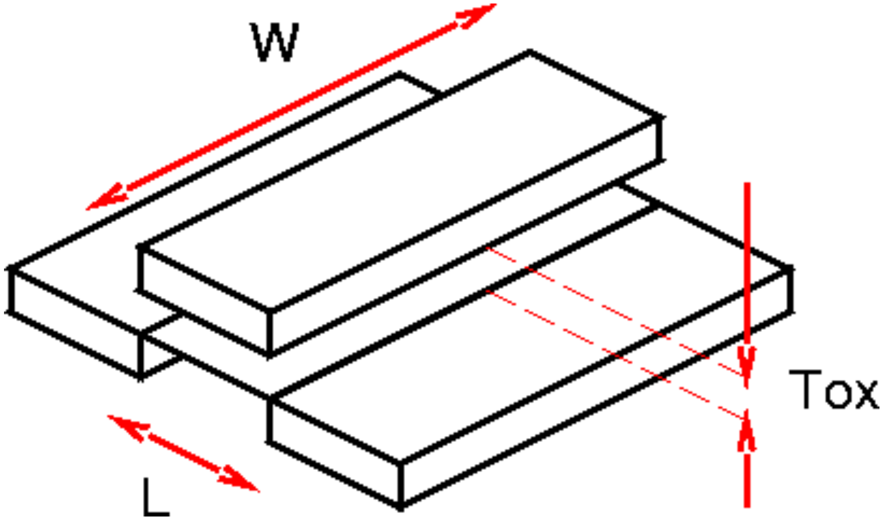
Reference:

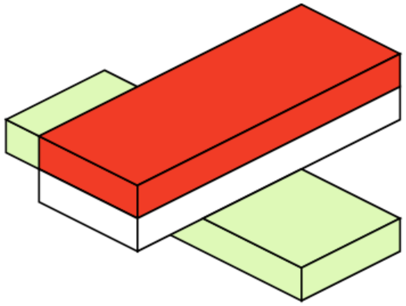




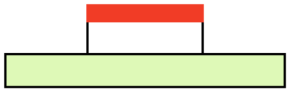
Dimensions

- Channel Length (L)
- Channel Width (W)
- Oxide Thickness (T_{ox})

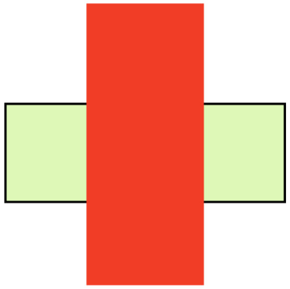




Oblique



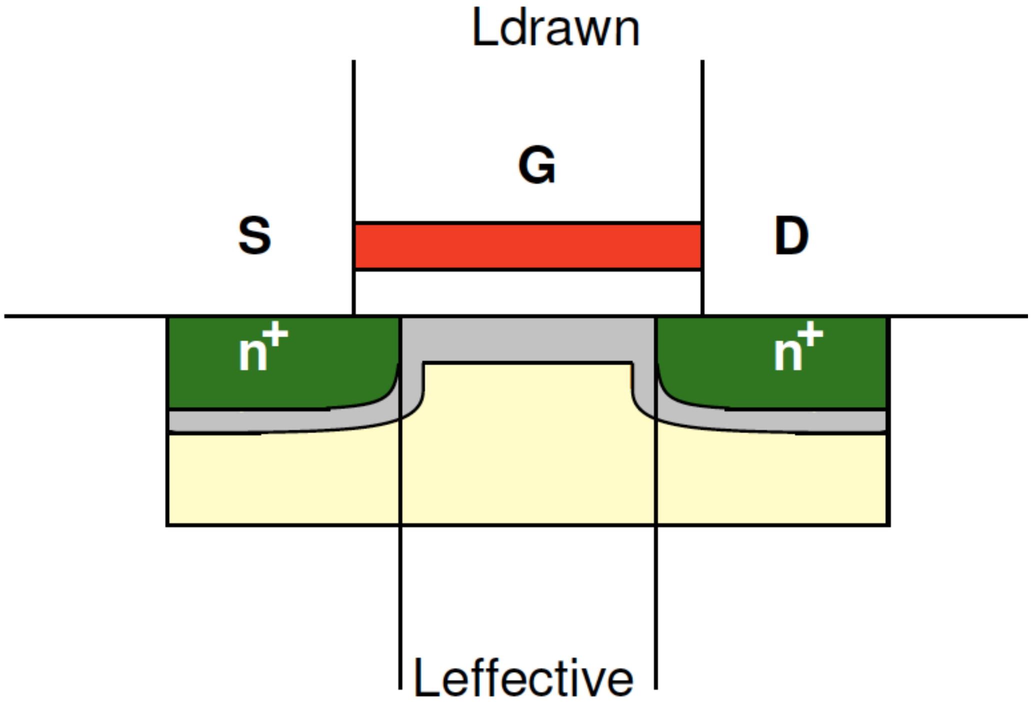
Side



Top

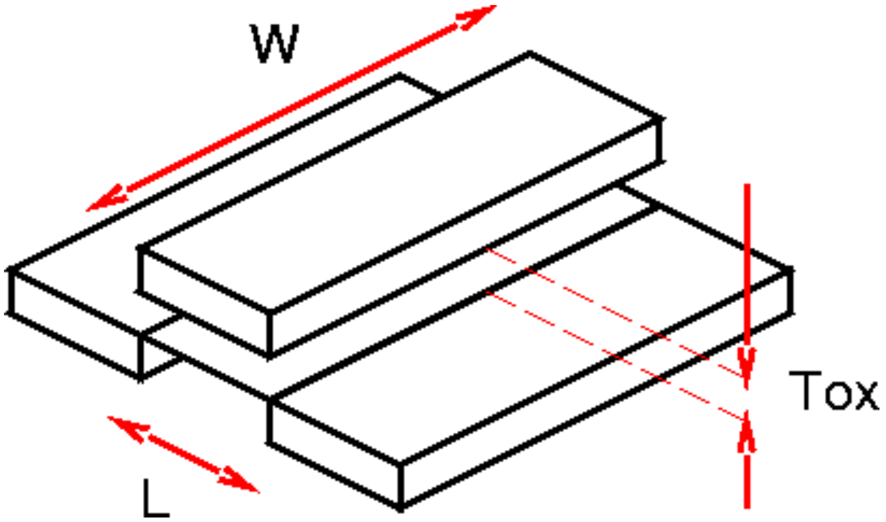
L_{drawn} vs. $L_{effective}$

- Doping not perfectly straight
- Spreads under gate
- Effective L smaller than draw gate width



Transistor Strength (W/L)

$$C_{OX} = \frac{\varepsilon_{OX}}{t_{OX}}$$



$$I_{DS} = \mu_n C_{OX}\left(\frac{W}{L}\right)\left[\left(V_{GS} - V_{th}\right)V_{DS} - \frac{V_{DS}^2}{2}\right]$$

Transistor Strength (W/L)

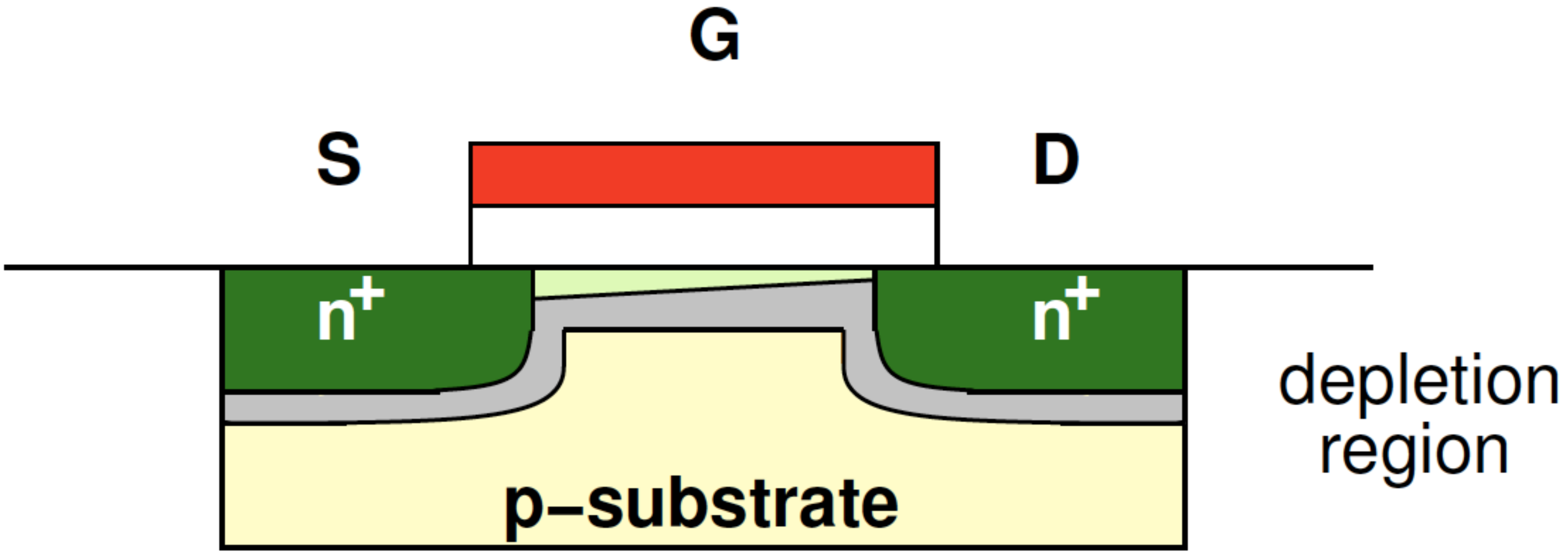
- Shape dependence match Resistance intuition
  - Wider = parallel resistors → decrease R
  - Longer = series resistors → increase R

$$R = \frac{\rho L}{A}$$

$$I_{DS} = \mu_n C_{OX} \left(\frac{W}{L} \right) \left[\left(V_{GS} - V_{th} \right) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

