ESE370: Circuit-Level Modeling, Design, and Optimization for Digital Systems

Lec 10: February 27, 2023

Energy and Power Tradeoffs

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Previously

- Three components of power
  - Static
  - Dynamic
  - Short circuit
- $P_{tot} = P_{static} + P_{dyn} + P_{sc}$

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Switching

Dynamic Power

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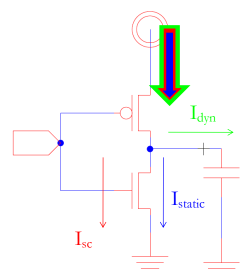
Switching Currents

- $I_{total}(t) = I_{static}(t) + I_{switch}(t)$
- $I_{switch}(t) = I_{sc}(t) + I_{dyn}(t)$



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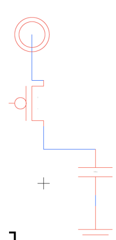
Charging

- $I_{dyn}(t)$ – why is it changing?
  - $I_{ds} = f(V_{ds}, V_{gs})$
  - and V_{gs}, V_{ds} changing



$$I_{DS} \approx v_{sat} C_{OX} W \left(V_{GS} - V_T - \frac{V_{DSAT}}{2} \right)$$

$$I_{DS} = \mu_n C_{OX} \left(\frac{W}{L} \right) \left[\left(V_{GS} - V_T \right) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

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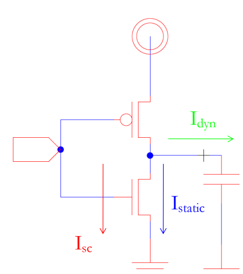
Switching Energy – focus on $I_{dyn}(t)$



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Switching (Dynamic)

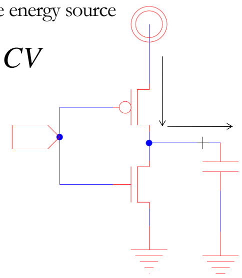
- CMOS circuit switching output from 0→1
  - Spends energy CV_{dd}^2 charging load
  - CV_{dd}^2 energy is pulled from the energy source

$$Q = \int I(t)\,dt = CV$$

$$E = \int P(t)dt$$

$$E = V_{dd} \int I(t)dt$$

$$E = CV_{dd}^2$$



Switching Power

- Every time output switches 0→1 pay:
  - $E = CV^2$
- P_{dyn} = (# 0→1 trans) × CV^2 / time

Charging Power

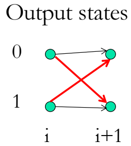
- P_{dyn} = (#0→1 trans) × CV^2 / time
- Often like to think about switching frequency
- Useful to consider per clock cycle
  - Frequency f = 1/clock-period = clock-cycles/time
- P_{dyn} = (#0→1 trans/clock-cycle) CV^2 f

Data Dependent Activity

- Consider an 8b counter
  - How often do each of the following switch?
    - Low bit?
    - High bit?
- Assuming random inputs
  - Activity at output of nand2?
  - Activity at output of xor2?

Gate Output Switching (random inputs)

Output states

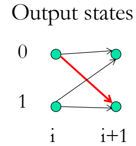


$P(out_i != out_{i+1}) = P(out_i=0)*P(out_{i+1}=1) + P(out_i=1)*P(out_{i+1}=0)$

Probablity of output switch of nand2?
Probablity of output switch of xor2?

Gate Output Switching (random inputs)

Output states



$P(out_i \rightarrow out_{i+1} = 0 \rightarrow 1) = P(out_i=0)*P(out_{i+1}=1)$

Dynamic Power

- P_{dyn} = (#0→1 trans/clock-cycle) CV^2 f
- Let a = activity factor
  - a = average $\#tran_{0\to1}$/clock
  - a = probability of $\#tran_{0\to1}$
- $P_{dyn} = aCV^2 f$

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Activity Factor

- Let a = activity factor
  - a = average $\#tran_{0\to1}$/clock
  - a = probability of $\#tran_{0\to1}$

$$a = p(out_i = 0)p(out_{i+1} = 1)$$

$$a = \frac{N_0}{2^N}\frac{N_1}{2^N} = \frac{N_0(2^N - N_0)}{2^{2N}}$$

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Switching

Short Circuit Power

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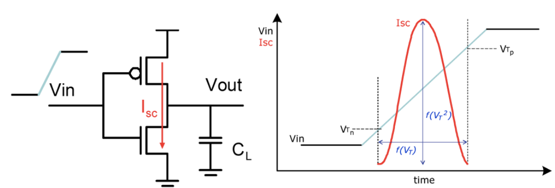
Short Circuit Power

