

ESE370: Circuit-Level Modeling, Design, and Optimization for Digital Systems

Lec 11: March 1, 2023
Ratioed Logic and Design-Space Exploration



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Today

- Ratioed Logic
 - Break all the rules... (lose our nice properties)
 - Not rail-to-rail signals, steady-state-current...
 - Correctness

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Ratioed Logic



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Previously

- Restoration and Noise Margins
 - Allows for gate abstraction
- CMOS Gates
 - Drive outputs rail-to-rail
 - Only one PDN/PUN turned on in steady state
 - Only subthreshold leakage current in steady state

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Today

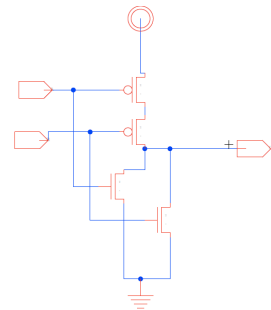
- Ratioed Gates
 - Break all the rules... (nice properties)
 - No rail-to-rail outputs, steady-state-current is not subthreshold...
 - Logic correctness
 - Performance
 - Power implications

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Idea

- Building both pull-up and pull-down can be expensive – many gates
- Seems wasteful to build logic function twice
 - Once in pullup, once in pulldown
 - Large gate capacitance

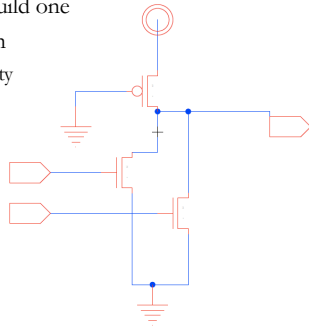


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Idea

- Maybe only need to build one
- Build NFET pulldown
 - Exploit high N mobility
 - traditional

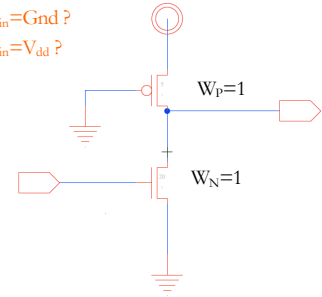


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Ratioed Inverter

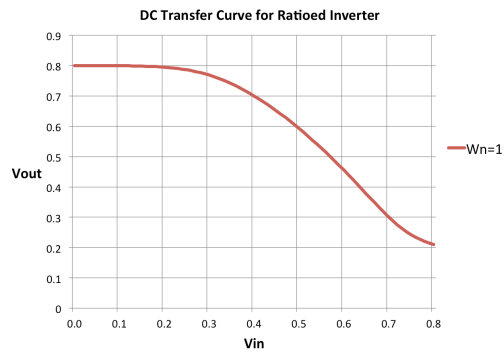
- Does this work?
 - What is V_{out} for $V_{in}=Gnd$?
 - What is V_{out} for $V_{in}=V_{dd}$?



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Ratioed Inverter in 22nm

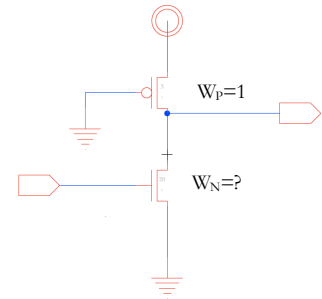


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Ratioed Inverter (Preclass 1)

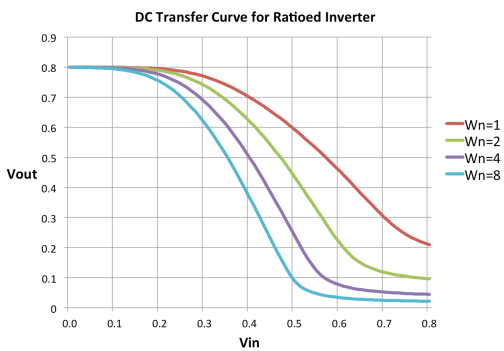
- How do we need to size N to make it “work”?
 - $V_{DD}=0.8$



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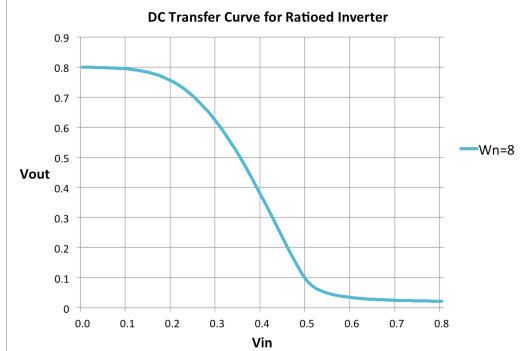
Ratioed Inverter in 22nm