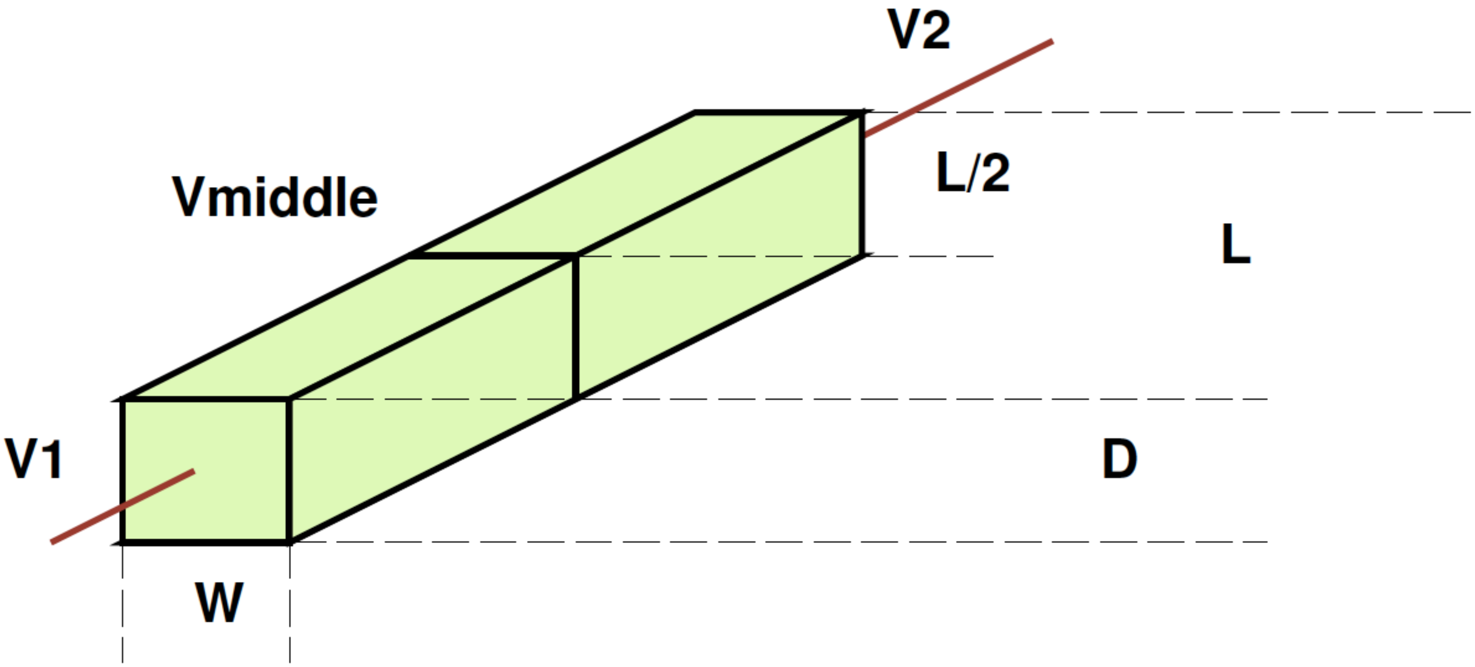
Channel Voltage

- Think of channel as resistor
- Voltage varies along channel



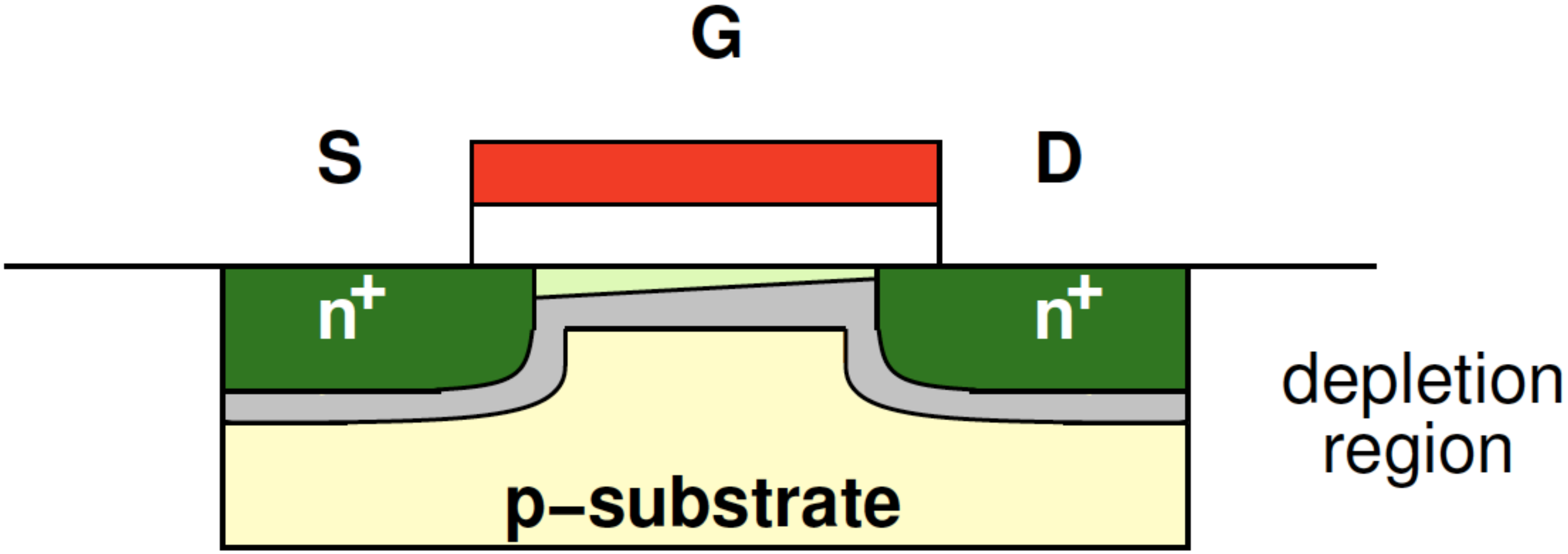
Preclass 2

- What is voltage in the middle of a resistive medium?
  - Relative to V1 and V2
  - halfway between terminals



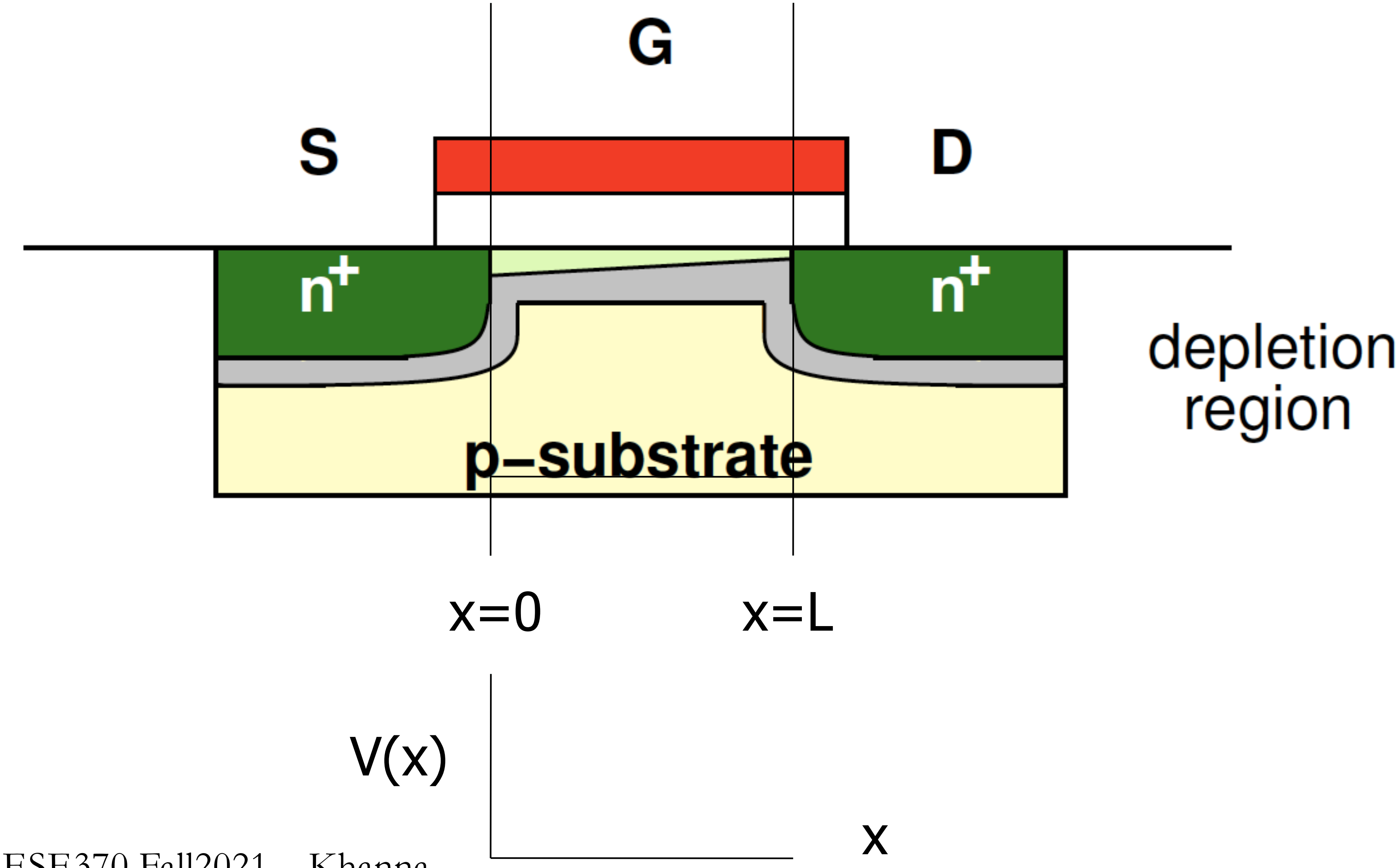
Channel Voltage

- Think of channel as resistor
- Voltage varies along channel
  - Serves as a voltage divider between V_S and V_D