- Between V_{TN} and $V_{dd} - V_{TP}$
  - Both N and P devices conducting



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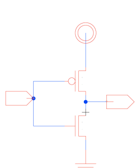
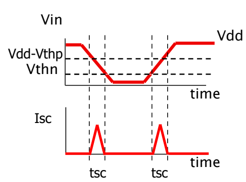
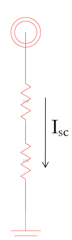
Short Circuit Power

- Between V_{TN} and $V_{dd} - V_{TP}$
  - Both N and P devices conducting
- Roughly:







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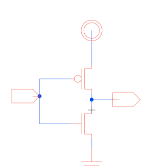
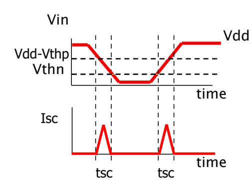
Peak Current

- I_{peak} around $V_{dd}/2$
  - If $|V_{TN}| = |V_{TP}|$ and sized equal rise/fall

$$I_{DS} \approx v_{sat} C_{OX} W\left(V_{GS} - V_T - \frac{V_{DSAT}}{2}\right)$$





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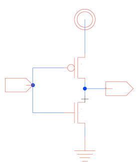
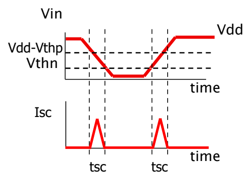
Peak Current

- I_{peak} around $V_{dd}/2$
  - If $|V_{TN}|=|V_{TP}|$ and sized equal rise/fall

$$I_{DS} \approx v_{sat} C_{OX} W \left(V_{GS} - V_T - \frac{V_{DSAT}}{2} \right)$$

$$\int I(t)dt \approx I_{peak} \times t_{sc} \times \left(\frac{1}{2}\right)$$





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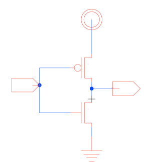
Peak Current

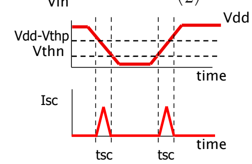
- I_{peak} around $V_{dd}/2$
  - If $|V_{TN}|=|V_{TP}|$ and sized equal rise/fall

$$I_{DS} \approx v_{sat} C_{OX} W \left(V_{GS} - V_T - \frac{V_{DSAT}}{2} \right)$$

$$\int I(t)dt \approx I_{peak} \times t_{sc} \times \left(\frac{1}{2}\right)$$

$$E = V_{dd} \times I_{peak} \times t_{sc} \times \left(\frac{1}{2}\right)$$





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Short Circuit Energy

- Make it look like switching an equivalent capacitance, C_{SC}

$$E = V_{dd} \times \left(I_{peak} \times t_{sc} \times \frac{1}{2} \right)$$

$$E = C_{SC} V_{dd}^2$$

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Short Circuit Energy

- Make it look like switching an equivalent capacitance, C_{SC}

$$E = V_{dd} \times \left(I_{peak} \times t_{sc} \times \frac{1}{2} \right)$$

$$E = C_{SC} V_{dd}^2$$

$$\boxed{C_{SC} = \frac{I_{peak} t_{sc}}{2V_{dd}}}$$

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Short Circuit Energy

- Every time switch (0→1 and 1→0)
  - Also dissipate short-circuit energy: $E = C_{sc}V^2$
  - C_{cs} “fake” capacitance (for accounting)

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Total Power

- $P_{tot} = P_{static} + P_{sc} + P_{dyn}$
- $P_{dyn} + P_{sc} = aC_{load}V^2f + 2aC_{sc}V^2f$
- $P_{tot} \approx a(C_{load} + 2C_{sc})V^2f + VI'_s(W/L)e^{-Vt/(nkT/q)}$

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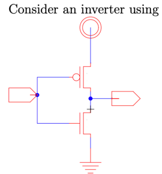
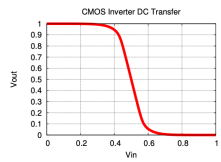
Preclass 1 (Done in last lecture)

