ESE370: Circuit-Level Modeling, Design, and Optimization for Digital Systems

Lec 15: March 27, 2024

Driving Large Capacitive Loads and Repeaters in Wiring

Penn

Today

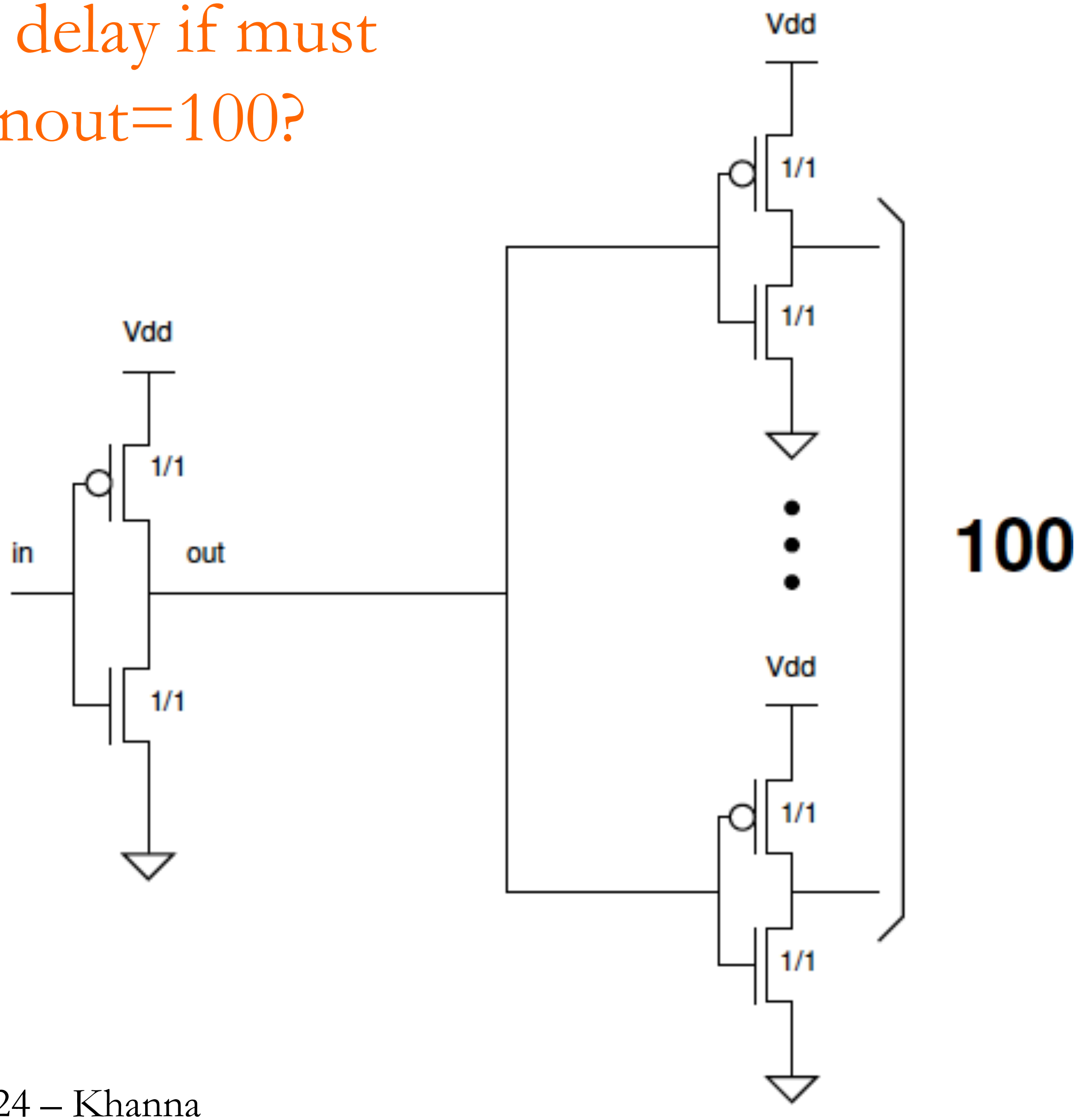
- Back to CMOS today
- How do we drive a large capacitive load?
  - Stages and buffer sizing
  - Minimum delay
- How can we reduce long wire delay?

Message

- To drive large loads
  - Scale buffers geometrically
  - Exponential scale up in buffer size
- Scale factor: 3—4 typically
  - One origin of FO4 target
- Drains contribute capacitance too (C_{diff})
- Can formulate sizing to optimize

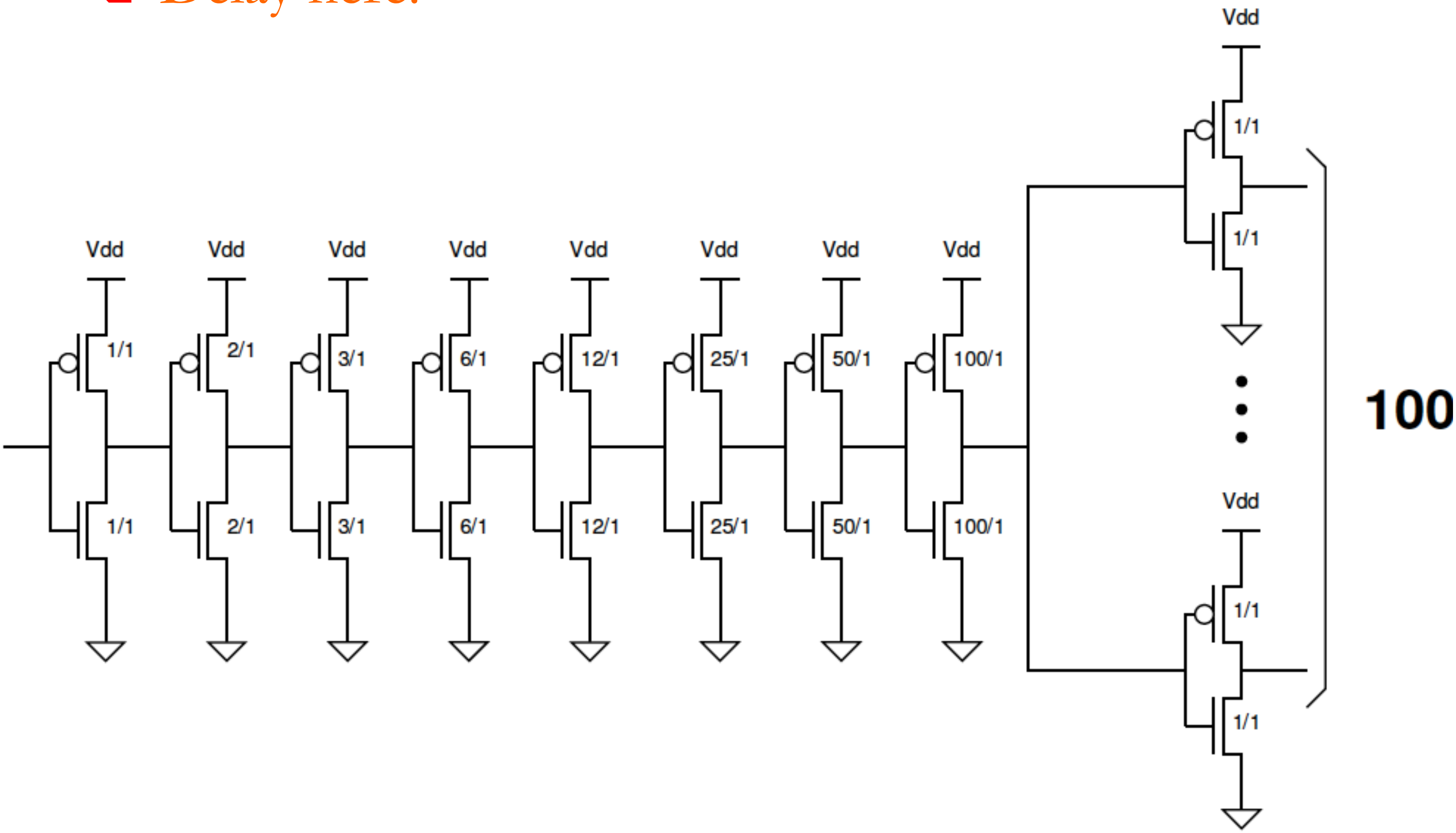
Call back: Large Fanout Delay

- What is delay if must drive fanout=100?



Call back: …and Again

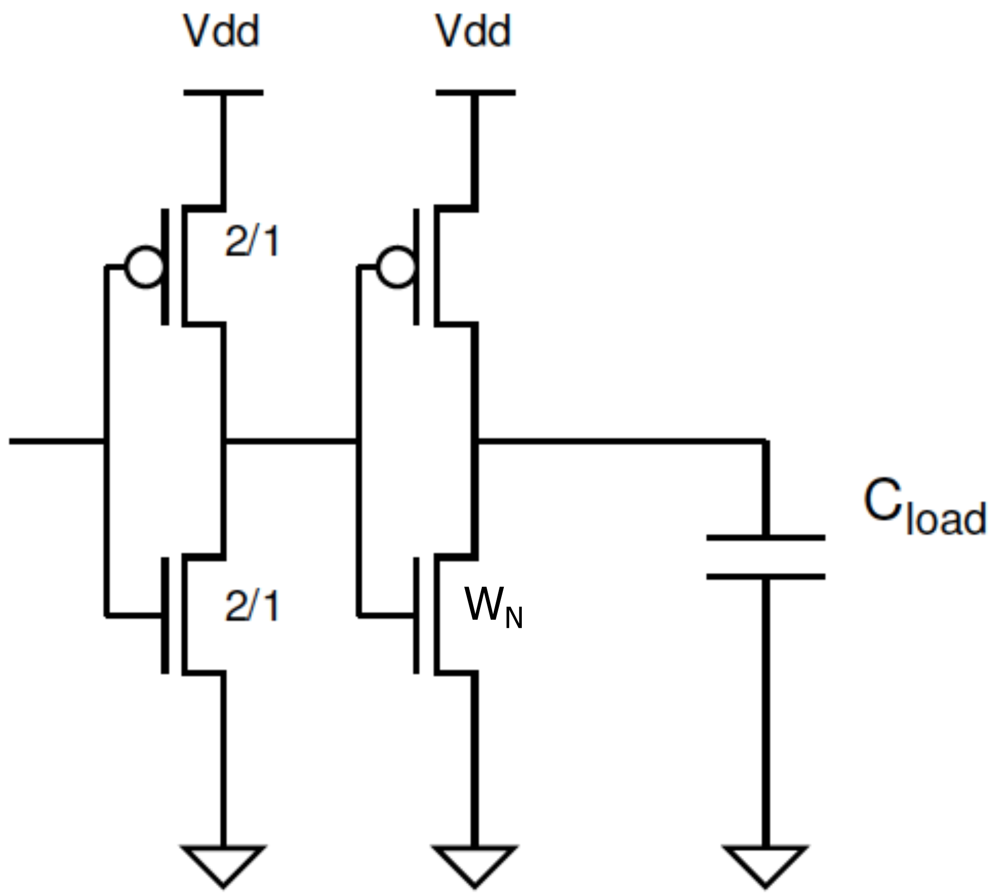
- Delay here?



Start C_{diff}=0

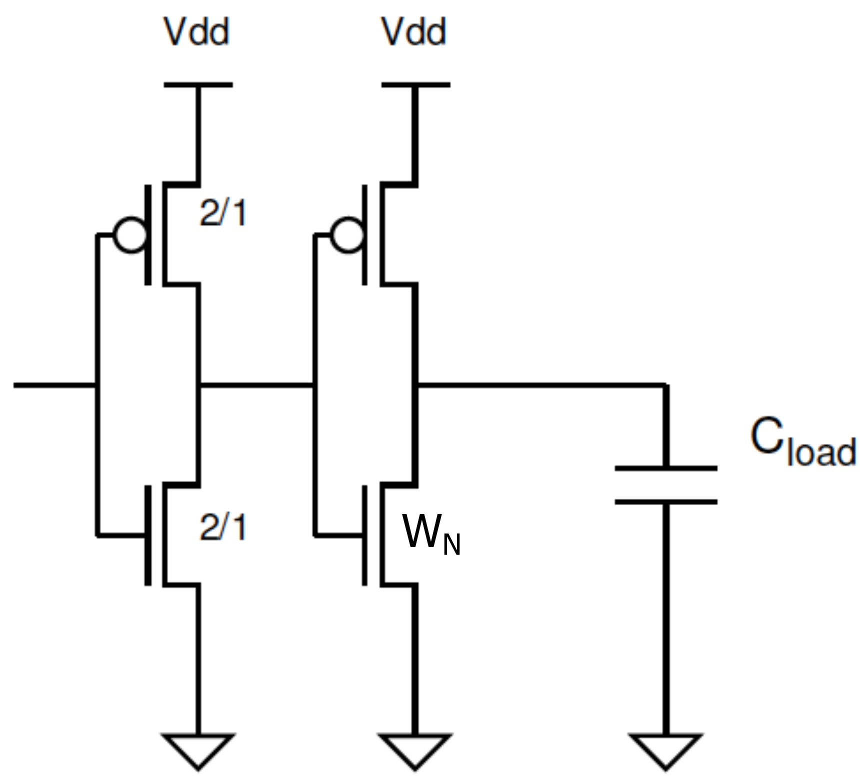
One Stage (Preclass 1)

- How do we size to minimize delay?



One Stage (Preclass 1)

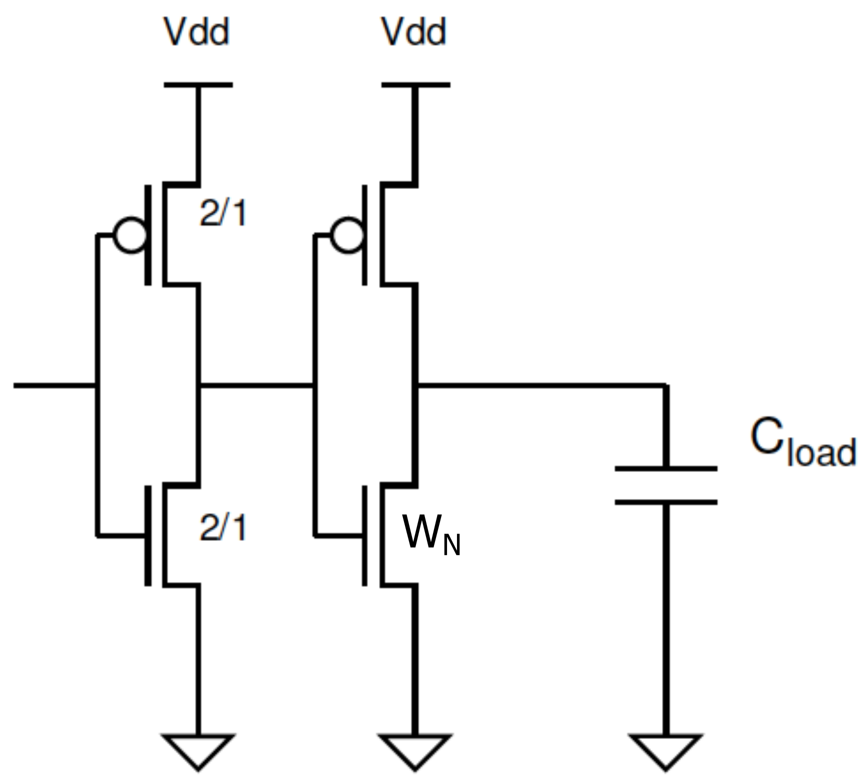
- Delay equation?