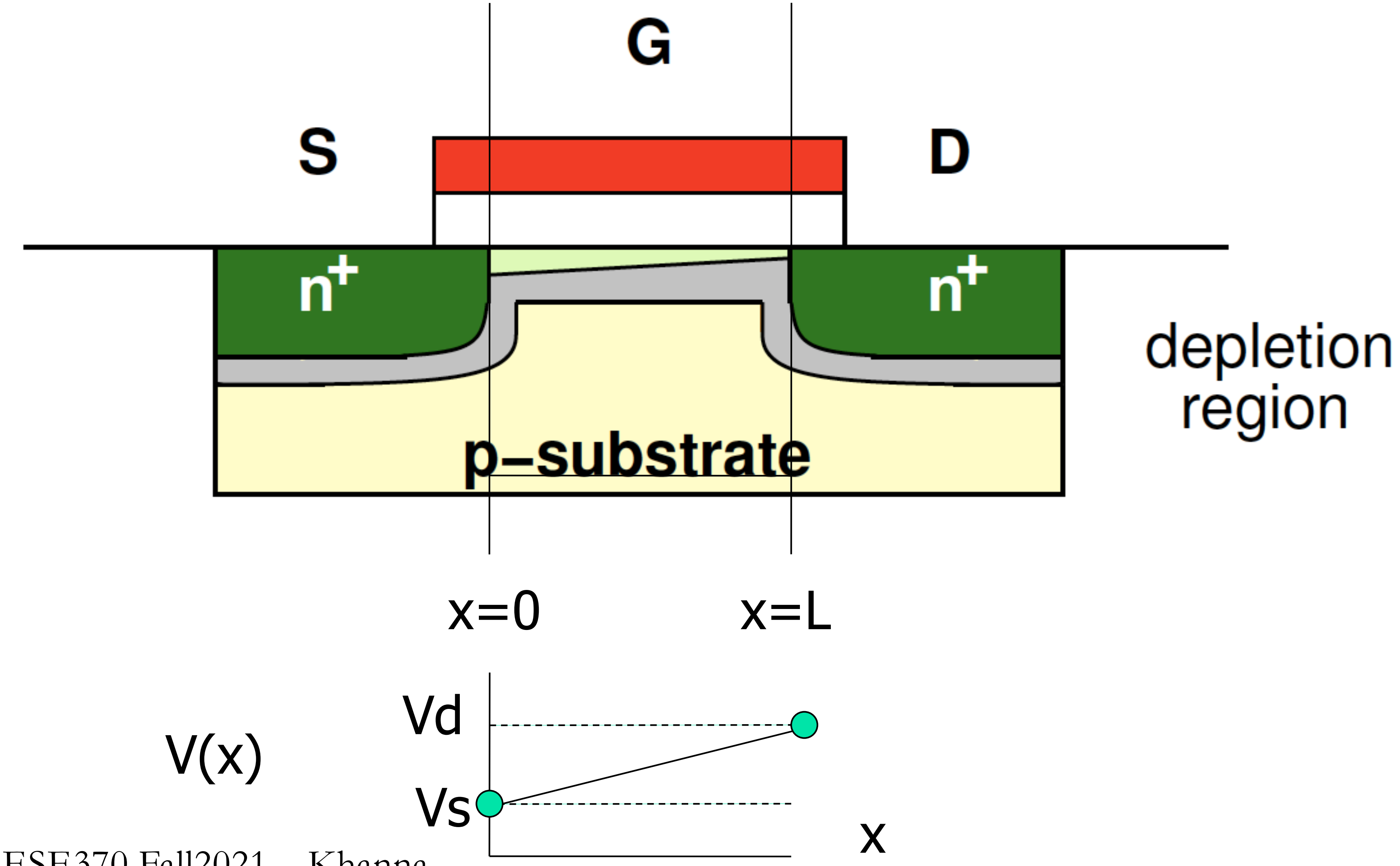
Voltage along Channel

- What does voltage along the channel look like?



Voltage along Channel

- What does voltage along the channel look like?



Voltage along Channel

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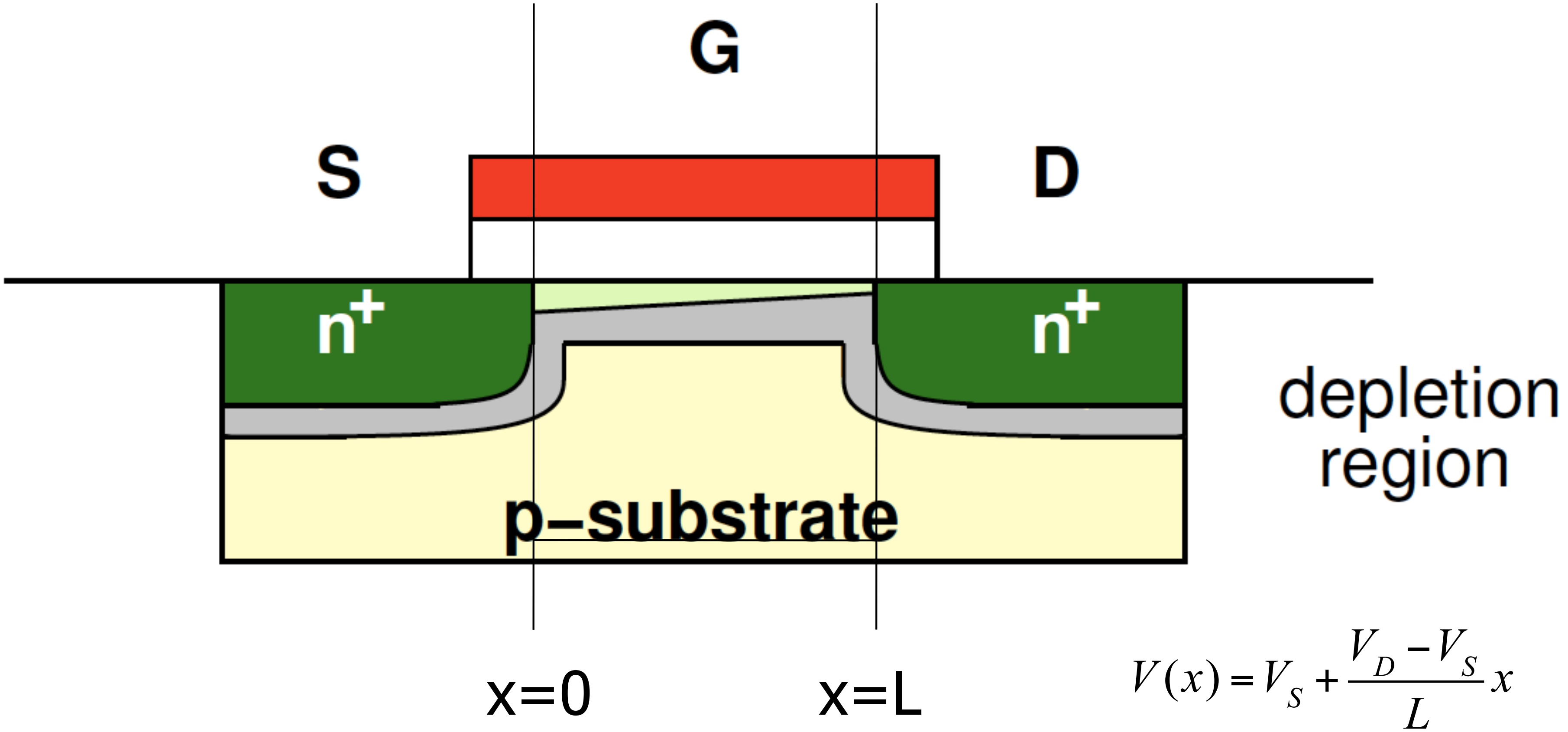
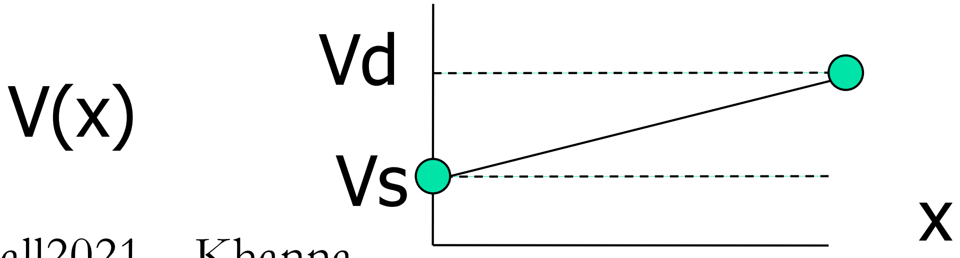




