ESE370: Circuit-Level Modeling, Design, and Optimization for Digital Systems

Lec 9: February 22, 2023

Energy and Power Basics

Penn

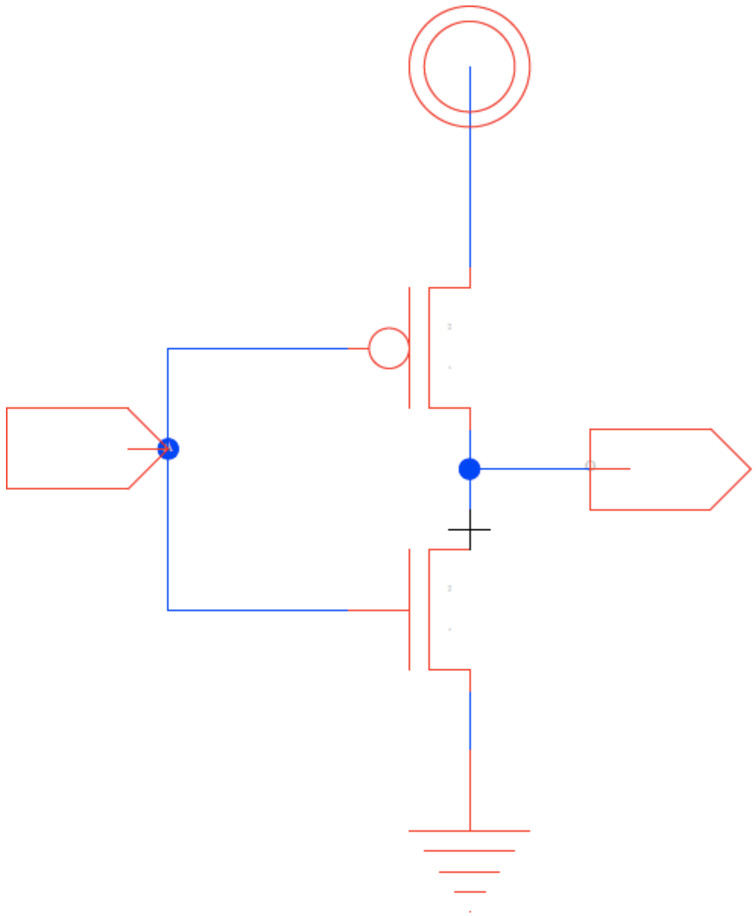
Today

- Power Sources
  - Static power
  - Switching power
    - Dynamic switching power
    - Short circuit power (if time)

Power

- $P = I \times V$

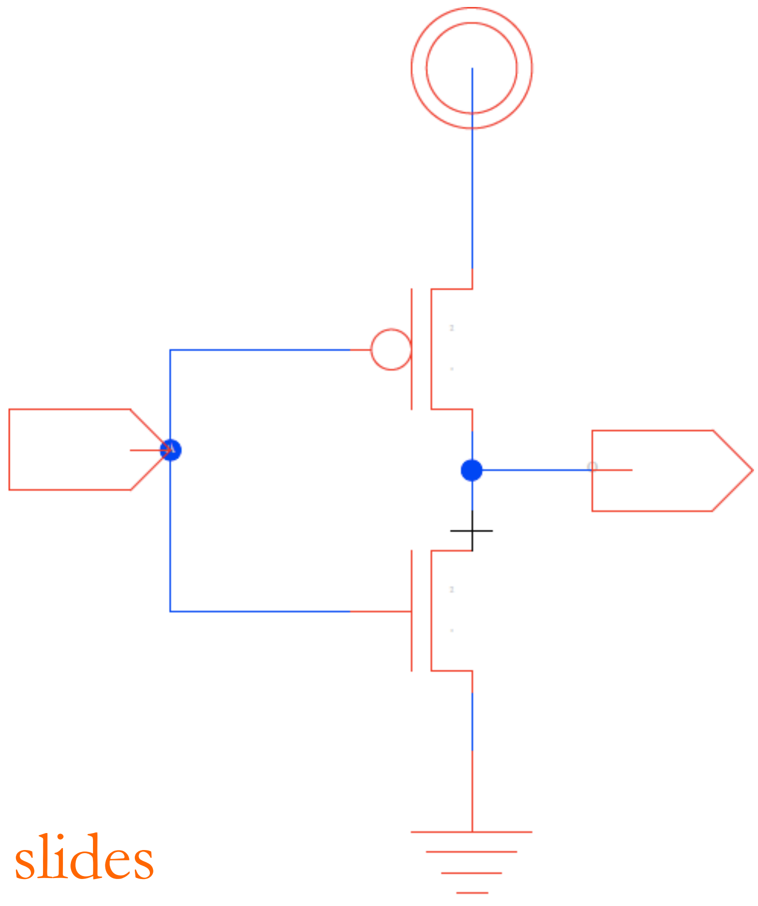
- Tricky part:
  - Understanding I
  - (pairing with correct V)



Inverter Current Simplification (preclass 1)

- What is $I_{pwr,gnd}$?
  - 0V
  - 140mV
  - 400mV
  - 500mV
  - 600mV
  - 860mV
  - 1V

  - From preclass – see next two slides



Preclass 1

Device	V_{gs}	I_d
NMOS	$V_{gs} < V_{thn}$	$(3 \times 10^{-7})\, e^{\frac{V_{gs}-V_{thn}}{40mV}}$
	$V_{gs} > V_{thn}$	$1.8 \times 10^{-4}\,(V_{gs} - V_{thn})$
PMOS	$V_{gs} > V_{thp}$	$(3 \times 10^{-7})\, e^{-\left(\frac{V_{gs}-V_{thp}}{40mV}\right)}$
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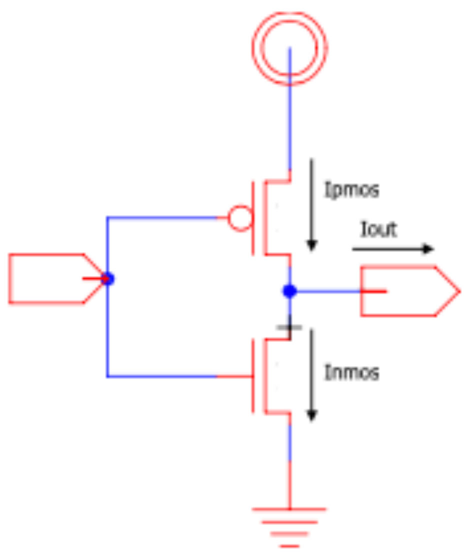
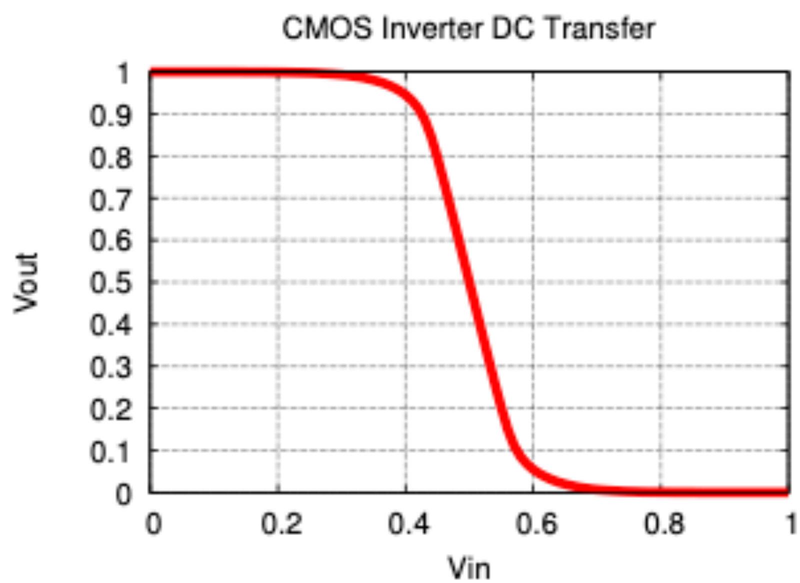
Consider an inverter:





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1. V_{dd}=1V, V_{thn}=300mV, V_{thp}=-300mV, assume steady-state operation at V_{in} given.

V_{in}	I_{pmos}	I_{nmos}	$\approx I_{pwr,gnd}$	
0V				A
140mV				B
400mV				C
500mV				D
600mV				E
860mV				F
1V				G

Approximate $I_{pwr,gnd} \approx \min(I_{nmos}, I_{pmos})$.

Useful: $e^{-1} \approx 0.37$, $e^{-4} \approx 0.02$, $e^{-7.5} \approx 6 \times 10^{-4}$,

Preclass 1

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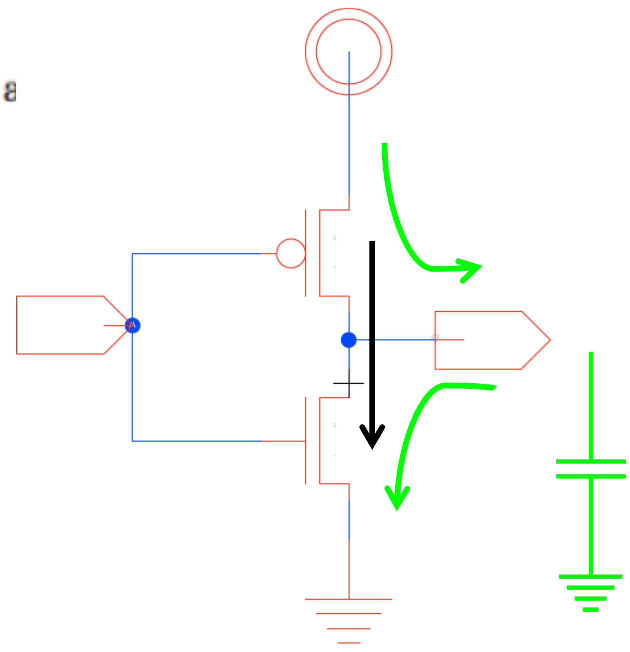

Preclass 2

Device	V_{gs}	V_{ds}	I_d
NMOS	$V_{gs} < V_{thn}$	any	$(3 \times 10^{-7})\, e^{\frac{V_{gs}-V_{thn}}{40mV}}$
	$V_{gs} > V_{thn}$	$V_{ds} < V_{gs} - V_{thn}$	$3.6 \times 10^{-4}\,(V_{gs} - V_{thn}) \times V_{ds}$
		$V_{ds} > V_{gs} - V_{thn}$	$1.8 \times 10^{-4}\,(V_{gs} - V_{thn})$
PMOS	$V_{gs} > V_{thp}$	any	$(3 \times 10^{-7})\, e^{-\left(\frac{V_{gs}-V_{thp}}{40mV}\right)}$
	$V_{gs} < V_{thp}$	$V_{ds} > V_{gs} - V_{thp}$	$-3.6 \times 10^{-4}\,(V_{gs} - V_{thp}) \times V_{ds}$
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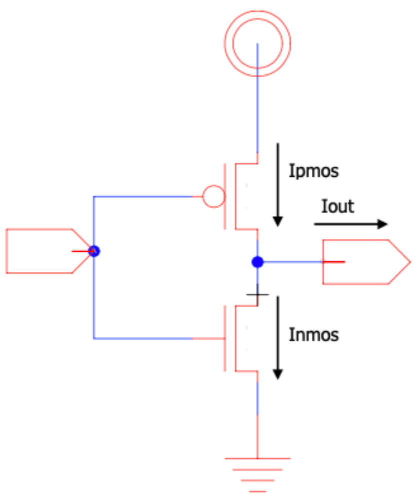