One Stage (Preclass 1)

- Delay equation?

$$delay = \frac{R_0}{2} 2W_N \cdot C_0 + \frac{R_0}{W_N} \cdot C_{load}$$



Minimize (Preclass 1)

$$delay = R_0 W_N \cdot C_0 + \frac{R_0}{W_N} \cdot C_{load}$$

- Differentiate and set to zero

$$R_0 C_0 - \frac{R_0}{W_N^2} \cdot C_{load} = 0$$

- What's W_N?

Minimize

$$delay = R_0 W_N \cdot C_0 + \frac{R_0}{W_N} \cdot C_{load}$$

- Differentiate and set to zero

$$R_0 C_0 - \frac{R_0}{W_N^2} \cdot C_{load} = 0$$

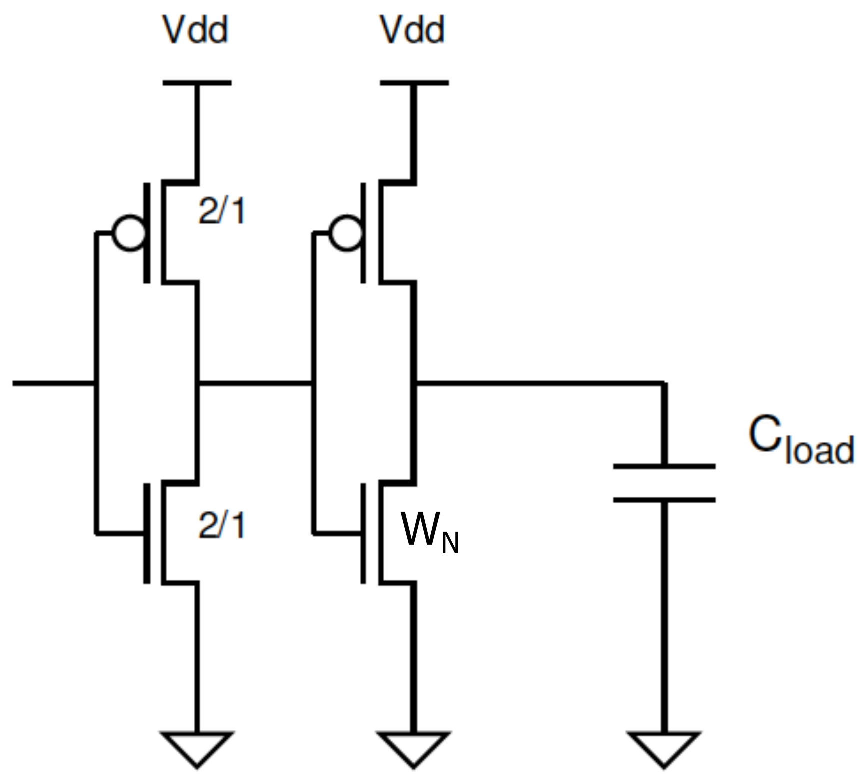
- What's W_N?

$$W_N^2 = \frac{C_{load}}{C_0} \qquad W_N = \sqrt{\frac{C_{load}}{C_0}}$$

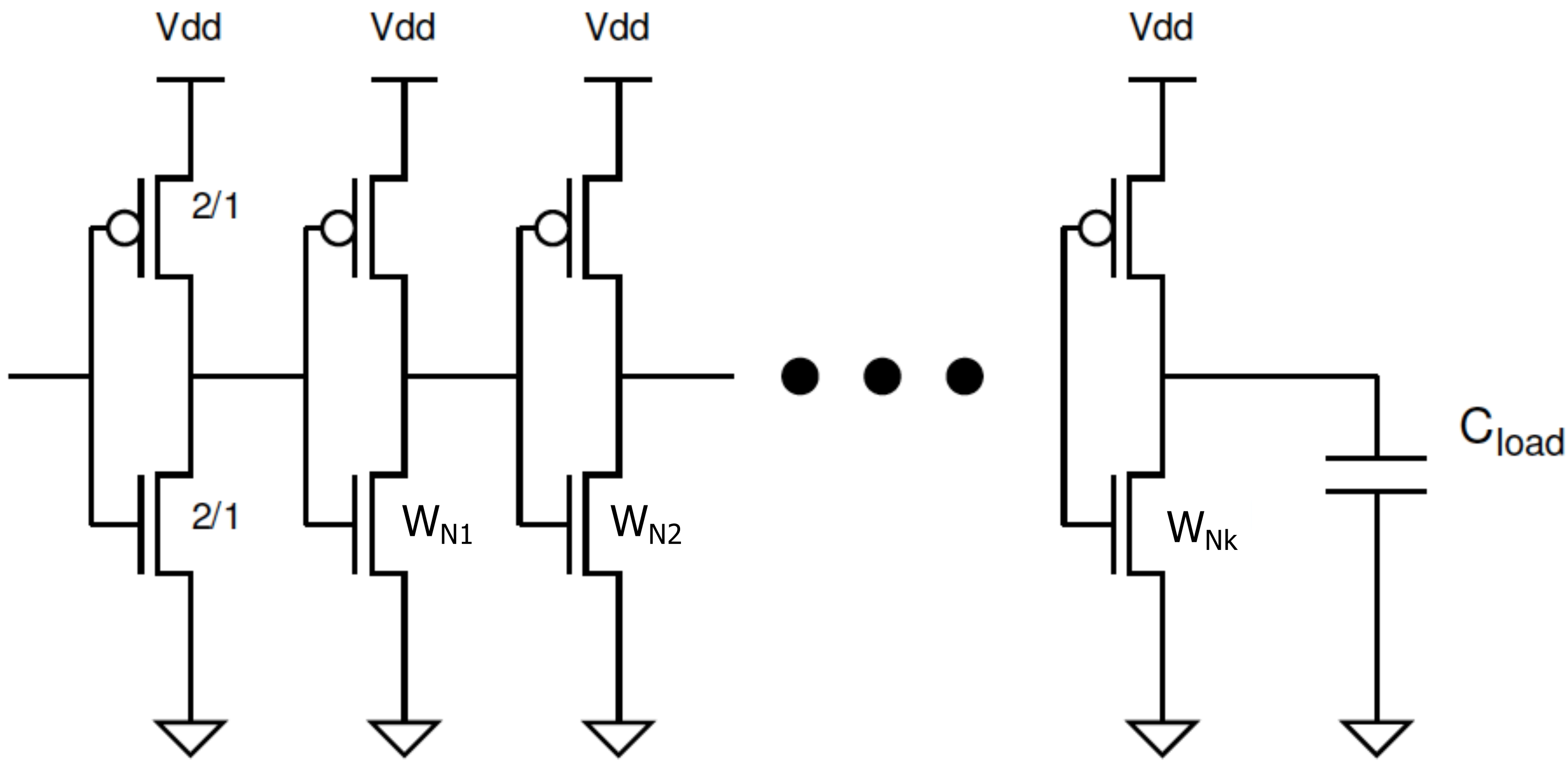
Concrete?

- What is W_N for $C_{load}=4x10^4C_0$?

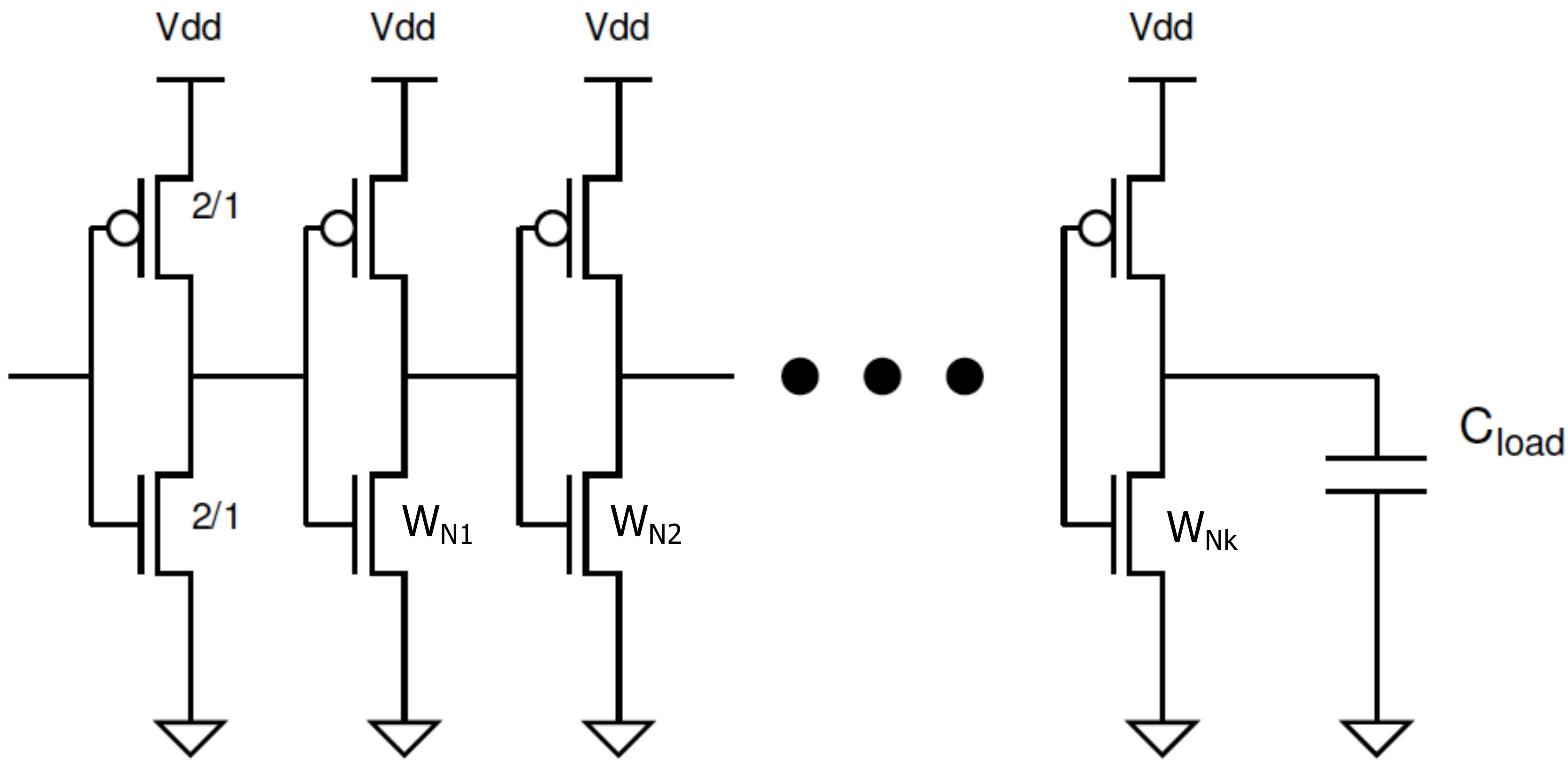
$$W_N = \sqrt{\frac{C_{load}}{C_0}}$$



k-stage Delay (Preclass 2a)



k-stage Delay (Preclass 2a)



$$2\tau\left(\frac{W_{N1}}{2}+\frac{W_{N2}}{W_{N1}}+\frac{W_{N3}}{W_{N2}}+\ldots+\frac{W_{Ni}}{W_{N(i-1)}}+\frac{W_{N(i+1)}}{W_{Ni}}+\ldots+\frac{W_{Nk}}{W_{N(k-1)}}\right)+\frac{R_0}{W_{Nk}}\times C_{load}$$

Size W_{Ni} to minimize delay (Preclass 2b)

- How do we minimize?

$$2\tau\left(\frac{W_{N1}}{2}+\frac{W_{N2}}{W_{N1}}+\frac{W_{N3}}{W_{N2}}+\ldots+\frac{W_{Ni}}{W_{N(i-1)}}+\frac{W_{N(i+1)}}{W_{Ni}}+\ldots+\frac{W_{Nk}}{W_{N(k-1)}}\right)+\frac{R_0}{W_{Nk}}\times C_{load}$$

Size W_{Ni} to minimize delay (Preclass 2b)

- Take partial derivative with respect to W_{Ni} and set = 0

$$2\tau\left(\frac{W_{N1}}{2}+\frac{W_{N2}}{W_{N1}}+\frac{W_{N3}}{W_{N2}}+\ldots+\frac{W_{Ni}}{W_{N(i-1)}}+\frac{W_{N(i+1)}}{W_{Ni}}+\ldots+\frac{W_{Nk}}{W_{N(k-1)}}\right)+\frac{R_0}{W_{Nk}}\times C_{load}$$

$$2\tau\left(0+0+0+\ldots+\frac{1}{W_{N(i-1)}}-\frac{W_{N(i+1)}}{\left(W_{Ni}\right)^2}+\ldots+0\right)+0=0$$

Size W_{Ni} to minimize delay (Preclass 2b)

- Take partial derivative with respect to W_{Ni} and set = 0

$$2\tau\left(\frac{W_{N1}}{2}+\frac{W_{N2}}{W_{N1}}+\frac{W_{N3}}{W_{N2}}+\ldots+\frac{W_{Ni}}{W_{N(i-1)}}+\frac{W_{N(i+1)}}{W_{Ni}}+\ldots+\frac{W_{Nk}}{W_{N(k-1)}}\right)+\frac{R_0}{W_{Nk}}\times C_{load}$$

$$2\tau\left(0+0+0+\ldots+\frac{1}{W_{N(i-1)}}-\frac{W_{N(i+1)}}{\left(W_{Ni}\right)^2}+\ldots+0\right)+0=0$$

$$\frac{1}{W_{N(i-1)}}=\frac{W_{N(i+1)}}{W_{Ni}^2}\rightarrow\frac{W_{Ni}}{W_{N(i-1)}}=\frac{W_{N(i+1)}}{W_{Ni}}$$

Delay

- **Conclude:** at optimal sizing, ratio of stages is same:

$$\frac{1}{W_{N(i-1)}} = \frac{W_{N(i+1)}}{W_{Ni}^2} \rightarrow \frac{W_{Ni}}{W_{N(i-1)}} = \frac{W_{N(i+1)}}{W_{Ni}}$$

Delay

- Call that ratio $\boldsymbol{\rho}$

$$\rho = \frac{W_{Ni}}{W_{N(i-1)}} = \frac{W_{N(i+1)}}{W_{Ni}}$$

Stage Delay

$$2\tau\left(\frac{W_{N1}}{2}+\frac{W_{N2}}{W_{N1}}+\frac{W_{N3}}{W_{N2}}+\ldots+\frac{W_{Ni}}{W_{N(i-1)}}+\frac{W_{N(i+1)}}{W_{Ni}}+\ldots+\frac{W_{Nk}}{W_{N(k-1)}}\right)+\frac{R_0}{W_{Nk}}\times C_{load}$$

Stage Delay

$$2\tau\left(\frac{W_{N1}}{2}+\frac{W_{N2}}{W_{N1}}+\frac{W_{N3}}{W_{N2}}+\ldots+\frac{W_{Ni}}{W_{N(i-1)}}+\frac{W_{N(i+1)}}{W_{Ni}}+\ldots+\frac{W_{Nk}}{W_{N(k-1)}}\right)+\frac{R_0}{W_{Nk}}\times C_{load}$$

$$2\tau\left(\frac{W_{N1}}{2}+\frac{W_{N2}}{W_{N1}}+\frac{W_{N3}}{W_{N2}}+\ldots+\frac{W_{Ni}}{W_{N(i-1)}}+\frac{W_{N(i+1)}}{W_{Ni}}+\ldots+\frac{W_{Nk}}{W_{N(k-1)}}+\frac{R_0}{2\tau\cdot W_{Nk}}\times C_{load}\right)$$