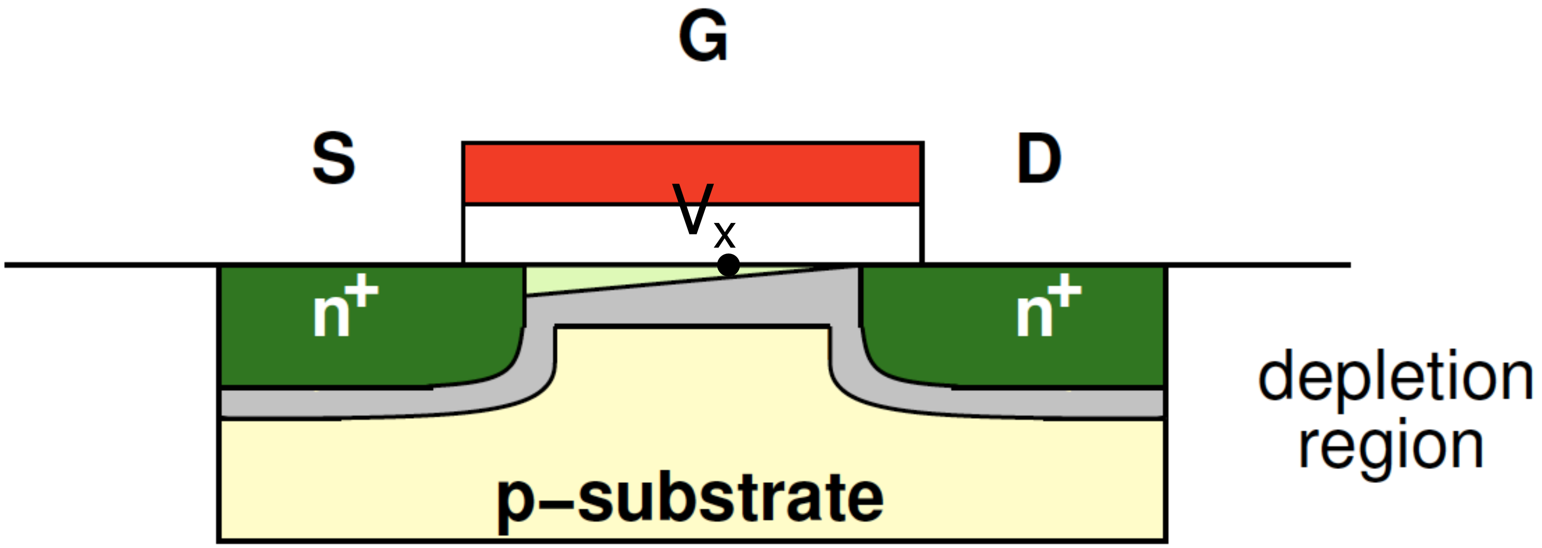
$$V(x) = V_S + \frac{V_D - V_S}{L} x$$

$$V_G - V(x) = V_G - \left(V_S + \frac{V_D - V_S}{L} x \right)$$

$$= V_{GS} - \frac{V_{DS}}{L} x$$

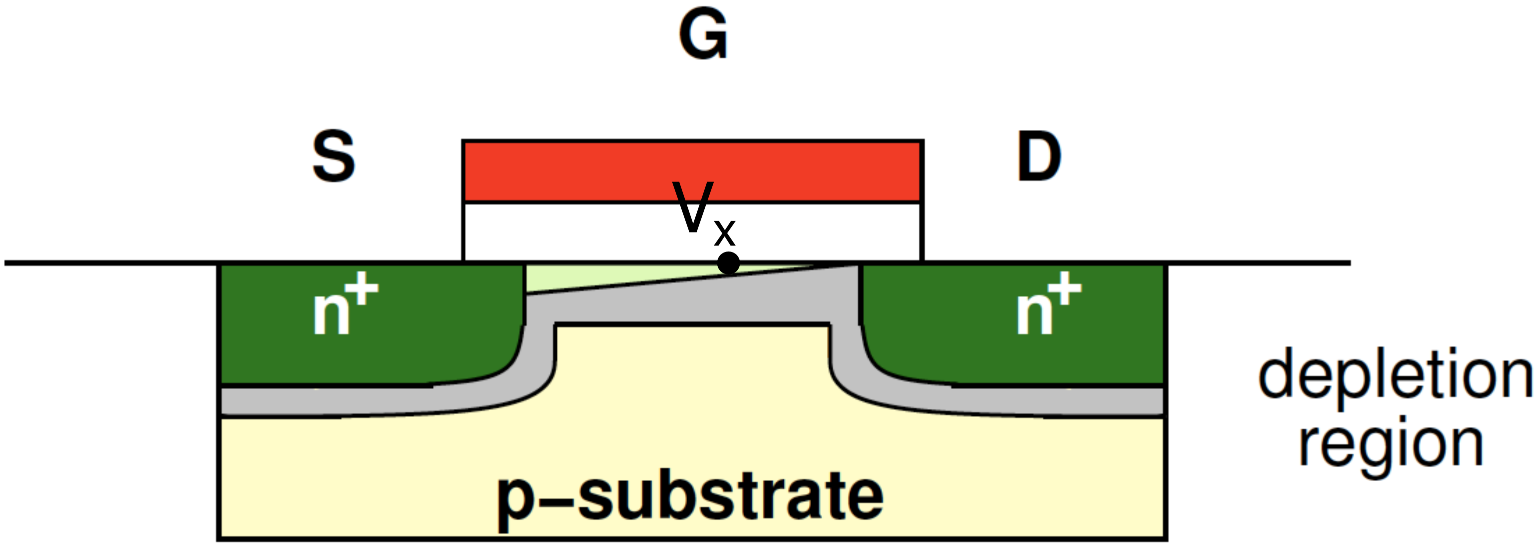
Channel Field

- When voltage gap V_G-V_x drops below V_{th}, drops out of inversion



Channel Field

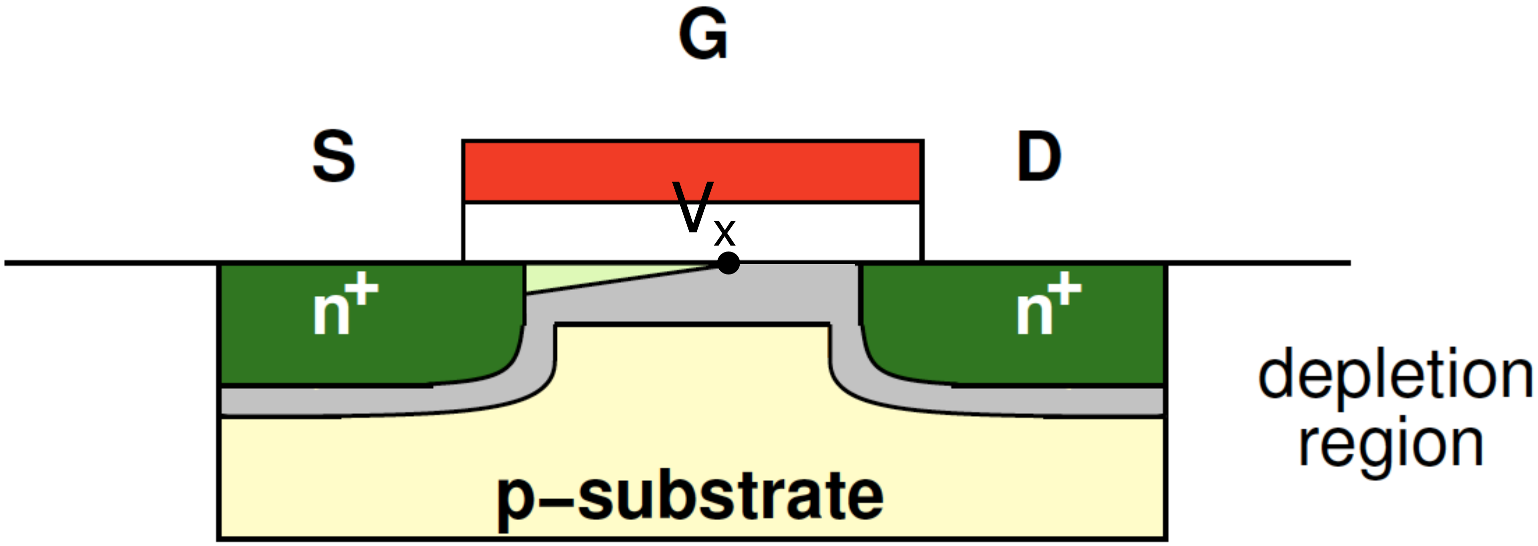
- When voltage gap V_G-V_x drops below V_{th}, drops out of inversion
  - Saturation Edge: $V_{DS} = V_{GS} – V_{th} \rightarrow V_G – V_X(@\ D) = ?$



$$\boxed{V_G - V(x) = V_{GS} - \frac{V_{DS}}{L}x}$$

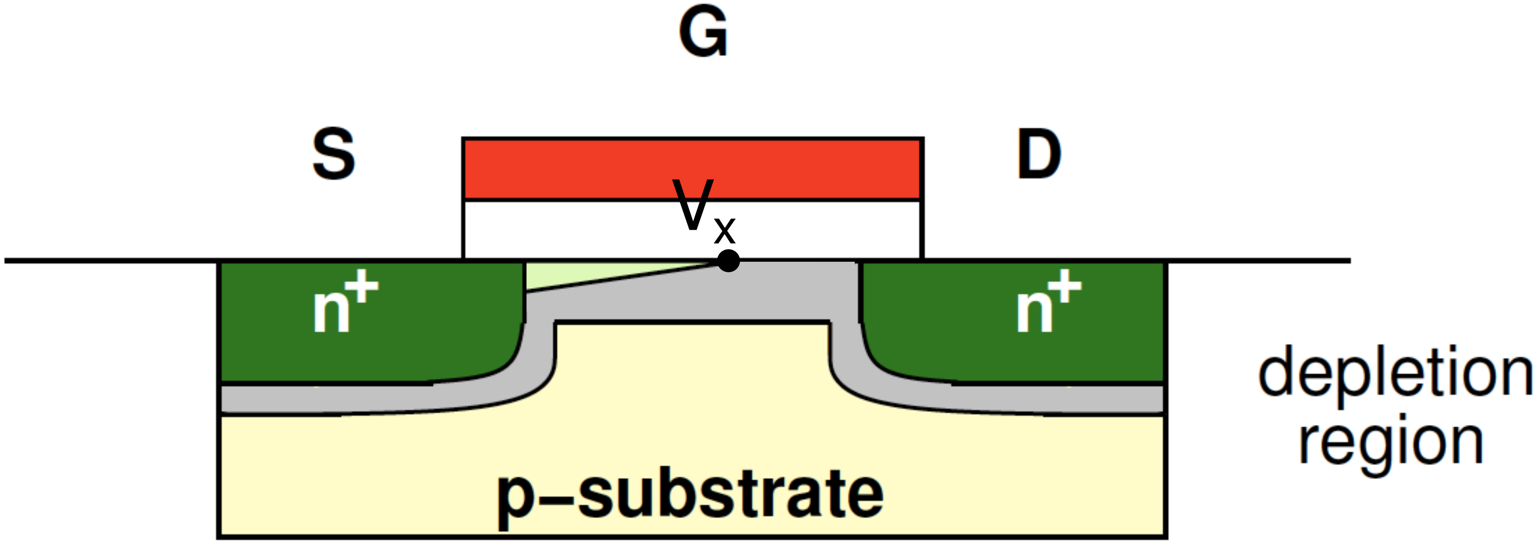
Channel Field

- When voltage gap V_G-V_x drops below V_{th}, drops out of inversion
  - Deep Saturation: $V_{DS} > V_{GS} - V_{th} \rightarrow V_G - V_X(@\, D) = ?$