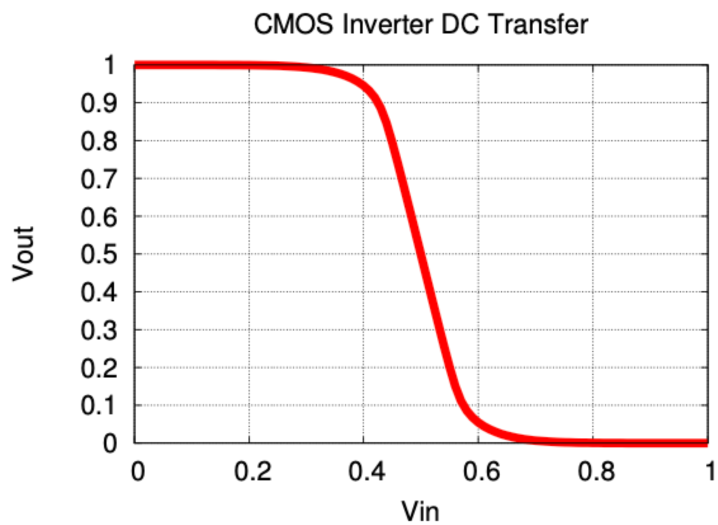

Preclass 2

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V_{in}	$I_{pmos} = I_{nmos} = I_{pwr,gnd}$	Vout	
0V			A
140mV			B
400mV			C
500mV			D
600mV			E
860mV			F
1V			G

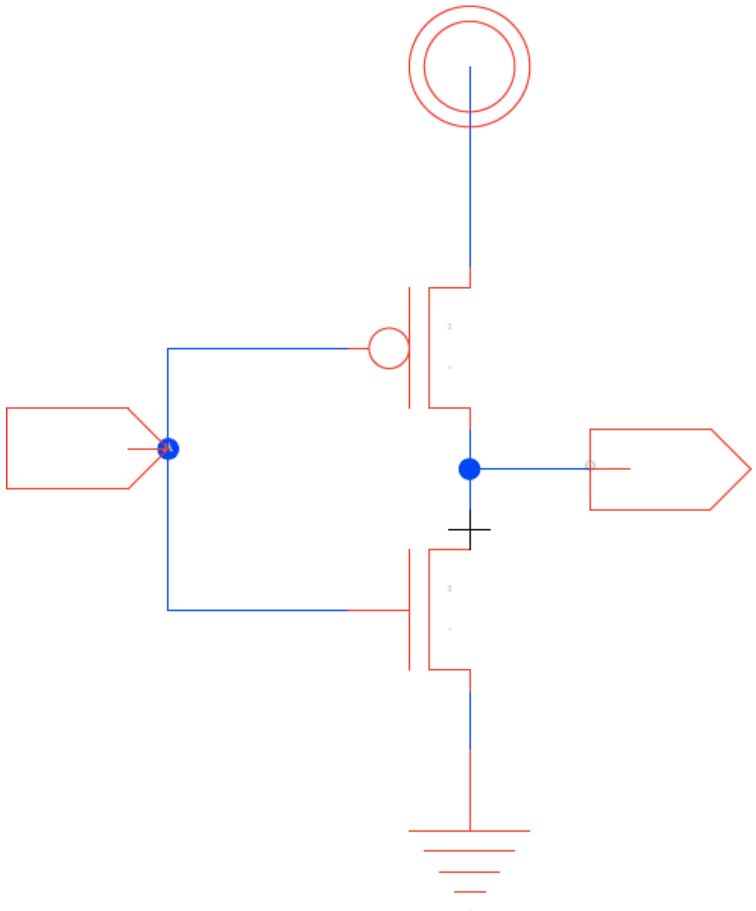
Approximate $I_{pwr,gnd} \approx \min(I_{nmos}, I_{pmos})$.

Understanding Currents

Static Power

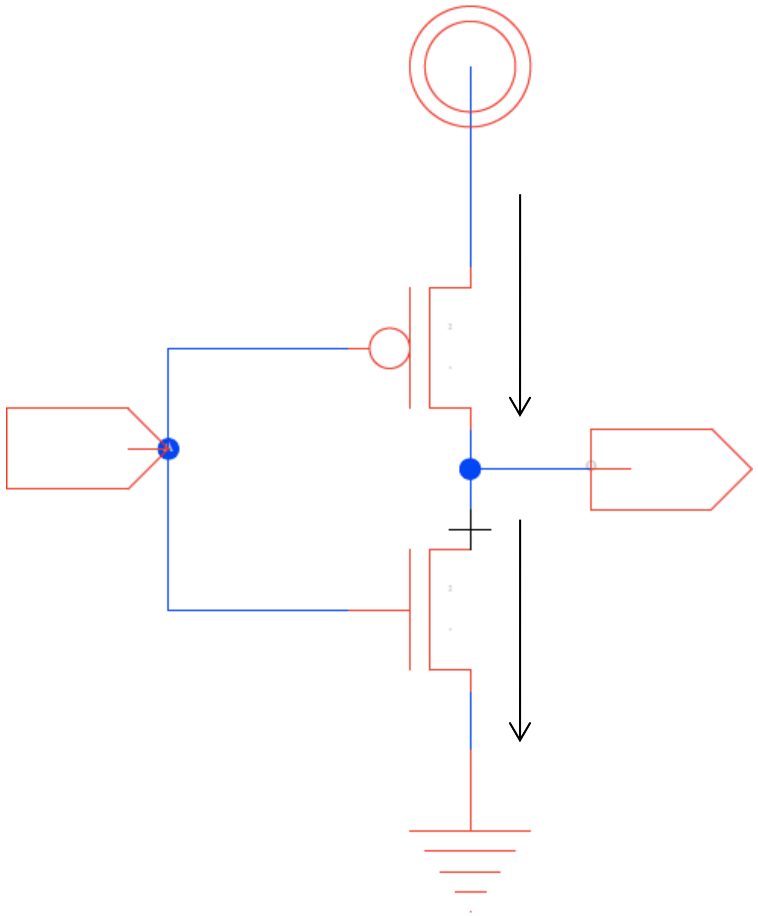
Operating Modes

- Steady-State: What modes are the transistors in?
  - $V_{in}=V_{dd}$
  - $V_{in}=Gnd$
- What current flows in steady state?



Operating Modes

- Steady-State: $V_{in}=V_{dd}$
  - PMOS: subthreshold
  - NMOS: resistive



Operating Modes

- Steady-State: $V_{in}=V_{dd}$
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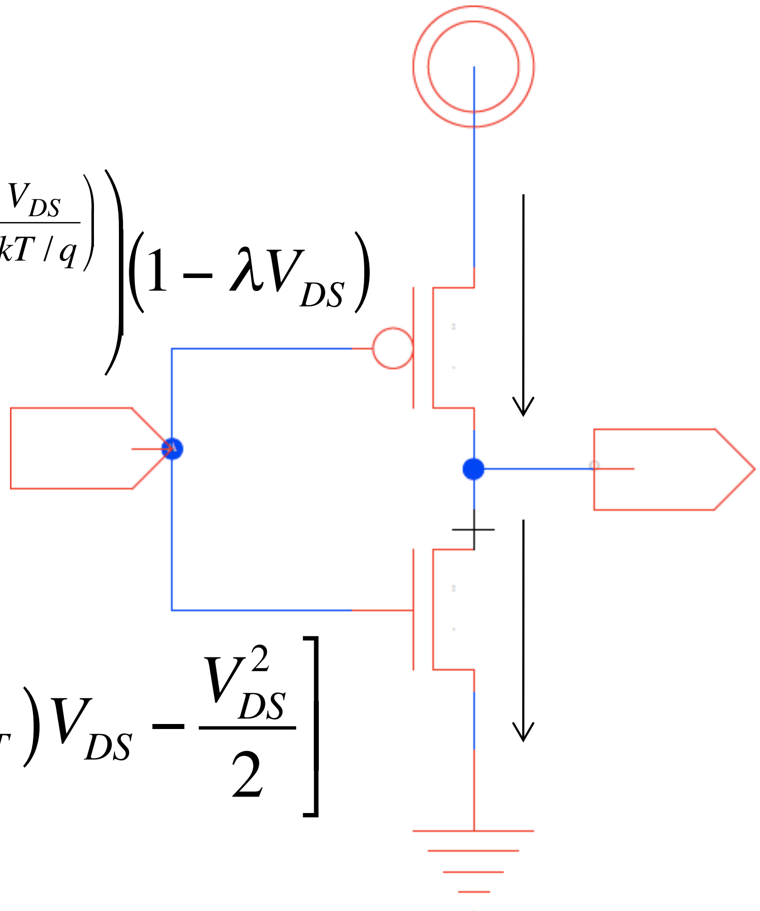
  - NMOS: resistive

$$I_{DSp} = -I_S'\left(\frac{W}{L}\right)e^{-\left(\frac{V_{GS}-V_T}{nkT/q}\right)}\left(1-e^{\left(\frac{V_{DS}}{kT/q}\right)}\right)(1-\lambda V_{DS})$$



$$I_{DSn} = \mu_n C_{OX}\left(\frac{W}{L}\right)\left[(V_{GS}-V_T)V_{DS}-\frac{V_{DS}^2}{2}\right]$$

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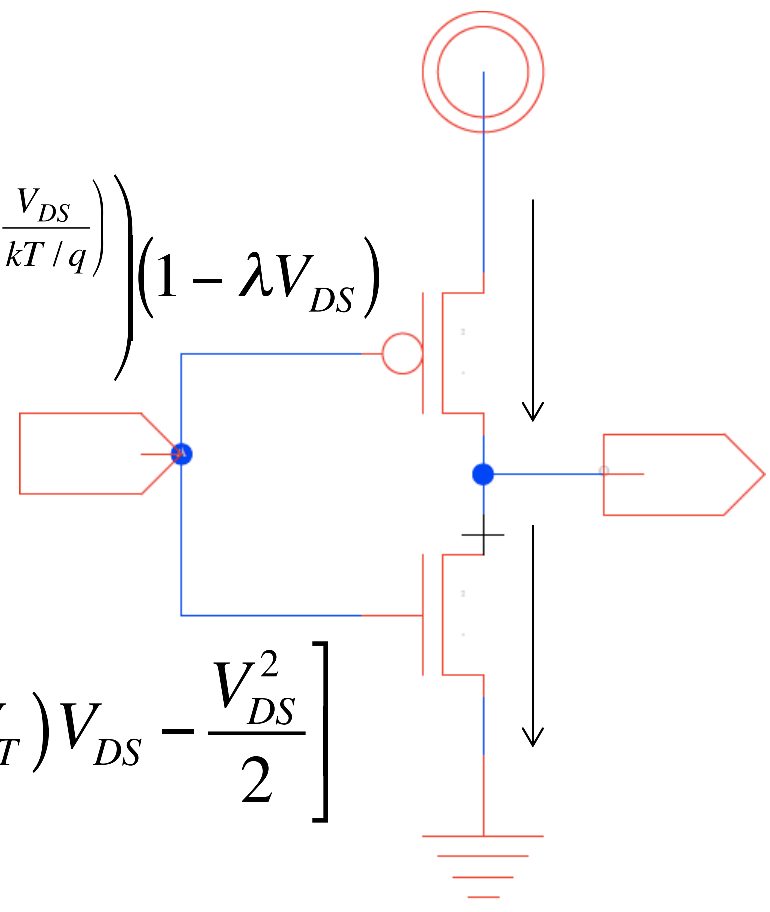
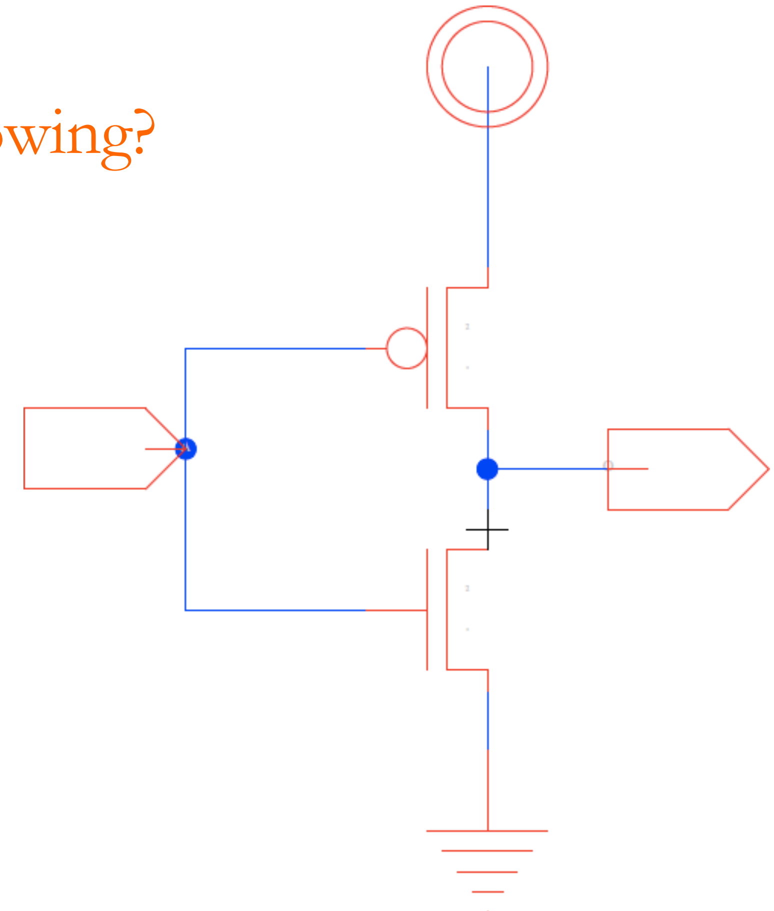