Stage Delay

$$2\tau\left(\frac{W_{N1}}{2}+\frac{W_{N2}}{W_{N1}}+\frac{W_{N3}}{W_{N2}}+\ldots+\frac{W_{Ni}}{W_{N(i-1)}}+\frac{W_{N(i+1)}}{W_{Ni}}+\ldots+\frac{W_{Nk}}{W_{N(k-1)}}\right)+\frac{R_0}{W_{Nk}}\times C_{load}$$

$$2\tau\left(\frac{W_{N1}}{2}+\frac{W_{N2}}{W_{N1}}+\frac{W_{N3}}{W_{N2}}+\ldots+\frac{W_{Ni}}{W_{N(i-1)}}+\frac{W_{N(i+1)}}{W_{Ni}}+\ldots+\frac{W_{Nk}}{W_{N(k-1)}}+\frac{R_0}{2\tau\cdot W_{Nk}}\times C_{load}\right)$$

$$\rho=\frac{W_{N1}}{2}=\frac{W_{Ni}}{W_{N(i-1)}}=\frac{W_{N(i+1)}}{W_{Ni}}=\frac{C_{load}}{W_{Nk}\left(2C_0\right)}$$

Stage Delay

$$\rho = \frac{W_{N1}}{2} = \frac{W_{Ni}}{W_{N(i-1)}} = \frac{W_{N(i+1)}}{W_{Ni}} = \frac{C_{load}}{W_{Nk}\left(2C_0\right)}$$

Two math simplifications: 1) to relate ρ and k, 2) total delay in terms of ρ and k

Stage Delay

$$\rho = \frac{W_{N1}}{2} = \frac{W_{Ni}}{W_{N(i-1)}} = \frac{W_{N(i+1)}}{W_{Ni}} = \frac{C_{load}}{W_{Nk}\left(2C_0\right)}$$

$$\left(\frac{W_{N1}}{2}\right)\left(\frac{W_{N2}}{W_{N1}}\right)\left(\frac{W_{N3}}{W_{N2}}\right)\cdots\left(\frac{W_{Ni}}{W_{N(i-1)}}\right)\left(\frac{W_{N(i+1)}}{W_{Ni}}\right)\cdots\left(\frac{W_{Nk}}{W_{N(k-1)}}\right)\frac{C_{load}}{W_{Nk}\left(2C_0\right)} = \rho^{k+1}$$

Stage Delay

$$\rho^{k+1} = \frac{C_{load}}{4C_0} \rightarrow \rho = \sqrt[k+1]{\frac{C_{load}}{4C_0}}$$

$$\rho = \left(\frac{C_{load}}{4C_0}\right)^{\left(\frac{1}{k+1}\right)}$$

Total Delay (Preclass 2c)

$$\rho = \frac{W_{N1}}{2} = \frac{W_{Ni}}{W_{N(i-1)}} = \frac{W_{N(i+1)}}{W_{Ni}} = \frac{C_{load}}{W_{Nk}\left(2C_0\right)}$$

$$2\tau\left(\frac{W_{N1}}{2} + \frac{W_{N2}}{W_{N1}} + \frac{W_{N3}}{W_{N2}} + \ldots + \frac{W_{Ni}}{W_{N(i-1)}} + \frac{W_{N(i+1)}}{W_{Ni}} + \ldots + \frac{W_{Nk}}{W_{N(k-1)}} + \frac{R_0}{2\tau \cdot W_{Nk}} \times C_{load}\right)$$

Total Delay (Preclass 2c)

$$\rho = \frac{W_{N1}}{2} = \frac{W_{Ni}}{W_{N(i-1)}} = \frac{W_{N(i+1)}}{W_{Ni}} = \frac{C_{load}}{W_{Nk}\left(2C_0\right)}$$

$$2\tau\left(\frac{W_{N1}}{2} + \frac{W_{N2}}{W_{N1}} + \frac{W_{N3}}{W_{N2}} + \ldots + \frac{W_{Ni}}{W_{N(i-1)}} + \frac{W_{N(i+1)}}{W_{Ni}} + \ldots + \frac{W_{Nk}}{W_{N(k-1)}} + \frac{R_0}{2\tau \cdot W_{Nk}} \times C_{load}\right)$$

$$TotalDelay = 2\tau(k+1)\rho$$

Total Delay

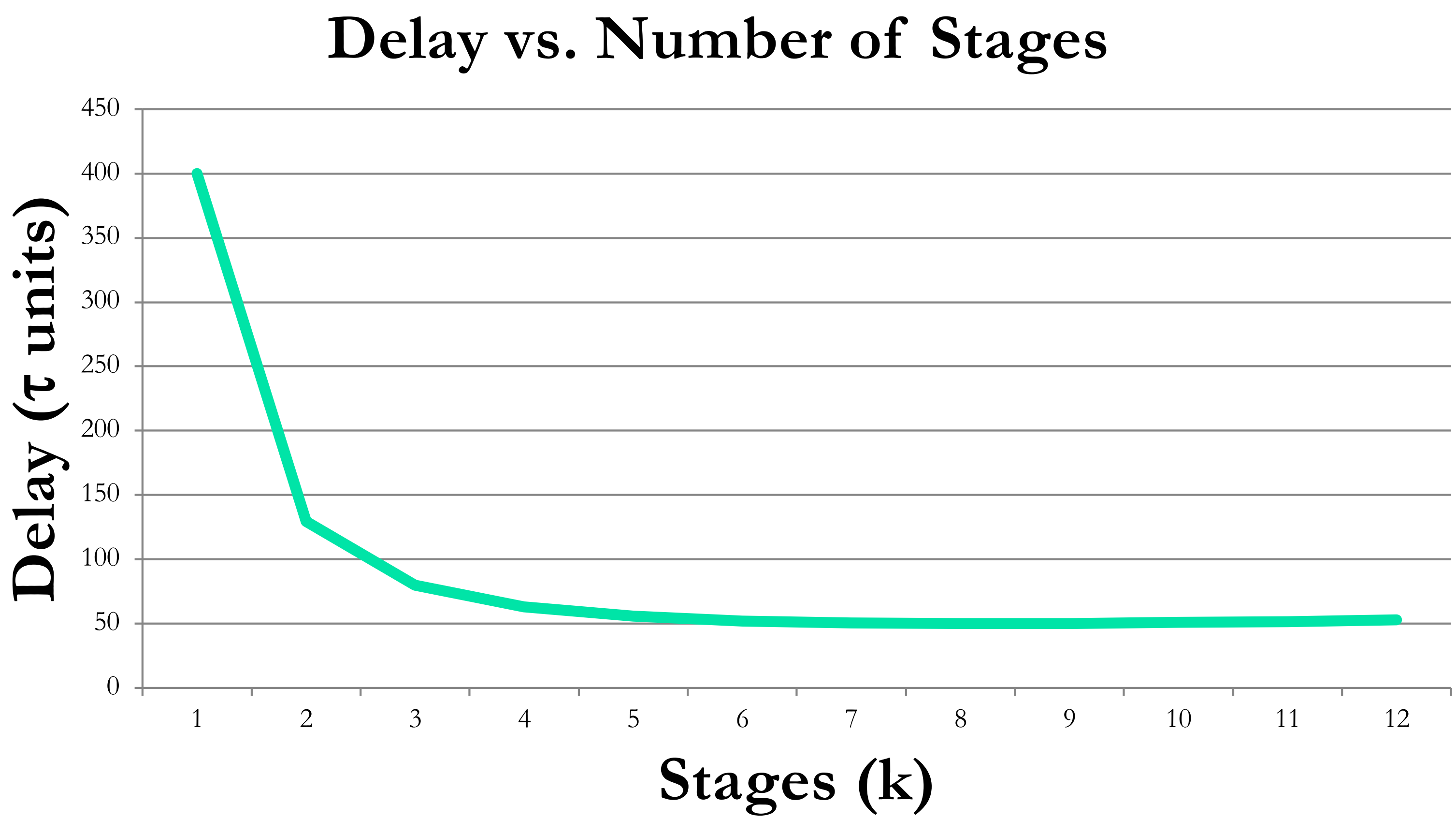
$$\rho = \left(\frac{C_{load}}{4C_0}\right)^{\left(\frac{1}{k+1}\right)}$$

$$TotalDelay = 2\tau(k+1)\rho$$

$$TotalDelay = 2\tau(k+1)\left(\frac{C_{load}}{4C_0}\right)^{\left(\frac{1}{k+1}\right)}$$

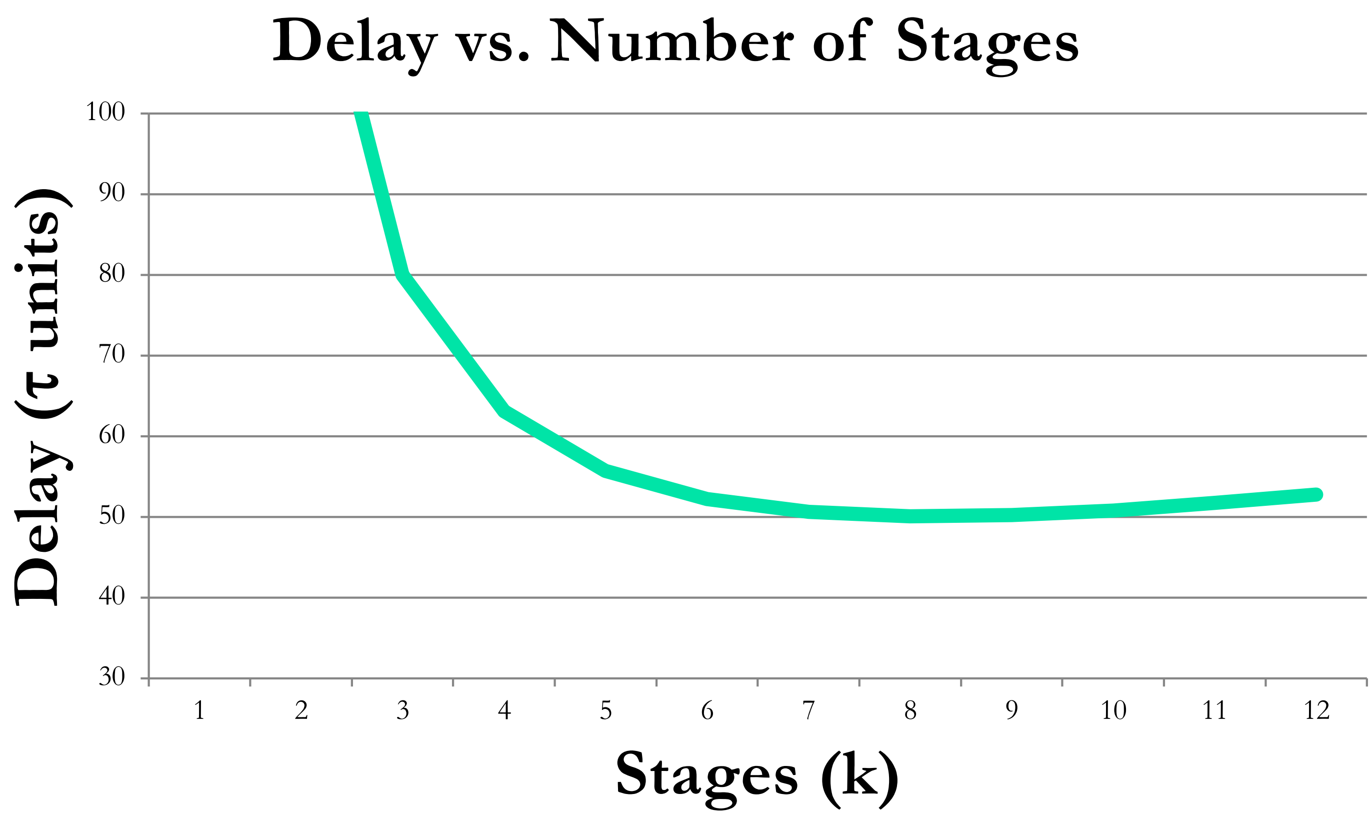
Plot Delay vs. k ($C_{load}=4x10^4C_0$)

$$TotalDelay = 2\tau(k+1)\left(\frac{C_{load}}{4C_0}\right)^{\left(\frac{1}{k+1}\right)}$$



Zoom: Plot Delay vs. k (C_{load}=4x$10^4$$C_0$)

$$TotalDelay = 2\tau(k+1)\left(\frac{C_{load}}{4C_0}\right)^{\left(\frac{1}{k+1}\right)}$$



Minimize

$$TotalDelay = 2\tau(k+1)\left(\frac{C_{load}}{4C_0}\right)^{\left(\frac{1}{k+1}\right)}$$

$$0 = 2\tau\left[\left(\frac{C_{load}}{4C_0}\right)^{\left(\frac{1}{k+1}\right)} - (k+1)\cdot\ln\left(\frac{C_{load}}{4C_0}\right)\left(\frac{C_{load}}{4C_0}\right)^{\left(\frac{1}{k+1}\right)}\left(\frac{1}{k+1}\right)^2\right]$$

$$\frac{d(b^x)}{dx} = \ln(b)\cdot b^x$$

$$\frac{d(1/x)}{dx} = -\frac{1}{x^2}$$

Minimize

$$TotalDelay = 2\tau(k+1)\left(\frac{C_{load}}{4C_0}\right)^{\left(\frac{1}{k+1}\right)}$$

$$0 = 2\tau\left[\left(\frac{C_{load}}{4C_0}\right)^{\left(\frac{1}{k+1}\right)} - (k+1)\cdot\ln\left(\frac{C_{load}}{4C_0}\right)\left(\frac{C_{load}}{4C_0}\right)^{\left(\frac{1}{k+1}\right)}\left(\frac{1}{k+1}\right)^2\right]$$

$$0 = 1 - \left(\frac{1}{k+1}\right)\ln\left(\frac{C_{load}}{4C_0}\right)$$

$$k = \ln\left(\frac{C_{load}}{4C_0}\right) - 1$$

$$\frac{d(b^x)}{dx} = \ln(b)\cdot b^x$$

$$\frac{d(1/x)}{dx} = -\frac{1}{x^2}$$

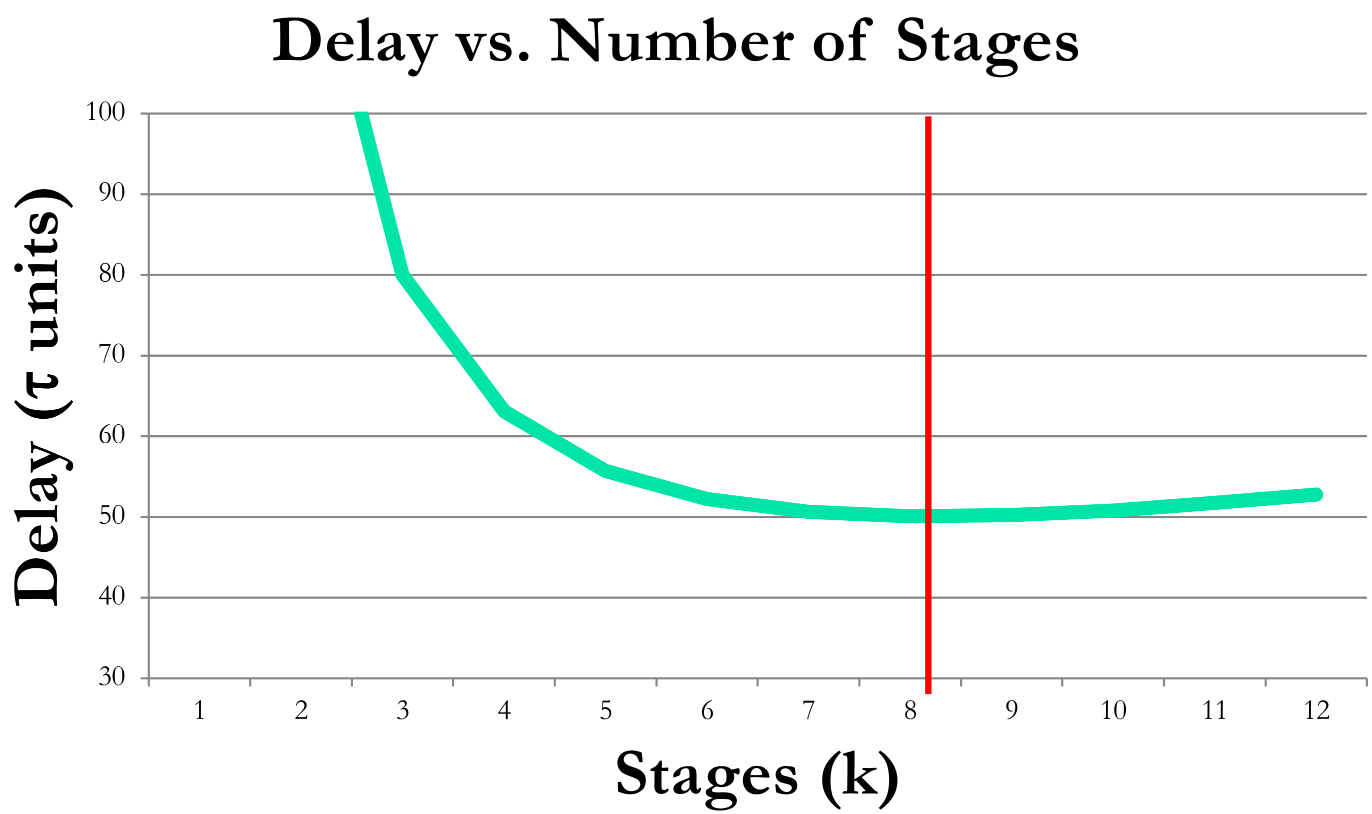
Concrete

- What is optimal k for $C_{load}=4\times10^4 C_0$?