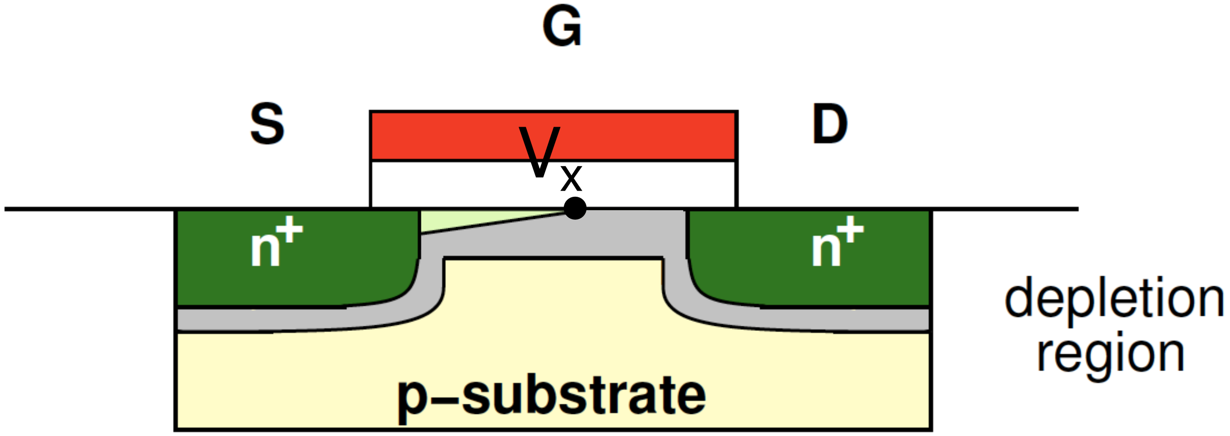
Channel Field

- When voltage gap V_G-V_x drops below V_{th}, drops out of inversion
  - Deep Saturation: $V_{DS} > V_{GS} - V_{th} \rightarrow V_G - V_X(@\,D) < V_{th}$
    Upper limit on current, channel is "pinched off"



Channel Field

- When voltage gap V_G-V_x drops below V_{th}, drops out of inversion
  - What if $V_{DS} > V_{GS} - V_{th}$?
    - Upper limit on current, channel is "pinched off"
    - For what x, is V_G-$V(x)$=V_T?

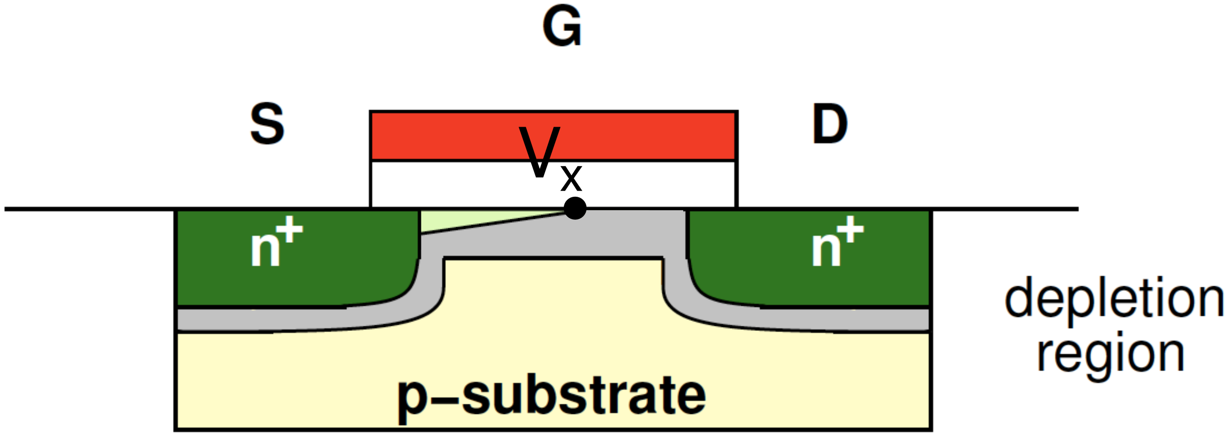


Channel Field

- When voltage gap V_G-V_x drops below V_{th}, drops out of inversion
  - What if $V_{DS} > V_{GS} - V_{th}$?
    - Upper limit on current, channel is "pinched off"
    - For what x, is V_G-$V(x)$=V_T?

$$V_G - V(x) = V_{GS} - \frac{V_{DS}}{L} x = V_T$$

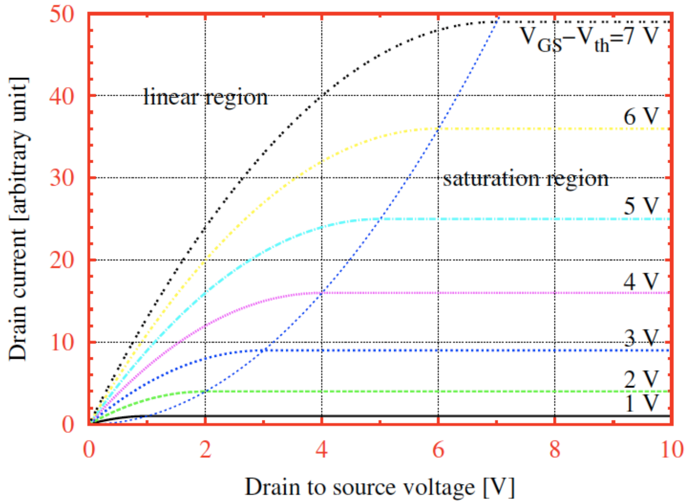
$$x = \frac{V_{GS} - V_T}{V_{DS}} L$$



Pinch Off

- When voltage along the channel drops below V_{th}, the channel drops out of inversion
  - Occurs when: $V_G - V_X(@\ D) < V_{th} \rightarrow V_{DS} > V_{GS} - V_{th}$

- **Conclusion:**
  - current cannot increase with V_{DS} once $V_{DS} > V_{GS}-V_T$
    - Not true! More later…



Saturation

- At edge of saturation, $V_{DS}= V_{GS}-V_T$

$$I_{DS} = \mu_n C_{OX}\left(\frac{W}{L}\right)\left[\left(V_{GS}-V_T\right)V_{DS}-\frac{V_{DS}^2}{2}\right]$$

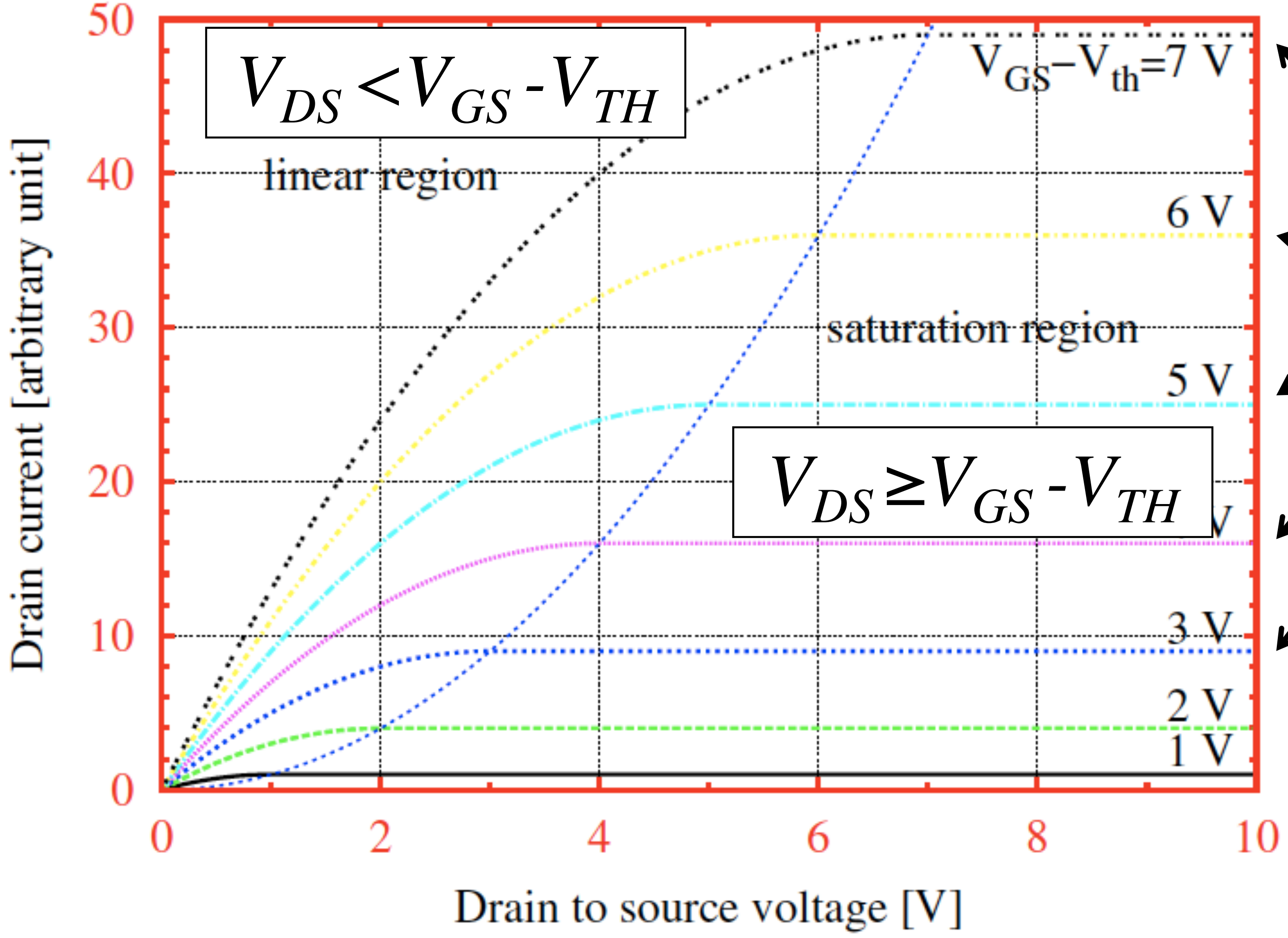
- Becomes:

$$I_{DS} = \mu_n C_{OX}\left(\frac{W}{L}\right)\left[\left(V_{GS}-V_T\right)^2-\frac{\left(V_{GS}-V_T\right)^2}{2}\right]$$

$$I_{DS} = \frac{\mu_n C_{OX}}{2}\left(\frac{W}{L}\right)\left[\left(V_{GS}-V_T\right)^2\right]$$

MOSFET – IV Characteristics

I_{DS}



$V_{DS} < V_{GS} - V_{TH}$

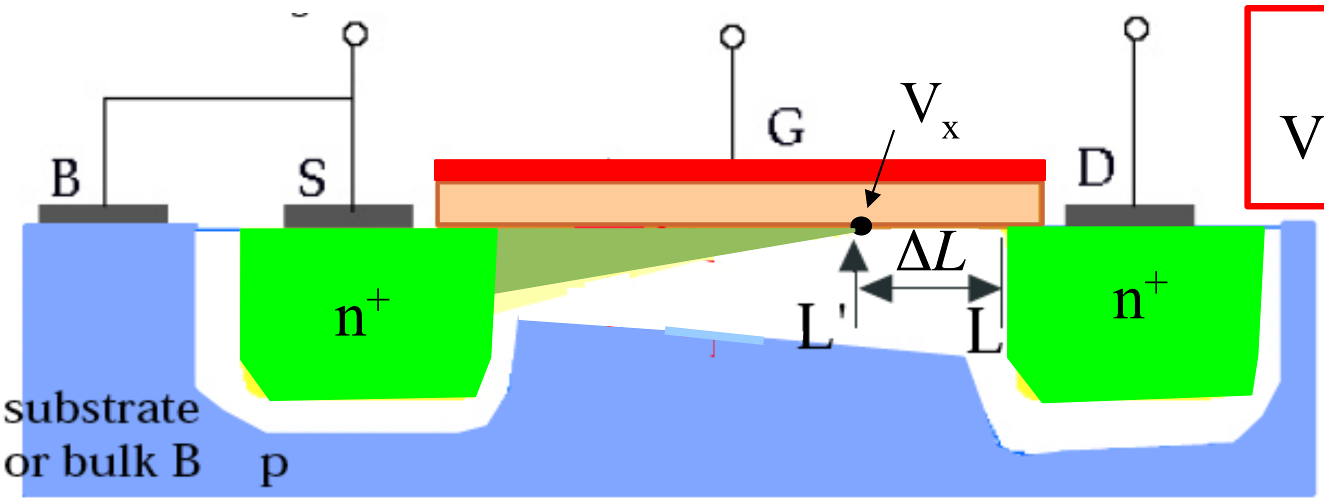
$V_{DS} \geq V_{GS} - V_{TH}$

$V_{GS} - V_{th}$

V_{DS}

Channel Length Modulation

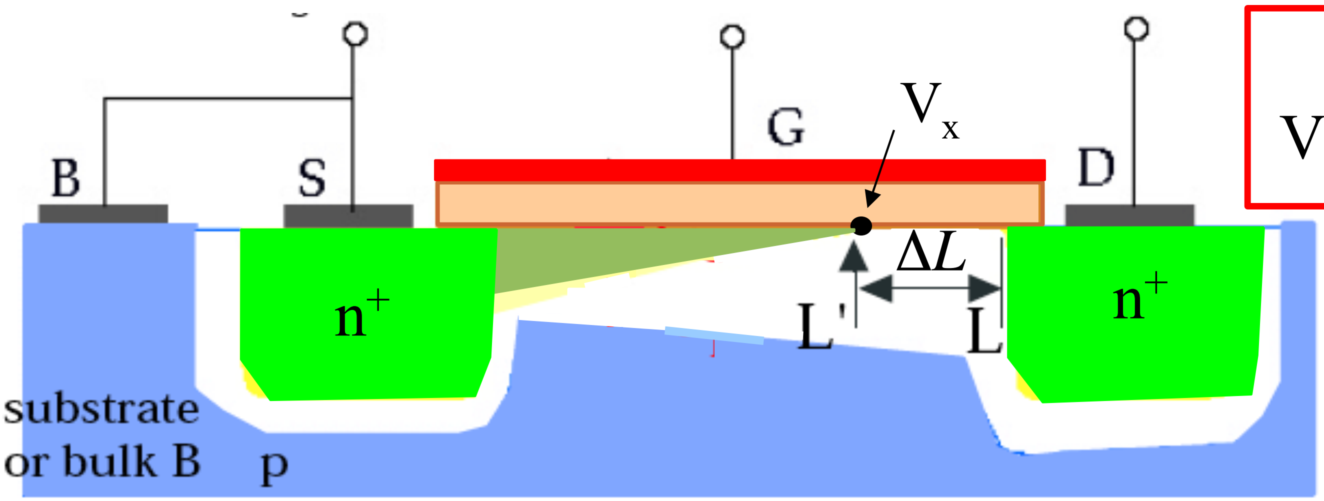
MOSFET IV Characteristics - Saturation



$V_{GS} > V_{T0}$
$V_{DS} > V_{GS} - V_{T0}$

$$I_{DS} = \frac{\mu_n \cdot C_{ox}}{2} \frac{W}{L'} \left(V_{GS} - V_{T0}\right)^2 = \frac{\mu_n \cdot C_{ox}}{2} \frac{W}{L - \Delta L} \left(V_{GS} - V_{T0}\right)^2$$

MOSFET IV Characteristics - Saturation



$V_{GS} > V_{T0}$
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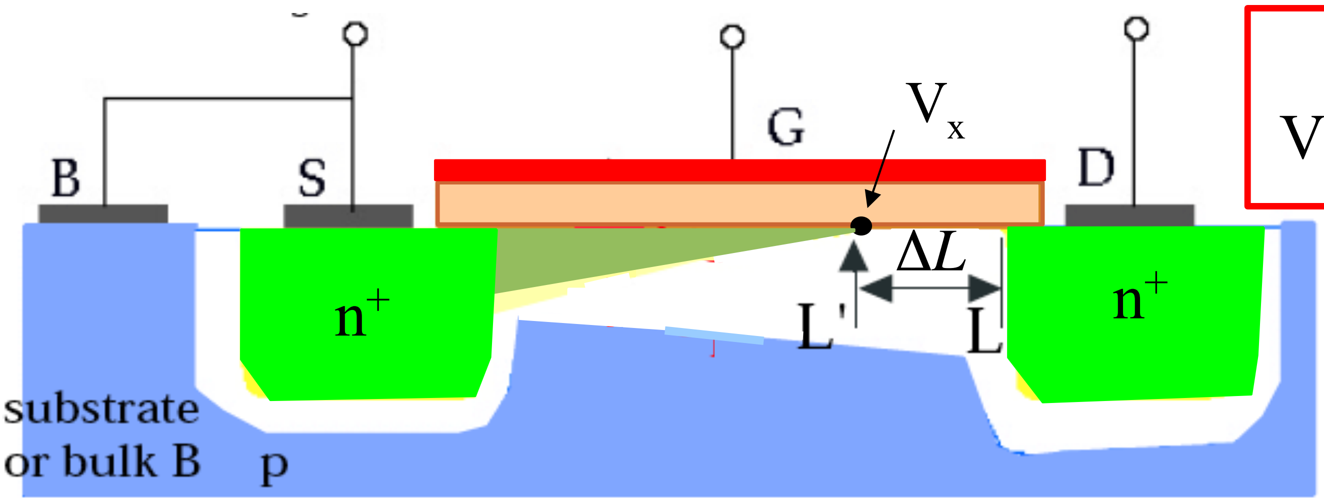
$$= \frac{\mu_n \cdot C_{ox}}{2} \frac{W}{L\left(1 - \frac{\Delta L}{L}\right)}\left(V_{GS} - V_{T0}\right)^2$$

$$= \frac{\mu_n \cdot C_{ox}}{2} \frac{W}{L}\left(V_{GS} - V_{T0}\right)^2 \frac{1}{\left(1 - \frac{\Delta L}{L}\right)}$$