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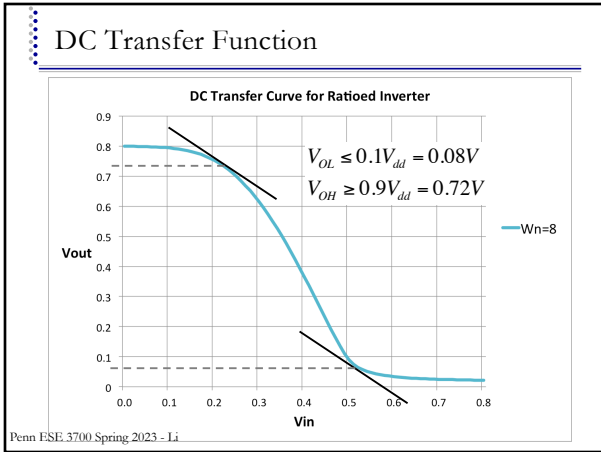
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DC Transfer Function



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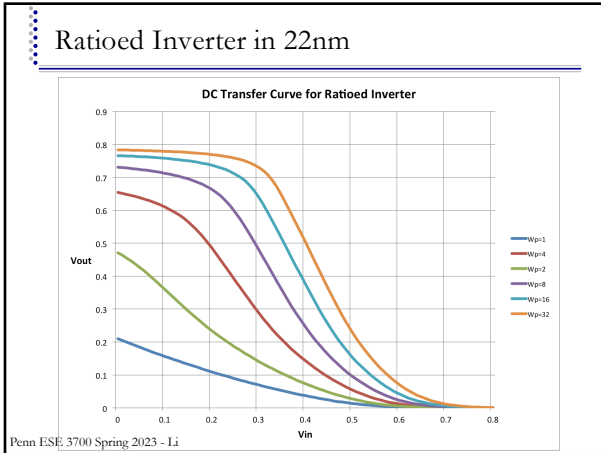
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Ratioed Inverter (Preclass 1)

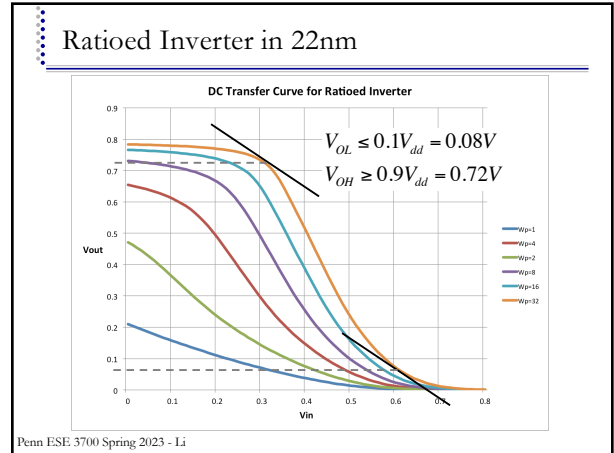
- How do we need to size P to make it work?
 - $V_{DD}=0.8$

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P vs. N

- Conclusion:** still prefer N to P for ratioed logic

DC Transfer Curve for Ratioed Inverter

Wp=1, Wp=4, Wp=8, Wp=16, Wp=32

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Noise Margin Tradeoff

- What is impact of increasing noise margin?
 - On size
 - On input capacitance

DC Transfer Curve for Ratioed Inverter

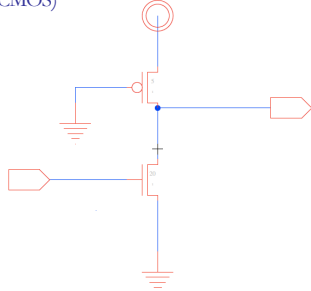
Wp=1, Wp=4, Wp=8, Wp=16, Wp=32

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Size for $R_0/2$ drive?

- How do we size for $R_0/2$ drive?
 - (assume $R_{0p}=R_{0n}$ for CMOS)



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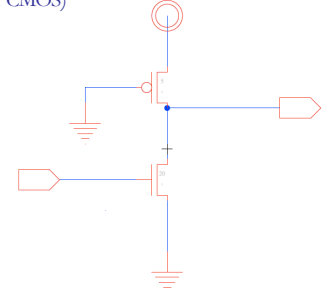
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Size for $R_0/2$ drive? (Preclass 2)

- How do we size for $R_0/2$ drive?
 - (assume $R_{0p}=R_{0n}$ for CMOS)

$$R_{drive} = \frac{R_{op}}{W_p} = \frac{R_0}{W_p} = \frac{R_0}{2}$$

$$\Rightarrow W_p = 2$$



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Size for $R_0/2$ drive?