Which current determines I_{static} ?

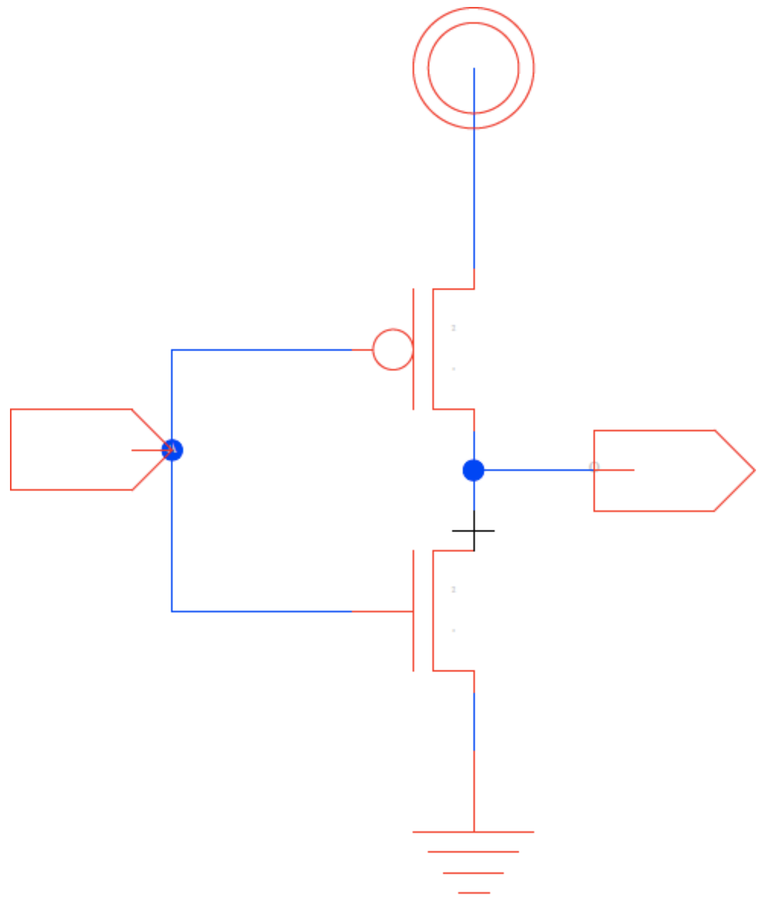
Static Power

- $P = I \times V$
- What V should we use?
  - Where is the static current flowing?



Data Dependent?

- How does the binary value of the input impact I_{static}?



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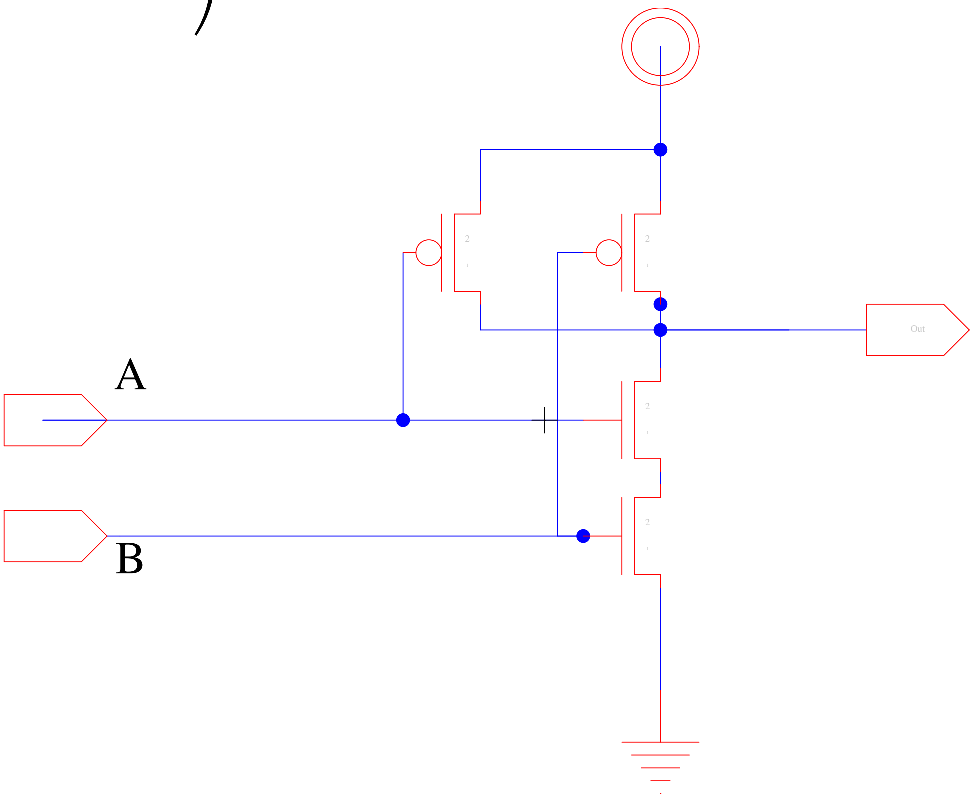

Data Dependent Leakage Current

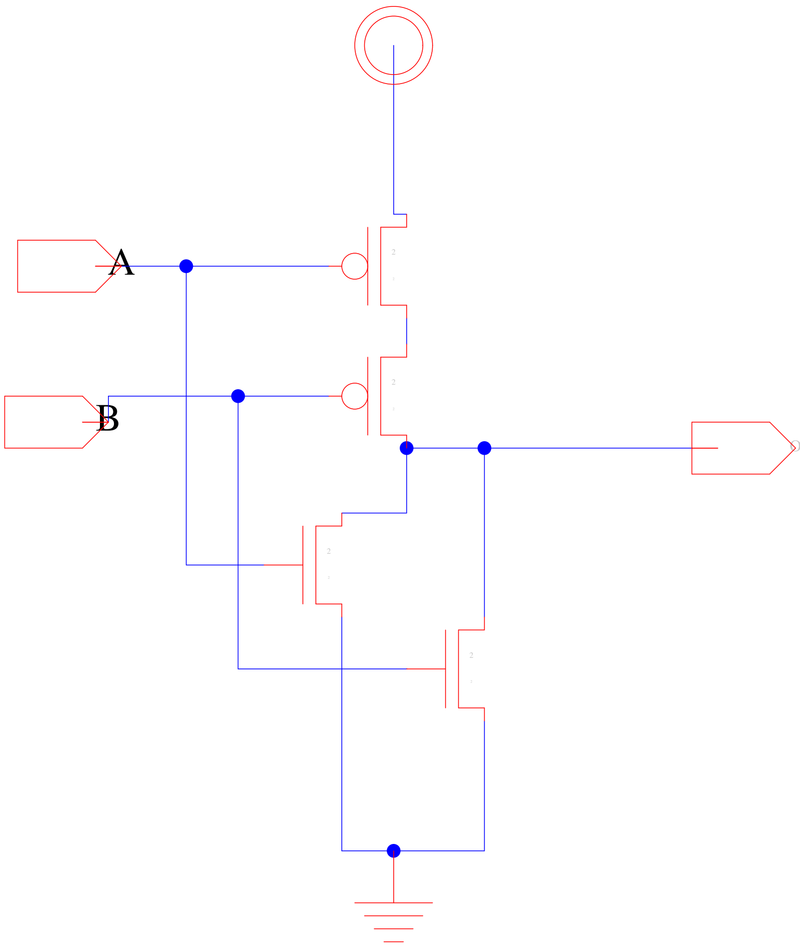

Table 1. Standard CMOS Gates Leakage Currents

	Temp °C	0	25	50	75	100
	Logic Lv.	Leakage Current (T°) [pA] @ 1,2 Vdd				
NOT	0	65,9	163,9	353,9	682,9	1203,9
	1	4, 4	10,4	23,5	49,0	93,3
NOR	0 0	131,9	327,7	707,9	1365,8	2405,7
	0 1	4, 4	10,4	23,5	48,1	90,1
	1 0	6,2	11,8	24,1	49,0	93,3
	1 1	2,3	4,4	9,3	19,3	37,4
NAND	0 0	9,4	24,5	57,5	120,9	231,2
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	1 0	52,0	128,4	279,7	545,8	972,8
	1 1	8,8	20,7	47,1	98,0	186,6
XOR	0 0	258,7	640,8	1388,7	2692,5	4768,1
	0 1	154,5	383,4	836,0	1633,8	2916,9
	1 0	140, 6	347,9	761,8	1496,7	2686,9
	1 1	135,6	333,8	727,8	1424,7	2549,0

ACM Great Lakes Symposium on VLSI Stresa, I. (n.d.). Analysis of data dependence of leakage current in CMOS cryptographic hardware. In GLSVLSI '07 proceedings of the 2007 ACM Great Lakes Symposium on VLSI : Stresa - Lago Maggiore, Italy, March 11-13, 2007 /. New York, N.Y. :: Association for Computing Machinery. https://doi.org/10.1145/1228784.1228808