Device	V_{gs}	I_d
NMOS	$V_{gs} < V_{thn}$	$(3 \times 10^{-7})\, e^{\frac{V_{gs}-V_{thn}}{40mV}}$
	$V_{gs} > V_{thn}$	$1.8 \times 10^{-4}\,(V_{gs} - V_{thn})$
PMOS	$V_{gs} > V_{thp}$	$(3 \times 10^{-7})\, e^{-\left(\frac{V_{gs}-V_{thp}}{40mV}\right)}$
	$V_{gs} < V_{thp}$	$-1.8 \times 10^{-4}\,(V_{gs} - V_{thp})$

Consider an inverter using the pmos and nmos devices described above:





Preclass 1

V_{in}	I_{static}	$I_{dynamic}$	I_{sc}
0V			
140mV			
400mV			
500mV			
600mV			
860mV			
1V			

Design Tradeoffs

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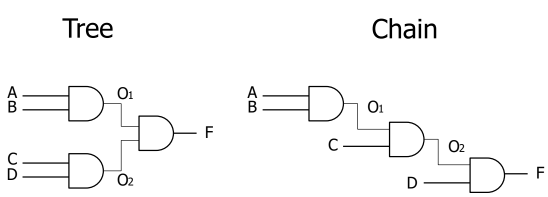
Reduce Dynamic Power?

- $P_{dyn} = aCV^2 f$
- How do we reduce dynamic power?

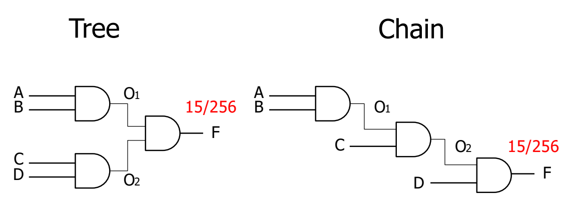
Reduce Activity Factor



$$a = p(out_i = 0)\, p(out_{i+1} = 1)$$

$$a = \frac{N_0}{2^N}\frac{N_1}{2^N} = \frac{N_0(2^N - N_0)}{2^{2N}}$$

Reduce Activity Factor



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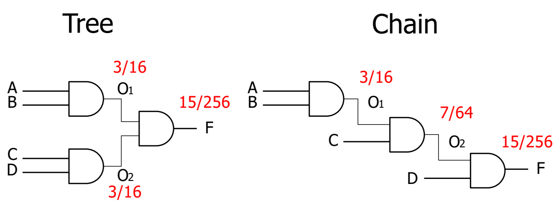
Reduce Activity Factor



$$a = p(out_i = 0)p(out_{i+1} = 1)$$

$$a = \frac{N_0}{2^N}\frac{N_1}{2^N} = \frac{N_0(2^N - N_0)}{2^{2N}}$$

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Reduce V_{dd} (Preclass 2)

- V_{dd}=520mV, V_{thn}=|V_{thp}|=300mV

V_{in}	I_{static}	$I_{dynamic}$	I_{sc}
0V			
140mV			
260mV			
380mV			
520mV			

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Reduce V_{dd} (Preclass 2)

- V_{dd}=520mV, V_{thn}=|V_{thp}|=300mV

V_{in}	I_{static}	$I_{dynamic}$	I_{sc}
0V	180pA	39.6uA	
140mV	6nA	14.4uA	
260mV	111nA		
380mV	6nA	14.4uA	
520mV	180pA	39.6uA	

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Reduce V_{dd}

- What happens as reduce V?
  - Energy?
    - Static
    - Switching
  - Delay?