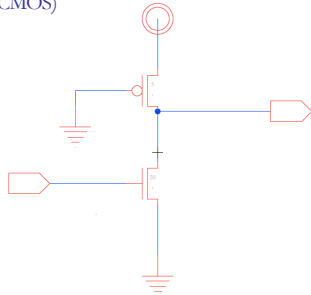
- How do we size for $R_0/2$ drive? What is C_{in} ?
 - (assume $R_{0p}=R_{0n}$ for CMOS)

$$R_{drive} = \frac{R_{op}}{W_p} = \frac{R_0}{W_p} = \frac{R_0}{2}$$

$$\Rightarrow W_p = 2$$

$$W_n > 8W_p$$

$$\Rightarrow W_n = 16$$

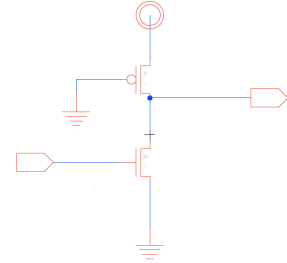


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Static Power

- I_{static} ?
- Input low-Output high?
 - I_{leak}
- Input high-Output low?
 - I_{pmos_on}
 - $\sim V_{dd}/(R_0/2)$ -- for our sample case



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Total Power

$$P_{tot} \approx a(C_{load} + 2C_{sc})V^2f$$

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Total Power

$$P_{tot} \approx a(C_{load} + 2C_{sc})V^2f$$

$$+ p(V_{out=low})V^2/R_{pon}$$

$$+ p(V_{out=high})VI_s(W/L)e^{-Vt/(nkT/q)}$$

$p(V_{out=low})$ – probability the output is low

$p(V_{out=high})$ – probability the output is high

$$p(V_{out=high})=1-p(V_{out=low})$$

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Compare Static CMOS

For $R_{drive} = R_0/2$ inverter (assume $R_{0p} = R_{0n}$ for CMOS)

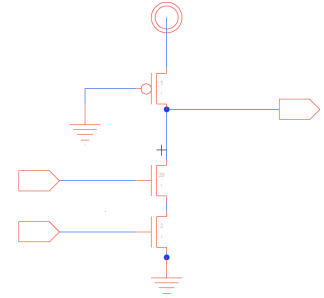
- Total Transistor Width?
- Input capacitance load?

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How do we size for $R_0/2$ drive?

- 2-input nand?



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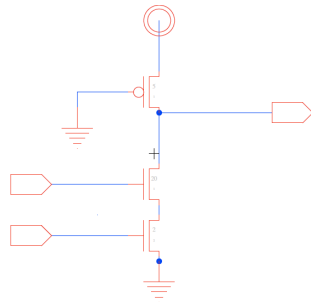
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How do we size for $R_0/2$ drive?

- 2-input nand?

$$R_{drive} = \frac{R_{op}}{W_p} = \frac{R_0}{W_p} = \frac{R_0}{2}$$

$$\Rightarrow W_p = 2$$



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How do we size for $R_0/2$ drive?

- 2-input nand? What is C_{in} ?

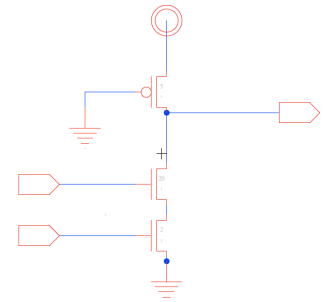
$$R_{drive} = \frac{R_{op}}{W_p} = \frac{R_0}{W_p} = \frac{R_0}{2}$$

$$\Rightarrow W_p = 2$$

$$R_{drive,pdn} = \frac{2R_0}{W_n} = \frac{R_0}{W_n/2}$$

$$W_n/2 > 8W_p$$

$$\Rightarrow W_n = 32$$

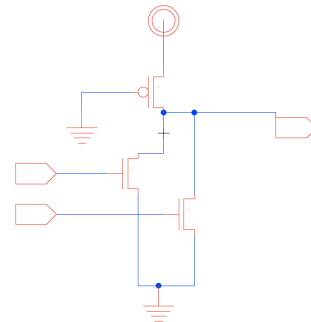


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How do we size for $R_0/2$ drive?

- 2-input nor?



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How do we size for $R_0/2$ drive?