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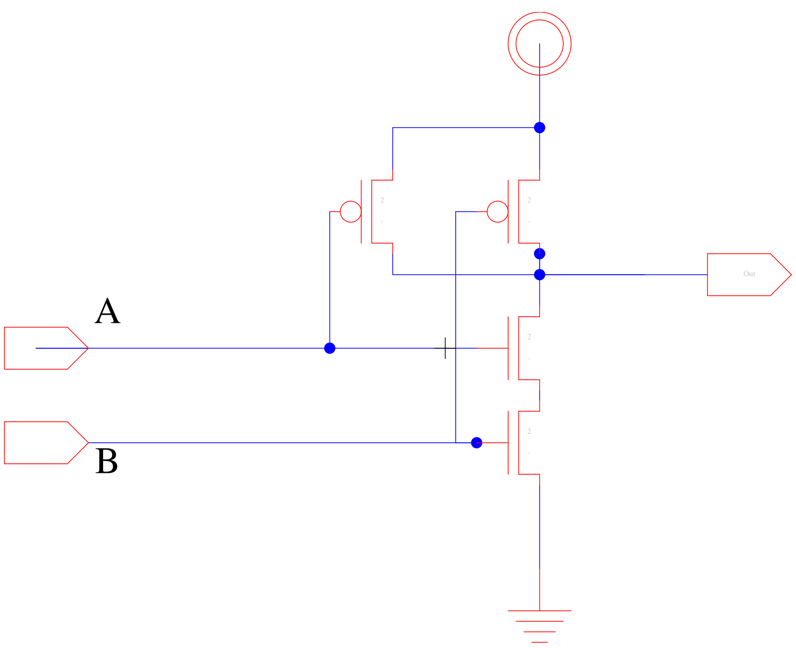

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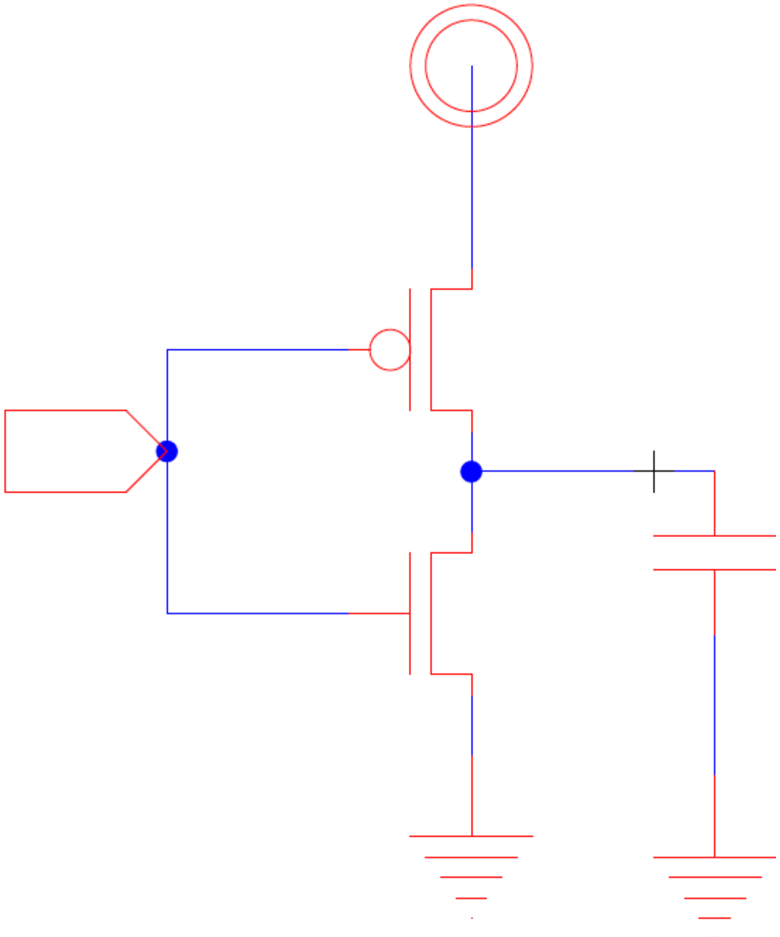
Understanding Currents

Dynamic Switching Currents

Power: During Switching

- $P = IV$
- Input switch: 1→0
- Where does *I* go?
  - V_{in}=Gnd



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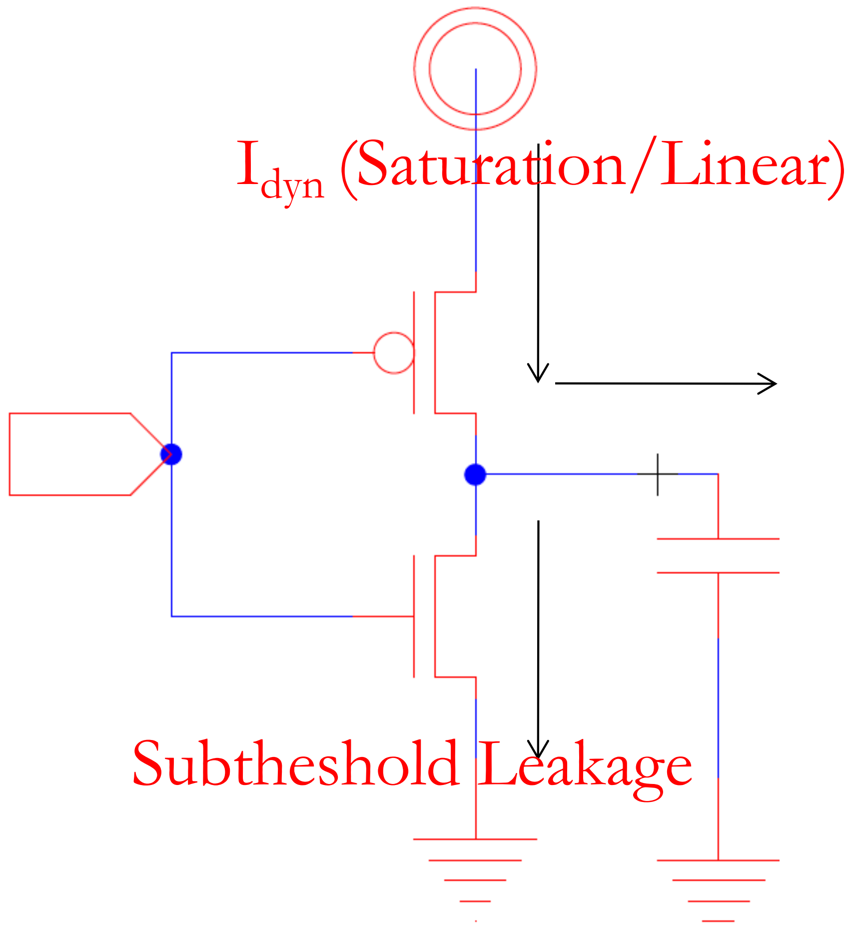
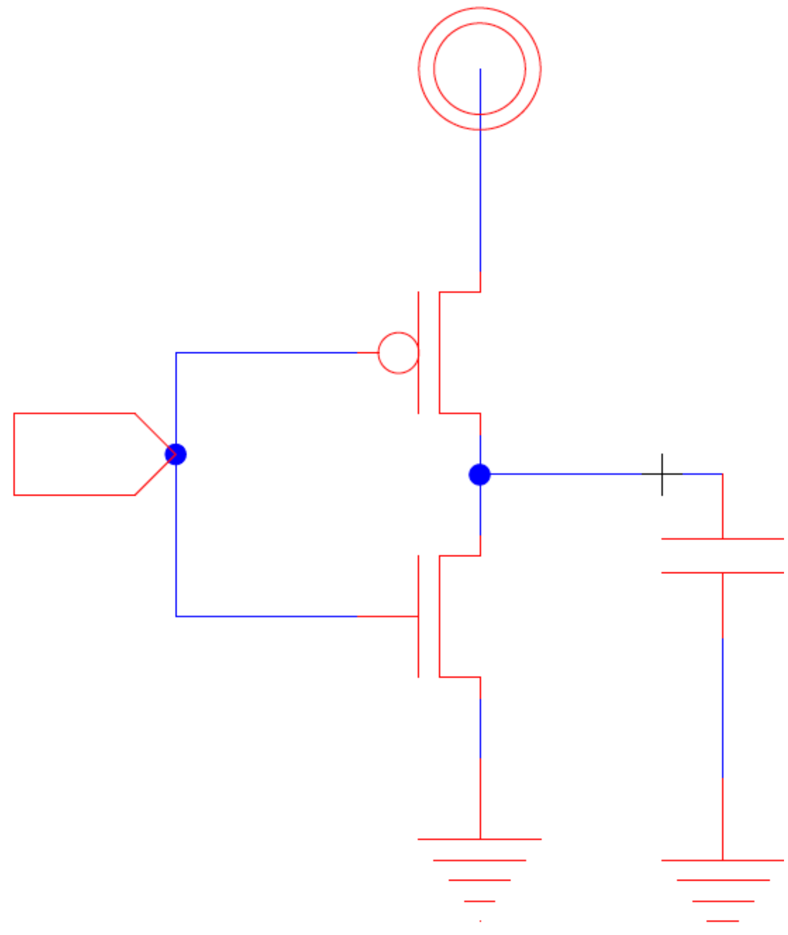


Power: During Switching

- $P = IV$
- Input switch 0→1
- Where does *I* go?
  - $V_{in}=V_{dd}$



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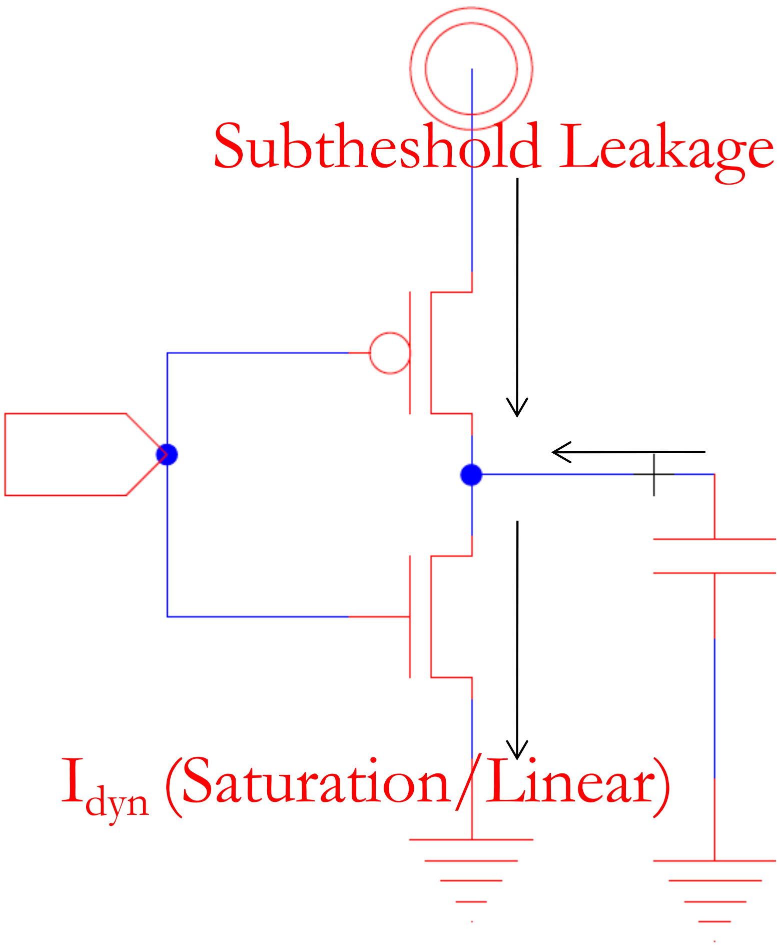
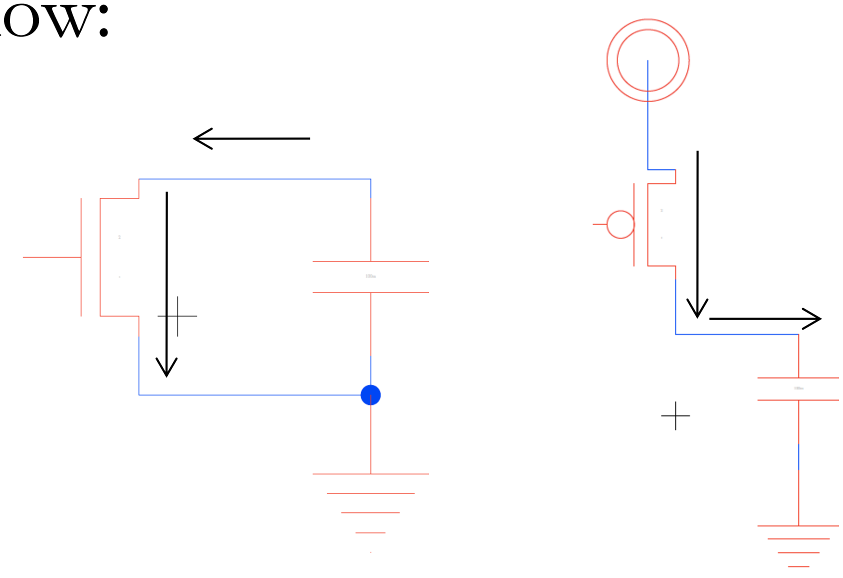