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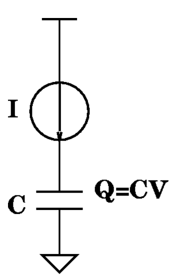
Reduce V_{dd}:

- τ_{gd}=Q/I=(CV)/I
- $I_d=(\mu C_{OX}/2)(W/L)(V_{gs}-V_{TH})^2$
- τ_{gd} impact?
- $\tau_{gd} \propto \frac{1}{V}$



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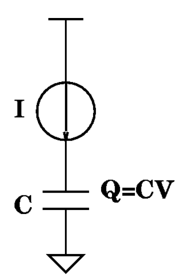
Reduce V_{dd}:

- τ_{gd}=Q/I=(CV)/I
- $I_d=(\mu C_{OX}/2)(W/L)(V_{gs}-V_{TH})^2$
- τ_{gd} impact?
- $\tau_{gd} \propto \frac{1}{V}$
- Ignoring leakage:

$$E \propto V^2$$



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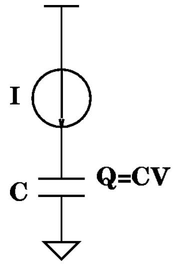
Reduce V_{dd}:

- $\tau_{gd}=Q/I=(CV)/I$
- $I_d=(\mu C_{OX}/2)(W/L)(V_{gs}-V_{TH})^2$
- τ_{gd} impact?
- $\tau_{gd} \propto \frac{1}{V}$
- Ignoring leakage:

$$E \propto V^2$$

$$E\tau^2 \approx Const$$



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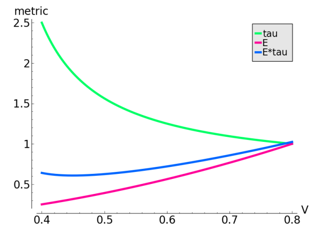
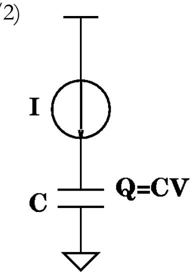
Reduce V_{dd}: Velocity Saturation

- $\tau_{gd}=Q/I=(CV)/I$
- $I_d=(\nu_{sat}C_{OX})(W)(V_{gs}-V_{TH}-V_{DSAT}/2)$
- τ_{gd} impact?





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Reduce V_{dd} (Preclass 3)

- $V_{thn}=|V_{thp}|$=300mV, $V_{in}=V_{dd}$, estimate Eτ

V_{dd}	I_{dyn}	$\tau/(\tau@V_{dd}=1)$	$E_{switch}/(E_{switch}@V_{dd}=1)$	Eτ
1V				
700mV				
500mV				
350mV				
260mV				

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Reduce V_{dd} (Preclass 3)

- $V_{thn}=|V_{thp}|$=300mV, $V_{in}=V_{dd}$, estimate Eτ

V_{dd}	I_{dyn}	$\tau/(\tau@V_{dd}=1)$	$E_{switch}/(E_{switch}@V_{dd}=1)$	Eτ
1V	126uA	1	1	1
700mV	72uA	1.225	0.49	.6
500mV	36uA	1.75	0.25	.437
350mV	9uA	4.9	0.12	.588
260mV	111nA	295	0.07	20.6

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Increase V_{th} (Preclass 4)

- What is impact of increasing threshold on
  - Delay?
  - Leakage?
- V_{dd}=1V, $V_{in}=V_{dd}$

$V_{thn}= -V_{thp}$	I_{dyn}	$\tau/(\tau@V_{th}$=300mV)	I_{static}	$I_{stat}/(I_{stat}@V_{th}$=300 mV)
300mV				
460mV				
600mV				

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Increase V_{th} (Preclass 4)

- What is impact of increasing threshold on
  - Delay?
  - Leakage?
- V_{dd}=1V, $V_{in}=V_{dd}$

$V_{thn}= -V_{thp}$	I_{dyn}	$\tau/(\tau@V_{th}$=300mV)	I_{static}	$I_{stat}/(I_{stat}@V_{th}$=300 mV)
300mV	126uA	1	180pA	1
460mV	97uA	1.3	3.6pA	0.02
600mV	72uA	1.75	108fA	0.0006

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Big Ideas

- Three components of power
  - Static
  - Dynamic
  - Short-circuit
- $P_{tot} = P_{static} + P_{dyn} + P_{sc}$
- Tradeoff (knobs: V_{dd}, V_{th}, a, etc.)
  - Speed
  - Switching energy
  - Leakage energy
  - Want to look at the energy-delay product for optimality
- Energy-Delay tradeoff: $E\tau^2$, $E\tau$

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Admin

- HW 5 due Friday 2/24
- Proj 1 release Friday 2/24
  - Design 8-bit adder (baseline design)
  - Due four weeks on Friday 3/24

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Acknowledgement

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