$$k = \ln\left(\frac{C_{load}}{4C_0}\right) - 1$$

Zoom: Plot Delay vs. k ($C_{load}=4x10^4C_0$)

$$TotalDelay = 2\tau(k+1)\left(\frac{C_{load}}{4C_0}\right)^{\left(\frac{1}{k+1}\right)}$$



Optimum Scale Up

- For optimum delay

$$k = \ln\left(\frac{C_{load}}{4C_0}\right) - 1$$

- What is ρ?

$$\rho = \left(\frac{C_{load}}{4C_0}\right)^{\left(\frac{1}{k+1}\right)}$$

Optimum Scale Up

$$\rho = \left(\frac{C_{load}}{4C_0}\right)^{\left(\frac{1}{\ln\left(\frac{C_{load}}{4C_0}\right)}\right)} = \left(Y\right)^{\left(\frac{1}{\ln\left(Y\right)}\right)}$$

Optimum Scale Up

$$\rho = \left(\frac{C_{load}}{4C_0}\right)^{\left(\frac{1}{\ln\left(\frac{C_{load}}{4C_0}\right)}\right)} = \left(Y\right)^{\left(\frac{1}{\ln\left(Y\right)}\right)}$$

$$\ln(\rho) = \frac{1}{\ln\left(Y\right)}\ln(Y) = 1$$

$$\rho = e$$

Call Back: Total Delay

$$\rho = \frac{W_{N1}}{2} = \frac{W_{Ni}}{W_{N(i-1)}} = \frac{W_{N(i+1)}}{W_{Ni}} = \frac{C_{load}}{W_{Nk}\left(2C_0\right)}$$

$$2\tau\left(\frac{W_{N1}}{2} + \frac{W_{N2}}{W_{N1}} + \frac{W_{N3}}{W_{N2}} + \ldots + \frac{W_{Ni}}{W_{N(i-1)}} + \frac{W_{N(i+1)}}{W_{Ni}} + \ldots + \frac{W_{Nk}}{W_{N(k-1)}}\right) + \frac{R_0}{W_{Nk}} \times C_{load}$$

$$2\tau\left(\frac{W_{N1}}{2} + \frac{W_{N2}}{W_{N1}} + \frac{W_{N3}}{W_{N2}} + \ldots + \frac{W_{Ni}}{W_{N(i-1)}} + \frac{W_{N(i+1)}}{W_{Ni}} + \ldots + \frac{W_{Nk}}{W_{N(k-1)}} + \frac{R_0}{2\tau \cdot W_{Nk}} \times C_{load}\right)$$

$$TotalDelay = 2\tau(k+1)\rho$$

Delay at Optimum (Preclass 3e)

$$k = \ln\left(\frac{C_{load}}{4C_0}\right) - 1 \qquad \rho = e$$

$$TotalDelay = 2\tau(k+1)\rho$$

Delay at Optimum (Preclass 3e)

$$k = \ln\left(\frac{C_{load}}{4C_0}\right) - 1 \qquad \rho = e$$

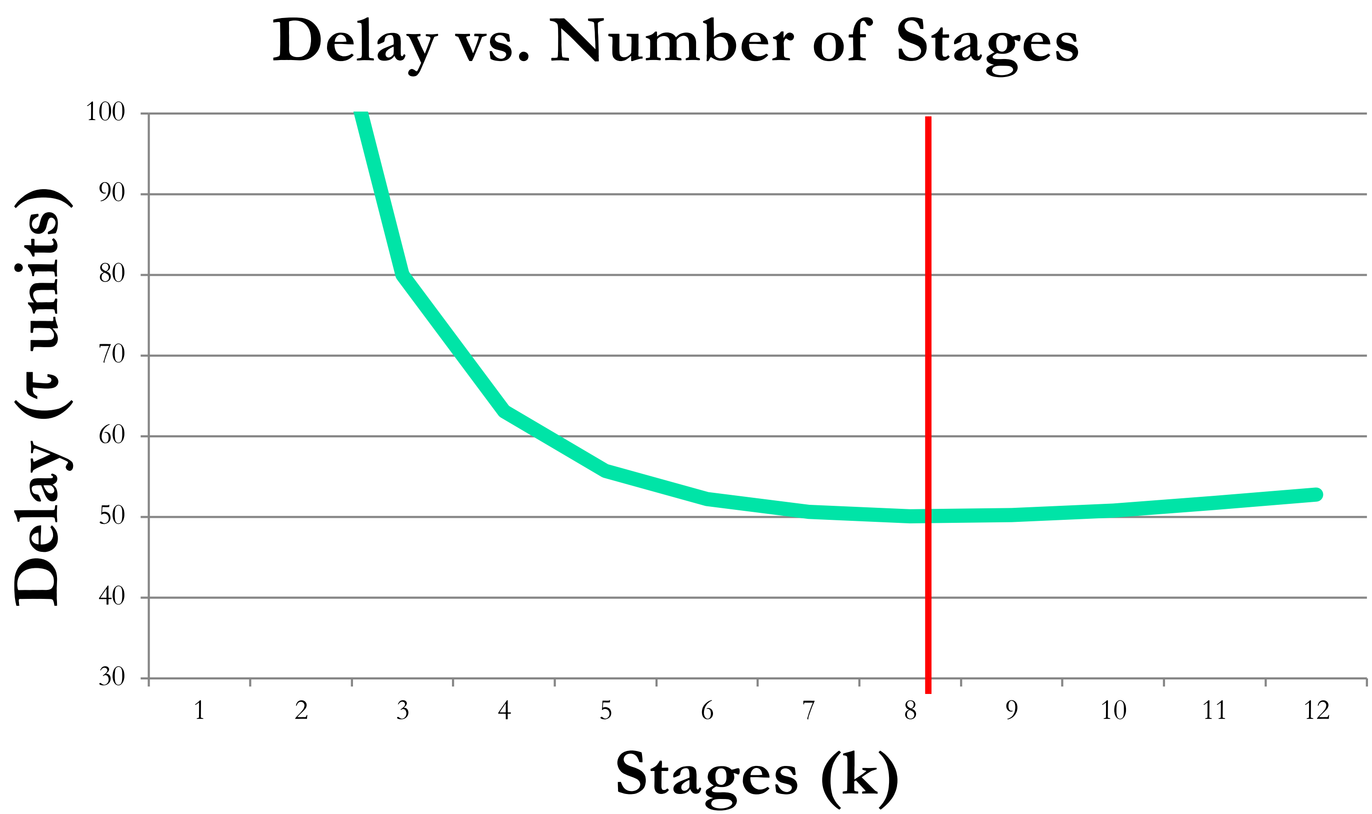
$$TotalDelay = 2\tau(k+1)\rho$$

$$TotalDelay = 2\tau \cdot \ln\left(\frac{C_{load}}{4C_0}\right) \cdot e$$

- What is optimal delay for $C_{load}=4x10^4C_0$ in tau units?

Zoom: Plot Delay vs. k (C_{load}=4x10^4C_0)

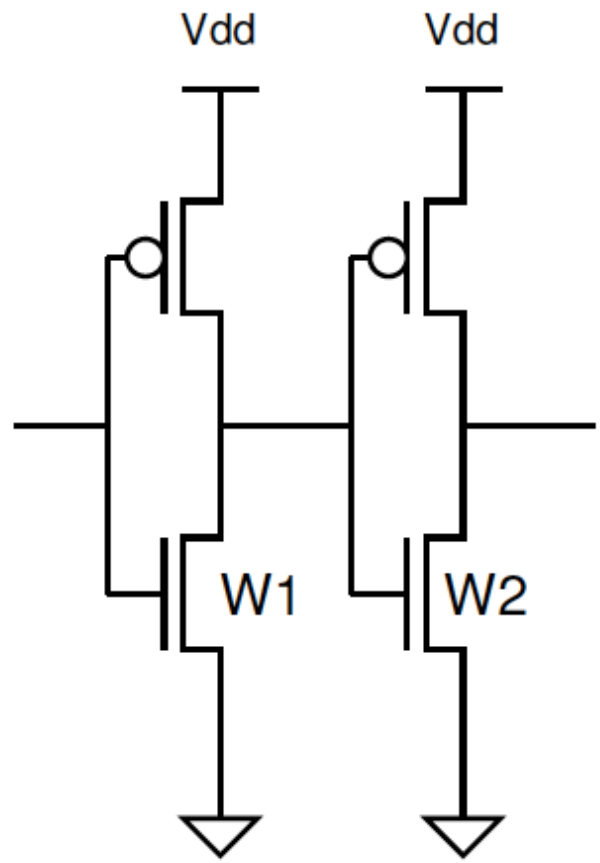
$$TotalDelay = 2\tau(k+1)\left(\frac{C_{load}}{4C_0}\right)^{\left(\frac{1}{k+1}\right)}$$



$$C_{diff}=\gamma C_{gate}$$

Diffusion Capacitance (Preclass 3)

- What does this do to τ model?
  - Delay of middle stage cascade?



Diffusion Capacitance (Preclass 3)

- What does this do to τ model?
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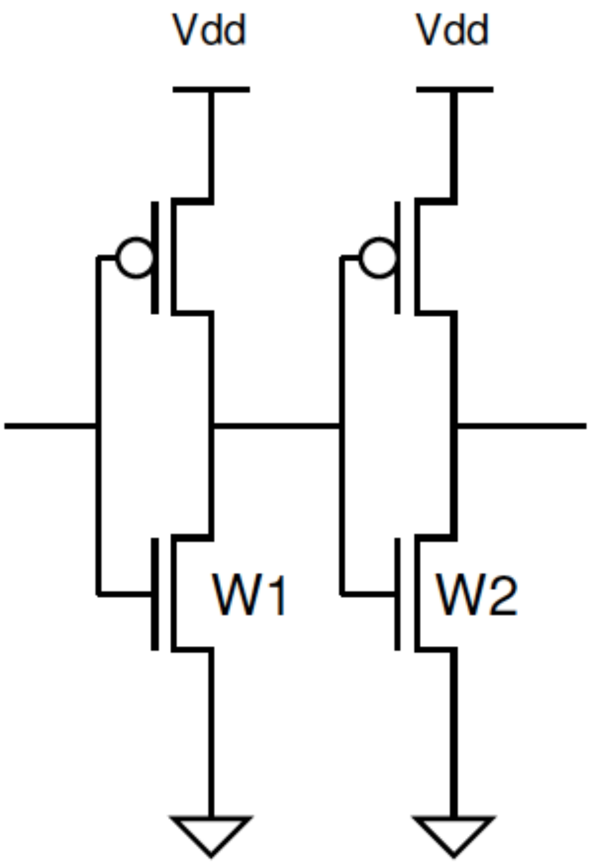


$$delay_{W1\rightarrow W2} = \left(\frac{R_0}{W_1}\right)\left(2W_1 \cdot C_{diff0} + 2W_2 \cdot C_0\right)$$

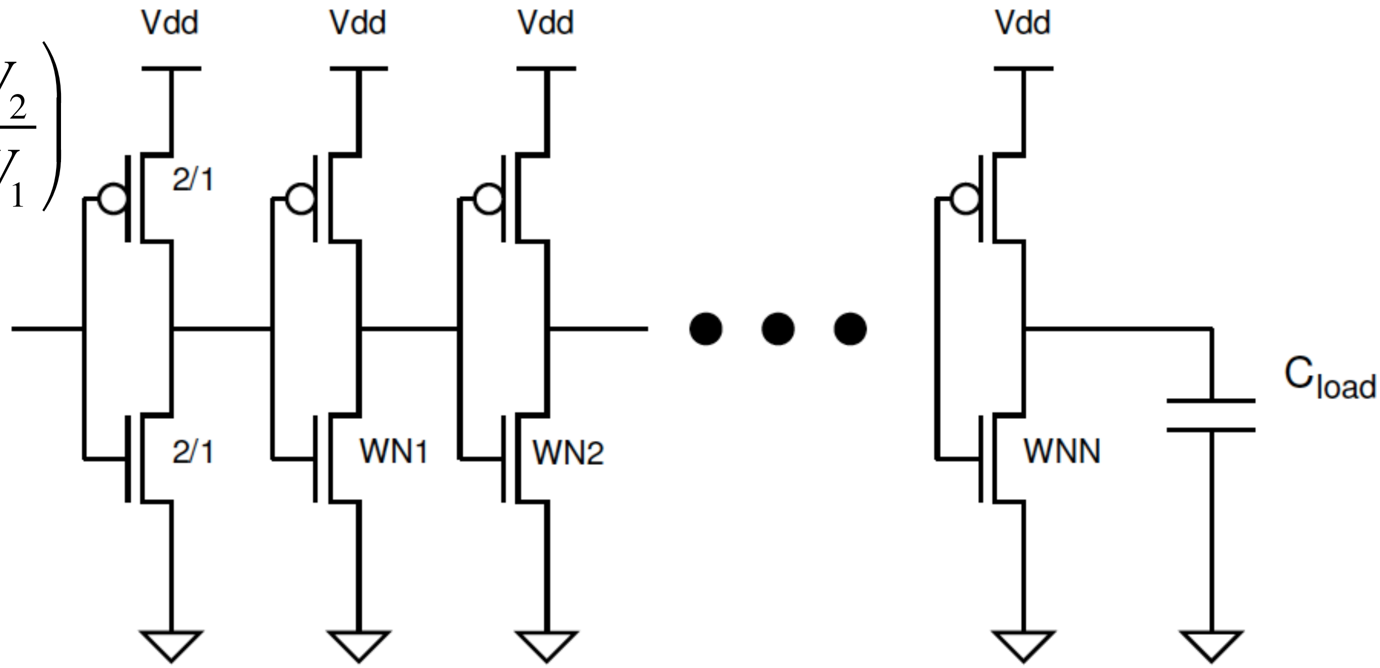
$$delay_{W1\rightarrow W2} = 2\left(\frac{R_0}{W_1}\right)\left(W_1 \cdot \gamma C_0 + W_2 \cdot C_0\right)$$

$$delay_{W1\rightarrow W2} = 2\tau\left(\gamma + \frac{W_2}{W_1}\right)$$

k-stage Delay

$$2\tau\left(\frac{W_{N1}}{2}+\frac{W_{N2}}{W_{N1}}+\frac{W_{N3}}{W_{N2}}+\ldots+\frac{W_{Ni}}{W_{N(i-1)}}+\frac{W_{N(i+1)}}{W_{Ni}}+\ldots+\frac{W_{Nk}}{W_{N(k-1)}}\right)+\frac{R_0}{W_{Nk}}\times C_{load}$$