MOSFET IV Characteristics - Saturation

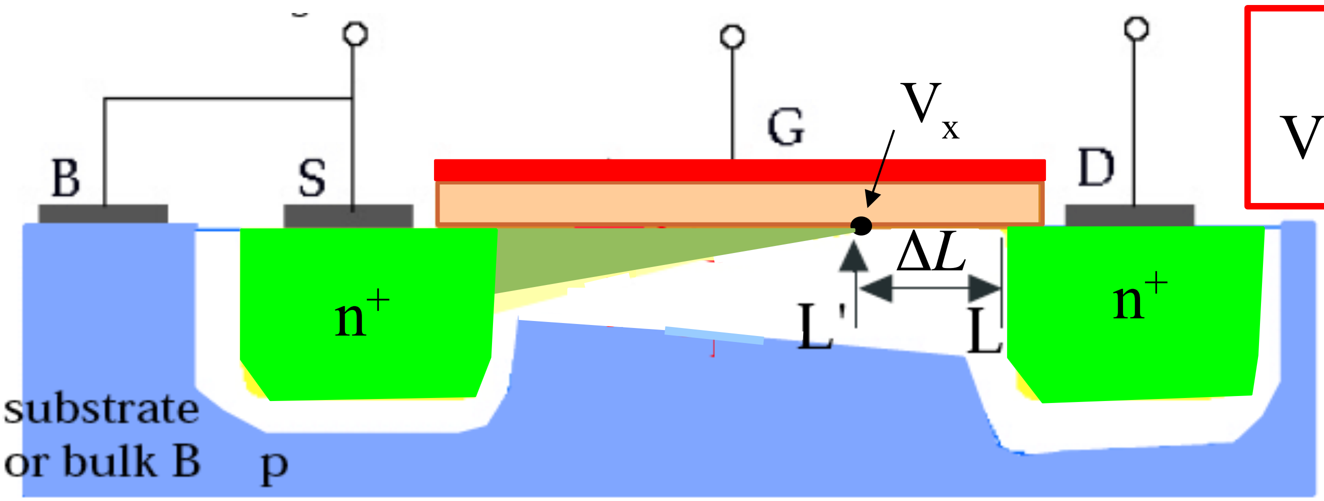


$V_{GS} > V_{T0}$
$V_{DS} > V_{GS} - V_{T0}$

$$I_{DS} = \frac{\mu_n \cdot C_{ox}}{2} \frac{W}{L} \left(V_{GS} - V_{T0}\right)^2 \frac{1}{\left(1 - \frac{\Delta L}{L}\right)}$$

$$\Delta L \propto \sqrt{V_{DS} - (V_{GS} - V_{T0})} \xrightarrow{\text{empirically}} 1 - \frac{\Delta L}{L} \approx 1 - \lambda \cdot V_{DS}$$

MOSFET IV Characteristics - Saturation



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$V_{DS} > V_{GS} - V_{T0}$

$$I_{DS} = \frac{\mu_n \cdot C_{ox}}{2} \frac{W}{L} \left(V_{GS} - V_{T0}\right)^2 \frac{1}{\left(1 - \frac{\Delta L}{L}\right)}$$

$$\Delta L \propto \sqrt{V_{DS} - (V_{GS} - V_{T0})} \quad \xrightarrow{\text{empirically}} \quad 1 - \frac{\Delta L}{L} \approx 1 - \lambda \cdot V_{DS}$$

If $\lambda \cdot V_{DS} << 1$,

$$\left(1 - \frac{\Delta L}{L}\right)^{-1} \approx \boxed{1 + \lambda \cdot V_{DS}}$$

MOSFET IV Characteristics - Saturation

$$I_{DS} = \frac{\mu_n \cdot C_{ox}}{2} \frac{W}{L} \left(V_{GS} - V_{T0}\right)^2 \frac{1}{\left(1 - \frac{\Delta L}{L}\right)}$$

$$1 - \frac{\Delta L}{L} \approx 1 - \lambda \cdot V_{DS}$$

MOSFET IV Characteristics - Saturation

$$I_{DS} = \frac{\mu_n \cdot C_{ox}}{2}\frac{W}{L}\left(V_{GS} - V_{T0}\right)^2 \frac{1}{\left(1 - \frac{\Delta L}{L}\right)}$$

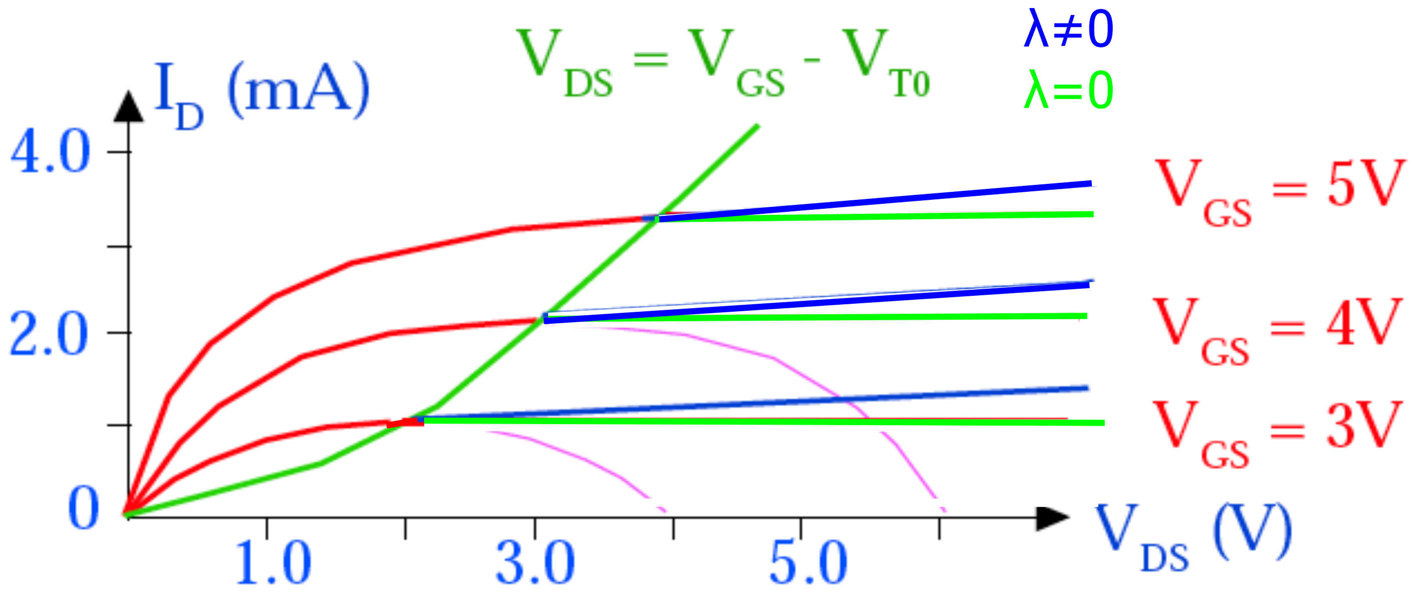
$$1 - \frac{\Delta L}{L} \approx 1 - \lambda \cdot V_{DS}$$

$$\boxed{I_D = \frac{\mu_n \cdot C_{ox}}{2}\frac{W}{L}\left(V_{GS} - V_{T0}\right)^2 (1 + \lambda \cdot V_{DS})}$$

MOSFET IV Characteristics

Linear Region: $$I_D = \mu_n \cdot C_{ox} \frac{W}{L}\left((V_{GS} - V_{T0})V_{DS} - \frac{V_{DS}^2}{2}\right)$$

Saturation Region: $$I_D = \frac{\mu_n \cdot C_{ox}}{2} \frac{W}{L}\left(V_{GS} - V_{T0}\right)^2 (1 + \lambda \cdot V_{DS})$$

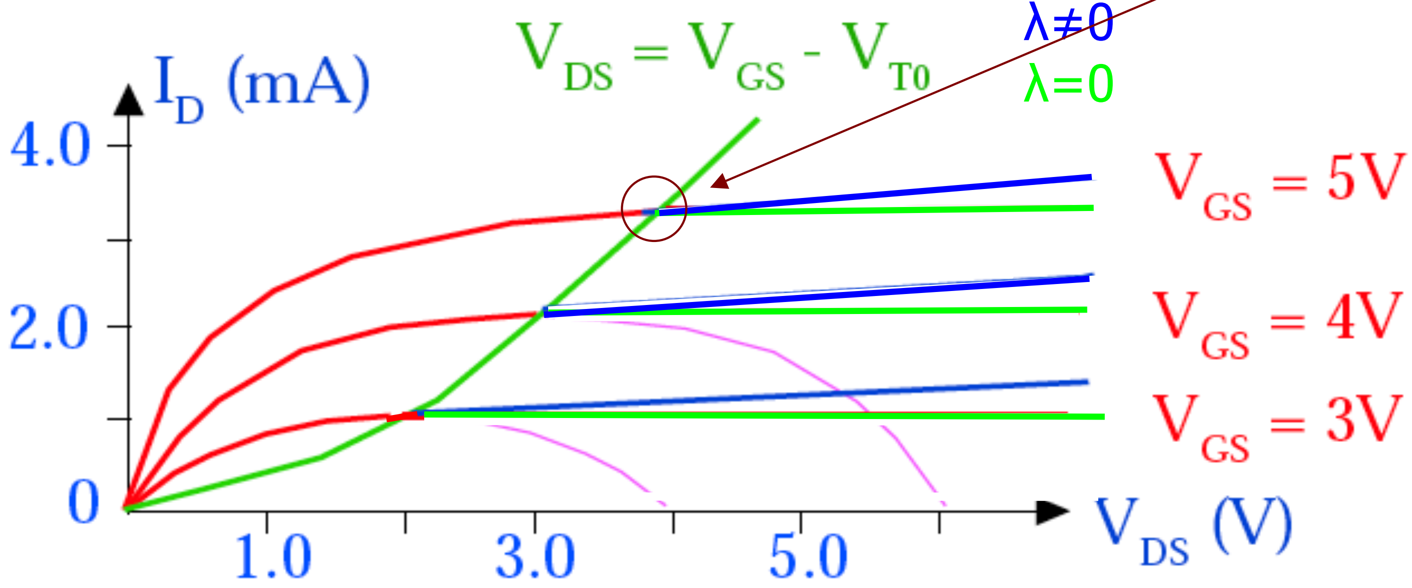


MOSFET IV Characteristics

Linear Region: $I_D = \mu_n \cdot C_{ox} \frac{W}{L}\left((V_{GS} - V_{T0})V_{DS} - \frac{V_{DS}^2}{2}\right)$