Switching Currents

- Dynamic current flow:

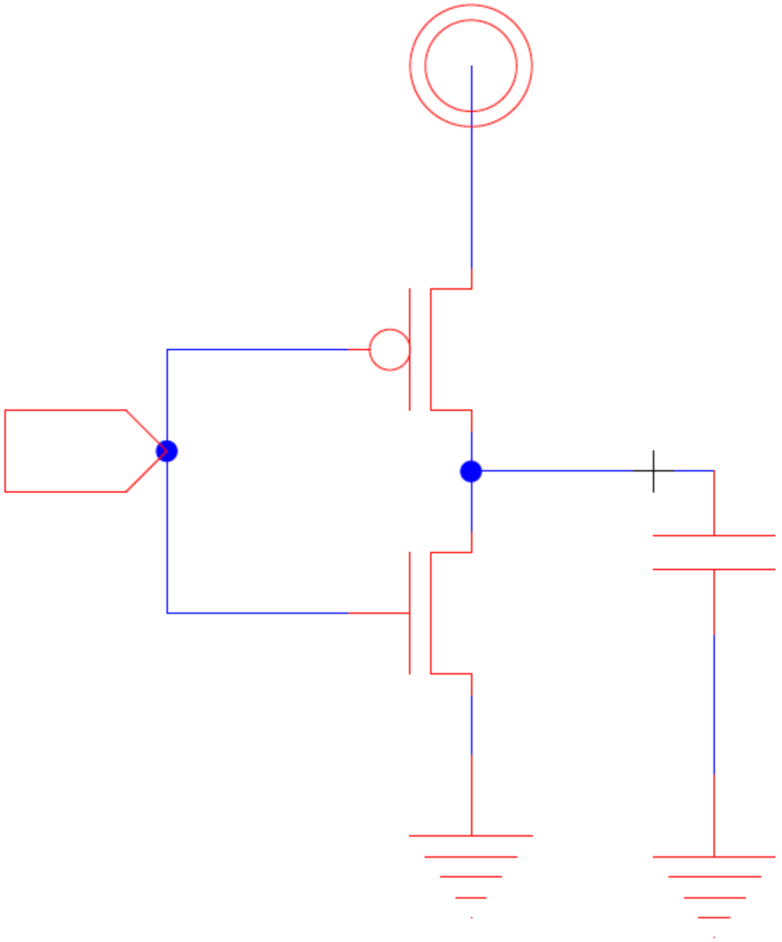


Understanding Currents

Short Circuit Currents

Power: During Switching

- $P = IV$
- Where does I go?
  - $V_{in}=V_{dd}/2$
    - And $V_{dd}>V_{thn}+|V_{thp}|$



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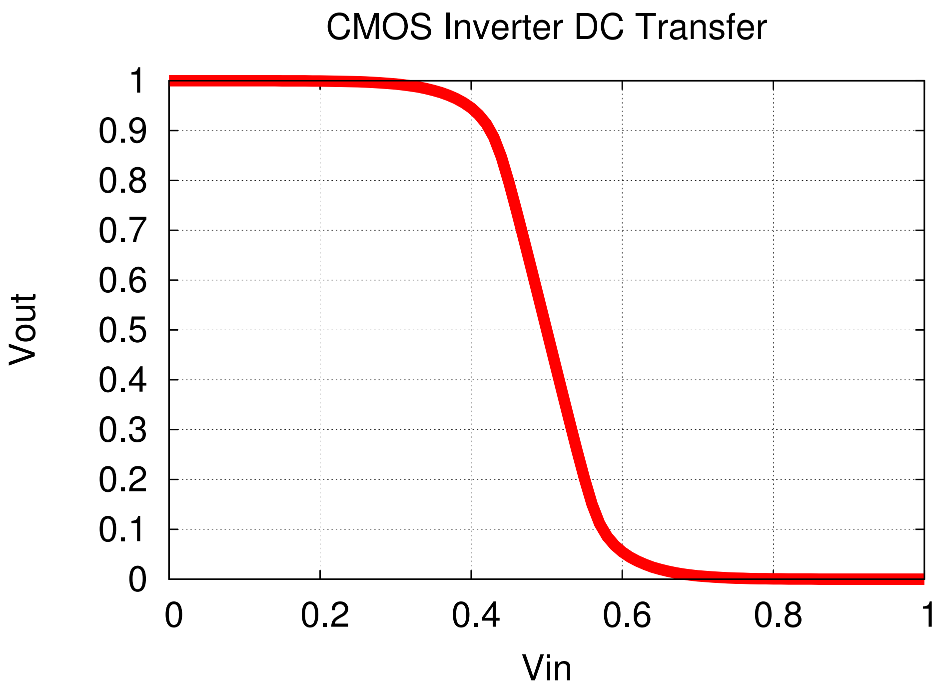

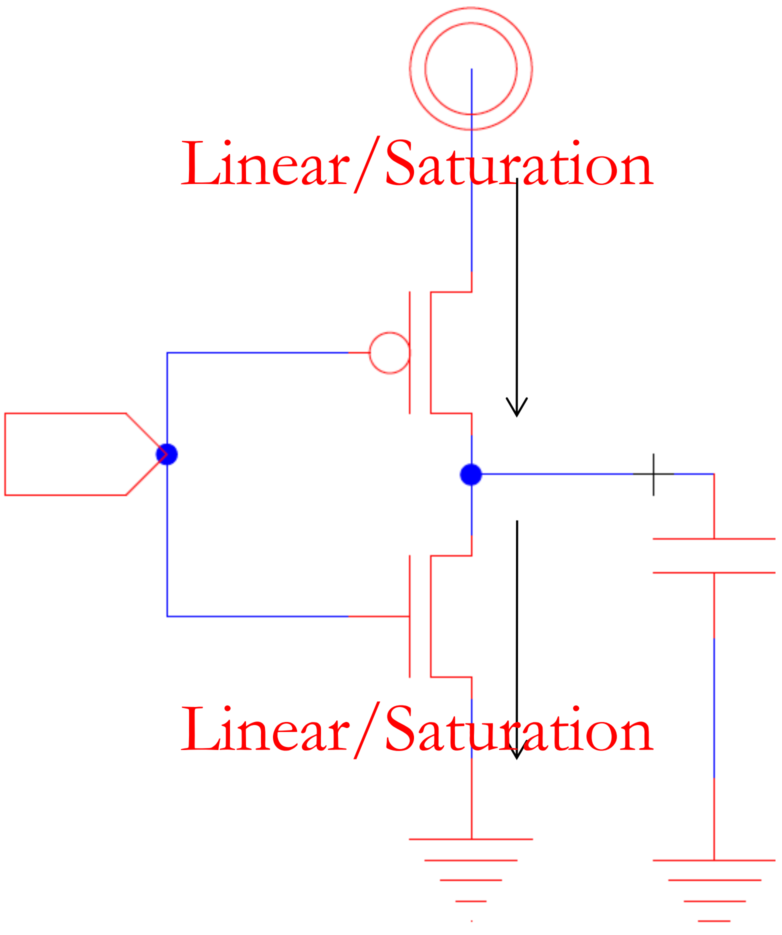

Switching Currents

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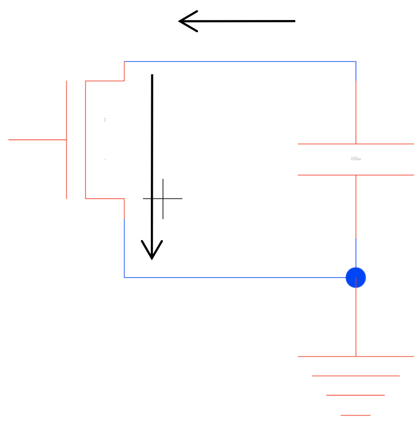
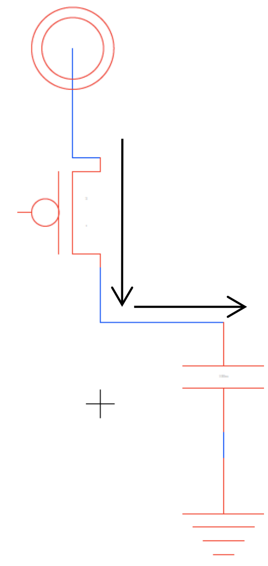
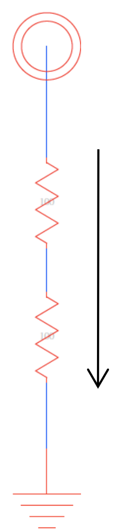


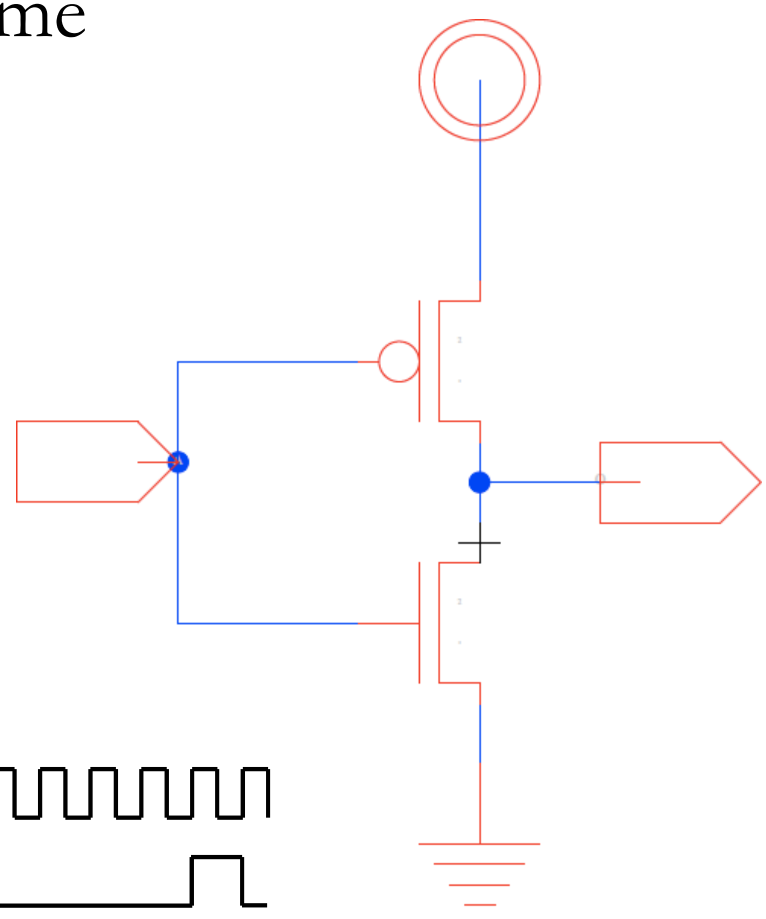


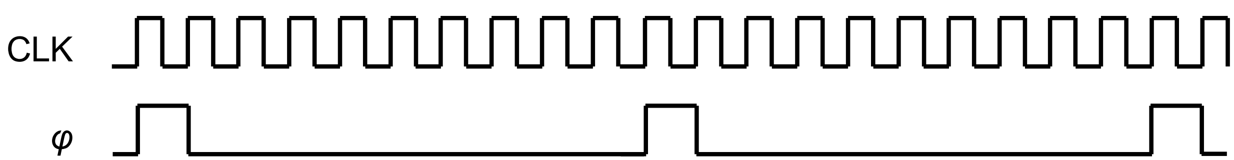
- If both transistor on:
  - Current path from V_{dd} to Gnd
  - Short circuit current