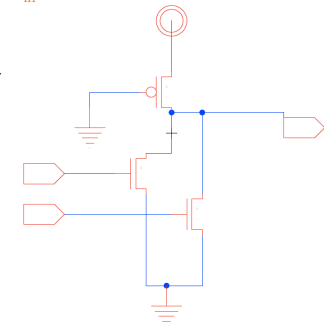
- 2-input nor? What is C_{in} ?

$$R_{drive} = \frac{R_{op}}{W_p} = \frac{R_0}{W_p} = \frac{R_0}{2}$$

$$\Rightarrow W_p = 2$$

$$W_n > 8W_p$$

$$\Rightarrow W_n = 16$$

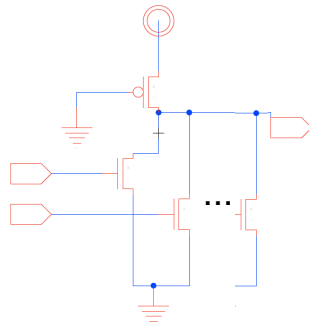


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How do we size for $R_0/2$ drive? (Preclass 3)

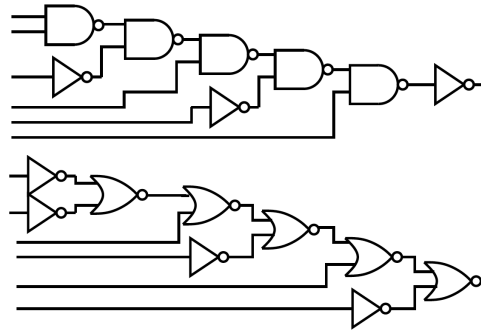
- k-input nor?
- What is the input capacitance for k-input nor?



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Which Implementation is faster in ratioed logic?



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Design Space Exploration



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Design Problem

- Function: Identify equivalence of two 32bit inputs
- Optimize: Minimize total energy
- Assumptions: Match case uncommon
 - I.e. Most of the time, the inputs won't be matched
- Deliberately focus on Energy to complement project
 - ...but will still talk about delay

Last page of preclass

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Idea: Design Space Explore

- Identify options
 - All the knobs you can turn
- Explore space systematically
- Formulate continuum where possible
 - i.e. formulate trends and tradeoffs quantitatively

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Problem Solvable (preclass 4.1)

- Is it feasible?
 - First, make sure we have a solution so we know our main goal is optimization
- How do we decompose the problem?

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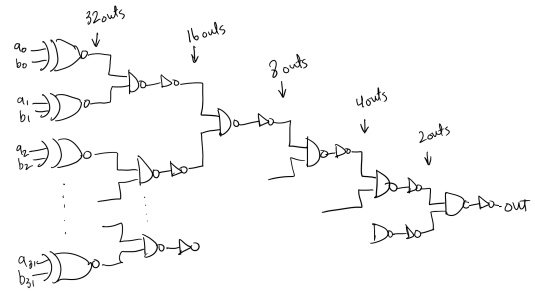
Problem Solvable (preclass 4.2)

- Is it feasible?
 - First, make sure we have a solution so we know our main goal is optimization
- How do we decompose the problem?
- What look like built out of nand2 gates and inverters?

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Nand2 and inverter implementation



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Single Gate Match Condition (preclass 4.3)

- Design a single gate for match comparison

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Total Power

- Static CMOS:
 - $P_{tot} \approx a(C_{load} + 2C_{sc})V^2f + VT_s(W/L)e^{-V_t/(nkT/q)}$
- Ratioed Logic:
 - $P_{tot} \approx a(C_{load} + 2C_{sc})V^2f$
 $+ p(V_{out=low})V^2/R_{pon}$
 $+ (1-p(V_{out=low}))VT_s(W/L)e^{-V_t/(nkT/q)}$
- What can we do to reduce power?

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Knobs (preclass 4.4)

- What are the options and knobs we can turn?