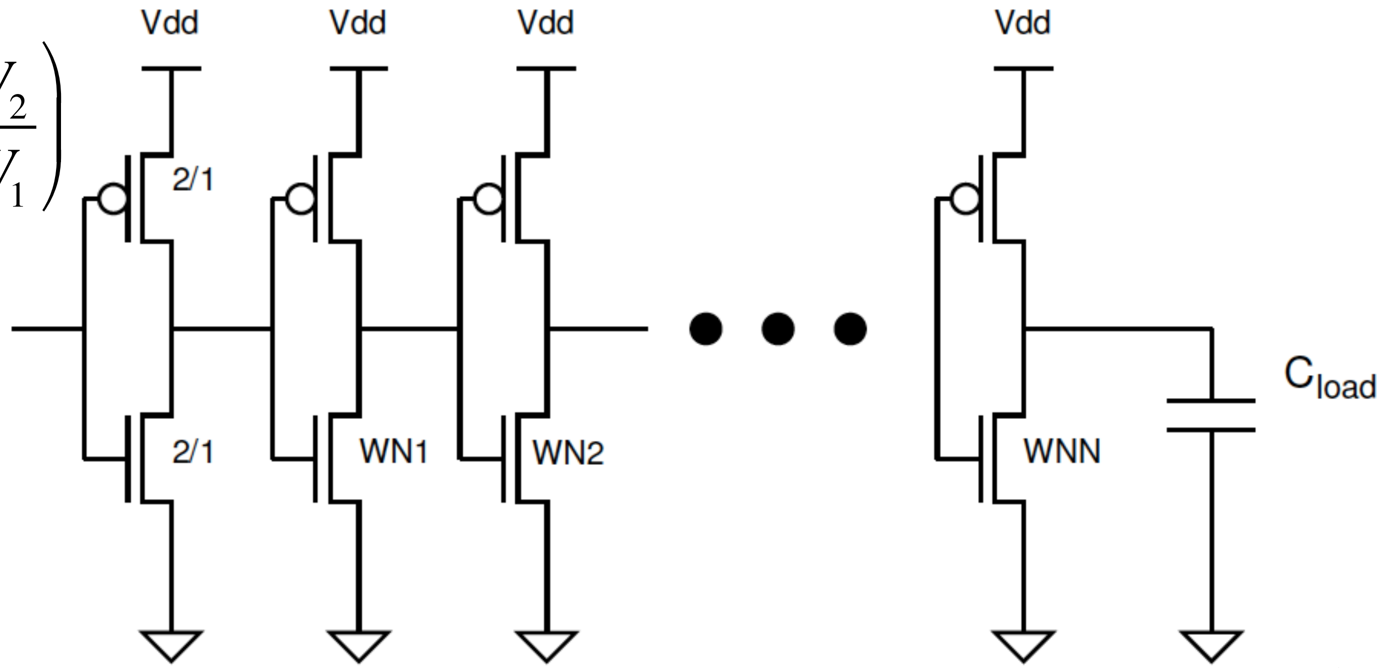
$$delay_{W1\rightarrow W2}=2\tau\left(\gamma+\frac{W_2}{W_1}\right)$$



k-stage Delay (Preclass 4a)

$$2\tau\left(\frac{W_{N1}}{2}+\frac{W_{N2}}{W_{N1}}+\frac{W_{N3}}{W_{N2}}+\ldots+\frac{W_{Ni}}{W_{N(i-1)}}+\frac{W_{N(i+1)}}{W_{Ni}}+\ldots+\frac{W_{Nk}}{W_{N(k-1)}}\right)+\frac{R_0}{W_{Nk}}\times C_{load}$$

$$delay_{W1\rightarrow W2}=2\tau\left(\gamma+\frac{W_2}{W_1}\right)$$



$$2\tau\left(\gamma+\frac{W_{N1}}{2}+\gamma+\frac{W_{N2}}{W_{N1}}+\gamma+\frac{W_{N3}}{W_{N2}}+\ldots+\gamma+\frac{W_{Ni}}{W_{N(i-1)}}+\gamma+\frac{W_{N(i+1)}}{W_{Ni}}+\ldots+\gamma+\frac{W_{Nk}}{W_{N(k-1)}}\right)$$

$$+\frac{R_0}{W_{Nk}}\left(C_{load}+2\gamma W_{Nk}C_0\right)$$

k-stage Delay

$$2\tau\left(\gamma+\frac{W_{N1}}{2}+\gamma+\frac{W_{N2}}{W_{N1}}+\gamma+\frac{W_{N3}}{W_{N2}}+\ldots+\gamma+\frac{W_{Ni}}{W_{N(i-1)}}+\gamma+\frac{W_{N(i+1)}}{W_{Ni}}+\ldots+\gamma+\frac{W_{Nk}}{W_{N(k-1)}}\right)$$

$$+\frac{R_0}{W_{Nk}}(C_{load}+2\gamma W_{Nk}C_0)$$

$$2\tau\left(\gamma k+\frac{W_{N1}}{2}+\frac{W_{N2}}{W_{N1}}+\frac{W_{N3}}{W_{N2}}+\ldots+\frac{W_{Ni}}{W_{N(i-1)}}+\frac{W_{N(i+1)}}{W_{Ni}}+\ldots+\frac{W_{Nk}}{W_{N(k-1)}}\right)+\frac{R_0}{W_{Nk}}(C_{load}+2\gamma W_{Nk}C_0)$$

k-stage Delay

$$2\tau\left(\gamma+\frac{W_{N1}}{2}+\gamma+\frac{W_{N2}}{W_{N1}}+\gamma+\frac{W_{N3}}{W_{N2}}+\ldots+\gamma+\frac{W_{Ni}}{W_{N(i-1)}}+\gamma+\frac{W_{N(i+1)}}{W_{Ni}}+\ldots+\gamma+\frac{W_{Nk}}{W_{N(k-1)}}\right)$$

$$+\frac{R_0}{W_{Nk}}(C_{load}+2\gamma W_{Nk}C_0)$$

$$2\tau\left(\gamma k+\frac{W_{N1}}{2}+\frac{W_{N2}}{W_{N1}}+\frac{W_{N3}}{W_{N2}}+\ldots+\frac{W_{Ni}}{W_{N(i-1)}}+\frac{W_{N(i+1)}}{W_{Ni}}+\ldots+\frac{W_{Nk}}{W_{N(k-1)}}\right)+\frac{R_0}{W_{Nk}}(C_{load}+2\gamma W_{Nk}C_0)$$

$$2\tau\left(\gamma k+\frac{W_{N1}}{2}+\frac{W_{N2}}{W_{N1}}+\frac{W_{N3}}{W_{N2}}+\ldots+\frac{W_{Ni}}{W_{N(i-1)}}+\frac{W_{N(i+1)}}{W_{Ni}}+\ldots+\frac{W_{Nk}}{W_{N(k-1)}}\right)+\frac{R_0}{W_{Nk}}C_{load}+2\tau\gamma$$

k-stage Delay

$$2\tau\left(\gamma+\frac{W_{N1}}{2}+\gamma+\frac{W_{N2}}{W_{N1}}+\gamma+\frac{W_{N3}}{W_{N2}}+\ldots+\gamma+\frac{W_{Ni}}{W_{N(i-1)}}+\gamma+\frac{W_{N(i+1)}}{W_{Ni}}+\ldots+\gamma+\frac{W_{Nk}}{W_{N(k-1)}}\right)$$

$$+\frac{R_0}{W_{Nk}}(C_{load}+2\gamma W_{Nk}C_0)$$

$$2\tau\left(\gamma k+\frac{W_{N1}}{2}+\frac{W_{N2}}{W_{N1}}+\frac{W_{N3}}{W_{N2}}+\ldots+\frac{W_{Ni}}{W_{N(i-1)}}+\frac{W_{N(i+1)}}{W_{Ni}}+\ldots+\frac{W_{Nk}}{W_{N(k-1)}}\right)+\frac{R_0}{W_{Nk}}(C_{load}+2\gamma W_{Nk}C_0)$$

$$2\tau\left(\gamma k+\frac{W_{N1}}{2}+\frac{W_{N2}}{W_{N1}}+\frac{W_{N3}}{W_{N2}}+\ldots+\frac{W_{Ni}}{W_{N(i-1)}}+\frac{W_{N(i+1)}}{W_{Ni}}+\ldots+\frac{W_{Nk}}{W_{N(k-1)}}\right)+\frac{R_0}{W_{Nk}}C_{load}+2\tau\gamma$$

$$2\tau\left(\gamma k+\frac{W_{N1}}{2}+\frac{W_{N2}}{W_{N1}}+\frac{W_{N3}}{W_{N2}}+\ldots+\frac{W_{Ni}}{W_{N(i-1)}}+\frac{W_{N(i+1)}}{W_{Ni}}+\ldots+\frac{W_{Nk}}{W_{N(k-1)}}+\frac{R_0}{2\tau W_{Nk}}C_{load}+\gamma\right)$$

k-stage Delay

$$2\tau\left(\gamma+\frac{W_{N1}}{2}+\gamma+\frac{W_{N2}}{W_{N1}}+\gamma+\frac{W_{N3}}{W_{N2}}+\ldots+\gamma+\frac{W_{Ni}}{W_{N(i-1)}}+\gamma+\frac{W_{N(i+1)}}{W_{Ni}}+\ldots+\gamma+\frac{W_{Nk}}{W_{N(k-1)}}\right)$$

$$+\frac{R_0}{W_{Nk}}(C_{load}+2\gamma W_{Nk}C_0)$$

$$2\tau\left(\gamma k+\frac{W_{N1}}{2}+\frac{W_{N2}}{W_{N1}}+\frac{W_{N3}}{W_{N2}}+\ldots+\frac{W_{Ni}}{W_{N(i-1)}}+\frac{W_{N(i+1)}}{W_{Ni}}+\ldots+\frac{W_{Nk}}{W_{N(k-1)}}\right)+\frac{R_0}{W_{Nk}}(C_{load}+2\gamma W_{Nk}C_0)$$

$$2\tau\left(\gamma k+\frac{W_{N1}}{2}+\frac{W_{N2}}{W_{N1}}+\frac{W_{N3}}{W_{N2}}+\ldots+\frac{W_{Ni}}{W_{N(i-1)}}+\frac{W_{N(i+1)}}{W_{Ni}}+\ldots+\frac{W_{Nk}}{W_{N(k-1)}}\right)+\frac{R_0}{W_{Nk}}C_{load}+2\tau\gamma$$

$$2\tau\left(\gamma k+\frac{W_{N1}}{2}+\frac{W_{N2}}{W_{N1}}+\frac{W_{N3}}{W_{N2}}+\ldots+\frac{W_{Ni}}{W_{N(i-1)}}+\frac{W_{N(i+1)}}{W_{Ni}}+\ldots+\frac{W_{Nk}}{W_{N(k-1)}}+\frac{R_0}{2\tau W_{Nk}}C_{load}+\gamma\right)$$

$$2\tau\left(\gamma(k+1)+\frac{W_{N1}}{2}+\frac{W_{N2}}{W_{N1}}+\frac{W_{N3}}{W_{N2}}+\ldots+\frac{W_{Ni}}{W_{N(i-1)}}+\frac{W_{N(i+1)}}{W_{Ni}}+\ldots+\frac{W_{Nk}}{W_{N(k-1)}}+\frac{R_0}{2\tau\cdot W_{Nk}}\cdot C_{load}\right)$$

Size W_{Ni} to minimize delay (Preclass 4b)

- Take partial derivative with respect to $W_{Ni} = 0$

$$2\tau\left(\gamma(k+1)+\frac{W_{N1}}{2}+\frac{W_{N2}}{W_{N1}}+\frac{W_{N3}}{W_{N2}}+\ldots+\frac{W_{Ni}}{W_{N(i-1)}}+\frac{W_{N(i+1)}}{W_{Ni}}+\ldots+\frac{W_{Nk}}{W_{N(k-1)}}+\frac{R_0}{2\tau\cdot W_{Nk}}\cdot C_{load}\right)$$

$$2\tau\left(0+0+0+\ldots+\frac{1}{W_{N(i-1)}}-\frac{W_{N(i+1)}}{\left(W_{Ni}\right)^2}+\ldots+0\right)+0=0$$

$$\frac{1}{W_{N(i-1)}}=\frac{W_{N(i+1)}}{W_{Ni}^2}\rightarrow\frac{W_{Ni}}{W_{N(i-1)}}=\frac{W_{N(i+1)}}{W_{Ni}}$$

Impact on Minimum W_{Ni}? (Preclass 4c)

- Partial derivative unchanged

$$2\tau\left(\gamma(k+1)+\frac{W_{N1}}{2}+\frac{W_{N2}}{W_{N1}}+\frac{W_{N3}}{W_{N2}}+\ldots+\frac{W_{Ni}}{W_{N(i-1)}}+\frac{W_{N(i+1)}}{W_{Ni}}+\ldots+\frac{W_{Nk}}{W_{N(k-1)}}+\frac{R_0}{2\tau\cdot W_{Nk}}\cdot C_{load}\right)$$

What does this say about ρ?