Currents Summary

- Current (I) changes over time
- At least two components
  - I_{static} – no switching
  - I_{switch} – when switching
    - I_{dyn} and I_{sc}





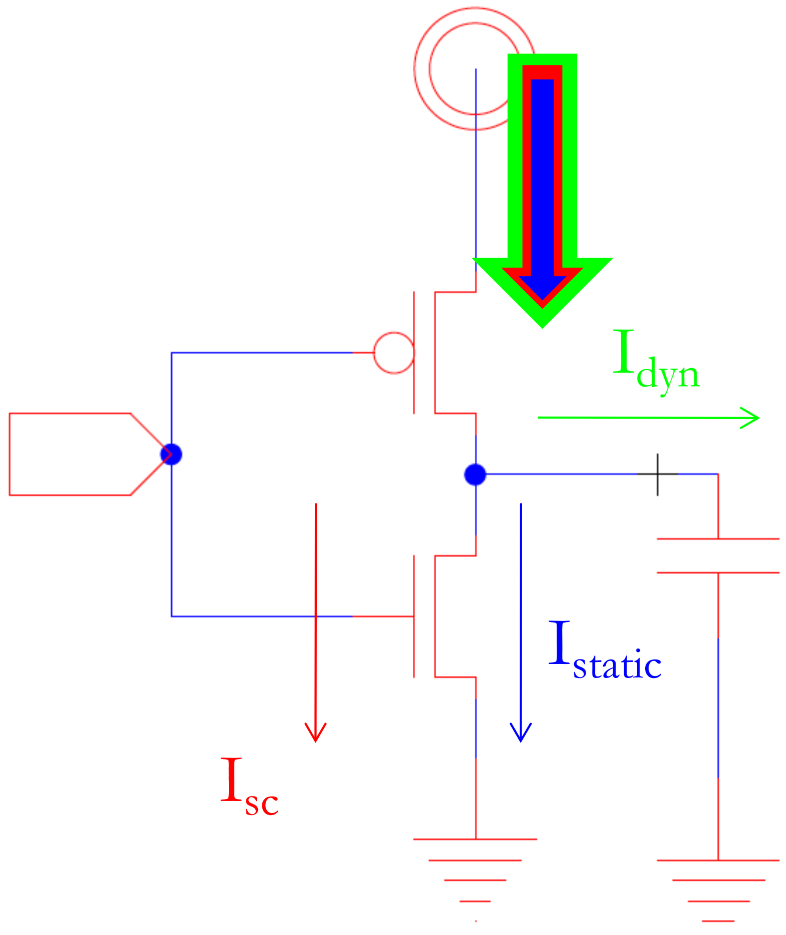
Switching

Dynamic Power

Switching Currents

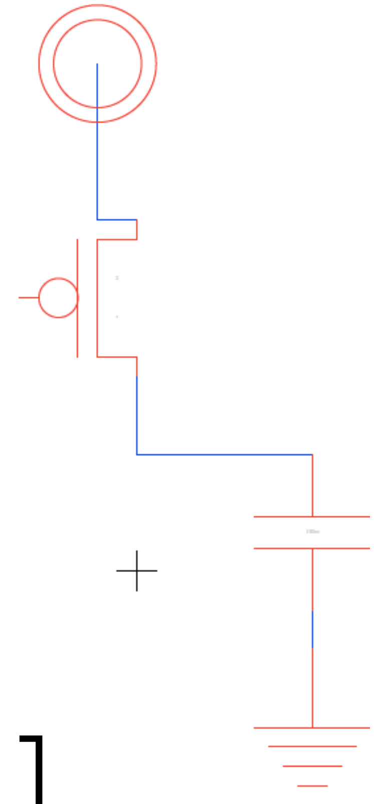
- $I_{total}(t) = I_{static}(t) + I_{switch}(t)$

- $I_{switch}(t) = I_{sc}(t) + I_{dyn}(t)$



Charging

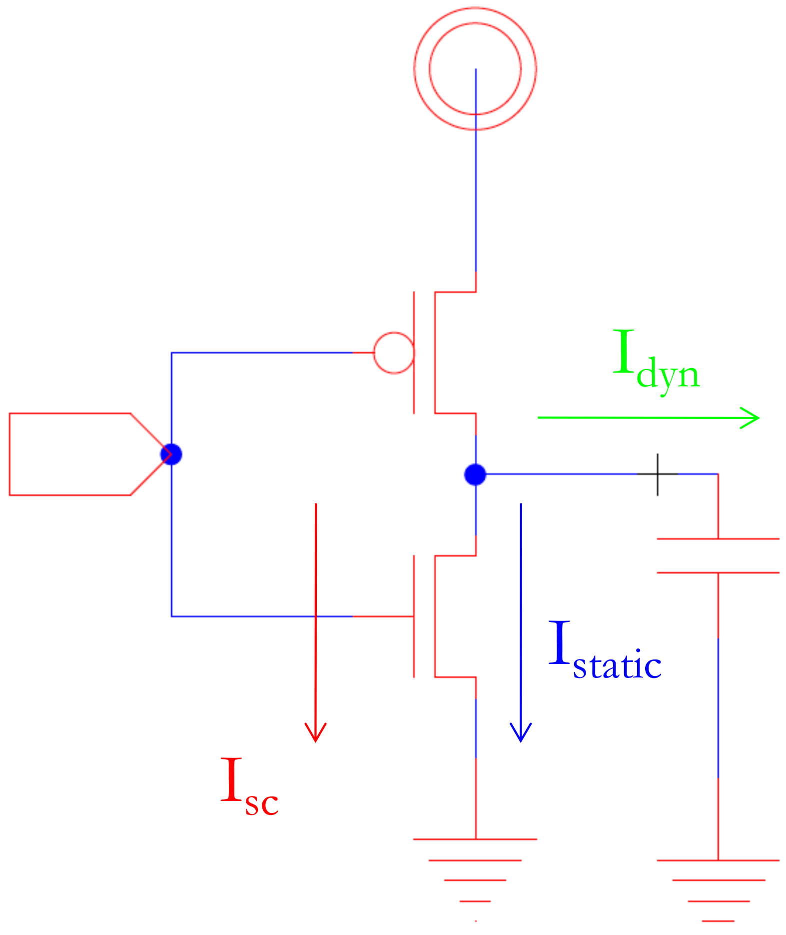
- $I_{dyn}(t)$ – why is it changing?
  - $I_{ds} = f(V_{ds}, V_{gs})$
  - and V_{gs}, V_{ds} changing



$$I_{DS} \approx \nu_{sat} C_{OX} W \left(V_{GS} - V_T - \frac{V_{DSAT}}{2} \right)$$

$$I_{DS} = \mu_n C_{OX} \left(\frac{W}{L} \right) \left[\left(V_{GS} - V_T \right) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

Switching Energy – focus on $I_{dyn}(t)$



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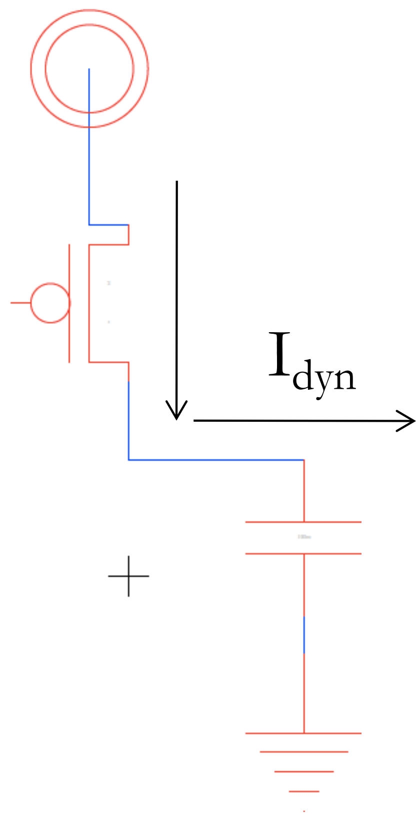


$$E = \int P(t)\,dt$$

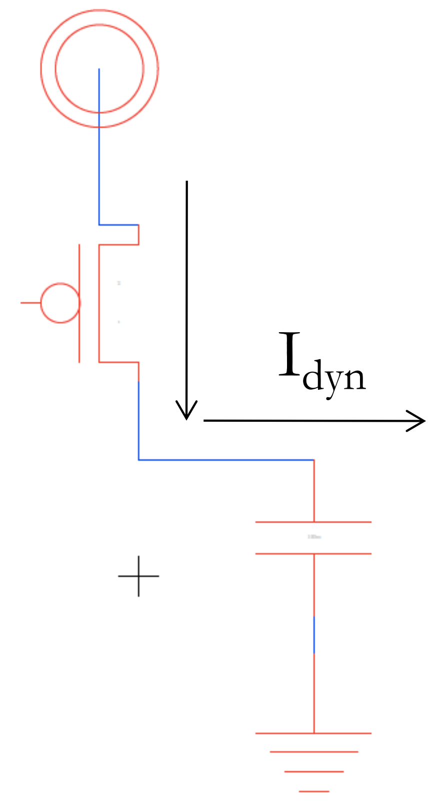
$$= \int I(t) V_{dd}\,dt$$

$$= V_{dd} \int I(t)\,dt$$

Switching Energy

- Do we know what this is?

$$\int I_{dyn}(t)\,dt$$

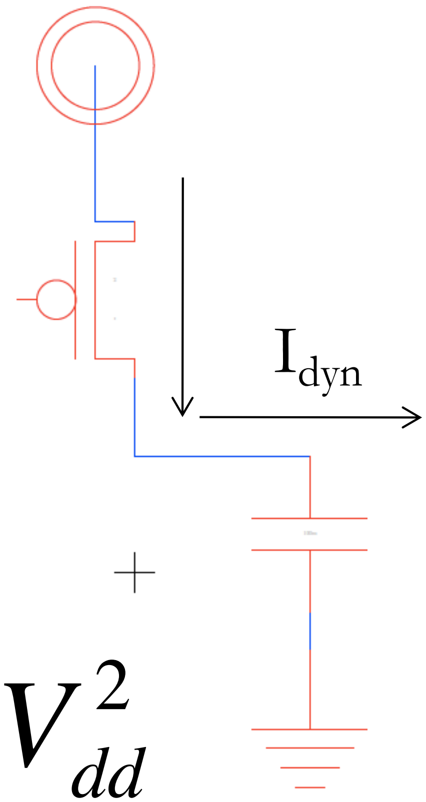


$$E = \int P(t)\,dt$$
$$= \int I(t) V_{dd}\,dt$$
$$= V_{dd} \int I(t)\,dt$$

Switching Energy

- Do we know what this is?

$$Q = \int I_{dyn}(t)\,dt$$
$$= CV$$



$$E = \int P(t)\,dt$$
$$= \int I(t) V_{dd}\,dt$$
$$= V_{dd} \int I(t)\,dt$$

$$\Rightarrow \quad E = C V_{dd}^2$$

Capacitor charging energy

Switching Power

- Every time output switches 0→1 pay:
  - $E = CV^2$

- $P_{dyn} = (\# \ 0 \rightarrow 1 \text{ trans}) \times CV^2 / \text{time}$

- $\# \ 0 \rightarrow 1 \text{ trans} = \frac{1}{2} \# \text{ of transitions}$

- $P_{dyn} = (\# \text{ trans}) \times \frac{1}{2} CV^2 / \text{time}$

Charging Power

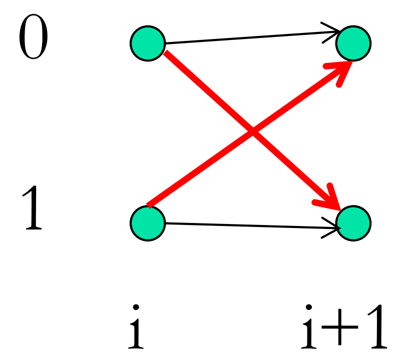
- P_{dyn} = (#0→1 trans) × CV^2 / time
- Often like to think about switching frequency
- Useful to consider per clock cycle
  - Frequency f = 1/clock-period = clock-cycles/time
- P_{dyn} = (#0→1 trans/clock-cycle) CV^2 f

Data Dependent Activity

- Consider an 8b counter
  - How often do each of the following switch?
    - Low bit?
    - High bit?
- Assuming random inputs
  - Activity at output of nand2?
  - Activity at output of xor2?