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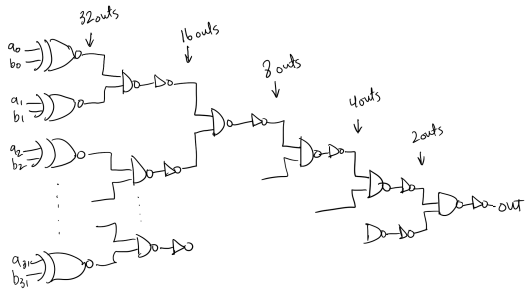
Design Space Dimensions

- Topology
 - (A) Gate choice, logical optimization
 - (B) Fanin, fanout, (C) Serial vs. parallel
- Gate style / logic family
 - (D) CMOS, Ratioed (N load, P load)
- (E) Transistor Sizing
- (F) Vdd
- (G) Vth

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Gate

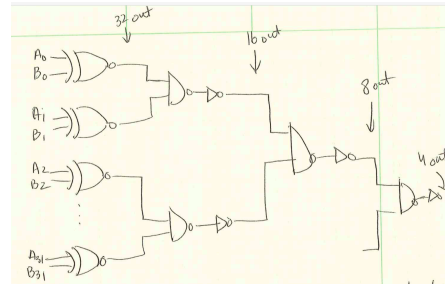


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Gate Choice

□ (A) What gates might we build?

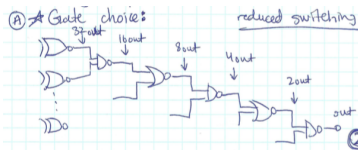


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Gate Choice

□ (A) What gates might we build?

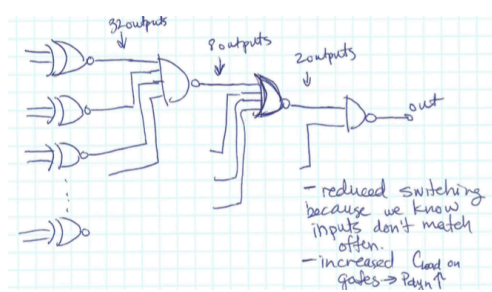


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Gate Fan-in

□ (B) High fan-in?

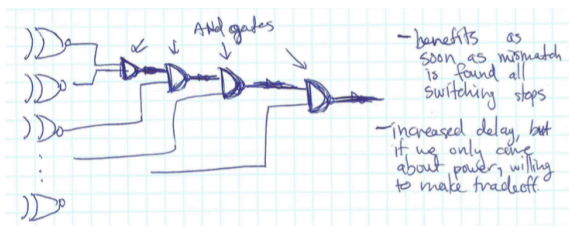


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Gate Topology

□ (C) Serial-Parallel?



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(D) Logic Family

□ Considerations for each logic family?

- CMOS
- Ratioed with PMOS load
- Ratioed with NMOS load

□ Ratioed Logic

- Reduced C_{loads} result in lower switching power (P_{dyn} ↓)
- Increased static power

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(E) Sizing

- How do we want to size gates?
 - Sizing transistors up will reduce delay →
 - Reduces short circuit power

$$E = V_{dd} \times \left(I_{peak} \times t_{sc} \times \left(\frac{1}{2} \right) \right)$$

- Increases dynamic power

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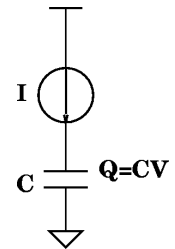
(F) Reduce Vdd

- What happens as reduce V?

- Energy?
 - Dynamic ↓
 - Static ↓
- Switching Delay? ↑

- $\tau_{gd} = Q/I = (CV)/I$
- $I_d = (\mu C_{OX}/2)(W/L)(V_{gs} - V_{TH})^2$
- τ_{gd} impact?
- $\tau_{gd} \propto 1/V$

- Limit on Vdd?



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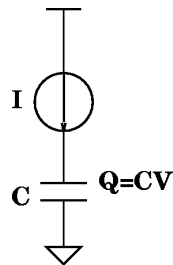
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(G) Increase V_{th}

- What is impact of increasing threshold on

- Dynamic Energy? ↓
- Leakage Energy? ↓
- Delay? ↑

- $\tau_{gd} = Q/I = (CV)/I$
- $I_{ds} = (v_{sat} C_{OX})(W)(V_{gs} - V_{TH} - V_{DSAT}/2)$



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Ideas

- There are other logic disciplines
- You have the tools to analyze
- Ratioed Logic
 - Tradeoff noise margin for
 - Reduced area? Capacitive load?
 - Dissipates non-leakage static power in one input case
- We know many things we can do to our circuits
- Design space is large
- Systematically identify dimensions
- Identify continuum (trends) tuning when possible
- Watch tradeoffs ...but don't over-tune

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Admin

- Project 1 out now
 - Design 8-bit ripple-carry adder
 - You already know how to do this
 - Refresh yourself on binary addition of 2 bits
 - Work **individually**
 - Full Report due F 3/24
- No in-class lecture (3/13)
 - Monday (after spring break)
 - Work on your project

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Acknowledgement

- Prof. André DeHon (University of Pennsylvania)
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