Saturation Region: $I_D = \frac{\mu_n \cdot C_{ox}}{2} \frac{W}{L}(V_{GS} - V_{T0})^2 (1 + \lambda \cdot V_{DS})$

DISCONTINUOUS!
@ $V_{DS} = V_{GS} - V_T$



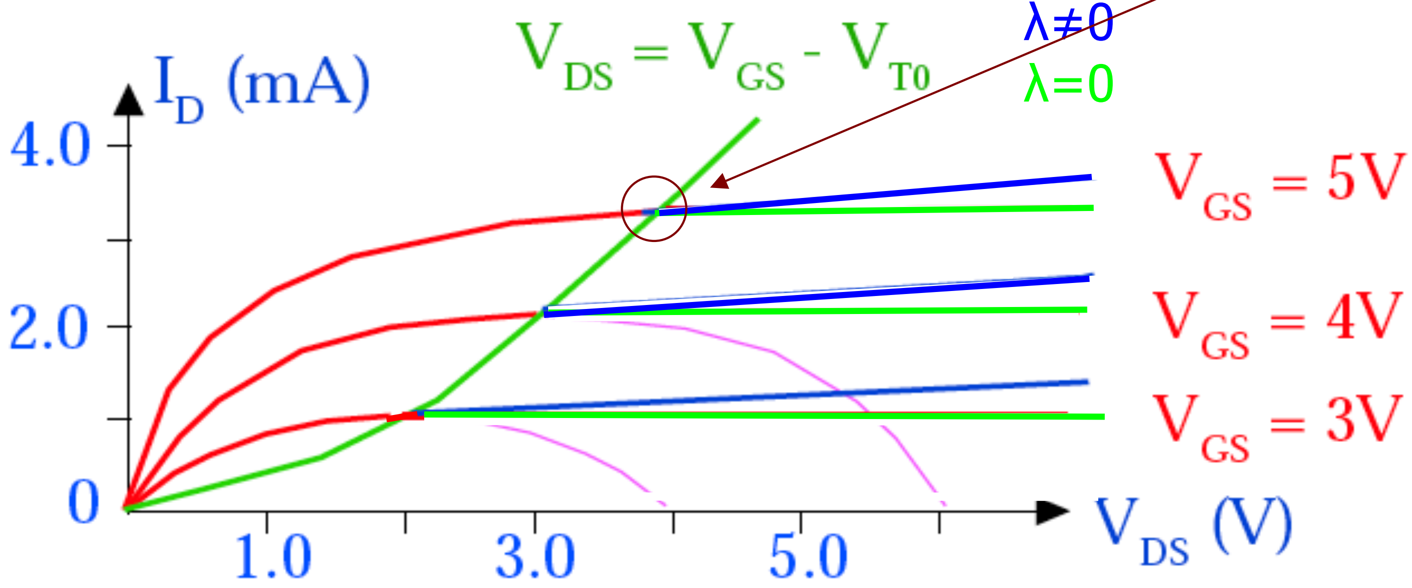
MOSFET IV Characteristics

Linear Region: $$I_D = \mu_n \cdot C_{ox} \frac{W}{L}\left((V_{GS} - V_{T0})V_{DS} - \frac{V_{DS}^2}{2}\right)\boxed{(1+\lambda \cdot V_{DS})}$$

Saturation Region: $$I_D = \frac{\mu_n \cdot C_{ox}}{2}\frac{W}{L}\left(V_{GS} - V_{T0}\right)^2(1+\lambda \cdot V_{DS})$$

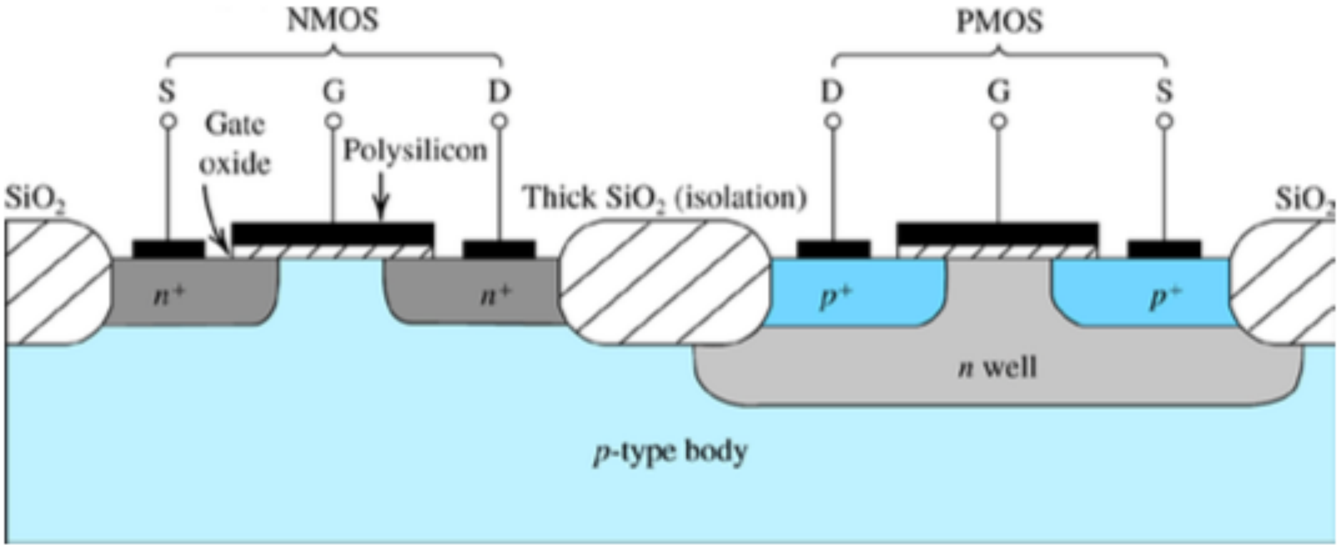
~~DIS~~CONTINUOUS!

@ $V_{DS} = V_{GS}\text{-}V_T$



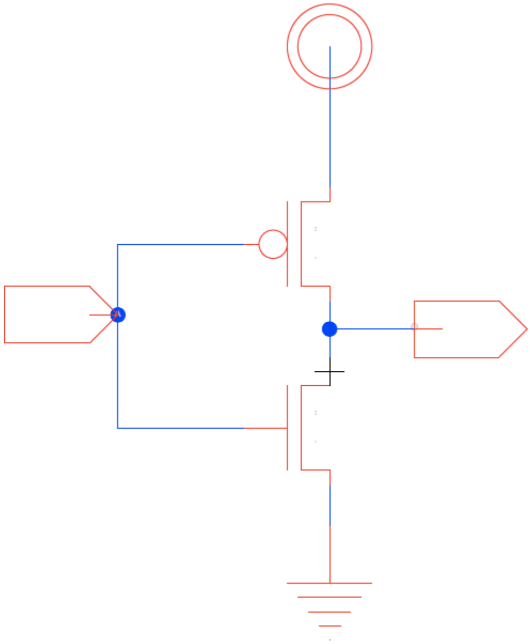
pMOS Device

- Analagous phenomena to NMOS
- Opposite polarity
  - Negative $V_{th,}$ λ
- Reason based on oppositely charged carriers



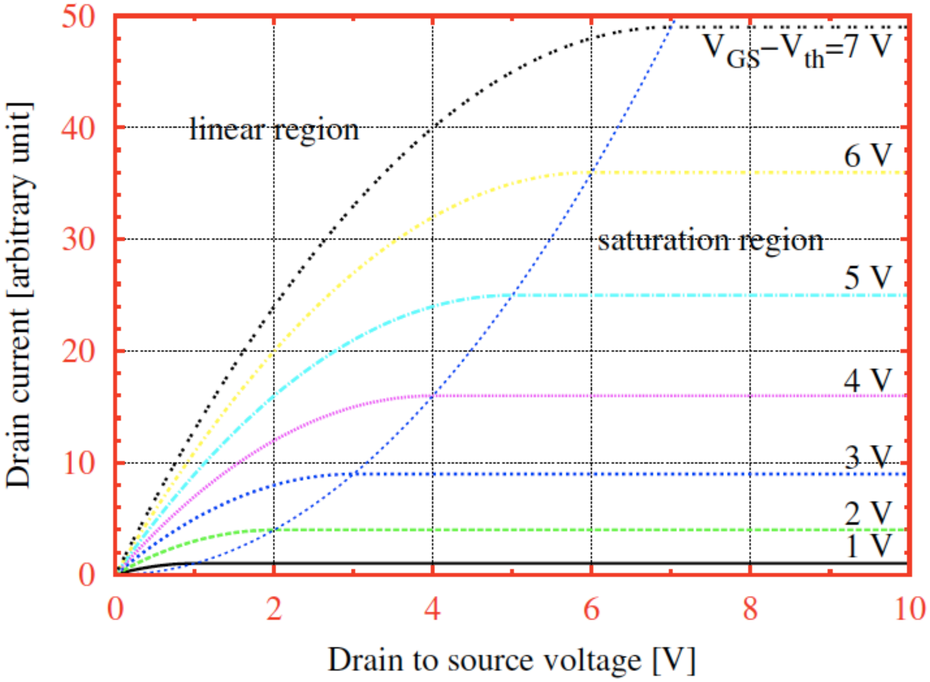
Approach

- Identify Region
- Drives governing equations
  - See preclass reference (pg 2)
- Use region and equations to understand operation



Big Idea

- 3 Regions of operation for MOSFET
  - Linear
  - Saturation
    - With channel length modulation
  - Subthreshold (next class)



Admin

- Text 3.3.2 – highly recommend read!!
  - Second half on Wednesday
- HW2 due tonight
- HW3 out
  - Get started now
  - Long and time-consuming