Gate Output Switching (random inputs)

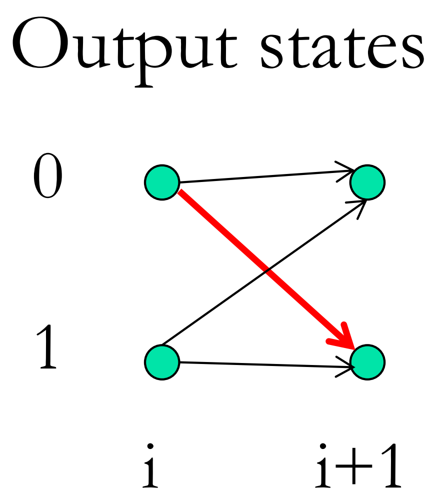
Output states



$$P(out_i != out_{i+1}) = P(out_i = 0) * P(out_{i+1} = 1) + P(out_i = 1) * P(out_{i+1} = 0)$$

Probablity of output switch of nand2?
Probablity of output switch of xor2?

Gate Output Switching (random inputs)



$$P(out_i \rightarrow out_{i+1}=0 \rightarrow 1)=P(out_i=0)*P(out_{i+1}=1)$$

Dynamic Power

- P_{dyn} = (#0→1 trans/clock-cycle) CV^2 f
- Let a = activity factor

  a = average $\#tran_{0\rightarrow1}$/clock

  a = probability of $\#tran_{0\rightarrow1}$

- $P_{dyn} = aCV^2 f$

Activity Factor

- Let a = activity factor
  - a = average $\#tran_{0 \rightarrow 1}$/clock
  - a = probability of $\#tran_{0 \rightarrow 1}$

$$a = p(out_i = 0)\,p(out_{i+1} = 1)$$

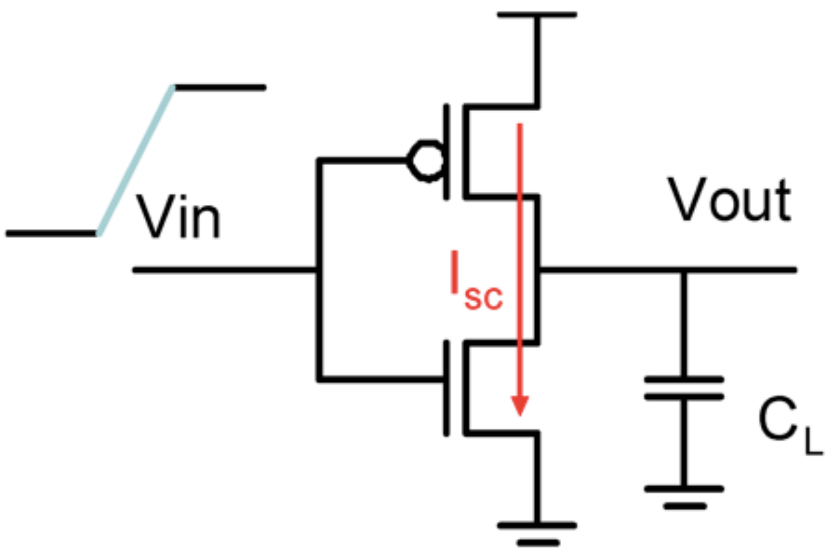
$$a = \frac{N_0}{2^N}\frac{N_1}{2^N} = \frac{N_0(2^N - N_0)}{2^{2N}}$$

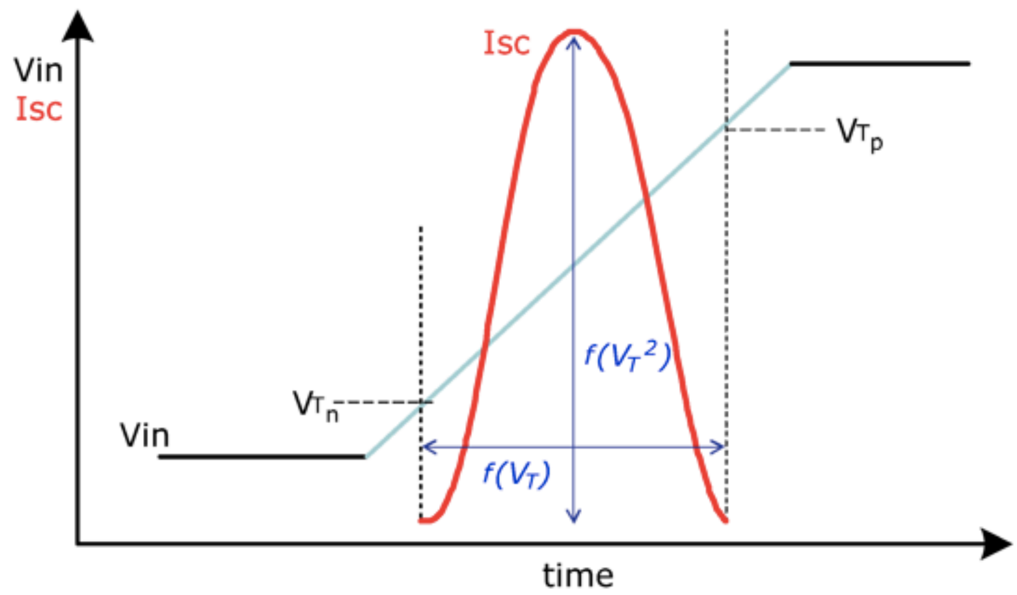
Switching

Short Circuit Power

Short Circuit Power

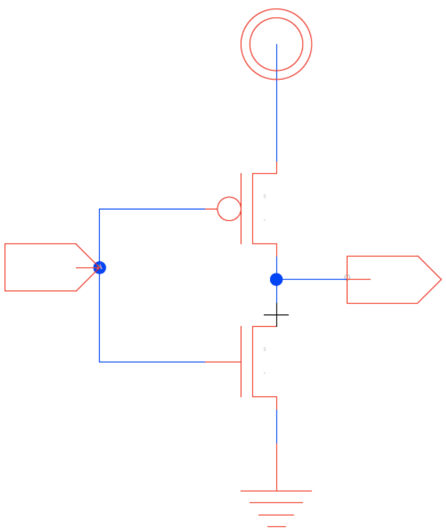
- Between V_{TN} and V_{dd} - V_{TP}
  - Both N and P devices conducting

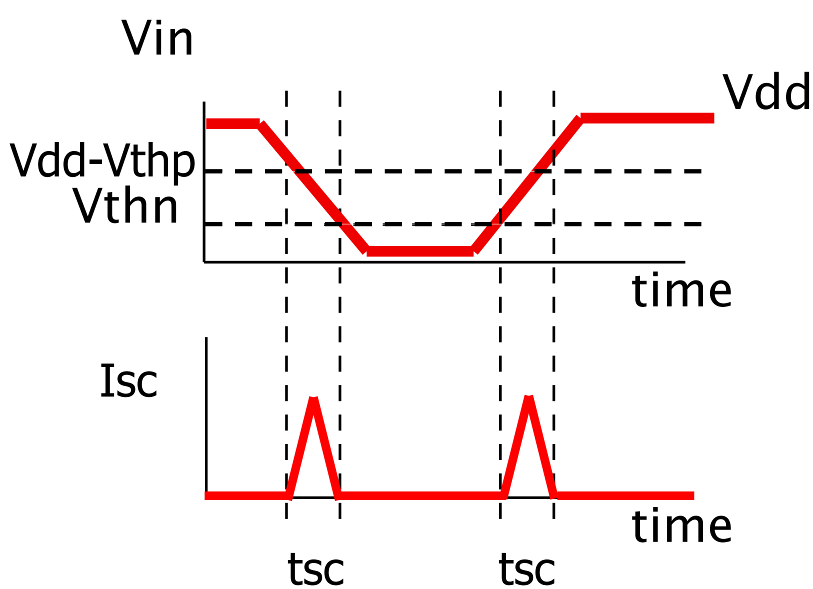


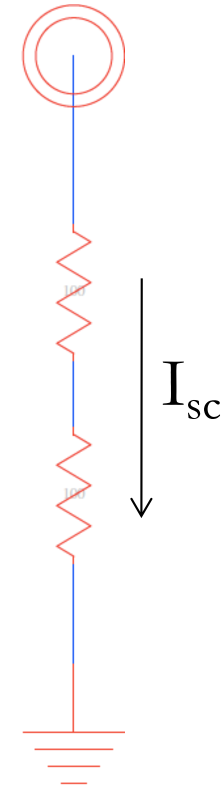


Short Circuit Power

- Between V_{TN} and V_{dd} - V_{TP}
  - Both N and P devices conducting
- Roughly:



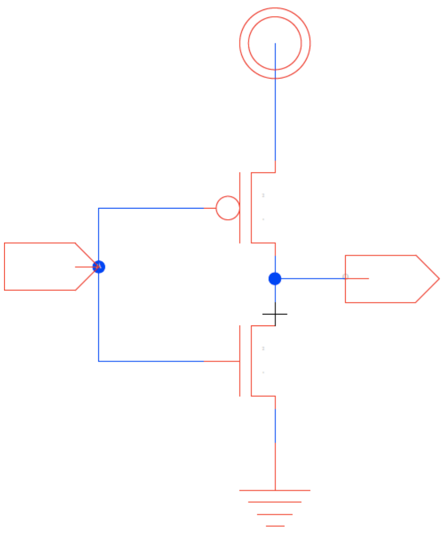


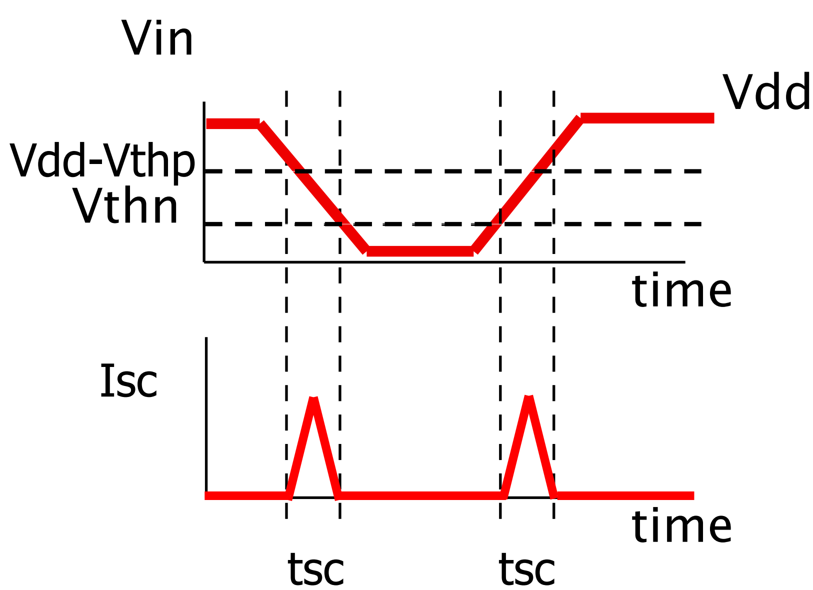


Peak Current

- I_{peak} around $V_{dd}/2$
  - If $|V_{TN}|=|V_{TP}|$ and sized equal rise/fall

$$I_{DS} \approx v_{sat} C_{OX} W \left(V_{GS} - V_T - \frac{V_{DSAT}}{2} \right)$$