Stage Delay: ρ unchanged (for fixed k)

$$\rho = \frac{W_{N1}}{2} = \frac{W_{Ni}}{W_{N(i-1)}} = \frac{W_{N(i+1)}}{W_{Ni}} = \frac{C_{load}}{W_{Nk}\left(2C_0\right)}$$

$$\rho = \left(\frac{C_{load}}{4C_0}\right)^{\left(\frac{1}{k+1}\right)}$$

Stage Delay: (Preclass 4d)

$$\rho = \frac{W_{N1}}{2} = \frac{W_{Ni}}{W_{N(i-1)}} = \frac{W_{N(i+1)}}{W_{Ni}} = \frac{C_{load}}{W_{Nk}\left(2C_0\right)}$$

$$\rho = \left(\frac{C_{load}}{4C_0}\right)^{\left(\frac{1}{k+1}\right)}$$

$$2\tau\left(\gamma(k+1) + \frac{W_{N1}}{2} + \frac{W_{N2}}{W_{N1}} + \frac{W_{N3}}{W_{N2}} + \ldots + \frac{W_{Ni}}{W_{N(i-1)}} + \frac{W_{N(i+1)}}{W_{Ni}} + \ldots + \frac{W_{Nk}}{W_{N(k-1)}} + \frac{R_0}{2\tau \cdot W_{Nk}} \cdot C_{load}\right)$$

Stage Delay: (Preclass 4d)

$$\rho = \frac{W_{N1}}{2} = \frac{W_{Ni}}{W_{N(i-1)}} = \frac{W_{N(i+1)}}{W_{Ni}} = \frac{C_{load}}{W_{Nk}\left(2C_0\right)}$$

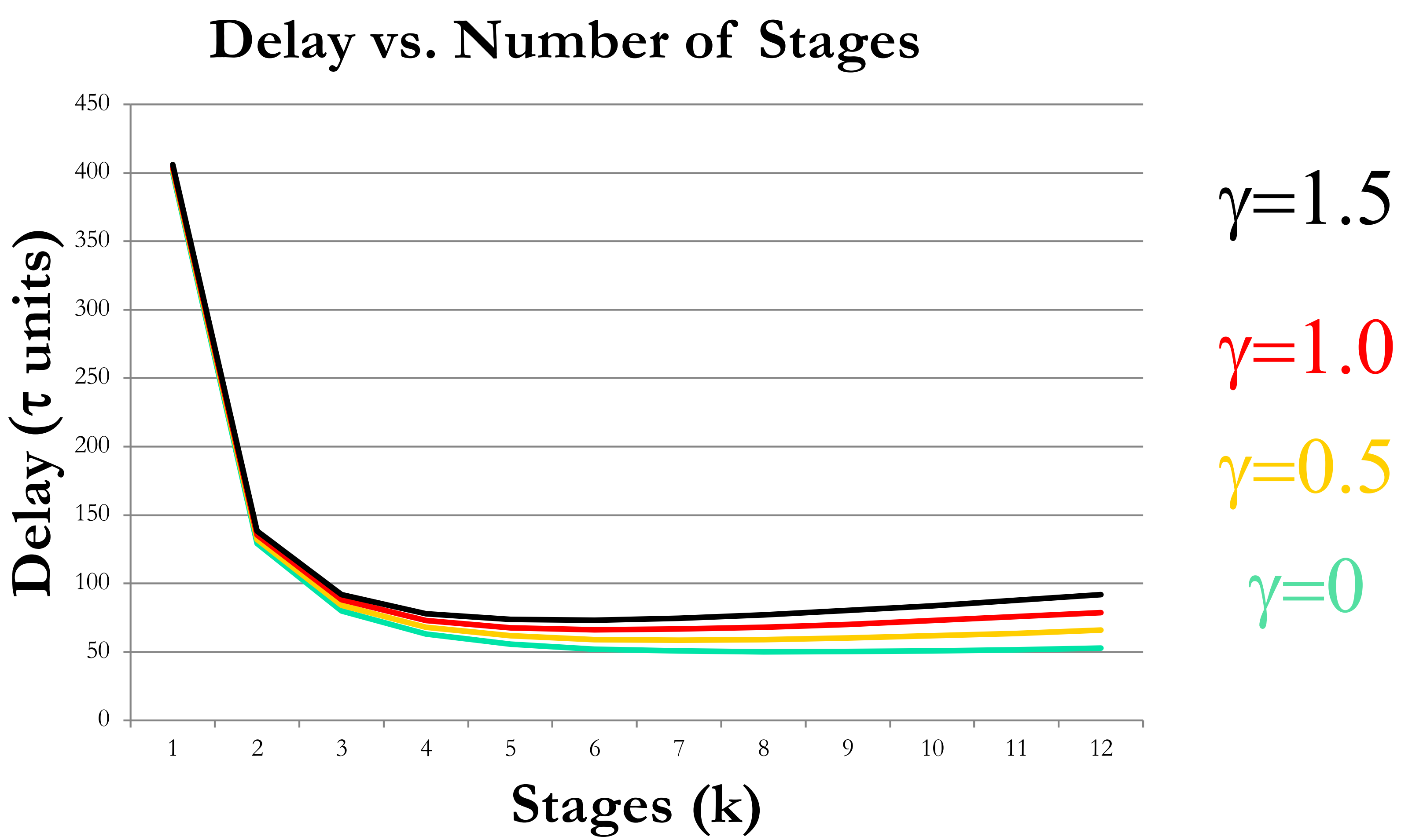
$$\rho = \left(\frac{C_{load}}{4C_0}\right)^{\left(\frac{1}{k+1}\right)}$$

$$2\tau\left(\gamma(k+1) + \frac{W_{N1}}{2} + \frac{W_{N2}}{W_{N1}} + \frac{W_{N3}}{W_{N2}} + \ldots + \frac{W_{Ni}}{W_{N(i-1)}} + \frac{W_{N(i+1)}}{W_{Ni}} + \ldots + \frac{W_{Nk}}{W_{N(k-1)}} + \frac{R_0}{2\tau \cdot W_{Nk}} \cdot C_{load}\right)$$

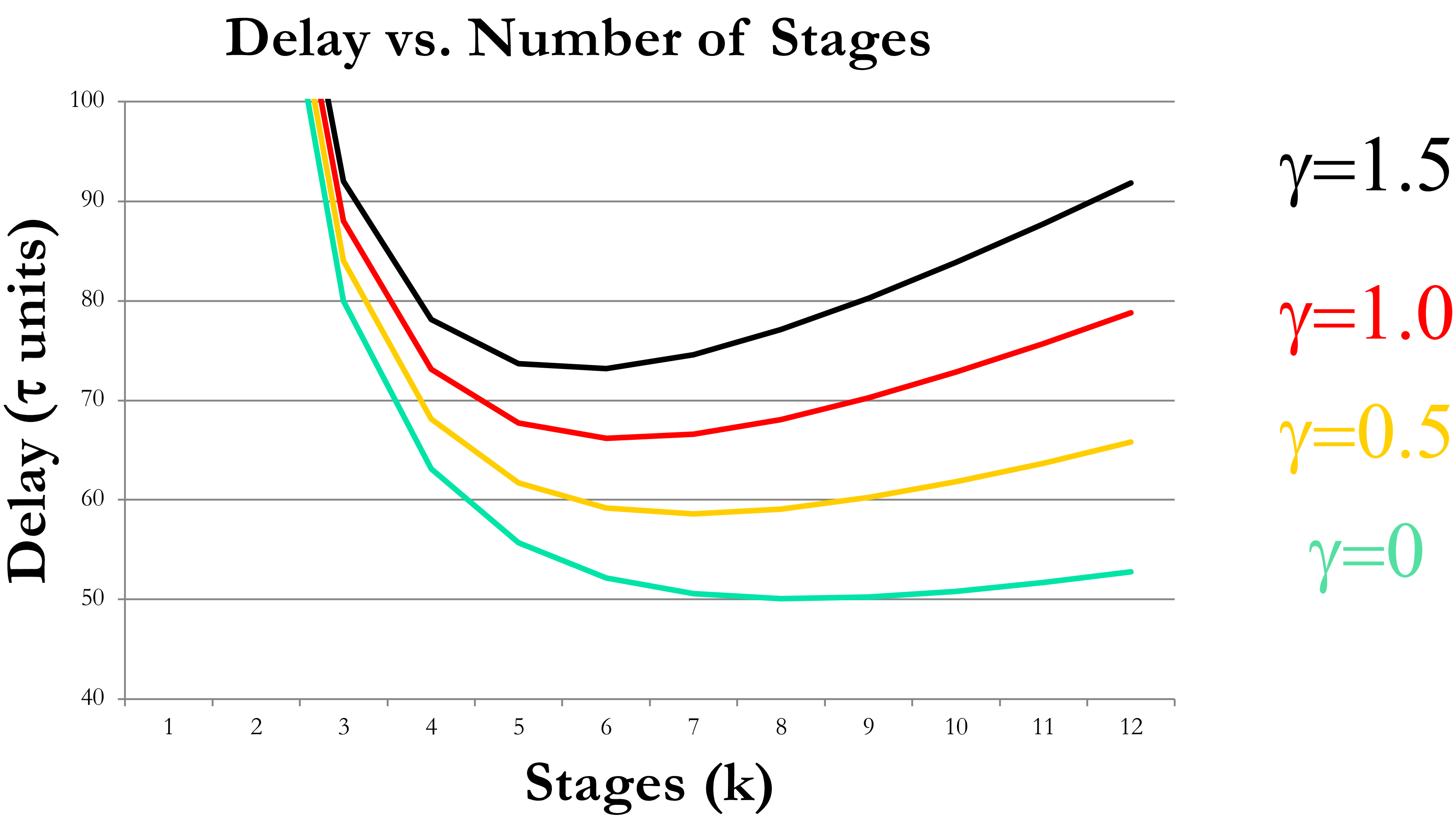
$$TotalDelay = 2\tau(k+1)(\rho + \gamma)$$

$$TotalDelay = 2\tau(k+1)\left(\left(\frac{C_{load}}{4C_0}\right)^{\left(\frac{1}{k+1}\right)} + \gamma\right)$$

Impact of Gamma



Impact of Gamma



Minimize

$$TotalDelay = 2\tau(k+1)(\rho+\gamma)$$

$$TotalDelay = 2\tau(k+1)\left(\left(\frac{C_{load}}{4C_0}\right)^{\left(\frac{1}{k+1}\right)} + \gamma\right)$$

$$0 = 2\tau\left[\gamma + \left(\frac{C_{load}}{4C_0}\right)^{\left(\frac{1}{k+1}\right)} - (k+1)\cdot\ln\left(\frac{C_{load}}{4C_0}\right)\left(\frac{C_{load}}{4C_0}\right)^{\left(\frac{1}{k+1}\right)}\left(\frac{1}{k+1}\right)^2\right]$$

$$0 = \gamma + \rho - (k+1)\cdot\ln\left(\frac{C_{load}}{4C_0}\right)\rho\left(\frac{1}{k+1}\right)^2$$

$$\gamma + \rho = \ln\left(\frac{C_{load}}{4C_0}\right)\left(\frac{1}{k+1}\right)\rho$$

Solve

$$\gamma + \rho = \ln\left(\frac{C_{load}}{4C_0}\right)\left(\frac{1}{k+1}\right)\rho$$

$$\frac{\gamma}{\rho} + 1 = \ln\left(\frac{C_{load}}{4C_0}\right)\left(\frac{1}{k+1}\right)$$

$$\frac{\gamma}{\rho} + 1 = \ln\left(\left(\frac{C_{load}}{4C_0}\right)^{\frac{1}{k+1}}\right)$$

$$\frac{\gamma}{\rho} + 1 = \ln(\rho)$$

Optimal Staging Any γ (Preclass 4e)

$$\rho = e^{\left(\frac{\gamma}{\rho}+1\right)}$$

ρ and γ? (Preclass 4f)

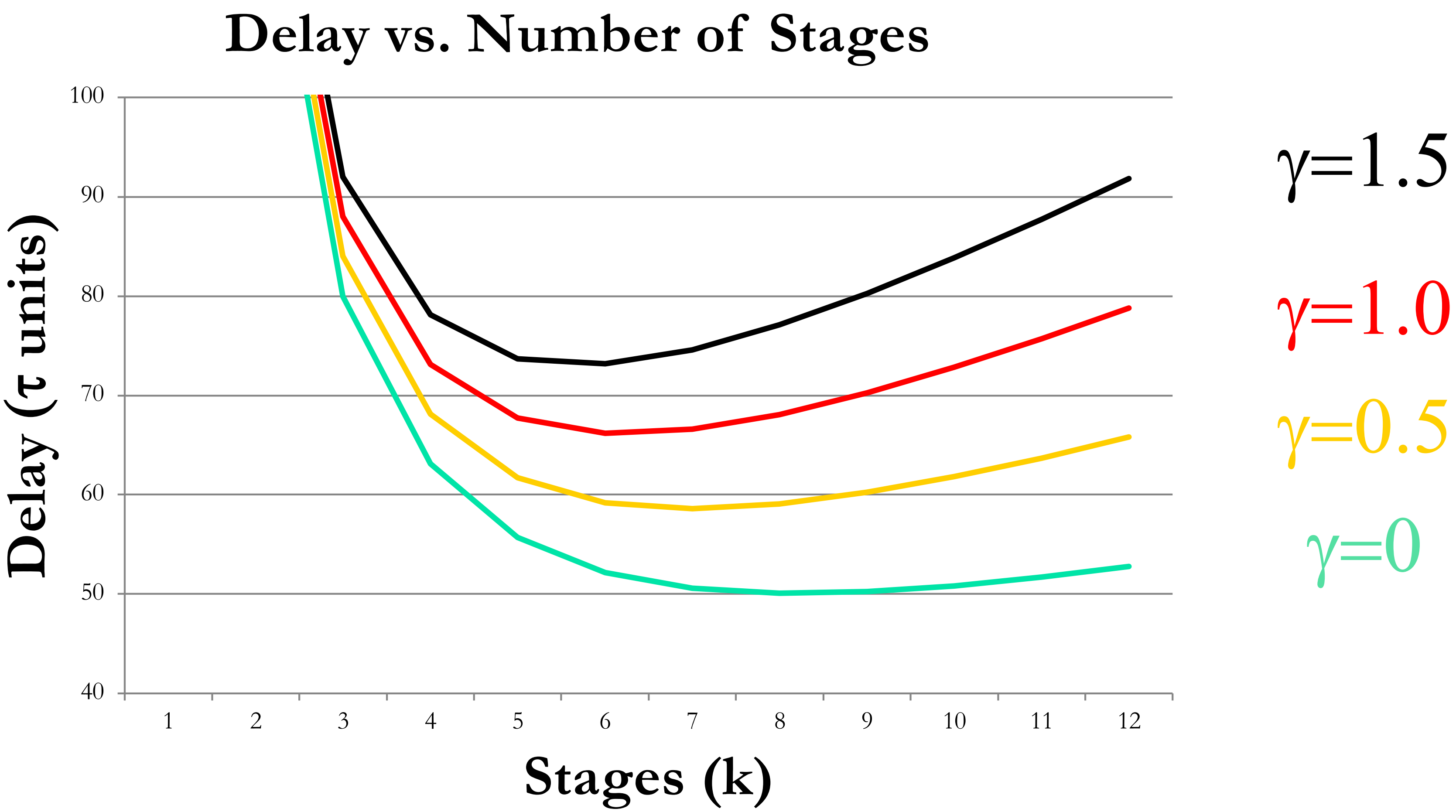
- ρ=4 is optimal for what γ?
- ρ=3 is optimal for what γ?

$$\rho = e^{\left(\frac{\gamma}{\rho}+1\right)}$$

$$\ln \rho = \frac{\gamma}{\rho} + 1$$

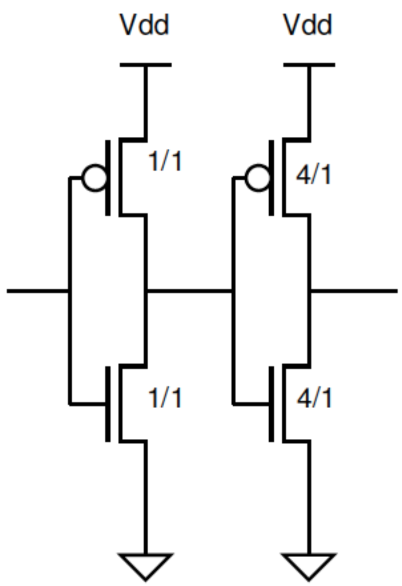
$$\rho \ln \rho - \rho = \gamma$$

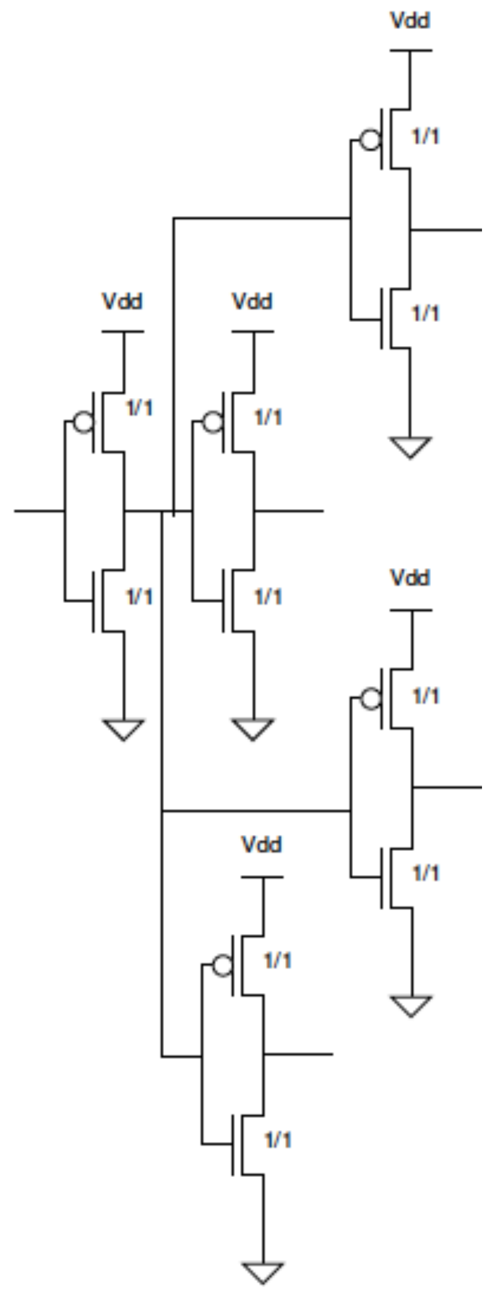
Impact of Gamma



Optimal Fanout

- Clearer why we use ρ=4 as our benchmark?





