# University of Pennsylvania Department of Electrical and System Engineering Circuit-Level Modeling, Design, and Optimization for Digital Systems 

- 4 problems with weights indicated.
- Parts within a problem will not be weighted equally.
- Calculators allowed.
- Closed book $=$ No text or notes allowed.

Name: Answers

Grade:

| Q1 |  |
| ---: | :--- |
| Q2 |  |
| Q3 |  |
| Q4 |  |
| Total | Mean: 92.4, Stdev: 7.2 |

1. (25 points) Identify if the following circuits are CMOS, why or why not, and their functions. [Show your work for partial credit consideration.]

(a)

| CMOS? (circle one) | Yes No |
| ---: | :--- |
| (if CMOS) Function (Out) |  |
| (if not CMOS) Why not? | Short from Vdd to <br> Gnd for $\mathrm{A}=\mathrm{B}=\mathrm{C}=1$ |



| CMOS? (circle one) | Yes No |
| ---: | :--- |
| (if CMOS) Function (Out) | $\overline{O u t}=A \cdot B \cdot C+\bar{A} \cdot \bar{B} \cdot \bar{C}$ |
| (if not CMOS) Why not? |  |



| CMOS? (circle one) | Yes No |
| ---: | :--- |
| (if CMOS) Function (Out) |  |
| (if not CMOS) Why not? | Short from Vdd to <br> Gnd when $\mathrm{A}=1$, <br> $\mathrm{B}=\mathrm{C}=0$ |

2. (25 points) Consider the following circuit:


Assume:

- all transistors are same size.
- all on transistors have resitance $R_{o n}$.
- all transistors have total gate capacitance $C_{g}$.
(a) Identify the output function.

(b) Lowest resistance driving output?

Case: $\mathrm{A}=\mathrm{B}=\mathrm{C}=\mathrm{D}=0$
Resistance: $\frac{10}{11} R_{o n}$
(c) Highest resistance driving output?
Case: $A=B=0, C=D=1$
Resistance: $2 R_{\text {on }}$
(d) Lowest capacitance of an input?

Which: C
Capacitance: $C_{g}$
(e) Highest capacitance of an input?

Which: B, D
Capacitance: $3 C_{g}$
(f) Worst-case 10-90 rise time for one of these gates driving a single input of another of these gates?
Case: with $\mathrm{B}=0, \mathrm{C}=\mathrm{D}=1, \mathrm{~A}: 1 \rightarrow 0$
Rise Time Expression: $2.2 * 2 R_{\text {on }} * 3 C_{g}=13.2 R_{\text {on }} C_{g}$
[Show calculation for partial credit consideration.]
3. (25 points) Consider the following circuit and its voltage transfer characteristic as well as the voltage at node X. Assume $V_{d d}=800 \mathrm{mV}$ and $V_{t h n}=-V_{t h p}=200 \mathrm{mV}$.


For the regions A-E labeled on the transfer function, identify the region of operation of the nMOS and both pMOS devices. Fill out the table below.

|  | N1 | P1 | P2 |
| :---: | :---: | :---: | :---: |
| A | Subthreshold | Linear | Saturation* |
| B | Subthreshold | Saturation | Saturation |
| C | Saturation | Saturation | Saturation |
| D | Linear | Saturation | Saturation* |
| E | Linear | Subthreshold | Saturation* |

Subthreshold and Saturation were both accepted.
For reference on the next page are the transistor current models and additional work space if needed:

NMOS:

| $V_{G S}$ | $V_{D S}$ | Mode | $I_{D S}$ |
| :---: | :---: | :---: | :---: |
| $\geq V_{\text {thn }}$ | $<V_{G S}-V_{\text {thn }}$ | Linear | $\mu_{n} C_{o x} \frac{W}{L} \cdot\left(\left(V_{G S}-V_{t h n}\right) \cdot V_{D S}-\frac{V_{D S}^{2}}{2}\right)$ |
|  | $\geq V_{G S}-V_{t h n}$ | Saturation | $\frac{1}{2} \mu_{n} C_{o x} \frac{W}{L} \cdot\left(V_{G S}-V_{t h n}\right)^{2}$ |
| $<V_{t h}$ |  | Subthreshold | 0 |

PMOS:

| $V_{G S}$ | $V_{D S}$ | Mode | $I_{D S}$ |
| :---: | :---: | :---: | :---: |
| $\leq V_{t h p}$ | $>V_{G S}-V_{t h p}$ | Linear | $\mu_{p} C_{o x} \frac{W}{L} \cdot\left(\left(V_{G S}-V_{t h p}\right) \cdot V_{D S}-\frac{V_{D S}^{2}}{2}\right)$ |
|  | $\leq V_{G S}-V_{t h p}$ | Saturation | $\frac{1}{2} \mu_{n} C_{o x} \frac{W}{L} \cdot\left(V_{G S}-V_{t h p}\right)^{2}$ |
| $>V_{t h}$ |  | Subthreshold | 0 |

4. (25 points) For this problem we consider a new technology that is not necessarily MOS. You can assume $V_{d d}=1 V$ We are told there are two "gates" available to us:

- $P$-gate: 1-input gate where:

$$
V_{\text {out }}=\left\{\begin{array}{cc}
1-0.5 V_{\text {in }} & V_{\text {in }}<0.3  \tag{1}\\
1.6-2.5 V_{\text {in }} & 0.3 \leq V_{\text {in }}<0.6 \\
0.25-0.25 V_{\text {in }} & 0.6 \leq V_{\text {in }}
\end{array}\right.
$$

- $Q$-gate: 2-input gate where: $V_{o u t}=\left|V_{A}-V_{B}\right|$
(a) What logical function does the restoring P-gate perform? Identify the noise margins that will provide restoration for this gate.
Function: Inverter

| $V_{O H}$ | 850 mV |
| :--- | :--- |
| $V_{I H}$ | 600 mV |
| $V_{I L}$ | 300 mV |
| $V_{O L}$ | 100 mV |
| $N M_{L}$ | 200 mV |
| $N M_{H}$ | 250 mV |

(b) What logical function does the non-restoring Q-gate perform and is it restoring? Draw a combination of $P$ and $Q$ gates that will serve as a restoring gate that performs the same logic function and identify the noise margins that will provide restoration for this new gate.
Function: XOR


| $V_{O H}$ | 750 mV |
| :--- | :--- |
| $V_{I H}$ | 600 mV |
| $V_{I L}$ | 300 mV |
| $V_{O L}$ | 150 mV |
| $N M_{L}$ | 150 mV |
| $N M_{H}$ | 150 mV |

$V_{I L}$ and $V_{I H}$ are the inflection points of the P-gates. The outputs of each P-gate will be $>0.85$ or $<0.1$. The worst-case low output will be when the P-gate outputs are 1 and 0.85 , thus $V_{O L}=0.15$; the worst-case high output will be when the P-gate outputs are 0.1 and $0.85, V_{O H}=0.75$