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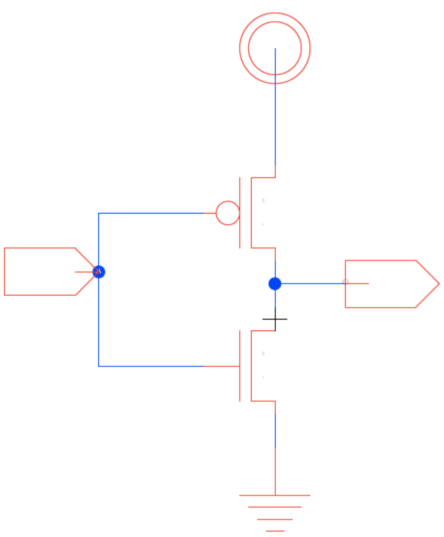
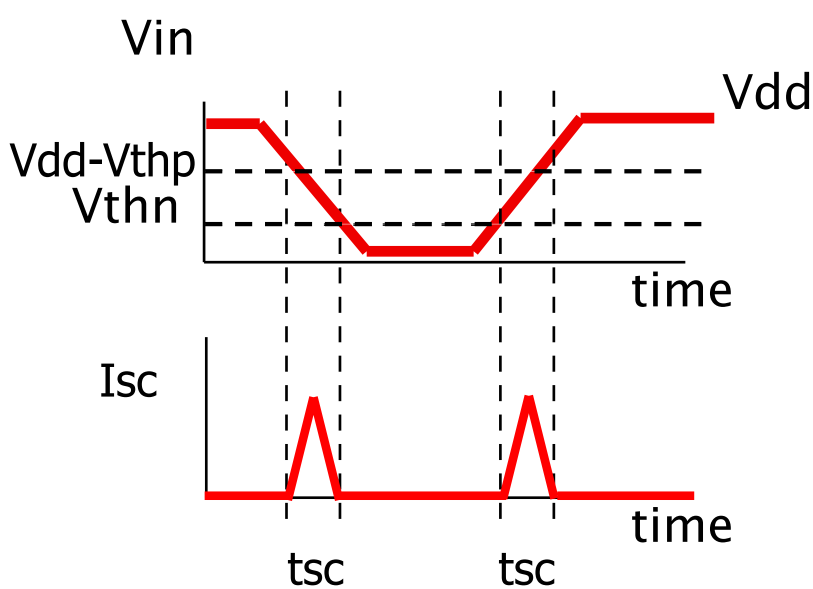
$$\int I(t)dt \approx I_{peak} \times t_{sc} \times \left(\frac{1}{2} \right)$$





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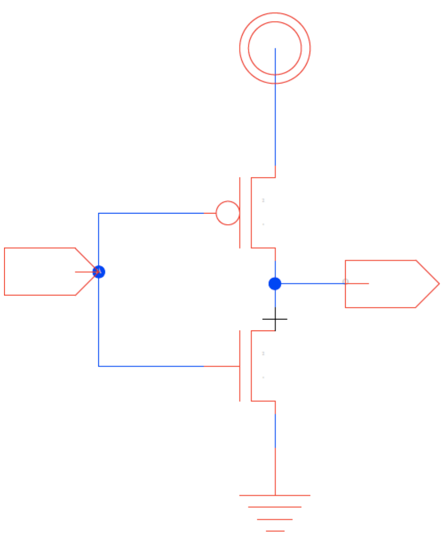
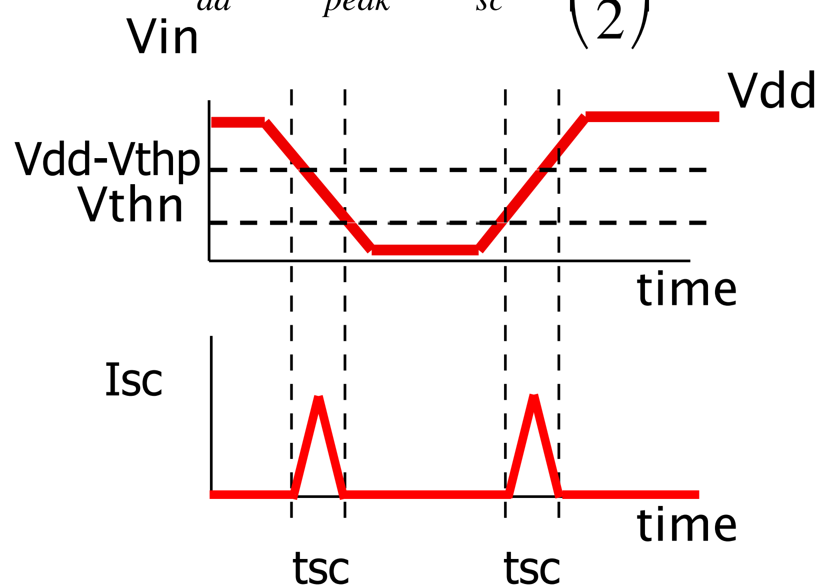
$$E = V_{dd} \times I_{peak} \times t_{sc} \times \left(\frac{1}{2} \right)$$





Short Circuit Energy

- Make it look like switching an equivalent capacitance, C_{SC}
  - $Q=I \times t$
  - $Q=CV$

$$E = V_{dd} \times \left(I_{peak} \times t_{sc} \times \left(\frac{1}{2} \right) \right)$$

$$E = V_{dd} \times Q_{SC}$$

$$E = V_{dd} \times (C_{SC} V_{dd}) = C_{SC} V_{dd}^2$$

Short Circuit Energy

- Make it look like switching an equivalent capacitance, C_{SC}
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$$E = V_{dd} \times Q_{SC}$$

$$E = V_{dd} \times (C_{SC} V_{dd}) = C_{SC} V_{dd}^2$$

$$C_{SC} = \frac{I_{peak} t_{sc}}{2V_{dd}}$$

Short Circuit Energy

- Every time switch (0→1 and 1→0)
  - Also dissipate short-circuit energy: $E = C_{sc}V^2$
  - C_{cs} “fake” capacitance (for accounting)

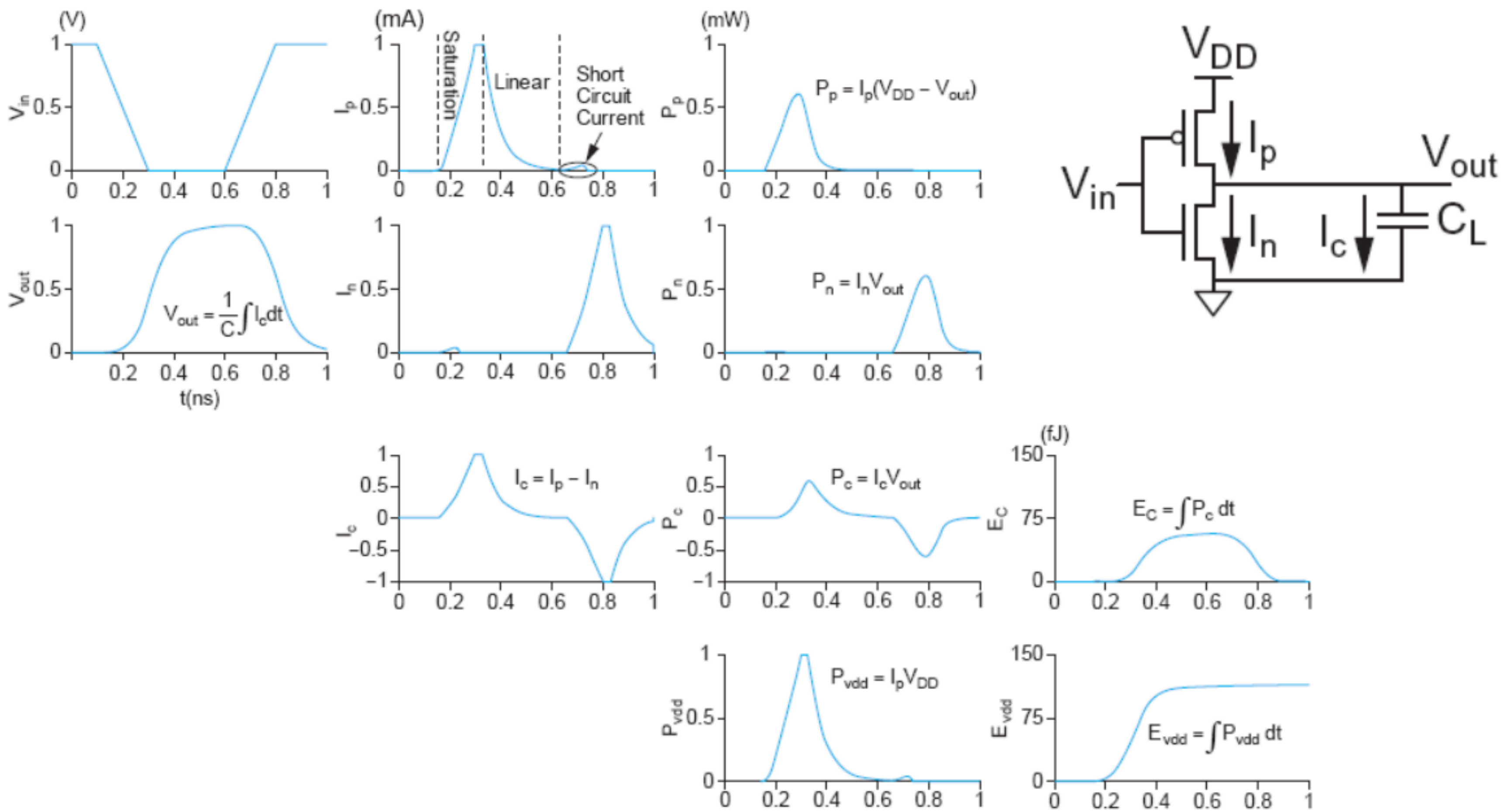
Total Power

- $P_{tot} = P_{static} + P_{sc} + P_{dyn}$

- $P_{dyn} + P_{sc} = aC_{load}V^2f + 2aC_{sc}V^2f$

- $P_{tot} \approx a(C_{load} + 2C_{sc})V^2f + VI'_s(W/L)e^{-Vt/(nkT/q)}$

Switching Waveforms



Ideas

- Three components of power
  - Static
  - Dynamic
  - Short-circuit
- $P_{tot} = P_{static} + P_{dyn} + P_{sc}$

Admin

- HW 5 due Friday 2/24
  - A lot of SPICE
  - Start early
  - Create your schematics, icons and test schematics with care to minimize the time spent

Acknowledgement

- Prof. André DeHon (University of Pennsylvania)
- Prof. Tania Khanna (University of Pennsylvania)