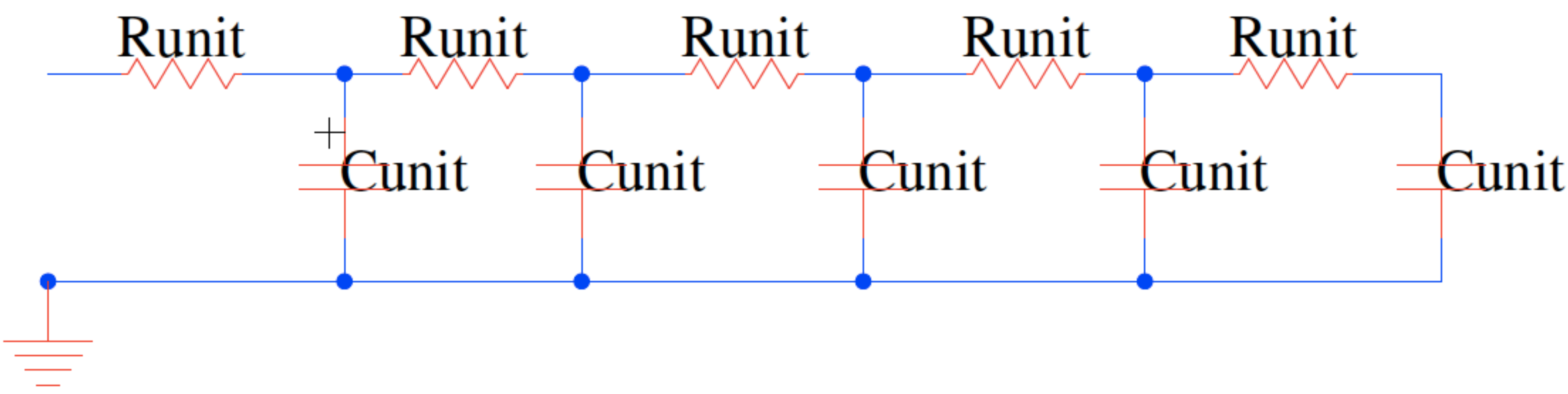
Repeaters in Wiring

Reminder: Wire Delay

- Wire N units long:
  - $=R_{unit}*C_{unit}*N^2/2$
- With
  - $R_{unit}=1k\Omega$
  - $C_{unit}=1pF$

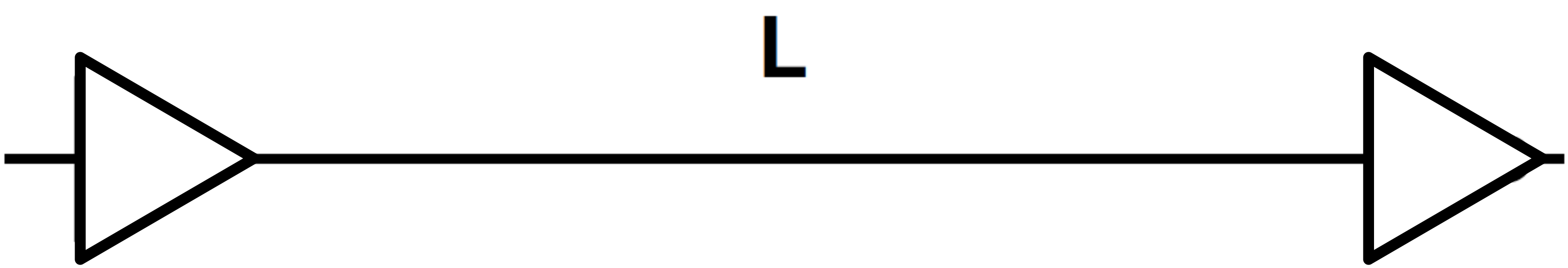
$$R_{wire} = N \times R_{unit}$$

$$C_{wire} = N \times C_{unit}$$

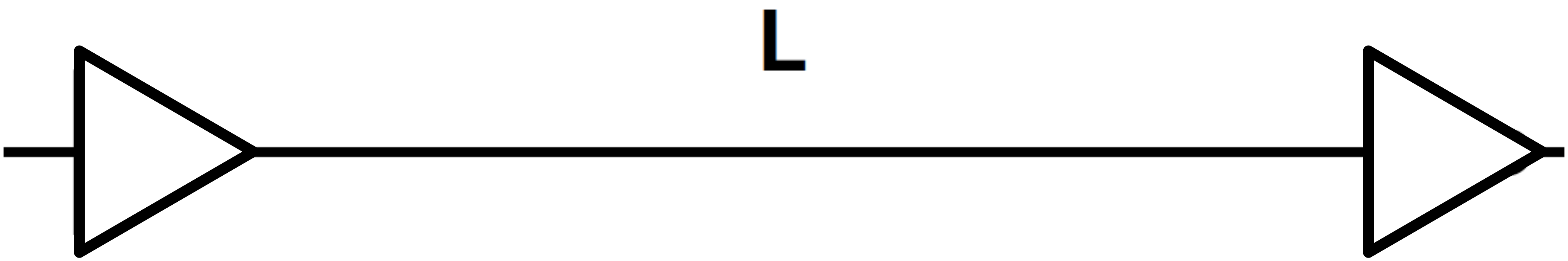


Delay of Wire (preclass 5)

- Long Wire: 1mm
- R_u = 60K Ω per 1mm of wire
- C_u = 0.16 pF per 1mm of wire
- Driven by buffer (R_0=25KΩ, C_0=0.01fF)
  - R_{buf} = 25K Ω
  - C_{self} = 0.02 fF
- Loaded by identical buffer



Formulate Delay (preclass 5)



Delay of buffer driving wire?

Calculate Delay

- $C_{load} = 2\ C_0 = .02fF$
- $R_{buf} = 25K\ \Omega$
- $C_{self} = 0.02fF$
- $C_{wire} = L*C_u = .16pF$
- $R_{wire} = L*R_u = 60K\ \Omega$

$$R_{buf} \times \left(C_{self} + C_{wire} + C_{load}\right) + 0.5 R_{wire} \times C_{wire} + R_{wire} \times C_{load}$$

Calculate Delay

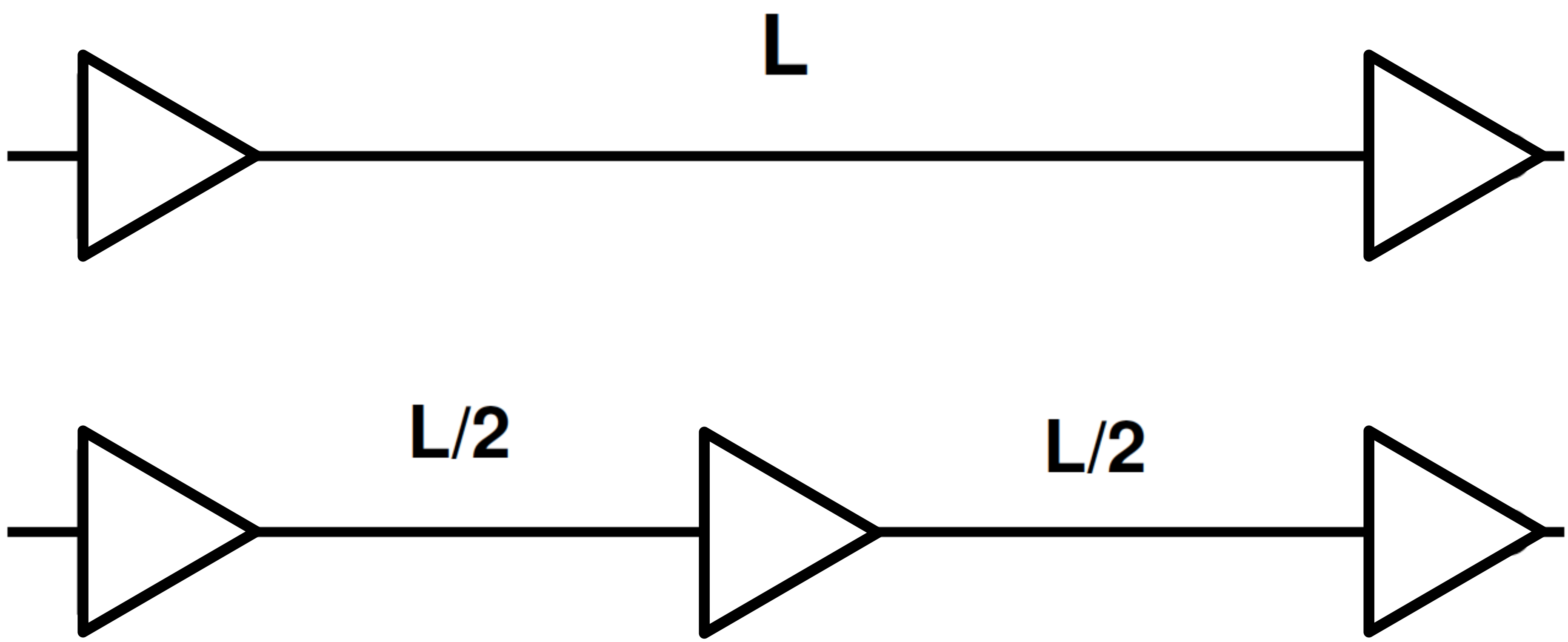
- $C_{load} = 2\ C_0 = .02fF$
- $R_{buf} = 25K\ \Omega$
- $C_{self} = 0.02fF$
- $C_{wire} = L*C_u = .16pF$
- $R_{wire} = L*R_u = 60K\ \Omega$

8.8ns

$$R_{buf} \times \left(C_{self} + C_{wire} + C_{load}\right) + 0.5 R_{wire} \times C_{wire} + R_{wire} \times C_{load}$$

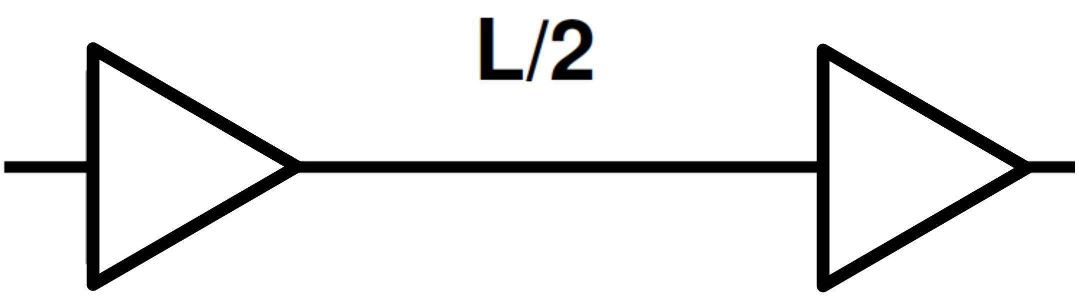
$$4ns + 4.8ns + 1.2ps$$

Buffering Wire



Buffering Wire: L/2

- $C_{load} = 2\ C_0 = .02fF$
- $R_{buf} = 25K\ \Omega$
- $C_{self} = .02fF$
- $C_{wire} = L/2*C_u = .08pF$
- $R_{wire} = L/2*R_u = 30K\ \Omega$

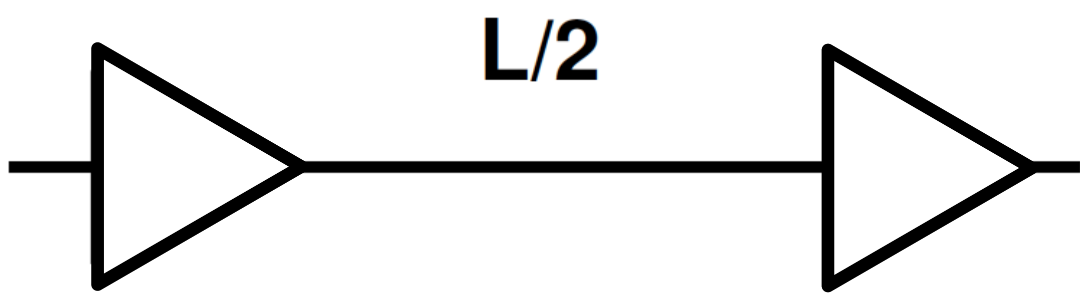


Buffering Wire: L/2

- C_{load} = 2 C_0 = .02fF
- R_{buf} = 25K Ω
- C_{self} = .02fF
- C_{wire} = L/2*C_u = .08pF
- R_{wire} = L/2*R_u = 30K Ω

$$R_{buf} \times \left(C_{self} + C_{wire} + C_{load}\right) + 0.5 R_{wire} \times C_{wire} + R_{wire} \times C_{load}$$

$$2ns + 1.2ns + .6ps = 3.2ns$$



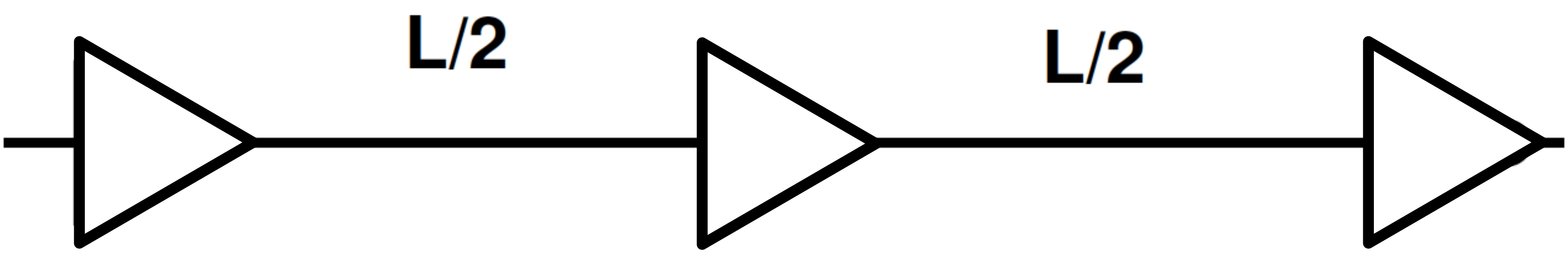
Buffering Wire: L/2

- $C_{load} = 2\ C_0 = .02fF$
- $R_{buf} = 25K\ \Omega$
- $C_{self} = .02fF$
- $C_{wire} = L/2*C_u = .08pF$
- $R_{wire} = L/2*R_u = 30K\ \Omega$

6.4ns

$$R_{buf} \times \left(C_{self} + C_{wire} + C_{load}\right) + 0.5R_{wire} \times C_{wire} + R_{wire} \times C_{load}$$

$$2ns + 1.2ns + .6ps = 3.2ns$$



Buffering Wire: L/N (preclass 6)

Wire of Length	Delay (ns)	Number in 1mm (N)	Total Delay for 1mm (ns)
1 mm	8.8ns	1	8.8ns
0.5mm	3.2ns	2	6.4ns
0.1mm		10	
0.01 mm		100	
0.001 mm		1000	

$$N\left(R_{buf}\left(C_{self}+\frac{C_{wire}}{N}+C_{load}\right)+0.5\left(\frac{R_{wire}}{N}\cdot\frac{C_{wire}}{N}\right)+\frac{R_{wire}}{N}\cdot C_{load}\right)$$

Buffering Wire: L/N (preclass 6)

Wire of Length	Delay (ns)	Number in 1mm (N)	Total Delay for 1mm (ns)
1 mm	8.8ns	1	8.8ns
0.5mm	3.2ns	2	6.4ns
0.1mm	0.45ns	10	4.5ns
0.01 mm	.041ns	100	4.1ns
0.001 mm	.005ns	1000	5ns

$$N\left(R_{buf}\left(C_{self}+\frac{C_{wire}}{N}+C_{load}\right)+0.5\left(\frac{R_{wire}}{N}\cdot\frac{C_{wire}}{N}\right)+\frac{R_{wire}}{N}\cdot C_{load}\right)$$

N Buffers (preclass 7)

$$R_{buf} \times \left(C_{self} + C_{wire} + C_{load}\right) + 0.5R_{wire} \times C_{wire} + R_{wire} \times C_{load}$$

- Delay Equation for N buffers?

$$N\left(R_{buf}\left(C_{self} + \frac{C_{wire}}{N} + C_{load}\right) + 0.5\left(\frac{R_{wire}}{N} \cdot \frac{C_{wire}}{N}\right) + \frac{R_{wire}}{N} \cdot C_{load}\right)$$

N Buffers

- Delay Equation for N buffers?

$$N\left(R_{buf}\left(C_{self}+\frac{C_{wire}}{N}+C_{load}\right)+0.5\left(\frac{R_{wire}}{N}\cdot\frac{C_{wire}}{N}\right)+\frac{R_{wire}}{N}\cdot C_{load}\right)$$

$$N\cdot R_{buf}\left(C_{self}+\frac{C_{wire}}{N}+C_{load}\right)+N\cdot 0.5\left(\frac{R_{wire}}{N}\cdot\frac{C_{wire}}{N}\right)+N\cdot\frac{R_{wire}}{N}\cdot C_{load}$$

$$N\cdot R_{buf}\left(C_{self}+C_{load}\right)+R_{buf}\times C_{wire}+0.5\left(\frac{1}{N}\right)R_{wire}C_{wire}+R_{wire}C_{load}$$

Minimize Delay (preclass 8)

- Minimize delay?

Minimize Delay (preclass 8)

- Minimize delay
- Derivative with respect to N

$$N \cdot R_{buf}\left(C_{self}+C_{load}\right)+R_{buf}\times C_{wire}+0.5\left(\frac{1}{N}\right)R_{wire}C_{wire}+R_{wire}C_{load}$$

$$0=R_{buf}\left(C_{self}+C_{load}\right)-0.5\left(\frac{1}{N^2}\right)R_{wire}C_{wire}$$

Solve for N (preclass 8)

$$0 = R_{buf}\left(C_{self} + C_{load}\right) - 0.5\left(\frac{1}{N^2}\right)R_{wire}C_{wire}$$

$$R_{buf}\left(C_{self} + C_{load}\right) = 0.5\left(\frac{1}{N^2}\right)R_{wire}C_{wire}$$

$$N^2 = \frac{0.5R_{wire}C_{wire}}{R_{buf}\left(C_{self} + C_{load}\right)}$$

$$N = \sqrt{\frac{0.5R_{wire}C_{wire}}{R_{buf}\left(C_{self} + C_{load}\right)}}$$

Minimize Delay (preclass 8)

Equalizes delays from buffer and wire

$$N = \sqrt{\frac{0.5R_{wire}C_{wire}}{R_{buf}\left(C_{self}+C_{load}\right)}}$$

$$\boxed{N \cdot R_{buf}\left(C_{self}+C_{load}\right)} + R_{buf}C_{wire} + \boxed{0.5\left(\frac{1}{N}\right)R_{wire}C_{wire}} + R_{wire}C_{load}$$

Delay with Optimal N (preclass 8)

$$N = \sqrt{\frac{0.5 R_{wire} C_{wire}}{R_{buf}\left(C_{self} + C_{load}\right)}}$$

$$N \cdot R_{buf}\left(C_{self} + C_{load}\right) + R_{buf} C_{wire} + 0.5\left(\frac{1}{N}\right) R_{wire} C_{wire} + R_{wire} C_{load}$$

Delay with Optimal N

$$N = \sqrt{\frac{0.5 R_{wire} C_{wire}}{R_{buf}\left(C_{self} + C_{load}\right)}}$$

$$N \cdot R_{buf}\left(C_{self} + C_{load}\right) + R_{buf} C_{wire} + 0.5\left(\frac{1}{N}\right) R_{wire} C_{wire} + R_{wire} C_{load}$$

$$\sqrt{\frac{0.5 R_{wire} C_{wire}}{R_{buf}\left(C_{self} + C_{load}\right)}} \cdot R_{buf}\left(C_{self} + C_{load}\right) + R_{buf} C_{wire} + 0.5\left(\sqrt{\frac{R_{buf}\left(C_{self} + C_{load}\right)}{0.5 R_{wire} C_{wire}}}\right) R_{wire} C_{wire} + R_{wire} C_{load}$$

Delay with Optimal N

$$N = \sqrt{\frac{0.5R_{wire}C_{wire}}{R_{buf}\left(C_{self}+C_{load}\right)}}$$

$$N \cdot R_{buf}\left(C_{self}+C_{load}\right)+R_{buf}C_{wire}+0.5\left(\frac{1}{N}\right)R_{wire}C_{wire}+R_{wire}C_{load}$$

$$\sqrt{\frac{0.5R_{wire}C_{wire}}{R_{buf}\left(C_{self}+C_{load}\right)}} \cdot R_{buf}\left(C_{self}+C_{load}\right)+R_{buf}C_{wire}+0.5\left(\sqrt{\frac{R_{buf}\left(C_{self}+C_{load}\right)}{0.5R_{wire}C_{wire}}}\right)R_{wire}C_{wire}+R_{wire}C_{load}$$

$$\sqrt{0.5R_{wire}C_{wire}\left(R_{buf}\left(C_{self}+C_{load}\right)\right)}+R_{buf}C_{wire}+\sqrt{0.5R_{wire}C_{wire}\left(R_{buf}\left(C_{self}+C_{load}\right)\right)}+R_{wire}C_{load}$$

Delay with Optimal N

$$N = \sqrt{\frac{0.5R_{wire}C_{wire}}{R_{buf}\left(C_{self}+C_{load}\right)}}$$

$$N \cdot R_{buf}\left(C_{self}+C_{load}\right) + R_{buf}C_{wire} + 0.5\left(\frac{1}{N}\right)R_{wire}C_{wire} + R_{wire}C_{load}$$

$$\sqrt{\frac{0.5R_{wire}C_{wire}}{R_{buf}\left(C_{self}+C_{load}\right)}} \cdot R_{buf}\left(C_{self}+C_{load}\right) + R_{buf}C_{wire} + 0.5\left(\sqrt{\frac{R_{buf}\left(C_{self}+C_{load}\right)}{0.5R_{wire}C_{wire}}}\right)R_{wire}C_{wire} + R_{wire}C_{load}$$

$$\sqrt{0.5R_{wire}C_{wire}\left(R_{buf}\left(C_{self}+C_{load}\right)\right)} + R_{buf}C_{wire} + \sqrt{0.5R_{wire}C_{wire}\left(R_{buf}\left(C_{self}+C_{load}\right)\right)} + R_{wire}C_{load}$$

$$2\sqrt{0.5R_{wire}C_{wire}\left(R_{buf}\left(C_{self}+C_{load}\right)\right)} + R_{buf}C_{wire} + R_{wire}C_{load}$$

Calculate: Delay at Optimum Stages (preclass 8)

- $R_u = 60K\,\Omega$ per 1mm of wire
- $C_u = 0.16$ pF per 1mm of wire
- $R_{buf}=25K\,\Omega$
- $C_{self}=C_{load}=0.02$fF

$$N = \sqrt{\frac{0.5R_{wire}C_{wire}}{R_{buf}\left(C_{self}+C_{load}\right)}}$$

$$2\sqrt{0.5R_{wire}C_{wire}\left(R_{buf}\left(C_{self}+C_{load}\right)\right)}+R_{buf}C_{wire}+R_{wire}C_{load}$$

Segment Length (preclass 8)

- $R_{wire} = L \times R_{unit}$
- $C_{wire} = L \times C_{unit}$

$$L^{*}_{seg} = \frac{L}{N}$$

$$N = \sqrt{0.5\left(\frac{R_{wire} \times C_{wire}}{R_{buf} \times \left(C_{self} + C_{load}\right)}\right)}$$

$$N = L\sqrt{0.5\left(\frac{R_{u} \times C_{u}}{R_{buf} \times \left(C_{self} + C_{load}\right)}\right)}$$

$$L^{*}_{seg} = \frac{L}{N} = \sqrt{2\left(\frac{R_{buf} \times \left(C_{self} + C_{load}\right)}{R_{u} \times C_{u}}\right)}$$

Optimal Segment Length

- Delay scales linearly with distance once optimally buffered

$$L_{seg}^{*} = \frac{L}{N} = \sqrt{2\left(\frac{R_{buf} \times \left(C_{self} + C_{load}\right)}{R_u \times C_u}\right)}$$

$$N = L\sqrt{0.5\left(\frac{R_u \times C_u}{R_{buf} \times \left(C_{self} + C_{load}\right)}\right)}$$

Buffer Size? (preclass 9)

- How big should buffer be?
  - $R_{buf} = R_0/W$
  - $C_{self} = 2\ W\ C_{dff} = 2\ W\ \gamma\ C_0$
  - $C_{load} = 2\ W\ C_0$

Buffer Size?

- How big should buffer be?
  - $R_{buf} = R_0/W$
  - $C_{self} = 2\ W\ C_{dff} = 2\ W\ \gamma\ C_0$
  - $C_{load} = 2\ W\ C_0$

$$2\sqrt{0.5R_{wire}C_{wire}\left(R_{buf}\left(C_{self}+C_{load}\right)\right)}+R_{buf}C_{wire}+R_{wire}C_{load}$$

$$2\sqrt{0.5R_{wire}C_{wire}\left(\frac{R_0}{W}\left(2WC_0\left(1+\gamma\right)\right)\right)}+\frac{R_0}{W}C_{wire}+R_{wire}\cdot 2WC_0$$

$$2\sqrt{0.5R_{wire}C_{wire}\left(2R_0C_0\left(1+\gamma\right)\right)}+\frac{R_0}{W}C_{wire}+R_{wire}\cdot 2WC_0$$

Optimal W (preclass 9)

- $R_{wire} = L \times R_{unit}$
- $C_{wire} = L \times C_{unit}$

$$2\sqrt{0.5 R_{wire} C_{wire} \left(2 R_0 C_0 \left(1+\gamma\right)\right)} + \frac{R_0}{W} C_{wire} + R_{wire} \cdot 2 W C_0$$

$$0 = 2 R_{wire} C_0 - R_0 C_{wire} \frac{1}{W^2}$$

$$W = \sqrt{\frac{R_0 C_{wire}}{2 R_{wire} C_0}} = \sqrt{\frac{R_0 C_{unit}}{2 R_{unit} C_0}}$$

Implication W

- $R_{wire} = L \times R_{unit}$
- $C_{wire} = L \times C_{unit}$

$$2\sqrt{0.5 R_{wire} C_{wire} \left(2 R_0 C_0 (1+\gamma)\right)} + \frac{R_0}{W} C_{wire} + R_{wire} \cdot 2 W C_0$$

$$0 = 2 R_{wire} C_0 - R_0 C_{wire} \frac{1}{W^2}$$

$$W = \sqrt{\frac{R_0 C_{wire}}{2 R_{wire} C_0}} = \sqrt{\frac{R_0 C_{unit}}{2 R_{unit} C_0}}$$

- → W independent of wire length L
  - Depends on process technology

Delay at Optimum W (preclass 9)

$$W = \sqrt{\frac{R_0 C_{wire}}{2R_{wire}C_0}}$$

$$2\sqrt{0.5R_{wire}C_{wire}\left(2R_0C_0\left(1+\gamma\right)\right)} + \frac{R_0}{W}C_{wire} + R_{wire}\cdot 2WC_0$$

Delay at Optimum W

$$W = \sqrt{\frac{R_0 C_{wire}}{2R_{wire}C_0}}$$

$$2\sqrt{0.5R_{wire}C_{wire}\left(2R_0C_0\left(1+\gamma\right)\right)} + \frac{R_0}{W}C_{wire} + R_{wire}\cdot 2WC_0$$

$$2\sqrt{0.5R_{wire}C_{wire}\left(2R_0C_0\left(1+\gamma\right)\right)} + \frac{R_0}{\sqrt{\frac{R_0C_{wire}}{2R_{wire}C_0}}}C_{wire} + R_{wire}\cdot 2\sqrt{\frac{R_0C_{wire}}{2R_{wire}C_0}}C_0$$

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$$2\sqrt{0.5R_{wire}C_{wire}\left(2R_0C_0\left(1+\gamma\right)\right)} + \sqrt{R_0C_{wire}\cdot 2R_{wire}C_0} + \sqrt{R_0C_{wire}\cdot 2R_{wire}C_0}$$

Delay at Optimum W

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$$2\sqrt{0.5R_{wire}C_{wire}\left(2R_0C_0\left(1+\gamma\right)\right)} + \sqrt{R_0C_{wire}\cdot 2R_{wire}C_0} + \sqrt{R_0C_{wire}\cdot 2R_{wire}C_0}$$

$$2\sqrt{0.5R_{wire}C_{wire}\left(2R_0C_0\left(1+\gamma\right)\right)} + 2\sqrt{2R_0C_0C_{wire}R_{wire}}$$

Delay at Optimum W

- If γ=1

$$2\sqrt{R_{wire}C_{wire}\left(R_0C_0\left(1+1\right)\right)}+2\sqrt{2R_0C_0C_{wire}R_{wire}}$$

$$2\sqrt{2R_0C_0C_{wire}R_{wire}}+2\sqrt{2R_0C_0C_{wire}R_{wire}}$$

$$4\sqrt{2R_0C_0C_{wire}R_{wire}}$$

- Optimal design equalizes all delays!

Delay at Optimum W

- If γ=1

$$2\sqrt{R_{wire}C_{wire}\left(R_0C_0\left(1+1\right)\right)}+2\sqrt{2R_0C_0C_{wire}R_{wire}}$$

$$2\sqrt{2R_0C_0C_{wire}R_{wire}}+2\sqrt{2R_0C_0C_{wire}R_{wire}}$$

$$4\sqrt{2R_0C_0C_{wire}R_{wire}}$$

- Optimal design equalizes all delays!

8.8ns → 0.27ns

Idea

- To drive large loads
  - Scale buffers geometrically
  - Exponential scale up in buffer size ($\rho = e$)
- Scale factor: 3—4 typically
  - One origin of fanout 4 target
- Wire delay linear once buffered optimally
- Optimal buffers equalizes delays
  - Buffer delay
  - Delay on wire between buffers
  - Delay of wire driving buffer

Admin

- Project 1
  - Final report due **Friday 3/29** midnight
- Wednesday 4/3 Midterm 2 (next week)
  - During class in Moore 216
  - Lectures 1-14
  - Closed note, calculator allowed
  - All old exams online
    - 2010-2021
  - TA review session
    - 4/1 7-9pm in Towne 307
- HW 6 release Wednesday 4/3

Acknowledgement

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